WAS URANUS OBSERVED BY HIPPARCHUS?

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1. *Introduction*

Although Uranus was 'officially' discovered by William Herschel (1738–1822) on 13 March 1781, it had previously been sighted on 22 occasions between 1690 and 1771 by four different astronomers.[1] But there is reason to think that a naked-eye observation of the planet was made in Antiquity.

The systematic error of –1° found in the longitudes of the *Almagest* star catalogue has raised many controversies about the identity of its author. Several ingenious theories[2] have been suggested to solve the problem, favouring either Ptolemy or Hipparchus. However the key to the mystery may lie in the description of a star pattern in the constellation Virgo.

All the versions of the *Almagest* agree about the presence of a quadrilateral in the left thigh of the Virgin, a part of the constellation Virgo located approximately three degrees northeast of Spica. While the coordinates given for the two stars forming the northern side of the quadrangle also agree, they do not for the two stars forming the southern side. The reason is obvious: there is only one star of the same brightness at the lower left corner of the figure. The purpose of this article is to show that the object (Baily 513 or Virgo 17) occupying the bottom right corner of the quadrangle was none other than the planet Uranus.

The fundamental argument of this paper is the following:
(1) There is a widespread consensus among experts (Toomer, Kunitzsch, Peters and Knobel, etc.) that the four stars in the quadrilateral in the left thigh of Virgo are as shown in Table 1.

TABLE 1. The Consensus solution.

Almagest	λ	β	Mag.	Name	Identity	V	Error
Virgo 16	$176\frac{1}{3}$	$+3\frac{1}{3}$	5	The northern star on the advance side	74 Vir	4.7	22′
Virgo 17	$177\frac{1}{4}$	$+\frac{1}{6}$	6	The southern star on the advance side	76 Vir	5.2	71′
Virgo 18	180	$+1\frac{1}{2}$	4.3	The northern star on the rear side	82 Vir	5.0	22′
Virgo 19	178	–3	5	The southern star on the rear side	68 Vir	5.2	13′

(2) There is an equally widespread realization within this consensus that this result is awkward and troubling. The quadrilateral does not look like a quadrilateral, and the coordinates do not match well for the supposed Virgo 17 = 76 Vir (see Figure 1 (left)).
(3) There is a simple way to fix things. For Virgo 19, read $\beta = -\frac{1}{3}$ rather than –3, in agreement with a couple of *Almagest* manuscripts. Then the Vir 19 coordinates match very well to 76 Vir.
(4) But then what matches to Virgo 17? There is a very good match in coordinates and magnitude to Virgo 17 if we assume that this celestial body was the planet Uranus

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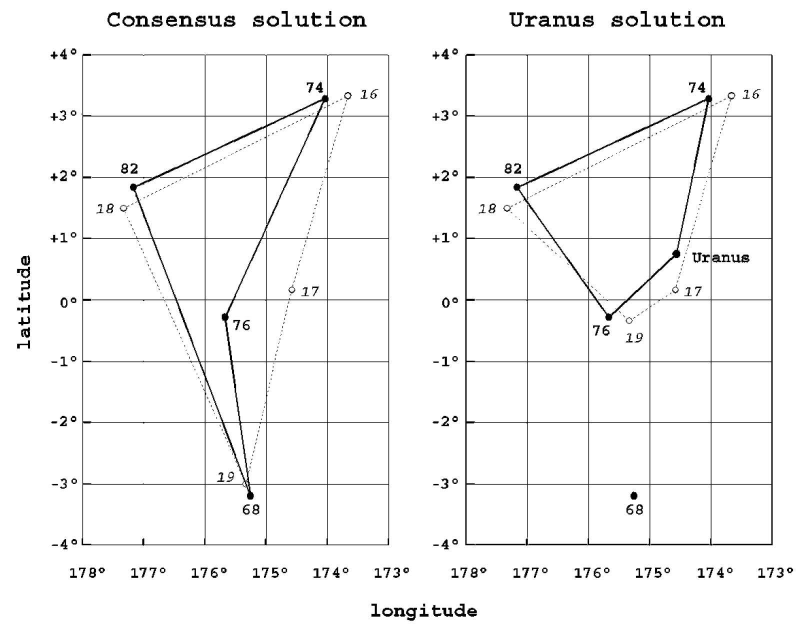

