

Times of inspiralling for interplanetary dust grains

J. Klačka¹★ and M. Kocifaj²★

¹Faculty of Mathematics, Physics, and Informatics, Comenius University, Mlynská dolina, 842 48 Bratislava, Slovak Republic

²Astronomical Institute, Slovak Academy of Sciences, Dúbravská cesta 9, 845 04 Bratislava, Slovak Republic

Accepted 2008 August 1. Received 2008 June 30; in original form 2008 May 21

ABSTRACT

There are uncharged spherical dust particles interacting with solar electromagnetic radiation moving in the gravitational fields of the Sun and the Earth. The Earth is supposed to be moving in a circular orbit, and the effect of solar electromagnetic radiation is given by the Poynting–Robertson effect. The time of particle inspiralling toward the Sun is analysed for the pure Poynting–Robertson effect and in order to include the gravitational effect of the Earth. It is shown that the exterior mean-motion orbital resonances with the Earth may increase the time of spiralling into the Sun by 50 per cent, compared with the case of neglecting the gravitational effect of the Earth, for eccentricities smaller than 0.8. The result holds for particles from 1 μm to tens of μm in radii.

Key words: scattering – methods: numerical – celestial mechanics – interplanetary medium – meteors, meteoroids.

1 INTRODUCTION

The action of electromagnetic radiation on spherical dust particles is known as the Poynting–Robertson effect (Robertson 1937; Klačka 2004). When the Poynting–Robertson effect acts on an interplanetary dust particle orbiting the Sun, the particle is decelerated and spirals in toward the Sun; a systematic decrease in the secular changes of particle’s semimajor axis and eccentricity exists (Robertson 1937; Wyatt & Whipple 1950; Klačka 2004; Sections 6.1 and 6.2.2) if the optical properties of the particle are constant.

If a planet is also taken into consideration, then the spiralling particle can be captured in mean-motion orbital resonances with the planet, for some time interval (Jackson & Zook 1989; Weidenschilling & Jackson 1993; Beaugé & Ferraz-Mello 1994; Marzari & Vanzani 1994; Šidlichovský & Nesvorný 1994; Liou & Zook 1995, 1997; Liou, Zook & Jackson 1995). As a consequence, there should be dust rings around planetary orbits. This has been found, and the observational situation is described and commented on by Dermott et al. (1994), Brownlee (1994) and Reach et al. (1995).

The effect of mean-motion orbital resonances can increase the particle’s time of inspiralling toward the Sun. We are interested in finding whether this increase is because of the presence of the Earth.

2 EQUATION OF MOTION

We are interested in the orbital evolution of spherical dust particles under the action of solar electromagnetic radiation, solar gravity

and gravity of the Earth. The presence of the Earth can cause a prolonging of the time of spiralling toward the Sun. The equation of motion of the particle is

$$\frac{d\mathbf{v}}{dt} = -\frac{GM_{\odot}}{r^3}\mathbf{r} + \beta\frac{GM_{\odot}}{r^2}\left[\left(1 - \frac{\mathbf{v}\cdot\mathbf{e}_R}{c}\right)\mathbf{e}_R - \frac{\mathbf{v}}{c}\right] - Gm_P\left(\frac{\mathbf{r}-\mathbf{r}_P}{|\mathbf{r}-\mathbf{r}_P|^3} + \frac{\mathbf{r}_P}{|\mathbf{r}_P|^3}\right),$$

$$\mathbf{e}_R \equiv \mathbf{r}/|\mathbf{r}|,$$

$$\beta \equiv \frac{L_{\odot}\pi R^2}{4\pi G M_{\odot} m} \frac{\bar{Q}'_{\text{pr}}}{c} = 7.6 \times 10^{-4} \bar{Q}'_{\text{pr}} \frac{\pi R^2 (\text{m}^2)}{m (\text{kg})}. \quad (1)$$

