

Distant future of the Sun and Earth revisited

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ABSTRACT

We revisit the distant future of the Sun and the Solar system, based on stellar models computed with a thoroughly tested evolution code. For the solar giant stages, mass loss by the cool (but not dust-driven) wind is considered in detail. Using the new and well-calibrated mass-loss formula of Schröder & Cuntz, we find that the mass lost by the Sun as a red giant branch (RGB) giant ($0.332 M_{\odot}$, 7.59 Gyr from now) potentially gives planet Earth a significant orbital expansion, inversely proportional to the remaining solar mass.

According to these solar evolution models, the closest encounter of planet Earth with the solar cool giant photosphere will occur during the tip-RGB phase. During this critical episode, for each time-step of the evolution model, we consider the loss of orbital angular momentum suffered by planet Earth from tidal interaction with the giant Sun, as well as dynamical drag in the lower chromosphere. As a result of this, we find that planet Earth will not be able to escape engulfment, despite the positive effect of solar mass loss. In order to survive the solar tip-RGB phase, any hypothetical planet would require a present-day minimum orbital radius of about 1.15 au. The latter result may help to estimate the chances of finding planets around white dwarfs.

Furthermore, our solar evolution models with detailed mass-loss description predict that the resulting tip-AGB (asymptotic giant branch) giant will not reach its tip-RGB size. Compared to other solar evolution models, the main reason is the more significant amount of mass lost already in the RGB phase of the Sun. Hence, the tip-AGB luminosity will come short of driving a final, dust-driven superwind, and there will be no regular solar planetary nebula (PN). The tip-AGB is marked by a last thermal pulse, and the final mass loss of the giant may produce a circumstellar (CS) shell similar to, but rather smaller than, that of the peculiar PN IC 2149 with an estimated total CS shell mass of just a few hundredths of a solar mass.

Key words: Sun: evolution – solar–terrestrial relations – stars: evolution – stars: mass-loss – supergiants – white dwarfs.

1 INTRODUCTION

Climate change and global warming may have drastic effects on the human race in the near future, over human time-scales of decades or centuries. However, it is also of interest, and of relevance to the far future of all living species, to consider the much longer term effects of the gradual heating of the Earth by a more luminous Sun as it evolves towards its final stage as a white dwarf star. This topic has been explored on several occasions (e.g. Sackmann, Boothroyd & Kraemer 1993; Rybicki & Denis 2001; Schröder, Smith & Apps 2001, hereafter SSA), and has been discussed very recently by Laughlin (2007).

Theoretical models of solar evolution tell us that the Sun started on the zero-age main sequence (ZAMS) with a luminosity only about

70 per cent of its current value, and it has been a long-standing puzzle that the Earth seems none the less to have maintained a roughly constant temperature over its lifetime, in contrast to what an atmosphere-free model of irradiation would predict. Part of the explanation may be that the early atmosphere, rich in CO₂ that was subsequently locked up in carbonates, kept the temperature up by a greenhouse effect which decreased in effectiveness at just the right rate to compensate for the increasing solar flux. The role of clouds, and their interaction with galactic cosmic rays (CR), may also be important: there is now some evidence (Svensmark 2007; but see Harrison et al. 2007 and Priest et al. 2007) that CRs encourage cloud cover at low altitudes, so that a higher CR flux would lead to a higher albedo and lower surface temperature. The stronger solar wind from the young Sun would have excluded galactic CRs, so cloud cover on the early Earth may have been less than now, allowing the full effect of the solar flux to be felt.

What of the future? Although the Earth's atmosphere may not be able to respond adequately on a short time-scale to the increased

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greenhouse effect of carbon dioxide and methane released into the atmosphere by human activity, there is still the possibility, represented by James Lovelock’s Gaia hypothesis (Lovelock 1979, 1988, 2006), that the biosphere may on a longer time-scale be able to adjust itself to maintain life. Some doubt has been cast on that view by recent calculations (Scaife, private communication; for details, see e.g. Betts et al. 2004; Cox et al. 2004) which suggest that, on the century time-scale, the inclusion of biospheric processes in climate models actually leads to an increase in carbon dioxide emissions, partly through a feedback that starts to dominate as vegetation dies back. In any case, it is clear that the time will come when the increasing solar flux will raise the mean temperature of the Earth to a level that not even biological or other feedback mechanisms can prevent. There will certainly be a point at which life is no longer sustainable, and we will discuss this further in Section 3.