FIG. 1. The Consensus solution (*left*) and the Uranus solution (*right*). *Almagest* stars (with numbers 16–19) are drawn in grey and real stars (with Flamsteed numbers) in black.

observed by Hipparchus about 128 B.C., and the quadrilateral is now more apparent (see Figure 1 (right)).

TABLE 2. The Uranus solution.

Almagest	λ	β	Mag.	Name	Identity	V	Error
Virgo 16	$176\frac{1}{3}$	$+3\frac{1}{3}$	5	The northern star on the advance side	74 Vir	4.7	22′
Virgo 17	$177\frac{1}{4}$	$+\frac{1}{6}$	6	The southern star on the advance side	Uranus	5.4	35′
Virgo 18	180	$+1\frac{1}{2}$	4.3	The northern star on the rear side	82 Vir	5.0	22′
Virgo 19	178	$-\frac{1}{3}$	5	The southern star on the rear side	76 Vir	5.2	21′

In Tables 1 and 2, Flamsteed numbers are used in column "Identity"; visual magnitudes are indicated in column "V"; and the resulting errors are given in arc minutes. As will be discussed in more detail below, the versions of the *Almagest* supporting the Uranus solution are the so-called Arabic versions while those favouring the Consensus solution are Greek.

In the remainder of this article, all dates are in the Julian calendar and unless otherwise stated, the years before Christ are written in the manner of astronomers (for instance 128 B.C. corresponds to –127), celestial positions are referred to the mean equinox of the year –128.0 and the time scale used is Universal Time.

2. *The Quadrilateral in the Left Thigh of the Virgin*

Before examining the appearance and the localization of this asterism in detail, let us first attempt to define the term *quadrilateral* (in Greek τετράπλευρον) which is rather puzzling because this shape may designate any convex polygon with four sides. The whole of the star catalogue found in Books 7 and 8 of the *Almagest* contains 14 such figures of which 12 are clearly identifiable. Among these 12 figures, 6 refer to a rectangle (Dra, Lep, Psc, Ser, UMa, UMi), 4 to a trapezium (Cet, Cnc, Ori, Sgr), 1 to a rhombus (Del), and 1 to a parallelogram (Tau) formed by four stars of nearly the same brightness. Apart from the unresolved quadrangle in Virgo, another ambiguous figure lies in the constellation Cetus. However, in that case, the problem is due the presence of several candidate stars in the vicinity.

This clarification made, let us now settle the position of the quadrangle with the help of two different celestial charts: an ancient and a modern one. The first (Figure 2) is an excerpt of the *Atlas coelestis* (edition of 1753) by John Flamsteed showing the outline of the constellation Virgo. The advantage of its iconography is to emphasize the position of the stars relative to the different parts of the body of the Virgin. The 32 *Almagest* stars belonging to that zodiacal constellation have been painted as black dots on their corresponding position on the chart. The numbering is faithful to