Here G is the gravitational constant, M_{\odot} is the mass of the Sun, \mathbf{r} is the position vector of the particle with respect to the Sun, $r = |\mathbf{r}|$, \mathbf{r}_P is the position vector of the planet with respect to the Sun, m_P is mass of the planet, m is mass of the particle and R is its effective radius. \bar{Q}'_{pr} is the efficiency factor for the radiation pressure integrated over the solar spectrum (averaged over all wavelengths and weighted by the solar spectrum) and calculated for the radial direction (as for the dimensionless factor of the effectiveness of radiation pressure; see also, for example, section 4.5 in Bohren & Huffman 1983 and equations (2)–(5) in Klačka & Kocifaj 2006a). We have also used the flux density of radiative energy $L_{\odot}/(4\pi r^2)$, where L_{\odot} is the solar luminosity. We consider the effect of the electromagnetic force to the first order in v/c , where v is the heliocentric velocity of the particle and c is the speed of light. The radiation term in equation (1) corresponds to the Poynting–Robertson effect. As for the effect of the gravity, the term $Gm_P\mathbf{r}_P/|\mathbf{r}_P|^3$ comes from the fact that we describe the motions of the particle and the planet with respect to the Sun (equations of motion hold in an inertial frame of reference and the Sun also moves in such a frame of reference because of the gravity force of the planet; this corresponds to the term $\mathbf{r}_P/|\mathbf{r}_P|^3$,

★E-mail: klacka@fmph.uniba.sk (JK); kocifaj@savba.sk (MK)

which is often ignored). We suppose that the planet moves in a circular orbit around the Sun.

3 TIME OF SPIRALLING – ANALYTICAL APPROACH: THE POYNTING–ROBERTSON EFFECT

If the planet were not present, then equation (1) reduces to the action of solar radiation and solar gravity alone. We suppose that the optical properties of the particle do not change. To analyse the inspiralling of the spherical particle from an initial value of the semimajor axis a_{in} to a final semimajor axis a we can introduce the integral quantity $F(e, e_{\text{in}})$:

$$F(e, e_{\text{in}}) = \frac{(1 - e_{\text{in}}^2)^2}{e_{\text{in}}^{8/5}} \int_e^{e_{\text{in}}} \frac{x^{3/5}}{(1 - x^2)^{3/2}} dx, \quad (2)$$

Here, e_{in} is the initial eccentricity of the particle based on the idea that central Keplerian acceleration is $-GM_{\odot}(1 - \beta)r/r^3$. The final eccentricity e is given by the initial values of the semimajor axis and eccentricity a_{in} and e_{in} , and also by the value of the final semimajor axis a :

$$\frac{1 - e_{\text{in}}^2}{1 - e^2} \left(\frac{e}{e_{\text{in}}} \right)^{4/5} = \frac{a}{a_{\text{in}}}. \quad (3)$$

Equations (2) and (3) are obtained on the basis of considerations presented in Robertson (1937) and Wyatt & Whipple (1950). Equation (2) comes from the differential equation $dp/dt = -2\beta(GM_{\odot}/c)(1 - e^2)^{3/2}/p$ (see p. 19 in Klačka 2004) and $p = p_{\text{in}}(e/e_{\text{in}})^{4/5}$ (Wyatt & Whipple 1950).

As a consequence of equation (3), we easily find that the value $e = 0$ corresponds to the value $a = 0$.

3.1 Semimajor axis: $a_{\text{in}} \rightarrow 0$

If the final semimajor axis equals 0, then the secular orbital evolution yields for the time of inspiralling toward the Sun

$$\tau_0 = \frac{2}{5} \left(\beta \frac{GM_{\odot}}{c} \right)^{-1} a_{\text{in}}^2 F(0, e_{\text{in}}), \quad (4)$$

where the function F is defined by equation (2). Frequently, the approximate time to spiral into the Sun is presented (e.g. Leinert & Grün 1990, p. 226):

$$\tau_{\text{circ}0} = \frac{1}{4} \left(\beta \frac{GM_{\odot}}{c} \right)^{-1} a_{\text{in}}^2. \quad (5)$$

The result in equation (5) can be obtained from equation (4) in the limit $e_{\text{in}} \rightarrow 0$; it is supposed that the particle moves in a ‘circular’ orbit.