After that, the fate of the Earth is of interest mainly insofar as it tells us what we might expect to see in systems that we observe now at a more advanced stage of evolution. We expect the Sun to end up as a white dwarf – do we expect there to be any planets around it, and in particular do we expect any small rocky planets like the Earth?

The question of whether the Earth survives has proved somewhat tricky to determine, with some authors arguing that the Earth survives (e.g. SSA) and others (e.g. Sackmann et al. 1993) claiming that even Venus survives, while general textbooks (e.g. Prialnik 2000) tend to say that the Earth is engulfed. A simple model (e.g. SSA), ignoring mass loss from the Sun, clearly shows that all the planets out to and including Mars are engulfed, either at the red giant branch (RGB) phase – Mercury and Venus – or at the later AGB phase – the Earth and Mars. However, the Sun loses a significant amount of mass during its giant branch evolution, and that has the effect that the planetary orbits expand, and some of them keep ahead of the advancing solar photosphere. The effect is enhanced by the fact (SSA) that when mass loss is included the solar radius at the tip of the AGB is comparable to that at the tip of the RGB, instead of being much larger; Mars certainly survives, and it appears (SSA) that the Earth does also.

The crucial question here is: what is the rate of mass loss in real stars? Ultimately, this must be determined from observations, but in practice these must be represented by some empirical formula. Most people use the classical Reimers’ formula (Reimers 1975, 1977), but there is considerable uncertainty in the value to be used for his parameter η , and different values are needed to reproduce the observations in different parameter regimes. In our own calculations (SSA), we used a modification of the Reimers’ formula, which has since been further improved and calibrated rather carefully against observation, so that we believe that it is currently the best available representation of mass loss from stars with non-dusty winds (Schröder & Cuntz 2005, 2007 – see Section 2, where we explore the consequences of this improved mass-loss formulation).

However, although we have considerably reduced the uncertainties in the mass-loss rate, there is another factor that works against the favourable effects of mass loss: tidal interactions. Expansion of the Sun will cause it to slow its rotation, and even simple conservation of angular momentum predicts that by the time the radius has reached some 250 times its present value (cf. Table 1) the rotation period of the Sun will have increased to several thousand years instead of its present value of under a month; effects of magnetic braking will lengthen this period even more. This is so much longer than the orbital period of the Earth, even in its expanded orbit, that the tidal bulge raised on the Sun’s surface by the Earth will pull the Earth back in its orbit, causing it to spiral inwards.

Table 1. Main physical properties of characteristic solar models.

Phase	Age (Gyr)	$L (L_{\odot})$	$T_{\text{eff}} (K)$	$R (R_{\odot})$	$M_{\text{Sun}} (M_{\odot})$
ZAMS	0.00	0.70	5596	0.89	1.000
Present	4.58	1.00	5774	1.00	1.000
MS:hottest	7.13	1.26	5820	1.11	1.000
MS:final	10.00	1.84	5751	1.37	1.000
RGB:tip	12.17	2730.	2602	256.	0.668
ZA-He	12.17	53.7	4667	11.2	0.668
AGB:tip	12.30	2090.	3200	149.	0.546
AGB:tip-TP	12.30	4170.	3467	179.	0.544

Note: 1.00 au = 215 R_{\odot} .

This effect was considered by Rybicki & Denis (2001), who argued that Venus was probably engulfed, but that the Earth might survive. An earlier paper by Rasio et al. (1996) also considered tidal effects and concluded on the contrary that the Earth would probably be engulfed. However, the Rybicki & Denis calculations were based on combining analytic representations of evolution models (of Hurley, Pols & Tout 2000) with the original Reimers’ mass-loss formula rather than on full solar evolution calculations with a well-calibrated mass-loss formulation. The Rasio et al. paper also employed the original Reimers’ formula, and both papers use somewhat different treatments of tidal drag. We have therefore re-considered this problem in detail, with our own evolutionary calculations and an improved mass-loss description as the basis; full details are given in Sections 2 and 4.

2 SOLAR EVOLUTION MODEL WITH MASS LOSS

In order to describe the long-term solar evolution, we use the Eggleton evolution code (Eggleton 1971, 1972, 1973) in the version described by Pols et al. (1995, 1998), which has updated opacities and an improved equation of state. Among other desirable characteristics, his code uses a self-adapting mesh and a ∇ -based prescription of ‘overshooting’, which has been well-tested and calibrated with giant stars in eclipsing binaries (for details see Pols et al. 1997; Schröder, Pols & Eggleton 1997; Schröder 1998). Because of the low mass and a non-convective core, solar evolution models are, however, not subject to any MS (main sequence) core overshooting. In use, the code is very fast, and mass loss is accepted simply as an outer boundary condition.