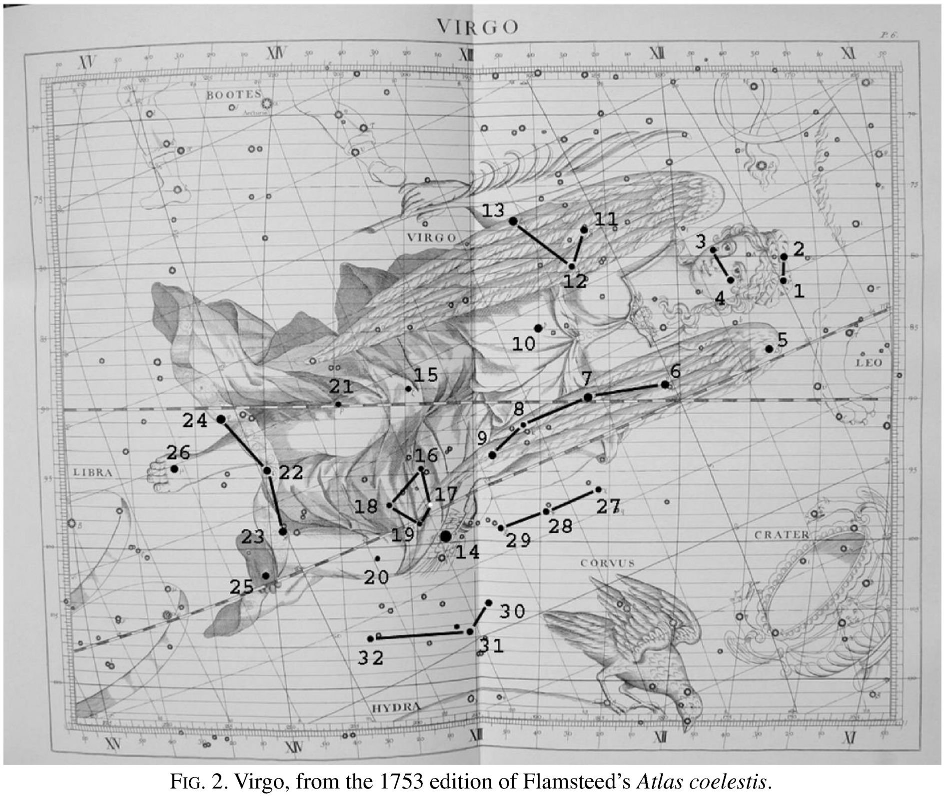

FIG. 2. Virgo, from the 1753 edition of Flamsteed's *Atlas coelestis*.

the medieval numbering of the *Almagest* star catalogue.

As can be seen, the quadrilateral symbolizing the left thigh of Virgo composed of the stars Virgo 16 to 19 (according to the Uranus solution given in Table 2) appears to the northeast of Spica (Virgo 14) and is naturally wedged between Virgo 9 (the End of the Left Wing = 51 Vir, θ Vir) and Virgo 20 (the Left Knee = 86 Vir). We will see later the relevance of this remark.

The second celestial chart (Figure 3) is a close-up of the field of interest (15° × 8°) made with a computer program taking into account the proper motion of the stars, which is not insignificant during the time lapse of 2 millennia considered here. The right ascension and declination lines are separated by intervals of 2 degrees and are referred to the equinox and equator of the year –128.0. Spica (α Vir), the bright star at bottom centre, and θ Vir, located at upper right, may help to localize the field on modern star atlases. Several small crosses indicate the position of *Almagest* stars. The path of Uranus, with the ticks marking the position of the planet on the first day (at 0h UT) of each month, is visible parallel and north of the ecliptic. The lower right corner of the quadrangle corresponds to the position of Uranus at the exact instant of its ecliptic conjunction with Virgo 17, namely on 9 April –127. Since the limiting magnitude of the star catalogue[3] used here is 8.0, the absence of any other star of equivalent brightness at the lower right corner is evident.

3. *A First Dating Attempt*

In order to identify unambiguously Virgo 17 with Uranus, it is necessary to know the date at which the measurements were made of the stars constituting the *Almagest* catalogue.

