Remarks on the mean orbits of the meteoroid stream

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Abstract

The practical formulae for calculating the mean orbit of the meteoroid stream, free from some conceptual faults, is given. Instead of Keplerian elements the heliocentric vectorial elements are averaged, using the least squares method with two constrains. However because due to simultaneous averaging of 7 variables, our approach is limited to the streams of 8 or more members only. In addition, a numerical example of the application of our method is also presented here.

1 Introduction: concepts of mean orbit of the stream

In astronomic literature it might be hard to find a paper in which explicit would be explain what a mean orbit of a meteoroid stream means, what it concerns and how it is calculated. Tacitly, a mean orbit is calculated as arithmetic mean of parameters obtained more or less directly from the observations.

In Williams et al. (2004) the authors distinguished two concepts of the mean orbit of the meteoroid stream. In the first concept, the mean orbit is determined by the use of orbits obtained directly from observations. This approach is often applied in the search of stream associations in meteor orbit databases. Meanwhile the stream modellers would use the concept of the mean of the whole sample of the test particles.

Therefore, both concepts contribute to the final result, which may be mutually different not only due to conceptional reasons but also, because of the method used for calculation of the mean parameters. This situation illustrates Fig. 1.

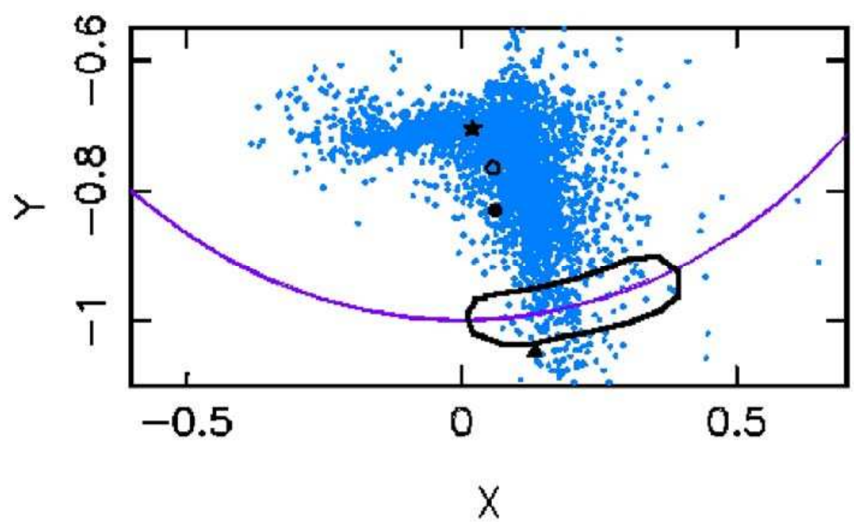

Figure 1: *The orbital nodes of members of meteoroid stream plotted on the ecliptic plane, (see Jopek et al., 2006). Also we plotted the nodes of three mean orbits obtained by: the arithmetic mean (filled circle), weighted mean (open circle) and by the method presented in this paper (star). Schematically we outlined the area coresponding to the observed meteors. This area contains the mean orbit determined by the observed sample.*

2 Problems and shortcomings

The mean orbit determination, in the frame of both concepts, particularly as an arithmetic means of the arbitrarily selected dynamical parameters, should be taken with care. Below, a short discussion of several remarks related to this topic is presented.

2.1 The choice of the averaged meteoroid parameters

The meteoroid orbit may be described or by the geocentric (α, δ, v_g, ...) or by the heliocentric parameters (a, q, e, ω, Ω, i). The averaging procedure of parameters from both groups separately unfortunately may lead to the numerical inconsistency between the mean heliocentric orbit and the mean geocentric radiant of the same stream. To avoid this we should calculate the mean parameters from one group only, and then use those results in determination of the mean parameters of the second group. However, we still have to arbitrarily decide, which group of parameters have priority over the second one. This leads to another difficulty - the final value of the mean orbit depends which group was chosen for averaging first.

Additionally, for averaging of the geocentric parameters we may recommend the method described by Voloshchuk and Kashcheev (1999), whereas the method for the heliocentric parameters is presented further in this paper.

2.2 The mean orbital elements have to satisfy equations of celestial mechanics

Separate averaging of the heliocentric elements (eg. a, a^{-1}, q, e, or p) may provide us results which are inconsistent with the well known laws of celestial mechanics. In particular it cannot be accepted as the scientific result the case when obtained mean orbit has $a_s < 0$, and simultaneously with eccentricity $e_s < 1$.