As already pointed out by VandenBerg (1991), evolution codes have the tendency to produce, with their most evolved models, effective temperatures that are slightly higher than the empirically determined values. The reason lies, probably, in an inadequacy of both low-temperature opacities and mixing-length theory (MLT) at low gravity. With the latter, we should expect a reduced efficiency of the convective energy transport for very low gravity because the largest eddies are cut out once the ratio of eddy-size to stellar radius has increased too much with g^{-1} . Hence, as described by Schröder, Winters & Sedlmayr (1999), our mixing-length parameter, normally $\alpha = 2.0$ for $\log g < 1.94$, receives a small adjustment in the form of a gradual reduction for supergiant models, reaching $\alpha = 1.67$ at $\log g = 0.0$. With this economical adjustment, our evolution models now give a better match to empirically determined effective temperatures of very evolved late-type giants and supergiants, such as α^1 Her (see Schröder & Cuntz 2007, fig. 4 in particular), and even later stages of stellar evolution (Dyck et al. 1996; van Belle et al. 1996; Van Belle et al. 1997).

The evolution model of the Sun presented here uses an opacity grid that matches the empirical solar metallicity of Anders & Grevesse (1989), $Z = 0.0188$, derived from atmospheric models with simple 1D radiative transfer – an approach consistent with our evolution models. Together with $X = 0.700$ and $Y = 0.2812$, there is a good match with present-day solar properties derived in the same way (see Pols et al. 1995). We note that the use of 3D-hydrodynamic modelling of stellar atmospheres and their radiative transfer may lead to a significantly lower solar abundance scale (e.g. Asplund, Grevesse & Sauval 2005, who quote $Z = 0.0122$), but these lower values are still being debated, and create some problems with helioseismology. Of course, using lower metallicities with an evolution code always results in more compact and hotter stellar models. Hence, if we used a lower Z our code would plainly fail to reproduce the present-day Sun, and the reliability of more evolved models with lower Z must therefore also be seriously doubted.

The resulting solar evolution model suggests an age of the present-day MS Sun of 4.58 Gyr (± 0.05 Gyr), counted from its zero-age MS start model, which is well within the range of commonly accepted values for the real age of the Sun and the Solar system (e.g. Sackmann et al. 1993). Our model also confirms some well-established facts: (1) The MS-Sun has already undergone significant changes, i.e. the present solar luminosity L exceeds the zero-age value by $0.30 L_{\odot}$, and the zero-age solar radius R was 11 per cent smaller than the present value. (2) There was an increase of effective temperature T_{eff} from, according to our model, 5596 to 5774 K (± 5 K). (3) The present Sun is increasing its average luminosity at a rate of 1 per cent in every 110 million years, or 10 per cent over the next billion years. All this is completely consistent with established solar models like the one of Gough (1981).

Certainly, the solar MS-changes and their consequences for Earth are extremely slow, compared to the current climate change driven by human factors. Nevertheless, solar evolution will force global warming upon Earth already in the ‘near’ MS future of the Sun, long before the Sun starts its evolution as a giant star [see our discussion of the habitable zone (HZ) in Section 3].

At an age of 7.13 Gyr, the Sun will have reached its highest T_{eff} of 5820 K, at a luminosity of $1.26 L_{\odot}$. From then on, the evolving MS Sun will gradually become cooler, but its luminosity will continue to increase. At an age of 10.0 Gyr, the solar effective temperature will be back at $T_{\text{eff}} = 5751$ K, while $L = 1.84 L_{\odot}$, and the solar radius then will be 37 per cent larger than today. Around that age, the evolution of the Sun will speed up, since the solar core will change from central hydrogen-burning to hydrogen shell-burning and start to contract. In response, the outer layers will expand, and the Sun will start climbing up the RGB (the ‘red’ or ‘first giant branch’ in the Hertzsprung–Russell diagram) – at first very gradually, but then accelerating. At an age of 12.167 Gyr, the Sun will have reached the tip of the RGB, with a maximum luminosity of $2730 L_{\odot}$.

