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The method of a reference selenocentric coordinate system construction for visible and far sides of the Moon referred to the lunar mass center and to its main inertia axes

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The global reference network construction on the Moon surface is one of the most important tasks of modern selenodesy. The basic methods of a reference coordinate system selenocentric creation in the center of gravity system and the principal axes of the Moon inertia by combining space and ground observations are described in this research.

Keywords: Selenodetic net, Systems of coordinates, Methods of reducing

1 Introduction

Nowadays, the Moon is the study object of many space experiments and the center of scientists' attention in astronomical and planetary science. The launching of American scientific satellites, such as "Clementine" and "Lunar Prospector", qualitatively changed the situation in Moon studies (Nefedjev 2004, 2006).

A powerful stream of high-precision and polyvalent information, which was taken from the modern space vehicles has caused a surge of interest in industrial, technical Moon reclamation to 2018 and the manned flight to the Mars in 2025–2030 years after the creation of long-term manned lunar bases. The development of space technology has particular requirements with reference to the results of coordinate-temporary support, which includes mutual base system and establishment of the inertial and dynamic coordinate system orientation, the study of dynamics and geometry of celestial bodies. This applies to the dynamic and geometric Moon operation factors, which were referred to the center of mass. However, the space Moon research, which was performed for scientific and practical purposes is not provided with selenodetic coordinate network – the directory of support objects which embrace the visible and

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the back/far sides of the Moon and have a center, which is close to the center of mass. The directory is constructed by observations with "Apollo" spaceship and the reference network on the Moon's western hemisphere which were obtained by processing pictures of the Moon's opposite side with "Zond-6", "Zond-8" missions and cover only part of the lunar surface. The detailed analysis of the internal accuracy of the "Apollo" data was carried out in the work (Dole, Ellasae, Lucas 1977).

We can formulate the following conclusions based on this analysis. Three ALSEP stations were used for Apollo topographic coordinate transform since the mean square errors (MSE) of transformation were less than ± 80 m and the measuring error was ± 60 m, so we can assume that the points which are close to and between three ALSEP stations have errors of less than ± 150 m. The deflection of the ALSEP location increases the supposed planned error up to ± 300 m and this is the majority of territories in the scope dimension area. The points location errors lying close to the borders of studied areas can reach ± 300 m and even exceed ± 1000 m. There are several coordinate systems for the visible side, the catalogue of 1162 objects (KSC-1162) (Nefedjev, 2003) is the more informative among them, it was constructed in the Engelhardt Astronomical Observatory (EAO) for the large-scale Moon's pictures with stars and the catalogue of 264 craters (Khabibullin, Rizvanov 1984) based on the same observations. The system of 4900 craters which was built in Golosiyivsk Observatory in Kiev by Gavrilov (Gavrilov, Kislyuk, Duma 1977) also should be mentioned. Kiev systems were obtained in quasidynamic coordinate system unlike Kazan directories which were built in the dynamic coordinate system.

It should be mentioned that despite the Moon being studied by space observations, the ground observations do not lose value, so the reasonable space and ground combination of the Moon observations methods are considered the optimal path of the implementation of selenodetic observations. Both ground and space astrometry are needed because they are complementary. The study in the presence of the base selenocentric catalogue of reference objects on the visible side of the Moon KSC-1162 and the series of objects catalogue in a libration zone and a far side of the Moon in the heterogeneous systems of a Unified Coordinates System construction with the center and axes which coincide with the center of the Moon mass and with the principal axes of its inertia, includes the following phases:

- The investigation of systematic and random errors in the KSC-1162 catalogue;
- Detailing and expansion of the KSC-1162 system catalogue in the visible far side of the Moon and the libration zone.