2.3 Averaging of the angles

During averaging of the angular parameters, among others things, the difficulties arise from the fact that their values are normalised into the interval 0° - 360°. This is well known property, which is especially apparent when members of the northern and southern branches of the same stream are averaged blindly.

In the literature, e.g. in Southworth and Hawkins (1963), Lindblad (1971) or Jopek and Froeschlé (1997), when such difficulties occurred, one of the meteor orbit (N or S type) was transferred into the opposite orbit type - prograde or retrograde. Other method is based on conversion of angular orbital elements, i.e. ω, Ω, and i, into the unit vectors P, Q, R, which are directed along with the line of apside, latus rectus and pointing to the pole the orbit, respectively. Instead of the angles, the components of these vectors are averaged separately, and then their mean values are transformed into the angular parameters again (see e.g. Jopek et al., 2003). However, because of the separate averaging of all components the resulting mean vectors are no longer mutually orthogonal as required by celestial mechanics. Therefore, by reason of this approach, we should be aware of some bias in such obtained mean angular orbital elements.

The method, free from the difficulties mentioned above, may be found in already mentioned paper of Voloshchuk and Kashcheev (1999), where the authors propose the method for averaging geocentric parameters of the meteoroid stream members. In the case of the angular elements they use a small variation of the approach described by Mardia (1972), which can be also used for calculating the mean values of ω, Ω and i.

2.4 Does the mean orbit of the stream have to satisfy the Earth's orbit crossing condition?

Some authors claim that the mean orbit of the meteoroid stream should fulfil the Earth's orbit crossing condition whereby at least one of its nodal distances, R_{Ω} or $R_{\mho}$, have to be contained in interval $[0.983, 1.017]$. These requirement is based on the conviction that such condition is fulfilled by the observed meteoroid orbits. However it is not accurate for all such orbits, as shows Fig. 2, where for the real observed low-inclined meteoroid orbit both nodal distances lay significantly outside the assumed interval. Therefore, we claim that there is no reason to impose on the mean orbit of the meteoroid stream the Earth's orbit crossing condition.

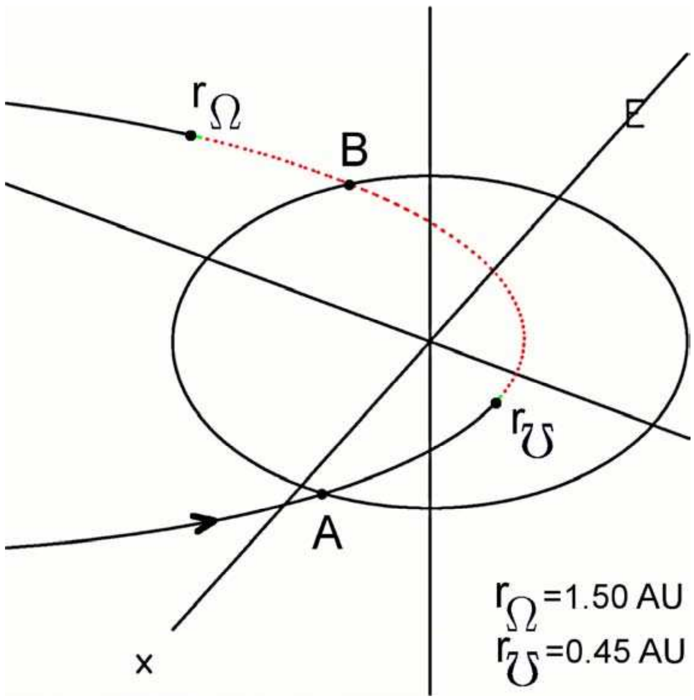

Figure 2: *Illustration of the Earth crossing condition for the low-inclined meteoroid orbit:* $\alpha_G = 14.5^{\circ}$, $\delta_G = 6.2^{\circ}$, $V_G = 29.19[\mathrm{km/s}]$ *and* $T_{OBS} = 1956\ 09\ 29.192$. *The meteoroid entered the atmosphere in point A, its orbit punctured the ecliptic plane in points* $r_{\mho}$ *and* r_{Ω}*, both are far away from the Earth orbit.*

2.5 The epoch of the mean parameters of the meteoroid stream

In the catalogues available at the IAU Meteor Data Center we have orbital parameters of meteors given at a time of their observations (osculating epochs). These data in a further process of a study may be used for averaging. However, in general, orbits of observed meteoroids are averaged ignoring the epochs which their correspond to. Even if we can average such orbits we must be aware that neglecting the differences in the epochs of the data would result in introducing (perhaps) small noise in the values of mean parameters.