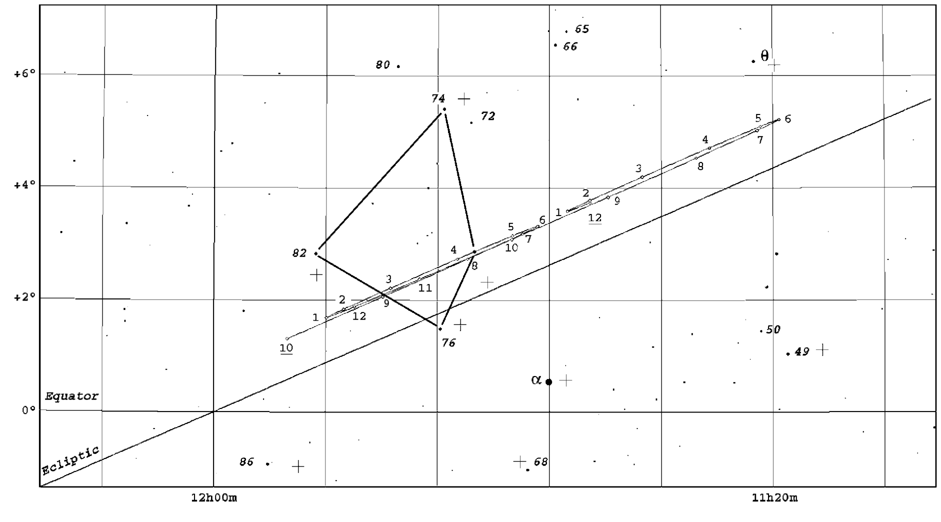

FIG. 3. The path of Uranus across the quadrilateral from 1 December –129 (the underlined n° 12 in the western loop) to 1 October –127 (the underlined n° 10 at extreme left).

Contrary to what Ptolemy claims [*Almagest* VII, 4], the coordinates given by him do not match with the first day of the Egyptian year in which the Roman emperor Antoninus began his reign, namely 20 July 137 A.D. Indeed, if we try to reproduce these coordinates on a celestial chart referenced to the equinox and equator of the standard epoch J2000.0 using current models for precession[4] and obliquity[5] we arrive at the conclusion that the catalogue dates from the middle of the first century after Christ.

As none of the astronomers that Claudius Ptolemy (*c.* A.D. 90–168) refers to in his *Mathematical compilation* lived at that moment, it has often been proposed that the catalogue that he represents as his own is really that of Hipparchus of Nicea (*c.* 190–120 B.C.). Two important clues can help us to circumscribe the time of creation of the catalogue. On the one hand Ptolemy [*Almagest* VII, 2] reports a measure of the longitude of the star Regulus (α Leo) made by Hipparchus himself during the 50th year of the 3rd Callippic Period, which corresponds to a date between the summer solstices of the years –128 and –127; on the other hand, Ptolemy states [*Almagest* VII, 3] that his epoch (A.D. 137) and that of Hipparchus are separated by an interval of 265 years, permitting us to set the latter around –128.

Since the two approaches agree fairly well, running the time back to the past, we see that Uranus was indeed located at the lower right corner of the quadrangle at that precise moment (see Figure 3). According to the coordinates given in Table 2 and considering these data as exact (which is doubtful), the conjunction in longitude between Uranus and Virgo 17 took place on 9 April –127. We will see later (Table 3) that the favourable viewing period extended from 7 to 19 April. Since midnight culmination of Spica occurred on 17 March, we may thus imagine that the goal of Hipparchus was to benefit from the best possible viewing conditions for Virgo.

Since the time interval between Hipparchus and Ptolemy was 265 years and the accepted value for precession [*Almagest* VII, 2] was then of one degree per century (the correct value being one degree in 71 years), it has been inferred that Ptolemy merely added 2°40′ to the longitudes of Hipparchus, leaving the latitudes unchanged.[6] This is the reason why the longitudes of the *Almagest* stars given in Tables 1 and 2 are greater than their values in Figure 1.

As can be seen in the fourth column of Table 2, Virgo 17 is classified as an object of the sixth magnitude. In a scale from 1 to 6, this class corresponds to the limit of naked-eye visibility. All versions of the *Almagest* agree in describing Virgo 17 as the faintest component of the quartet, which plainly conforms to reality since Uranus was then of magnitude 5.4 compared to the three other stars whose magnitudes were between 4.7 and 5.2 (see column 7 of the same Table).