2 The brief survey of KSC 1162 catalogue

The support selenocentric network KSC-1162 on the lunar surface was formed on the basis of the large-scale Moon pictures with stars which were obtained the unique separate plates method (Khabibullin, Rizvanov, Bystrov 1974). Unlike the methods of processing Moon pictures without stars, we have the absolute definition of the zero orientation, the point of the coordinate system and its scale, in the case of binding to the stars. When choosing lunar craters, which are included in the basic KSC-1162 network, the following criteria were used. Firstly, the regularly-round shaped craters were taken; secondly the craters were of small size; thirdly, the selected objects have been well observed, and fourthly, the craters of a network, had been included in the objects listings of other known selenodetic catalogues and met the IAU recommendations. The required parameters were determined from 2m equations:

$$\mathbf{A}\Theta + \varepsilon = \mathbf{Z},$$

where $A = (A_{ij})_m$ is structured matrix, $\Theta = (\Delta\xi, \Delta\eta, \Delta\zeta)$ is column vector of the required parameters, $\mathbf{Z} = (\Delta X, \Delta Y)$ is column vector of the observation, ε is column vector of MSE. The solution of the required parameters $\hat{\Theta} = (\Delta\hat{\xi}, \Delta\hat{\eta}, \Delta\hat{\zeta})$ will be:

$$\hat{\Theta} = (A^T\mathbf{PA}) + (A^T\mathbf{PZ}),$$

And the errors are put by covariance matrix:

$$\mathbf{D}(\hat{\Theta}) = \frac{V^T\mathbf{PV}}{2m-3}(A^T\mathbf{PA}),$$

where $\mathbf{V}$ is the vector of residual deviations. The KSC-1162 catalogue analysis has shown that it meets the following demands: it includes a sufficient number of control points for the lunar figures study and for the realization of fine bindings to them. It contains the objects with coordinates, which are related to the lunar mass ephemeris center and these objects cover a large lunar surface area. The accuracy points coordinates is ±30 m in the planning coordinates and ±80 m in height.

In the construction of basic network KSC-1162 the algorithms developed for binding of lunar and star plates are used. At least, two of tasks of a method of the least squares (MLS) considered at it can be solved today more precisely. As over-treatment the base catalogue will demand bulky over-calculations, it is necessary to investigate and to estimate numerically as far as expedient. At the decision of a standard task of definition of constants of a star plate the method of six constants (Turner's method) was used. We shall consider three possible updatings this way based on regressional modelling (Valeev 1991).

3 The redefining of the coordinates of selenocentric KSC-1162 directory

The worked out algorithms (Bystrov, Rizvanov 1973; Khabibullin, Rizvanov, Bystrov 1974) for the lunar and star plates binding were used by KSC-1162 base network building. Today, two problems of the least square method (LSM) can be solved more precisely. Since the base directory reprocessing will require cumbersome recalculation, it is necessary to investigate and estimate numerically how it will be appropriate.

The 6th permanent method (Turner method) was used to solve the standard problem of permanent stellar plate determination. We consider three possible modifications of this method which are based on the regression modeling (Valeev 1991).

- The method of the complete exhaustion structures. The second and third degrees polynomials can be used instead of a polynomial expansion of the standard star X and Y coordinates of the first degree by measured star and y coordinates. The optimal structure of the transformation model for each coordinate is determined by the complete exhaustion structures under the minimum "outside" MSE σ_Δ. This model of the "floating" structure for each plate provides increased accuracy of the mark coordinate determination and of course the catalogue objects from a few tens of per cent or higher.

- Orthogonalization method for two-dimensional case. The coordinate transformation problem is considered as a Turner's problem with the additional condition of orthogonality transition from the measured coordinates system in the standard one which is an adaptation to a breach of the LSM condition of the measured x and y coordinates independence.
- The method of the independence X and Y coordinates standard accounting (the solution of the simultaneous equations system SSE). In this case the interdependency influence between the standard X and Y coordinates is eliminated. Detecting it, the one of the coordinates comes into the right part of the polynomial by another coordinate with a factor which is subject to assessment. The second LSM problem is solved for the system in (Rizvanov 1985).

$$\mathbf{A}\Theta + \varepsilon = \mathbf{Z}, \tag{1}$$

where $A = (A_{ij})_m$ is previously calculated matrix of coordinates conversion for each m plate, $\Theta = (\Delta\xi, \Delta\eta, \Delta\zeta)$ is the vector of estimated corrections to the adopted values of the coordinates craters (catalogue objects), $\mathbf{Z} = (\Delta X, \Delta Y)_K^T$ is the observation vector.