The epochs problem has two components. Apart from already mentioned averaging of the orbits corresponding to different osculating epochs, the second, quite probable bias, coresponds to different epochs of the origin of the observed meteoroids. Whereas the first component of the problem can be removed by preintegration of the data in a catalogue onto the common osculating epoch, elimination of the second factor is rather impossible.

3 Averaging of the heliocentric parameters - our method

In the paper Jopek et al. (2006, 2008), to find the mean heliocentric orbit of a meteor stream, we propose an approach based on the vectorial elements of the stream members. We average these quantities by the least square method with two constrains. Using the mean values of the vectorial elements, one may calculate the mean Keplerian orbital elements, and in turn, they may be used to find the mean geocentric parameters.

As the vectorial elements we take the components of the expanded vector $(h,e,E)^T$, which consists the angular momentum vector h, the Laplace vector e and the energy constant E. In the units of AU, AU/day and the mass of the Sun, those quantities are defined by the following equations:

$$h = (h_1, h_2, h_3)^T = r \times r \tag{1}$$

$$e = (e_1, e_2, e_3)^T = \frac{1}{\mu} r \times h - \frac{r}{|r|} \tag{2}$$

$$E = \frac{1}{2} r^2 - \frac{\mu}{|r|} \tag{3}$$

where: $\mu = k^2$, k is the Gauss constant, whereas $r = (x, y, z)^T$ and $\dot{r} = (\dot{x}, \dot{y}, \dot{z})^T$ are the heliocentric vectors of the position and velocity of the meteoroid, respectively.

The vectors h and e, by definition, are mutually orthogonal ($h \cdot e = 0$), whereas their lengths and the energy E are related by equation $e^2 - 2Eh^2\mu^{-2} - 1 = 0$. Including these constrains into the least-squares method, in Jopek et al. (2006) we have developed an iterative procedure which gives the mean orbital elements of the meteoroid stream consistent with the laws of the celestial mechanics, namely

$$O_{i+1} = O_i + \Delta O_i = O_i + R_i^{-1} t_i \qquad i = 0,1,2,... \tag{4}$$

where

$$\mathbf{R}_i^{-1}\mathbf{t}_i = \begin{pmatrix} N & 0 & 0 & 0 & 0 & 0 & 0 & -e_{1i} & -\frac{4h_{1i}E_i}{\mu^2} \\ 0 & N & 0 & 0 & 0 & 0 & 0 & -e_{2i} & -\frac{4h_{2i}E_i}{\mu^2} \\ 0 & 0 & N & 0 & 0 & 0 & 0 & -e_{3i} & -\frac{4h_{3i}E_i}{\mu^2} \\ 0 & 0 & 0 & N & 0 & 0 & 0 & -h_{1i} & -2e_{1i} \\ 0 & 0 & 0 & 0 & N & 0 & 0 & -h_{2i} & -2e_{2i} \\ 0 & 0 & 0 & 0 & 0 & N & 0 & -h_{3i} & -2e_{3i} \\ 0 & 0 & 0 & 0 & 0 & 0 & N & 0 & \frac{2h_i^2}{\mu^2} \\ -e_{1i} & -e_{2i} & -e_{3i} & -h_{1i} & -h_{2i} & -h_{3i} & 0 & 0 & 0 \\ -\frac{4h_{1i}E_i}{\mu^2} & -\frac{4h_{2i}E_i}{\mu^2} & -\frac{4h_{3i}E_i}{\mu^2} & -2e_{1i} & -2e_{2i} & -2e_{3i} & \frac{2h_i^2}{\mu^2} & 0 & 0 \end{pmatrix} \begin{pmatrix} \sum_{k=1}^{N}(h_{1i} - h_{1k}) \\ \sum_{k=1}^{N}(h_{2i} - h_{2k}) \\ \sum_{k=1}^{N}(h_{3i} - h_{3k}) \\ \sum_{k=1}^{N}(e_{1i} - e_{1k}) \\ \sum_{k=1}^{N}(e_{2i} - e_{2k}) \\ \sum_{k=1}^{N}(e_{3i} - e_{3k}) \\ \sum_{k=1}^{N}(E_i - E_k) \\ h_i \cdot e_i \\ e_i^2 - \frac{2E_i}{\mu^2} h_i^2 - 1 \end{pmatrix} \tag{5}$$