If the time is measured in years and the semimajor axis in astronomical units (au), then $GM_{\odot} = 4\pi^2 \text{ au}^3 \text{ yr}^{-2}$ and the speed of light is $c = 6.31 \times 10^4 \text{ au yr}^{-1}$. Equations (4) and (5) can be written as

$$\tau_0(\text{yr}) = \frac{2}{5} \frac{1.6 \times 10^3}{\beta} [a_{\text{in}}(\text{au})]^2 F(0, e_{\text{in}}), \quad (6)$$

$$\tau_{\text{circ}0}(\text{yr}) = \frac{4.0 \times 10^2}{\beta} [a_{\text{in}}(\text{au})]^2. \quad (7)$$

3.2 Semimajor axis: $a_{\text{in}} \rightarrow a$

A more general case corresponds to the situation when the spherical particle spirals from the initial value of the semimajor axis a_{in} to the final semimajor axis equal to a . The secular orbital evolution yields for the time of inspiralling from a_{in} to a

$$\tau_a = \frac{2}{5} \left(\beta \frac{GM_{\odot}}{c} \right)^{-1} a_{\text{in}}^2 F(e, e_{\text{in}}), \quad (8)$$

where the value of a is given by equation (3) and equation (2) is also important.

Again, the results are based on the idea that central Keplerian acceleration is $-GM_{\odot}(1 - \beta)r/r^3$.

If we want to use the approximation of a ‘circular’ orbit represented by equation (5), then we would obtain the following result instead of equation (8):

$$\tau_{\text{circ}a} = \frac{1}{4} \left(\beta \frac{GM_{\odot}}{c} \right)^{-1} (a_{\text{in}}^2 - a^2). \quad (9)$$

Equations (8) and (9) can also be written in the forms:

$$\tau_a(\text{yr}) = \frac{2}{5} \frac{1.6 \times 10^3}{\beta} [a_{\text{in}}(\text{au})]^2 F(e, e_{\text{in}}), \quad (10)$$

$$\tau_{\text{circ}a}(\text{yr}) = \frac{4.0 \times 10^2}{\beta} \{ [a_{\text{in}}(\text{au})]^2 - [a(\text{au})]^2 \}. \quad (11)$$

Also, equation (3) has to be used in equation (10).

4 NUMERICAL RESULTS

The numerical study concentrates on the evolution of dust particles with β equal to 0.01 and 0.10, and with the following initial values of orbital elements: (i) 1–2.5 au in the semimajor axis; (ii) 0–0.8 in eccentricity. We suppose that distributions in a_{in} and e_{in} are uniform. These considerations can be taken as an approximation for near-Earth asteroids.

4.1 Semimajor axis: $a_{\text{in}} \rightarrow 0$

The average time of spiralling toward the Sun ($\tau_{\text{p}0}$) is analysed in this section, where the index indicates that the Earth is taken into account. The result is compared with the average time of spiralling without the gravity of the Earth; that is, with the average times (τ_0) and ($\tau_{\text{circ}0}$) corresponding to equations (4) and (5), or equations (6) and (7), for the same initial values of orbital elements. The obtained results are presented in Table 1 for the case when the ecliptic plane is identical to the plane of the particle’s motion.

Table 1. Comparison of the average times of spiralling toward the Sun for particles with the given values of β and various initial conditions in orbital elements. $\tau_{\text{circ}0}$ and τ_0 are given by equations (2), (6) and (7) and $\tau_{\text{p}0}$ denotes the time taken to spiral into the Sun when the Earth is also included. Averaging is calculated over all initial values of the semimajor axis a_{in} and eccentricity e_{in} and various initial positions of the particle with respect to the Earth.