Unlike the first problem which aims to predict, expression (1) is used to obtain estimates, making it necessary to verify LSM application conditions, to diagnose the regression (RA)–LSM analysis conditions (Valeev 1991). Carrying them out, we can state that these estimates are the best linear estimates (consistent, unbiased and efficient) within the capabilities of used observations amount. The adaptation to the breach of the independence conditions can be carried put by the orthogonalization method or SSE solution. The adapted methods to other breaches are considered in the monograph (Valeev 1991).

4 The selenodesy directories reduction in the KSC-1162 catalogue system

Detailing and expansion of the selenocentric catalogue to the visible and far side of the Moon and to its libration zone are included in the study. The determining problem of the matrix transition elements between KSC-1162 baseline, intermediate systems and reducible directory must be solved accurately.

The methods for the two-dimensional case can be investigated for its precision solutions. We will give a detailed consideration of one of them; its analytic version has been applied to the expansion of the visible Moon hemisphere coordinate network to the back one, according to the pictures measuring results with "Zond-6, -8" missions (Valeev 1991). When transforming coordinates from one (X) Cartesian coordinate system into another (Y), the model of affine transformation is used

$$\mathbf{X} = \mathbf{AY} + \mathbf{X}_0, \tag{2}$$

where $\mathbf{X} = (X_1 X_2 X_3)^T, \mathbf{Y} = (Y_1 Y_2 Y_3)^T$ are coordinate vectors in M_x and M_y systems, $\mathbf{A} = (A_{ij})$ is a matrix of orientation, $\mathbf{X}_0 = (X_{01} X_{02} X_{03})^T$ is the vector of M_x center system displacement relative to M_y.

LSM is used for determining elements by the general objects and for displacing. LSM is carried to each of three equations subsystems either individually or to the joint system. (2) transformation does not always provide a satisfactory accuracy. Because of coordinate errors in M_x and M_y systems and the probable estimates interdependence matrix can not satisfy the conditions of the orthogonal transition from M_x to M_y:

$$\mathbf{A}^T \mathbf{A} = \mathbf{E}, det\mathbf{A} = 1, \tag{3}$$

where $\mathbf{E}$ is identity matrix.

In view of it, the (2) expression is a model, which competes with (2) model and others. (2) expression is considered together with (3) conditions. This model has been used in Valeev (1991). Within the framework of constrained minimization theory, the model parameters can be evaluated by analytical or numerical problem solution of the minimum search of quadratic form (absolute or relative) $S = \varepsilon^T \varepsilon$ with non-linear contingency in the equalities form

$$\begin{array}{c} min\varepsilon^T \varepsilon, \\ \mathbf{A}, \mathbf{X}_0 \in \mathbf{G} \\ \mathbf{A}^T \mathbf{A} = \mathbf{E}, det\mathbf{A} = 1, \end{array} \tag{4}$$

where ε is the vector of errors for (2) model, $\varepsilon^T \varepsilon = \Sigma_{1 \leq i \leq n} \Sigma_{1 \leq j \leq 3} \varepsilon_{ij}, n$ is number of objects; $\mathbf{G}$ is accessible region.

Conclusion

The choice of method of coordinate transformation should be made by the thorough investigations of the comparative effectiveness of the following approaches:

- affine transformation
- optimal polynomial approximation
- orthogonal transformation with and without the account of systematic errors
- a solution of the simultaneous system equations.

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