4. *Viewing Conditions*

Let us now establish the conditions necessary for an observer located at mid-northern latitudes to succeed in making an observation of Uranus with the naked-eye. These conditions can be summarized as follows: (a) Uranus near the southern meridian, where its altitude above horizon is greatest; (b) Sun more than 18° below the horizon, to guarantee total darkness; (c) no Moon glare; (d) solar elongation as large as possible; (e) Uranus in the northern celestial hemisphere; (f) no bright star or planet in the immediate vicinity; and (g) Uranus as close as possible to the perihelion of its orbit, so that its intrinsic brightness is near maximum.

While conditions (a) to (d) can be met quite often because they depend on daily, monthly and yearly cycles, condition (g) is more restrictive because it is determined by Uranus's relatively long period of revolution (84 years). Although the precession cycle of 26 000 years does not affect the maximum declination attainable by Uranus, it plays a role in the declination reached by a given constellation. Nevertheless, as we shall see, Hipparchus could benefit from extremely favourable circumstances.

On the one hand, given that Uranus is typically 0.4 magnitude brighter at perihelion than at aphelion and that the previous passage at perihelion took place on 7 January –134 (i.e. barely 7 years earlier), the planet was still close enough to the Sun to be at its maximum brightness.

On the other hand, thanks to precession, during the second century B.C. the constellation Virgo was more favourably placed than at present since its altitude at southern meridian was 11° higher than today, allowing the Southern Cross and Alpha Centauri to be visible from the island of Rhodes (36°N, 28°E) where Hipparchus conducted his observations.[7] Around the year –127, Uranus's declination of +3° allowed the planet to pass at only 33° from the zenith.

While nowadays (2013) the magnitude of Uranus remains constantly between 5.7 and 5.9 depending on its position relative to the Earth, Hipparchus could witness the planet shining between magnitudes 5.3 and 5.5. Since an experienced observer can detect a 6.5 magnitude star with the unaided eye near the zenith under ideal viewing conditions, it would have been possible for the 5.4 magnitude Uranus to be spotted by the patient and keen-eyed Greek astronomer under his clear and unpolluted Mediterranean sky.

5. *A Second Dating Attempt*

The first dating attempt allowed us to conclude that it was possible to observe Uranus in mid-April –127. Since this date is well inside the 50th year of the 3rd Callippic Period and the quadrilateral was reconstructed at the right place, we are tempted to satisfy ourselves with these results and stop our investigations at this point. However, for the sake of completeness, we will examine another option.

If Hipparchus's habits of work consisted in establishing a precise watching program several days before, or in checking his measurements several days later, we may wonder if the retrograde motion of Uranus, being then 2.3 arc minutes per day, could have escape his vigilance. Besides, Figure 3 shows that the trapezoidal shape obtained so far is a little more pronounced compared to the other figures of the same type described in the *Almagest* star catalogue.

A closer look at the track of Uranus (Figure 3) shows that the planet had performed two stations a few dozen arc minutes south of the point for which a perfectly symmetrical figure was obtained. The first of these stationary points occurred on 31 December –129 (before the planet began its retrograde motion) and the second on 4 June –127 (before the planet resumed its direct motion). During a time lapse of about two weeks on either side of these dates, Uranus remained practically motionless and, at these moments, there was nothing to distinguish it from any ordinary star.

Consequently, if we consider as more probable that Uranus was observed by Hipparchus during one of these two stationary points, how can we then explain such large discrepancies (respectively 114′ and 80′) between the actual and the measured position of the planet? Before searching for a specific cause, we may first point out that such large errors are not uncommon in the *Almagest* star catalogue. According to the Toomer/Grasshoff classification,[8] 4% of the stars in the northern hemisphere have errors larger than 114′ and 6% larger than 80′, while for the southern hemisphere these numbers are respectively 8% and 14%. However, the case of Virgo 17 is particular because Uranus was never seen again at the position recorded by Hipparchus. As a consequence, a transcription error is more likely to have occurred during one of the many copies of the original manuscript than for the other stars. Nevertheless, a more concrete explanation can be floated.

