SIDEREAL PHOTOMETRIC ASTROMETRY AS EFFICIENT INITIAL SEARCH FOR SPIN VECTOR

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I describe how to use the Sidereal Photometric Astrometry method of spin vector determination (Drummond *et al.*, 1988) to identify the specific plausible candidate combinations of sidereal periods and pole solution regions that are consistent with a lightcurve epoch data set, to use as initial inputs to the convex inversion method of shape determination (Kaasalainen *et al.*, 2001).

Introduction

The means to calculate asteroid spin vectors and model shapes from asteroid lightcurves is accessible to the community of observers as available code implementing the powerful and elegant convex inversion approach (Kaasalainen *et al.*, 2001), which uses an iterative nonlinear algorithm that requires initial values for sidereal period and pole location.

Constraining the sidereal period is discussed in the pair of papers (Slivan, 2012) and (Slivan, 2013), hereafter "paper I" and "paper II" respectively, which describe a robust approach to establish the best constraints by deliberately managing the possible ambiguities. Its results can help in judging whether a set of epoch data are sufficient to proceed with spin vector analyses, and they also identify the candidate period solutions once there are enough data.

This paper addresses the remaining issue of the initial values for the pole location. An exhaustive approach is to scan a grid of trial poles and look for minima in some goodness-of-fit metric, but a 1°-resolution grid of candidate poles on the celestial sphere numbers over 64,000 poles, and running that many trials of convex inversion is computationally expensive. Choosing a 10° resolution instead, as has been used by authors publishing analyses in the *Minor Planet Bulletin*, reduces the needed computation by a factor of 100×, but the lower resolution makes it difficult to understand the behavior of the search space, and thus to distinguish true possible pole regions from spurious solutions and other fluctuations in the metric. It is preferable and more robust instead to use where possible some other method to locate the pole regions before invoking convex inversion shape modeling. In this paper I describe how to efficiently identify the regions of possible pole solutions and exclude spurious results using Sidereal Photometric Astrometry (Drummond *et al.*, 1988).

Method

Sidereal Photometric Astrometry (SPA) is a statistical least-squares fitting approach to deduce the spin axis orientation and also the direction of spin from a set of epochs t_i, making the assumption that every epoch corresponds to the same rotational phase of the asteroid to within a possible half-rotation ambiguity, an assumption which must be made with caution as is discussed in Paper II.

SPA makes use of the fact that the number of rotations elapsed between any two epochs includes a fraction of a rotation that depends on the change in viewing aspect angle with respect to the asteroid's spin vector, an effect that was ignored in Paper II. Including all of the contributions, the number of sidereal rotations N_i (integer number of half-rotations) that elapse during a time interval T_i between epochs, here correcting a typographic error in Drummond *et al.* (1988, Eq. 2), is

$$N_i = 0.5\ \mathrm{INT}\ (2T_i/P_{\mathrm{sid}} - 2k_i + 0.5) \quad \text{(Eq. 1)}$$

The candidate sidereal period values P_{sid} are directly available from the method described in Paper II. The observed intervals T_i since the earliest epoch are $t_i - t_1$ where $i = 1$ is the index of the earliest epoch. Each corresponding fraction of a rotation k_i is how much the body would have to turn between epochs t_1 and t_i for the same asterocentric longitude to be facing the observer—it's the difference between the asterocentric longitudes L_{SP} of the sub-observer point at the two epochs, and it includes contributions from both the direction vector change and the polar angle change. For longitudes in radians this is

$$k_i = (L_{\mathrm{SP},i} - L_{\mathrm{SP},1})/2\pi \quad \text{(Eq. 2)}$$

Calculation of (B_{SP}, L_{SP}), the pole-dependent asterocentric latitude and longitude of the sub-observer point, will require the ecliptic coördinates of the sub-observer direction (β_{SP}, λ_{SP}) which are the reflex of the asteroid direction. Here the asteroid direction (β, λ) is represented by the phase angle bisector (Harris, 1984) to allow for the effect of nonzero solar phase angle on observed times of epochs:

$$\beta_{\mathrm{SP}} = -\beta \quad \text{(Eq. 3)}$$

$$\lambda_{\mathrm{SP}} = \lambda + \pi \quad (0 \le \lambda_{\mathrm{SP}} < 2\pi) \quad \text{(Eq. 4)}$$