For $i = 0$ the procedure starts with the approximate values

$$O_0 = (h_0, e_0, E_0)^T = (h_{10}, h_{20}, h_{30}, e_{10}, e_{20}, e_{30}, E_0)^T$$

obtained as the arithmetic means of the vectorial elements of N stream members: $O_k = (h_{1k}, h_{2k}, h_{3k}, e_{1k}, e_{2k}, e_{3k}, E_k)^T$, $k = 1 \ldots N$.

Having the mean values of the vectorial elements $O_s = (h_s, e_s, E_s)^T = (h_{1s}, h_{2s}, h_{3s}, e_{1s}, e_{2s}, e_{30}, E_s)^T$, we may easily convert them into Keplerian orbital parameters. Namely, for the eccentricity e_s and the semi-latus rectum p_s we have

$$e_s = \sqrt{1 + \frac{2E_s}{\mu^2} h_s^2} \qquad p_s = \frac{h_s^2}{\mu} \tag{6}$$

The perihelion distance q_s and the semi-major axis we calculate only when $E_s \neq 0$ and $h_s = |h_s| \neq 0$

$$q_s = \frac{h_s^2}{\mu(1+e_s)} \qquad a_s = -\frac{\mu}{2E_s} \tag{7}$$

If $h_s = |h_s| = |(h_{1s}, h_{2s}, h_{3s})| \neq 0$, the inclination of the mean meteoroid orbit i may be computed by

$$\cos i = \frac{h_{3s}}{h_s}, \quad \sin i = \frac{\sqrt{h_{1s}^2 + h_{2s}^2}}{h_s} \tag{8}$$

If $i \neq 0$ and $i \neq \pi$, for the longitude of the ascending node we have equations

$$\cos\Omega = \frac{-h_{2s}}{\sqrt{h_{1s}^2 + h_{2s}^2}}, \quad \sin\Omega = \frac{h_{1s}}{\sqrt{h_{1s}^2 + h_{2s}^2}} \tag{9}$$

Finally, if $e_s = |e_s| = |(e_{1s}, e_{2s}, e_{3s})| \neq 0$, $i \neq 0$ and $i \neq \pi$, the argument of perihelion ω one can find from

$$\cos\omega = \frac{h_{1s} e_{2s} - h_{2s} e_{1s}}{e_s \sqrt{h_{1s}^2 + h_{2s}^2}}, \quad \sin\omega = \frac{h_s e_{3s}}{e_s \sqrt{h_{1s}^2 + h_{2s}^2}} \tag{10}$$

4 Numerical example

To make deployment of our method easier, we give an numerical example. Choosing the true anomaly $f = 0$, by means of formulae

$$\begin{aligned}
x &= q\,(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) \\
y &= q\,(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) \\
z &= q\,\sin\omega\sin i \\
\dot{x} &= \sqrt{\mu(1+e)q^{-1}}\,(-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i) \\
\dot{y} &= \sqrt{\mu(1+e)q^{-1}}\,(-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i) \\
\dot{z} &= \sqrt{\mu(1+e)q^{-1}}\,\cos\omega\sin i \\
r &= \sqrt{x^2 + y^2 + z^2}
\end{aligned} \tag{11}$$

$$\begin{aligned}
h_1 &= y\,\dot{z} - z\,\dot{y} & e_1 &= \mu^{-1}(\dot{y}\,h_3 - \dot{z}\,h_2) - xr^{-1} \\
h_2 &= z\,\dot{x} - x\,\dot{z} & e_2 &= \mu^{-1}(\dot{z}\,h_1 - \dot{x}\,h_3) - xr^{-1} \\
h_1 &= x\,\dot{y} - y\,\dot{x} & e_3 &= \mu^{-1}(\dot{y}\,h_2 - \dot{y}\,h_1) - xr^{-1} \\
E &= 0.5\,(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \mu r^{-1}
\end{aligned} \tag{12}$$

we calculated vectorial elements for $N = 11$ orbits of the Lyrids identified in Jopek et al. (2003). Next, after 3 iterations we determined the mean vectorial elements and using equations (6-10) the values of the mean Keplerian orbital parameters, see Table 1.