We have no knowledge of the kind of instrument(s) used by Hipparchus to perform his measurements. The ecliptic armillary sphere described by Ptolemy (*Almagest* V, 1) was designed to measure lunar or planetary positions and to establish a set of reference stars, but was inappropriate for determining the position of a great number of stars dispersed over the entire celestial sphere.

As recent papers suggest,[9] Hipparchus probably measured a fairly complete star catalogue in *equatorial* coordinates and this was later converted to *ecliptic* coordinates. Must we thus conclude that he used exclusively an equatorial armillary sphere? These cumbersome instruments, with their numerous circles, had several major drawbacks: because of the thickness of the rings composing the device, the sight to the target star and the access to the eyepiece could be impossible. Since the stars composing the catalogue are uniformly distributed over the entire celestial sphere, it is likely that Hipparchus used a different technique.

Right ascensions and declinations are semi-natural coordinates. Knowing the latitude (φ) of the site and the altitude (h) of the target star when it is due south, the declination is obtained by the simple formula $\delta = \varphi + h - 90°$ (with minor modifications for high declination stars passing between the north pole and the zenith). In the same way, the right ascension, which corresponds at this moment to the local sidereal time, can be obtained by adding the hour angle (H) of any reference star to its known right ascension (α_0), so that the final value is found to be $\alpha = \alpha_0 + H$. Of course, in practice, Hipparchus could avoid any calculation for the declination by rotating his meridian circle in the correct direction (i.e. with graduation +36° towards the zenith).

To sum up, with the help of two different instruments, the first of them being a meridian circle intended to give the declination of the target star and the second a rudimentary equatorial armillary sphere that served to determine the hour angle of a reference star, the coordinates of any star could be obtained without complex equipment (e.g. reliable day-round clocks) or difficult trigonometric formulas.

In this perspective, we may imagine that while Hipparchus was occupied in observing the target star crossing the meridian, an assistant kept the armillary sphere locked on the reference star in order to secure the current sidereal time. If a third person was busy transcribing the coordinates and a precise watching program was established beforehand, a large part of the celestial sphere could be covered on a single night (as Hipparchus would no doubt have wished, given his awareness of problems resulting from the precession of the equinoxes).

As shown in Figure 3, at both stationary points performed by Uranus in the lower right corner of the quadrangle, the planet was practically in equatorial conjunction with Spica. Considering its brightness (mag. 1.0) and its proximity to the equator ($\delta = +0°32'$), this neighbouring star was very well suited to be used as a reference star. On this assumption and knowing the drawbacks of the armillary sphere, it is plausible that during the transit of Uranus at southern meridian, the mobile ring turned toward Spica interfered with the fixed ring positioned in the meridian of this device, resulting in a poor estimation of Spica's hour angle and thus of Uranus's final right ascension.

Given the visibility conditions set out in the previous paragraph, and considering the hypothesis made about the likely technique used by Hipparchus (meridian transits) and the trajectory of Uranus across the quadrilateral (which could not differ too much from a rectangle), we show in Table 3 the five windows within which a naked-eye observation of Uranus could have been performed.

TABLE 3. The five favourable viewing windows.

Start	End	Mag.	Elongation
30 Dec –129	11 Jan –128	5.4	103–115°W
26 Jan –128	8 Feb –128	5.4	130–144°W
22 Feb –128	5 Mar –128	5.3	158–171°W
7 Apr –127	19 Apr –127	5.4	161–149°E
4 May –127	12 May –127	5.4	134–126°E

6. *Conclusion*

The presence of Uranus in the left thigh of the Virgin can explain the great confusion that reigned for centuries (if not for millennia) about the identification of the two objects constituting the southern side of the quadrangle. If today a pair of binoculars is sufficient to convince ourselves that there is only one star at the lower left corner of the figure, we may wonder how keen observers like the Persian Al Sufi (903–86), more than a thousand years after Hipparchus, still maintained Ptolemy's original coordinates since his own description of the quadrangle tends to prove that he was aware of the absence of Virgo 17 at the place recorded in the *Almagest*.[10]