β	$\langle \tau_{\text{circ}0} \rangle$ (yr)	$\langle \tau_0 \rangle$ (yr)	$\langle \tau_{\text{p}0} \rangle / \langle \tau_0 \rangle$
0.01	1.37×10^5	1.27×10^5	1.50
0.10	1.37×10^4	1.27×10^4	1.42

Table 2. Comparison of the average times of spiralling toward the Sun for particles with the given values of β and various initial conditions in orbital elements. $\tau_{\text{circ } 1}$ and τ_1 are given by equations (2), (3), (10) and (11) and τ_{p1} denotes the time taken to spiral toward the Sun when the Earth is also included. Averaging is calculated over all initial values of the semimajor axis a_{in} and eccentricity e_{in} and various initial positions of the particle with respect to the Earth.

β	$\langle\tau_{\text{circ } 1}\rangle$ (yr)	$\langle\tau_1\rangle$ (yr)	$\langle\tau_{p1}\rangle/\langle\tau_1\rangle$
0.01	9.7×10^4	9.0×10^4	1.71
0.10	9.7×10^3	9.0×10^3	1.59

4.2 Semimajor axis: $a_{\text{in}} \rightarrow a = 1$ au

The time of spiralling toward the Sun (τ_{p1}) is calculated for particles with semimajor axes decreasing from various initial values a_{in} to the final value $a = 1$ au. The Earth is taken into account. This reflects the fact that only exterior mean-motion orbital resonances with the Earth are significant in determining the time of inspiralling of interplanetary dust grains toward the Sun (capture times in interior mean-motion orbital resonances are negligibly small). The result is compared with the average time of spiralling without the gravity of the Earth; that is, with the average times $\langle\tau_1\rangle$ and $\langle\tau_{\text{circ } 1}\rangle$ corresponding to equations (8), (3) and (9), or equations (3), (10) and (11), for the same initial values of orbital elements. The obtained results are presented in Table 2 for the case when the plane of the Earth's motion is identical to the plane of the particle's motion.

4.3 Resonant captures

Captures into mean-motion orbital resonances increase the particle's time of spiralling toward the Sun. The increase is about 60–70 per cent of the Poynting–Robertson spiralling times before the particles reach the orbit of the Earth (see Table 2). The result is practically independent of the size of the particles. In reality, any particle may be captured into several different resonances (characterized by the ratio n_p/n , where n_p is the mean motion of the Earth and n is the ('instantaneous') mean motion of the particle during its spiralling toward the Sun. Fig. 1 depicts the mean times that particles spend in various resonances; the two values of β and the initial conditions defined above are taken into account. In accor-

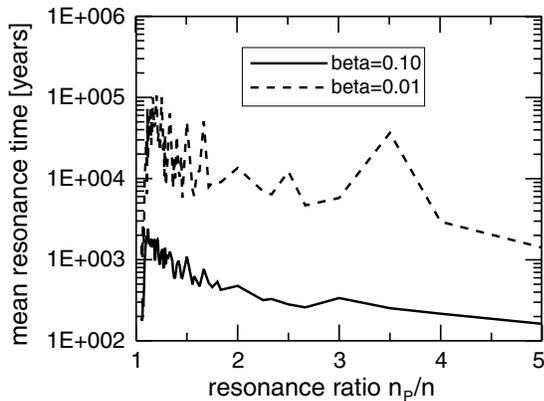


Figure 1. Relative occurrence (in per cent) of the ratios of capture times $T_{\beta=0.01}$ and $T_{\beta=0.1}$ for exterior mean-motion orbital resonances with the Earth. On average, particles with $\beta = 0.01$ exhibit larger capture times than particles with $\beta = 0.1$.