In order to quantify the mass-loss rate of the evolved, cool solar giant at each time-step, we use the new and well-calibrated mass-loss formula for ordinary cool winds (i.e. not driven by dust) of Schröder & Cuntz (2005, 2007). This relation is, essentially, an improved Reimers’ law, physically motivated by a consideration of global chromospheric properties and wind energy requirements:

$$\dot{M} = \eta \frac{L_* R_*}{M_*} \left(\frac{T_{\text{eff}}}{4000 \text{ K}} \right)^{3.5} \left(1 + \frac{g_{\odot}}{4300 g_*} \right) \quad (1)$$

with $\eta = 8 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$, g_{\odot} = solar surface gravitational acceleration, and L_* , R_* and M_* in solar units.

This relation was initially calibrated by Schröder & Cuntz (2005) with the total mass loss on the RGB, using the blue-end (i.e. the least massive) horizontal-branch (HB) stars of globular clusters with different metallicities. This method avoids the interfering problem of temporal mass-loss variations found with individual giant stars and leaves an uncertainty of the new η -value of only 15 per cent, just under the individual spread of RGB mass loss required to explain the width of HBs.

Later, Schröder & Cuntz (2007) tested their improved mass-loss relation with six nearby galactic giants and supergiants, in comparison with four other, frequently quoted mass-loss relations. All but one of the tested giants are AGB stars, which have (very different) well-established physical properties and empirical mass-loss rates, all by cool winds *not* driven by radiation-pressure on dust. Despite the afore-mentioned problem with the inherent time-variability of this individual-star approach, the new relation equation (1) was confirmed to give the best representation of the cool, but not ‘dust-driven’ stellar mass loss: it was the only one that agreed within the uncertainties (i.e. within a factor of 1.5 to 2) with the empirical mass-loss rates of *all* giants. Hence, since the future Sun will not reach the critical luminosity required by a ‘dust-driven’ wind (see Section 5), we here apply equation (1) to describe its AGB mass loss as well as its RGB mass loss.

The exact mass loss suffered by the future giant Sun has, of course, a general impact on the radius of the solar giant, since the reduced gravity allows for an even larger (and cooler) supergiant. The luminosity, however, is hardly affected because it is mostly set by the conditions in the contracting core and the hydrogen-burning shell. In total, our solar evolution model yields a loss of $0.332 M_{\odot}$ by the time the tip-*RGB* is reached (for $\eta = 8 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$). This is a little more than the $0.275 M_{\odot}$ obtained by Sackmann et al. (1993), who used a mass-loss prescription based on the original, simple Reimers’ relation. Furthermore, our evolution model predicts that at the very tip of the RGB, the Sun should reach $R = 256 R_{\odot} = 1.2 \text{ au}$ (see Fig. 1), with $L = 2730 L_{\odot}$ and $T_{\text{eff}} = 2602 \text{ K}$. More details are given in Table 1.

By comparison, a prescription of the (average) RGB mass-loss rate with $\eta = 7 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$, near the lower error limit of the mass-loss calibration with HB stars, yields a solar model at the very

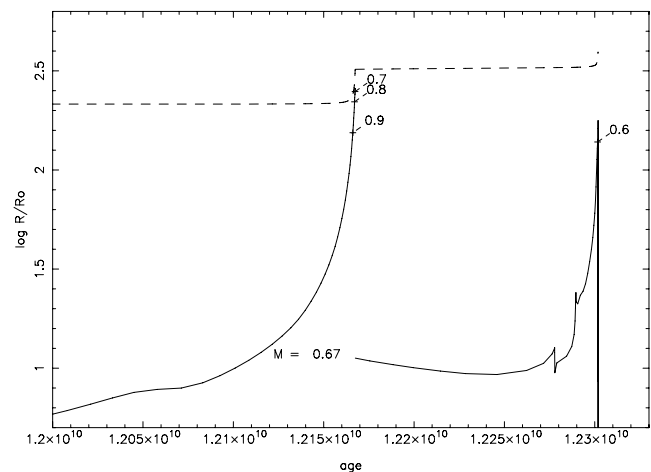


Figure 1. Solar radius evolution during the RGB and AGB phases. Included for comparison (dashed curve) is the potential orbital radius of planet Earth, taking account of solar mass loss but neglecting any loss of orbital angular momentum. The labels on the curve for the solar radius show the mass of the Sun in units of its present-day mass.

tip of the RGB with $R = 249 R_{\odot}$, $L = 2742 L_{\odot}$, $T_{\text{eff}} = 2650$ K and a total mass lost on the RGB of $0.268 M_{\odot}$. With $\eta = 9 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$, on the other hand, the Sun would reach the very tip of the RGB with $R = 256 R_{\odot}$, $L = 2714 L_{\odot}$, $T_{\text{eff}} = 2605$ K and will have lost a total of $0.388 M_{\odot}$. While these slightly different possible outcomes of solar tip-RGB evolution – within the uncertainty of the mass-loss prescription – require further discussion, which we give in Section 4.3, the differences are too small to be obvious on the scale of Fig. 1.