The equations to transform these ecliptic coördinates to B_{SP} and L_{SP} referenced to the asteroid pole location (β_p, λ_p) are from Taylor (1979), here correcting a typographic error in case (f):

$$B_{\mathrm{SP}} = \sin^{-1}[\sin\beta_{\mathrm{SP}}\sin\beta_p + \cos\beta_{\mathrm{SP}}\cos\beta_p\cos(\lambda_{\mathrm{SP}} - \lambda_p)] \quad \text{(Eq. 5)}$$

$$L'_{\mathrm{SP}} = \sin^{-1}[\cos\beta_{\mathrm{SP}}\sin(\lambda_{\mathrm{SP}} - \lambda_p)/\cos B_{\mathrm{SP}}] \quad \text{(Eq. 6)}$$

$$L_{\mathrm{SP}} = \begin{cases} 0, & \text{if } \sin L'_{\mathrm{SP}} = 0 \text{ and } Q < 0 \quad \text{(a)} \\ |L'_{\mathrm{SP}}|, & \text{if } \sin L'_{\mathrm{SP}} > 0 \text{ and } Q < 0 \quad \text{(b)} \\ \pi/2, & \text{if } \sin L'_{\mathrm{SP}} > 0 \text{ and } Q = 0 \quad \text{(c)} \\ \pi - |L'_{\mathrm{SP}}|, & \text{if } \sin L'_{\mathrm{SP}} > 0 \text{ and } Q > 0 \quad \text{(d)} \\ \pi, & \text{if } \sin L'_{\mathrm{SP}} = 0 \text{ and } Q > 0 \quad \text{(e)} \\ \pi + |L'_{\mathrm{SP}}|, & \text{if } \sin L'_{\mathrm{SP}} < 0 \text{ and } Q > 0 \quad \text{(f)} \\ 3\pi/2, & \text{if } \sin L'_{\mathrm{SP}} < 0 \text{ and } Q = 0 \quad \text{(g)} \\ 2\pi - |L'_{\mathrm{SP}}|, & \text{if } \sin L'_{\mathrm{SP}} < 0 \text{ and } Q < 0 \quad \text{(h)} \end{cases} \quad \text{(Eq. 7)}$$

where determining the quadrant of the longitude involves the sign of the auxiliary value Q:

$$Q = (\sin\beta_{\mathrm{SP}} - \sin\beta_p\sin B_{\mathrm{SP}})/\cos\beta_p\cos B_{\mathrm{SP}} \quad \text{(Eq. 8)}$$

The straight line model fitted to the epochs is

$$T_i = P'_{\mathrm{sid}}(N_i + k_i) + T'_1 \quad \text{(Eq. 9)}$$

where T_i is the dependent variable, $N_i + k_i$ is the independent variable, and the primes indicate the fitted parameters. The T'_1 result is discarded as it will be simply the error in the first epoch $T_1 = 0$.

Given a candidate sidereal period, the algorithm to locate pole solution regions by generating a graph of the goodness of fit of trial pole locations is:

for all trial poles (λ_p, β_p) on celestial sphere **do**
  for all epochs (T_i, λ_i, β_i) in input data **do**
    calculate k_i and N_i (Eqs. 1–8)
  end for
  fit the straight line model $T_i = P'_{sid}(N_i + k_i) + T'_1$ (Eq. 9)
  if fitted P'_{sid} is within initial period constraint range **then**
    include RMS error of this fit on graph as function of (λ_p, β_p)
  end if
end for

As an example of the method, I continue the analysis begun in Paper II of the lightcurve epoch observations of main-belt asteroid (1223) Neckar given in Table I by checking the three candidate sidereal periods for possible pole solution regions. The epochs were weighted equally for the fitting, and the resulting three RMS error graphs are shown in Fig. 1.

The graphs corresponding to candidate periods 7.82133 h retrograde (Fig. 1a) and 7.82124 h prograde (Fig. 1b) reveal that in both cases there are indeed well-behaved minima corresponding to possible pole solutions, which are summarized in Table II.

In contrast, the graph corresponding to candidate period 7.82115 h retrograde (Fig. 1c) has a fragmented appearance. Discontinuities in the RMS error pattern indicate that the best epoch fits for adjacent trial pole orientations are requiring different numbers of rotations N between some pair of epochs, which would not be the case near a possibly correct pole solution. Based on the lack of any well-behaved minima regions in this graph I confidently reject this period; this is not a surprising outcome because it was already flagged in Paper II as suspiciously sensitive to the uncertainties estimated for the epochs.

Table I: Summary of (1223) Neckar lightcurve epoch observations 1977–1996, one epoch t per observed apparition as measured using the Fourier filtering approach described by Slivan et al. (2003). λ and β are ecliptic longitude and latitude (J2000) of the phase angle bisector. Lightcurve data references: a, Tedesco (1979); b, Binzel (1987); c, Slivan and Binzel (1996); d, Michałowski *et al.* (2000).

t (MJD)	λ(°)	β(°)	UT date	Ref.
43187.0000	132.4	+3.2	1977 Feb 13	a
45469.2539	230.3	-0.5	1983 May 15	b
46885.0147	158.1	+2.6	1987 Mar 31	c
47776.1156	328.4	-3.0	1989 Sep 07	c
48210.1221	71.9	+1.4	1990 Nov 15	c
49131.0887	253.0	-1.6	1993 May 24	c
50098.0010	87.5	+2.5	1996 Jan 16	d