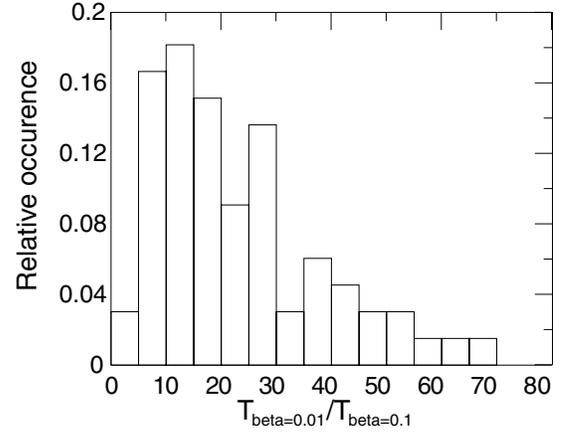


Figure 2. Probability of the resonance occurrence among all the cases when particles are captured in resonance. The most probable are captures into the resonances 2 : 1, 3 : 2, 3 : 1 and 5 : 2 with the Earth.

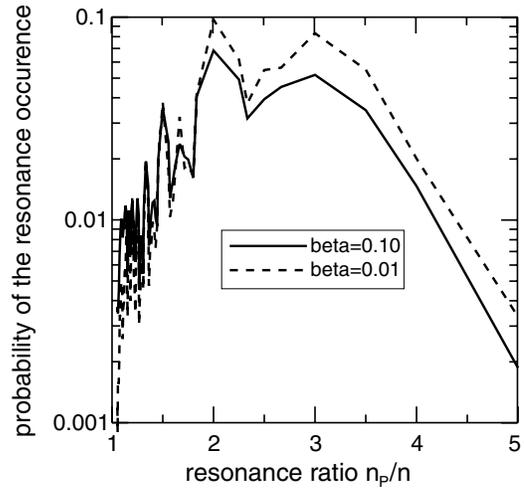


Figure 3. Mean resonance time as a function of the type of exterior mean-motion orbital resonance with the Earth. Spherical particles with $\beta = 0.01$ exhibit longer captures than particles with $\beta = 0.1$, on average.

dance with the results of Table 2, the capture times are larger, in general, for smaller β . Fig. 2 presents a summary of all exterior mean-motion orbital resonances with the Earth. Because the process of the Poynting–Robertson spiralling toward the Sun is slower for smaller β , we expect that the possibility of being captured into a resonance is higher for smaller β . The real situation is different, and the result can be obtained from Fig. 3, if the values on the vertical axis are multiplied by the numerical factors 2.94 for $\beta = 0.01$ and by 7.61 for $\beta = 0.1$ (if we want to obtain the results in per cent, then multiplication by 100 also has to be carried out).

4.4 Eccentricities greater than 0.8

We have also performed numerical calculations for the set of initial eccentricities lying in the interval 0.8–0.99; the initial values of the semimajor axes were unchanged. The mean spiralling times, calculated according to equations (2) and (6), are 6.97×10^3 yr for $\beta = 0.1$, and 6.97×10^4 yr for $\beta = 0.01$. The speed of inspiralling

is more rapid than for more circular orbits (compare the values with the values of $\langle\tau_0\rangle$ presented in Table 1), and thus we expect a lower possibility of being captured in a mean-motion orbital resonance with the Earth. The numerical results show that this situation really occurs and the values of the increase of the time of spiralling toward the Sun, because of the capture in the resonances, are very small, less than 1 per cent for both values of β : $\langle\tau_{p0}\rangle/\langle\tau_0\rangle = 1.009$ for $\beta = 0.01$ and $\langle\tau_{p0}\rangle/\langle\tau_0\rangle = 1.003$ for $\beta = 0.1$.

5 DISCUSSION

Tables 1 and 2 present the important ratios $\langle\tau_{p0}\rangle/\langle\tau_0\rangle$ and $\langle\tau_{p1}\rangle/\langle\tau_1\rangle$ for the two values of β . It can immediately be seen that the ratios are not very sensitive to the value of β . The smaller β is, the greater the ratios discussed above.