Unlike modern optical instruments whose diameter can be increased at will, the human eye has limits (the diameter of the pupil rarely exceeds 7mm even after full darkness adaptation). Furthermore, the epithet of *Almagest* (*The Greatest*) indicates the reverence in which Ptolemy's work was held in the Arab world during the first millennium after Christ. In the same manner, given the high respect with which Ptolemy regarded Hipparchus's observations, it is unlikely that he would have dared to contradict his illustrious predecessor. But if this reasoning applied during the Early Islamic Period (600–1000), the same was no longer the case afterwards. Since no one star was ever seen again at the place vacated by Uranus, the distant successors of Hipparchus, perhaps during the Late Islamic Period (1050–1450), had to decide at some moment to find a substitute for Virgo 17.

While Al Sufi merely estimated Ptolemy's magnitudes anew, Ulugh Beg (1394–1449), who observed from Samarkand around A.D. 1437, was the first to produce a star catalogue with positions based on new, independent measurements. According to Frank Verbunt and Robert van Gent,[11] Ulugh Beg reconstructed the quadrangle using 74 Vir for Virgo 16, 76 Vir for Virgo 17, 82 Vir for Virgo 18 and 86 Vir for Virgo 19. However, he then had no other solution but to use an obscure star of magnitude 5.9 bearing the designation "y" Vir (Yale BSC 5106, slightly out of the field of Figure 3 and located halfway between 68 and 86 Vir) to place at the position of Virgo 20, the Left Knee, which in view of Figure 2 is an anatomical aberration.

The general trend of the oldest (Arabic) versions of the *Almagest* that reached Europe via Spain with the Arab conquerors in the twelfth century was to leave intact Ptolemy's original description of the two stars (Virgo 17 and 19) forming the southern side of the quadrangle despite the fact that one of them remained unidentified. Besides the version of Al Sufi, typical examples are found in the copies based on the Arabic-to-Latin translation made by Gerard of Cremona.

On the other hand, Greek versions of the *Almagest* reached Western Europe via Constantinople. The trend of these latter versions (a typical example is the Greek-to-Latin translation made by George of Trebizond) was to identify Virgo 17 with 76 Vir and Virgo 19 with 68 Vir. Unfortunately, this solution was worse than that of Ulugh Beg because 68 Vir is not only outside of the left thigh of the Virgin and separated from the quadrangle by more than 3° but, additionally, the bright star Spica was interposed between them (see Figure 3), which is a nonsense. Last but not least, the concave polygon obtained in this manner can no longer be called a quadrilateral (see left part of Figure 1). Strangely, this was accepted by most of the modern authors, such as Toomer, Kunitzsch, and Peters and Knobel. Similarly meaningless is the attempt[12] to associate Virgo 17 with the star 80 Vir (mag. 5.7) located north of 74 Vir, for this identification does not respect the original name of Virgo 17, which is clearly called the "the Southern Star on the Advance Side".

Given that Uranus returns nearly to the same celestial position after a time lapse of 84 years, Table 4 shows the approximate moments during which the first stationary point, conjunction with Virgo 17, and the second stationary point as described in the text, repeat under almost the same conditions. As the observations cited by Ptolemy as his own span the time interval A.D. 127–141,[13] we find ourselves outside of this range. As to the pre-Hipparchan era, although the Alexandrian astronomers

TABLE 4. Passages of Uranus in the quadrangle.

First station (January)	Conj. (April)	Second station (June)
–296	–295	–295
–212	–211	–211
–128	–127	–127
–44	–43	–43
+40	+41	+41
+124	+125	+125

Aristillus and Timocharis were active during the middle of the first Callippic Period (i.e. around 294 B.C.), it is hardly conceivable that their archaic equipment was able to provide better measurements than those of Hipparchus; moreover the presence of Jupiter in the quadrangle in January –296 would have prevented any observation of Uranus. Given what we know about the duration of Hipparchus's active period, namely 26 September –146 to 7 July –126,[14] it seems that the long debate about the author of the *Almagest* star catalogue looks more than ever as favouring Hipparchus.