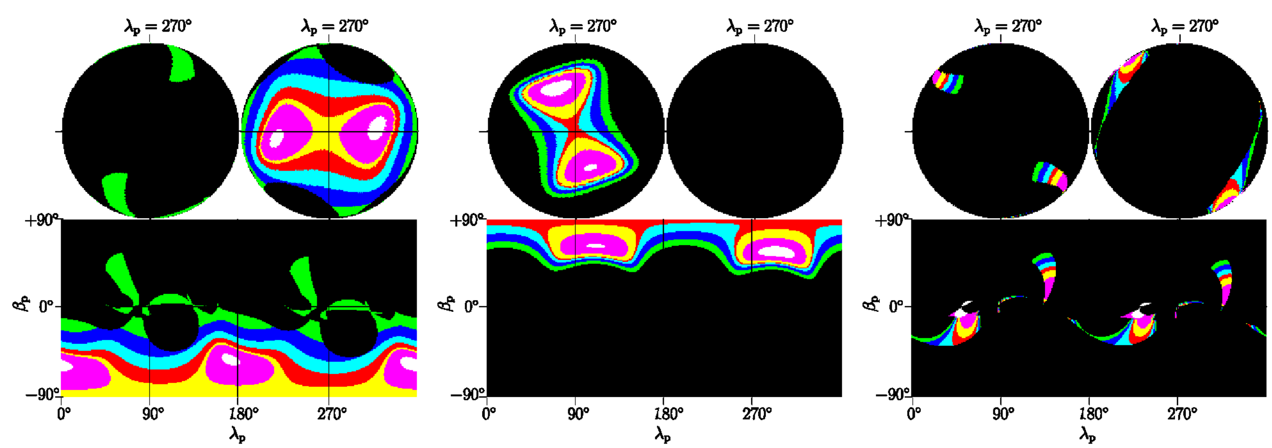
Table II: Candidate spin vector solution results for (1223) Neckar lightcurve epoch observations given in Table I.

sidereal period (h)	poles (ecliptic J2000)
7.82133	(170°;-50°) and (355°;-55°)
7.82124	(110°;+65°) and (295°;+60°)

Discussion

Because SPA is a spin vector analysis approach that makes use of epoch information, its success depends on having enough epochs for fitting and counting elapsed rotations, sufficient aspect coverage, and the total observed time span being long enough. SPA in particular also depends on how well the epochs satisfy the underlying assumption of unchanging asterocentric longitude.

The example epochs are sufficient in number, aspect coverage, and total length of time spanned to narrow the solution possibilities to only two candidate periods, one retrograde and one prograde. Each period has two possible pole solution regions about 180° apart in ecliptic longitude, whose differences in RMS errors are not significant and cannot be used to distinguish between the pair of solution regions.



(a) (b) (c)

Fig. 1: (a) Contour graph of the RMS error "goodness of fit" of trial poles of (1223) Neckar from SPA analysis of the epoch observations in Table I for candidate sidereal period 7.82133 h. The resolution of trial poles is 1° and regions of best fit are colored white. The lower half of each graph represents the celestial sphere projected on a rectangular grid for easy measurement of longitude and latitude coördinates. The same data in a polar format undistorted near the ecliptic poles appear in the upper half of each graph, where north- and south-hemisphere views are plotted separately. For this sidereal period, retrograde pole regions are found near (170°;−50°) and (355°;−55°). (b) Graph for sidereal period 7.82124 h, locating prograde pole regions near (110°;+65°) and (295°;+60°). (c) Graph for sidereal period 7.82115 h. Here the regions of lowest RMS error are not well-behaved minima and they do not correspond to possible pole solutions.

SPA doesn't use lightcurve amplitudes or brightnesses, which contain the pole longitude information that's needed to identify which of the two example period solutions is the correct one, and to more accurately locate the pole within its region. Here this isn't a problem because SPA is used only to identify the pole solution regions that are consistent with the epochs, to then be checked using convex inversion which uses both epoch and amplitude information for analysis. After SPA narrowed the example possible solutions to only one pair of poles for each of two sidereal periods, convex inversion had no trouble definitively identifying the prograde period and poles as the correct solution (Slivan *et al.*, 2003).

Conclusion

SPA is a computationally efficient approach to identify plausible pole solution regions, and is quite robust because the only assumption is unchanging asterocentric longitudes of the epochs. It permits pole searches of sufficient resolution to be able to see the behavior of the search space near RMS error minima, and thus reveal whether the data are in fact sufficient for spin vector solutions, identify true possible pole regions, and exclude spurious solutions and other fluctuations in the metric. Using SPA speeds determination of creditable poles by reducing the number of trial poles that have to be tested through convex inversion, and provides the needed initial values for the pole location.

After Paper II was published, its algorithm to constrain sidereal period was implemented as a Web application at the site *http://www.koronisfamily.com*. A Web application implementing the SPA algorithm described in this paper is presently in development for the same site.

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