Why does the ratio $[\langle\tau_{p0}\rangle/\langle\tau_0\rangle(\beta = 0.01)] : [\langle\tau_{p0}\rangle/\langle\tau_0\rangle(\beta = 0.1)]$ practically equal the ratio $[\langle\tau_{p1}\rangle/\langle\tau_1\rangle(\beta = 0.01)] : [\langle\tau_{p1}\rangle/\langle\tau_1\rangle(\beta = 0.1)]$, and why is the value 1.066 ± 0.010 ?

In order to be able to understand the obtained results, we have to use some ideas from orbital evolution in mean-motion orbital resonances. According to equations (73) and (83) in Klačka & Kocifaj (2006a) – compare also Klačka & Kocifaj (2006b) and Liou & Zook (1997) – the secular change of eccentricity of the particle is

$$\frac{de}{dt} = \frac{\beta}{(1-\beta)^{2/3}} \left(1 + \frac{\eta}{Q_{pr,1}}\right) \frac{GM_\odot}{c} \left(\frac{n_p}{n}\right)^{-4/3} \times \frac{\sqrt{1-e^2}}{a_p^2 e} \left[1 - \frac{n}{n_p} \frac{1+3e^2/2}{(1-e^2)^{3/2}}\right]. \quad (12)$$

Here, a_p is the semimajor axis of the planet, n_p and n are mean motions of the planet and the particle and the constant $\eta = 1/3$ considers the effect of the solar wind. Although we have not considered the solar wind in our simulations, we have kept it in equation (12) in order to show that it does not influence the result pointed out in the questions presented above.

Equation (12) yields that the secular evolution of eccentricity in mean-motion orbital resonance, defined by the ratio n_p/n , is a monotonous function of time. The evolution of e tends to a limiting value, which is never reached, and thus we are not able to find the real capture time. However, we can obtain information on the capture time dependence on β , on the basis of equation (12). We expect that

$$T_{\text{capture}} \propto \frac{(1-\beta)^{2/3}}{\beta}. \quad (13)$$

If we are interested in the ratio $(T_{\text{spiral}} + T_{\text{capture}})/T_{\text{spiral}}$, we immediately obtain, on the basis of equations (4) and (13) (or equations 8 and 13):

$$\frac{T_{\text{spiral}} + T_{\text{capture}}}{T_{\text{spiral}}} = 1 + \frac{T_{\text{capture}}}{T_{\text{spiral}}} = 1 + \frac{(1-\beta)^{2/3}}{\beta} \left(\frac{1}{\beta}\right)^{-1} Z. \quad (14)$$

Here, Z represents all quantities except for β . We can assume that the value of Z is practically constant for an averaging in calculating T_{capture} for a large set of initial conditions, as used in our simulations. Equation (14) then yields:

$$\frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_1) \left[\frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_2) \right]^{-1} = \left(\frac{1-\beta_1}{1-\beta_2} \right)^{2/3}. \quad (15)$$

On the basis of the results presented in Tables 1 and 2, and equation (15), we can summarize:

$$\begin{aligned} \frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_1) \left[\frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_2) \right]^{-1} &= \left[\left(\frac{\langle\tau_{p0}\rangle}{\langle\tau_0\rangle} - 1 \right) (\beta_1) \right] \left[\left(\frac{\langle\tau_{p0}\rangle}{\langle\tau_0\rangle} - 1 \right) (\beta_2) \right]^{-1} \\ &= \frac{50}{42} = 1.19, \end{aligned}$$

$$\begin{aligned} \frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_1) \left[\frac{T_{\text{capture}}}{T_{\text{spiral}}}(\beta_2) \right]^{-1} &= \left[\left(\frac{\langle\tau_{p1}\rangle}{\langle\tau_1\rangle} - 1 \right) (\beta_1) \right] \left[\left(\frac{\langle\tau_{p1}\rangle}{\langle\tau_1\rangle} - 1 \right) (\beta_2) \right]^{-1} \\ &= \frac{71}{59} = 1.20, \end{aligned}$$

$$\left(\frac{1-\beta_1}{1-\beta_2} \right)^{2/3} = 1.07,$$

$$\beta_1 = 0.01 \quad \beta_2 = 0.1. \quad (16)$$

The physical considerations presented in this section show that they are not very different from reality, as the obtained values 1.19, 1.20 and 1.07 are of the same order, and moreover they do not significantly differ from each other.