The fact that the magnitude of Uranus was correctly estimated has another interesting implication. Even if it is now widely accepted that the *Almagest* star catalogue originated in some sense with Hipparchus, that conclusion applied mainly to the coordinates, so until now it was at least possible that the magnitudes were added later, perhaps by Ptolemy himself (although this seems unlikely, considering that he barely mentions the magnitudes in the *Almagest*). For example, we know that during the first century A.D., Marcus Manilius [*Astronomica* V, 710–17] mentions stars of magnitudes between 3 and 6. Brighter stars were probably discussed in the now lost lines just preceding the lines that survive. So let us consider the following scenario: at some time later than Hipparchus someone decides to add the magnitudes. But when they look for Virgo 17, it is extremely unlikely that they will find anything (i.e. Uranus is unlikely to be there), much less the correct answer (i.e. the Uranus magnitude), so they would have nothing to enter for the magnitude. Thus the present result is at least a hint that Hipparchus did himself observe and record the magnitudes.

A brief paper entitled "Ancient Uranus?"[15] likewise refers to the possible identification of Virgo 17 with Uranus. The author of the present article came independently to the same conclusion by analysing the starry field enclosing the planet. While stellar coordinates can be contaminated by measurements, conversions and copying errors, a geometrical figure cannot. It is thus noteworthy to underline that without the heavy and impractical notation used by Hipparchus to designate his stars, the likely observation of Uranus made by him more than two millennia ago could never be advocated.

Finally, what is the answer to the question that opened this paper: Did Hipparchus unwittingly observe the seventh planet of the solar system? Considering the accumulation of concordant indications, namely: convergence of the dates, reestablishment of the quadrilateral at the right place with respect of the name of the stars, agreement on the magnitudes, favourable viewing conditions, and absence of any other star of the same brightness (variable or not) in the vicinity, we conclude that geometrical reasoning and common sense invite us to respond affirmatively. Unfortunately, since the manuscripts containing the dates and times of Hipparchus's observations are irretrievably lost, we will probably never be able to provide definitive proof.

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REFERENCES

1. D. C. Wright, "Uranus in Antiquity", *Quarterly journal of the Royal Astronomical Society*, xxviii (1987), 79.
2. James Evans, "On the origin of the Ptolemaic star catalogue: Part 1", *Journal for the history of astronomy*, xviii (1987), 155–72.
3. SKY2000 Master Catalog, Version 4 (Myers, 2002).
4. Jay H. Lieske, "Precession matrix based on IAU (1976) system of astronomical constants", *Astronomy and astrophysics*, lxxiii (1979), 282–4.
5. *Explanatory supplement to the Astronomical Almanac*, 1992, mean obliquity based on IAU (1980) expression 3.222-1, p. 114.
6. Evans, *op. cit.* (ref. 2), 155.
7. Gerd Grasshoff, *The history of Ptolemy's star catalogue* (New York, 1990), 7.
8. Grasshoff, *op. cit.* (ref. 7), appendix B, 270–316.
9. Dennis Duke, "The measurement method of the Almagest stars", *DIO*, xii (2002), 35–50.
10. H. C. F. C. Schjellerup, "Description des étoiles fixes composée au milieu du dixième siècle de notre ère par l'astronome persan Abd-al-Rahman al Sûfi" (St Petersburg, 1874), 160, 164–5.
11. Frank Verbunt and Robert van Gent, "The star catalogues of Ptolemaios and Ulugh Beg", *Astronomy and astrophysics*, dxliv (2012), A31–65.
12. Keith A. Pickering, "A re-identification of some entries in the ancient star catalog", *DIO*, xii (2002), 61–2.
13. James Evans, "On the origin of the Ptolemaic star catalog: Part 2", *Journal for the history of astronomy*, xviii (1987), 233–78, p. 241.
14. Grasshoff, *op. cit.* (ref. 7), 7.
15. Keith P. Hertzog, "Ancient Uranus?", *Quarterly journal of the Royal Astronomical Society*, xxix (1988), 277–9.