With the reduced solar mass and, consequently, lower gravitational attraction, all planetary orbits – that of the Earth included – are bound to expand. This is simply a consequence of the conservation of angular momentum $\Lambda_E = M_E v_E r_E$, while the orbital radius (i.e. r_E) adjusts to a new balance between centrifugal force and the reduced gravitational force of the Sun, caused by the reduced solar mass $M_{\text{Sun}}(t)$. Substituting $v_E = \sqrt{GM_{\text{Sun}}(t)/r_E}$ in Λ_E yields $r_E \propto \Lambda_E^2 / M_{\text{Sun}}(t)$. For this conservative case, we find that r_E is 1.50 au for the case $\eta = 8 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$. For the smaller (7×10^{-14}) and larger (9×10^{-14}) values of η , we find, respectively, $r_E = 1.37$ and 1.63 au, so in all cases the orbital radius is comfortably more than the solar radius, when angular momentum is conserved.

Section 4.1 provides a treatment of the more realistic case, in which angular momentum is *not* conserved. We have taken great care in determining the mass loss and other parameters for our models because the best possible models of the evolution of solar mass and radius through the tip-RGB phase are required to provide reliable results.

The significant solar RGB mass loss will also shape the later solar AGB evolution. Compared with models without mass loss, the AGB Sun will not become as large and luminous, and will be shorter-lived, because it lacks envelope mass for the core and its burning shells to ‘eat’ into. In fact, the solar tip-AGB radius ($149 R_{\odot}$) will never reach that of the tip-RGB (see Fig. 1), and AGB thermal pulses (TPs) are no threat to any planet which would have survived the tip-RGB. Our evolution code resolved only the two final and most dramatic TPs (cf. Section 5).

The regular tip-AGB luminosity of $2090 L_{\odot}$ will not exceed the tip-RGB value, either. Hence, as will be discussed in Section 5, the tip-AGB Sun will not develop a sustained dust-driven superwind but will stay short of the critical luminosity required by dust-driven winds (see Schröder et al. 1999). The very tip of the AGB coincides with a TP, after which the giant briefly reaches a peak luminosity of $4170 L_{\odot}$, but at a higher $T_{\text{eff}} = 3467$ K than on the RGB (see Table 1 and Section 5), keeping the radius down to $179 R_{\odot}$. Again, the best possible treatment of all prior mass loss from the giant Sun is essential for modelling this phase reliably.

3 EVOLUTION OF THE HABITABLE ZONE

The Earth currently sits in the ‘habitable zone’ in the Solar system, that is, the region in which conditions on the Earth – in particular the average planetary temperature – are favourable for life. There are various precise definitions of ‘habitability’ in the literature, and a useful overview of HZs in the wider context of extrasolar planetary systems is given by Franck et al. (2002). For the current paper, a convenient definition is that a planet is habitable if the conditions on it allow the presence of liquid water on its surface. This may allow extremes of temperature that would make life uncomfortable if not impossible for humans, but the argument is that life of any kind (at least any kind we know about at present) requires water at some stage in its life cycle. We will adopt that definition in this

paper, but note that even with that apparently simple definition it is not straightforward to calculate the width of the HZ.

It may be instructive to begin with a calculation of the mean planetary temperature in terms of a spherical blackbody by assuming that the planetary body absorbs the solar flux intercepted by its (circular) cross-sectional area and re-emits it spherically symmetrically at a blackbody temperature T . Then, (cf. SSA) T is given by

$$T = (1 - A)^{1/4} \left(\frac{R}{2D} \right)^{1/2} T_{\text{eff}} \\ = 0.0682 (1 - A)^{1/4} \left(\frac{R}{R_{\odot}} \right)^{1/2} \left(\frac{1 \text{ au}}{2D} \right)^{1/2} T_{\text{eff}}, \quad (2)$$

where D is the distance of the body from the centre of the Sun, R is the radius of the Sun, A is the Bond albedo of the Earth and T_{eff} is the effective temperature of the Sun. On that basis, taking $T_{\text{eff}} = 5774$ K and $R = R_{\odot}$ (Table 1), and $A = 0.3$ (Kandel & Viollier 2005), we find $T(1 \text{ au}) = 255$ K. But the actual mean temperature of the Earth at present is 33 K warmer, at $T = 288$ K. This demonstrates the warming effect of our atmosphere, which becomes significantly more important with higher temperature (see below).