6 SUMMARY AND CONCLUSIONS

The motion of spherical interplanetary dust grains with radii between 1 μm and several tens of μm is driven by the gravity of the Sun, and its electromagnetic radiation in the form of the Poynting–Robertson effect. These two forces are the dominant forces. The other non-negligible force is produced by the gravity of planets. If we consider dust grains in the inner part of the Solar system, the gravitational effect of the Earth may play a significant role. In our model, the Earth is supposed to be moving in a circular orbit. The times of inspiralling toward the Sun are compared for the Poynting–Robertson inspiralling and for the case of inclusion of the gravitational effect of the Earth. The exterior mean-motion orbital resonances with the Earth may increase the time of spiralling into the Sun by 50 per cent, compared with the case of neglecting the gravitational effect of the Earth, if a uniform distribution for the particles' semimajor axes and eccentricities are supposed: $1 \leq a_{\text{in}} \leq 2.5$ au and $0 \leq e_{\text{in}} \leq 0.8$. The real time taken to spiral into the orbit of the Earth is increased by 60–70 per cent, compared with the case of neglecting the gravitational effect of the Earth. These results hold for particles from 1 μm to tens of μm in radii.

ACKNOWLEDGMENTS

This work has been supported by the Scientific Grant Agency VEGA (grant No. 1/3074/06) and by the Slovak Academy of Sciences (grant No. 350/OMS/Fun/07, as part of DAAD project No. D/07/01266).

REFERENCES

- Beaugé C., Ferraz-Mello S., 1994, *Icarus*, 110, 239
 Bohren C. F., Huffman D. R., 1983, *Absorption and Scattering of Light by Small Particles*. Wiley, New York

- Brownlee D. E., 1994, *Nat*, 369, 706
Dermott S. F., Jayaraman S., Xu Y. L., Gustafson B. A. S., Liou J. C., 1994, *Nat*, 369, 719
Jackson A. A., Zook H. A., 1989, *Nat*, 337, 629
Klačka J., 2004, *Celest. Mech. Dynam. Astron.*, 89, 1
Klačka J., Kocifaj M., 2006a, in Maravell N. S., ed., *Space Science: New Research*. Nova Science, New York, p. 245
Klačka J., Kocifaj M., 2006b, *J. Quant. Spectrosc. Radiat. Transfer*, 100, 187
Leinert Ch., Grün E., 1990, in Schwen R., Marsch E., eds, *Physics of the Inner Heliosphere I*. Springer-Verlag, Berlin, p. 207
Liou J.-Ch., Zook H. A., 1995, *Icarus*, 113, 403
Liou J.-Ch., Zook H. A., 1997, *Icarus*, 128, 354
Liou J.-Ch., Zook H. A., Jackson A. A., 1995, *Icarus*, 116, 186
Marzari F., Vanzani V., 1994, *A&A*, 283, 275
Reach W. T. et al., 1995, *Nat*, 374, 521
Robertson H. P., 1937, *MNRAS*, 97, 423
Šidlichovský M., Nesvorný D., 1994, *A&A*, 283, 972
Weidenschilling S. J., Jackson A. A., 1993, *Icarus*, 104, 244
Wyatt S. P., Whipple F. L., 1950, *ApJ*, 111, 558

This paper has been typeset from a $\text{\TeX}/\text{\LaTeX}$ file prepared by the author.