q	e	ω	Ω	i	$h_1 \cdot 10^2$	$h_2 \cdot 10^2$	$h_3 \cdot 10^3$	$e_1 \cdot 10^1$	$e_2 \cdot 10^1$	$e_3 \cdot 10^1$	$E \cdot 10^5$
0.918	1.153	213.0	31.4	81.5	1.246	-2.042	3.575	-7.770	-5.830	-6.211	2.466
0.911	0.927	216.3	31.4	79.0	1.166	-1.910	4.349	-5.831	-4.786	-5.387	-1.186
0.925	0.984	213.1	32.4	80.1	1.230	-1.938	4.007	-6.465	-5.197	-5.294	-0.2559
0.928	0.994	212.3	33.6	80.4	1.277	-1.922	3.902	-6.508	-5.387	-5.237	-0.09566
0.919	0.906	215.1	32.8	78.5	1.209	-1.875	4.539	-5.668	-4.888	-5.105	-1.513
0.915	0.968	215.3	32.9	79.1	1.231	-1.903	4.365	-6.059	-5.179	-5.493	-0.5174
0.930	0.971	212.2	34.1	81.1	1.290	-1.905	3.603	-6.355	-5.269	-5.112	-0.4614
0.917	0.897	215.5	31.1	79.3	1.152	-1.909	4.212	-5.753	-4.600	-5.118	-1.662
0.936	0.860	211.7	32.0	80.4	1.186	-1.898	3.785	-5.806	-4.517	-4.456	-2.213
0.911	0.985	215.7	31.3	76.0	1.166	-1.918	5.596	-6.112	-5.344	-5.577	-0.2436
0.924	0.987	213.1	32.3	80.0	1.227	-1.940	4.047	-6.489	-5.209	-5.308	-0.2082
0.9203	0.9661	213.90	32.31	79.58	1.2166	-1.9233	4.1831	-6.2560	-5.1098	-5.2998	-0.54495
0.9213	0.9665	213.94	32.30	79.58	1.2162	-1.9236	4.1801	-6.2560	-5.1098	-5.22998	-0.53546

Table 1: *The Keplerian orbital elements and the vectorial elements of 11 Lyrids identified amongst 1830 photographic data. The first row of the second part of table gives the mean values obtained by our method and the second row gives the mean values calculated as arithmetic means.*

5 Conclusions

The concept of the mean orbit of the meteoroid stream is a bit conceptually and computationally unclear, and should be taken with some care. In both concepts of the mean (the mean of the observed sample, and the mean of the population) is still not certain which parameters should be used to obtain the mean, and how they should be averaged. Nevertheless, the method for calculating the mean orbits of the meteoroid stream described in this paper gives some progress to the problem.

The method:

- is based on the vectorial elements and two constrains and gives the mean elements also satisfying these constrains, which, as consequence, means that the orbital elements also satisfying the laws of celestial mechanics,

- due to simultaneous averaging of 7 vectorial elements by the least square method, our approach is limited to the streams of 8 or more members only,

- 7 vectorial elements fully describe the size, shape and the spatial orientation of the orbits (the ellipses, parabolas, and hyperbolas). Actually they are calculated firstly from the meteoroid geocentric parameters, the Earth position and velocity vectors, and then are used for estimating the Keplerian orbital elements,

- the mean geocentric parameters may be calculated by the theoretical radiant approach, however to obtain the geo- and helio- parameters mutually consistent and referred to the well defined epoch, before averaging all members of the stream should be preintegrated to the common epoch of time,

- the results of the numerical example given in this paper has shown, that the orbital elements, calculated by the arithmetic means and by our method, differ by small values, more significantly for q, and e. For obvious reasons, the largest difference occurs for the semi-major axes - using the values of E from the last two rows of Table 1 we found that for Lyrids it amounted $\Delta a = 0.52$ (AU).

Acknowledgement

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Q&A

Ms G. Ryabova:

I see that you presented results of very complicated study. But I have a question: how are you going to use these nice averaged meteoroid stream orbits? For what purpose?

The authors:

The mean orbit of the stream may be considered as an approximation which gives some typical values of the orbital elements of the stream members. We can then compare the mean orbit of a well established meteor stream with an orbit of a newly observed meteor (or meteors) to confirm its origin. This saves us time which we would need to spend for making separate comparisons with all meteors from our data.