In fact, there are various complex, partly antagonistic, atmospheric feedback mechanisms (e.g. the greenhouse effect, the variation of planetary albedo with the presence of clouds, snow and ice, and the carbonate–silicate cycle which determines the amount of carbon dioxide in the atmosphere) that act to change the surface temperature from what it would be in the absence of an atmosphere. These mechanisms have been carefully discussed by Kasting, Whitmire & Reynolds (1993), who conclude that a conservative estimate of the current HZ stretches from 0.95 to 1.37 au. We will adopt their result for the limited purposes of this paper. It can be adjusted in a simple-minded way to allow for the evolution of the Sun by scaling the inner and outer HZ radii $r_{\text{HZ},i}$, $r_{\text{HZ},o}$ with the changing solar luminosity $L_{\text{Sun}}(t)$: $r_{\text{HZ}} \propto \sqrt{L_{\text{Sun}}(t)}$. In this way, the respective critical values of solar irradiance derived by Kasting et al. (1993) for the inner and outer edge of the HZ are maintained.

Certainly, with the 10 per cent increase of solar luminosity over the next 1 Gyr (see previous section), it is clear that Earth will come to leave the HZ already in about a billion years time, since the inner (hot side) boundary will then cross 1 au. By the time the Sun comes to leave the main sequence, around an age of 10 Gyr (Table 1), our simple model predicts that the HZ will have moved out to the range 1.29 to 1.86 au. The Sun will have lost very little mass by that time, so the Earth’s orbital radius will still be about 1 au – left far behind by the HZ, which will instead be enveloping the orbit of Mars.

By the time the Sun reaches the tip of the RGB, at 12.17 Gyr, the Earth’s orbital radius will only have expanded to at most 1.5 au, but the HZ will have a range of 49.4 to 71.4 au, reaching well into the Kuiper Belt! The positions of the HZ boundaries are not as well determined as these numbers suggest, because in reality the scaling for the boundaries of the HZ almost certainly depends also on how clouds are affected by changes in the solar irradiance. These effects are complex and uncertain (cf. Kasting 1988), and may increase or decrease the speed at which the HZ drifts outwards. But, none the less it seems clear that the HZ will move out past the Earth long before the Sun has expanded very much, even if the figure of one billion years is a rather rough estimate of how long we have before the Earth is uninhabitable.

In other planetary systems around solar-type stars, conditions may be different, and it may even be possible for life to start during a star’s post-main-sequence evolution, if a planet exists at a suitable distance from the star. This possibility is discussed by Lopez,

Schneider & Danchi (2005), who also give a general discussion of the evolution of HZs with time. However, they use the evolution models of Maeder & Meynet (1988), which do not agree as well as ours with the colours and observed T_{eff} of the red giants in star clusters (see e.g. illustrations given by Meynet, Mermilliod & Maeder 1993), and which predict a very different behaviour for the solar radius; so their results are not directly comparable with ours.

What will happen on the Earth itself? Ignoring for the moment the short-time-scale (decades to centuries) problems currently being introduced by climate change, we may expect to have about one billion years before the solar flux has increased by the critical 10 per cent mentioned earlier. At that point, neglecting the effects of solar irradiance changes on the cloud cover, the water vapour content of the atmosphere will increase substantially and the oceans will start to evaporate (Kasting 1988). An initially moist greenhouse effect (Laughlin 2007) will cause runaway evaporation until the oceans have boiled dry. With so much water vapour in the atmosphere, some of it will make its way into the stratosphere. There, solar UV will dissociate the water molecules into OH and free atomic hydrogen, which will gradually escape, until most of the atmospheric water vapour has been lost. The subsequent dry greenhouse phase will raise the surface temperature significantly faster than would be expected from our very simple blackbody assumption, and the ultimate fate of the Earth, if it survived at all as a separate body (cf. Section 4), would be to become a molten remnant.

4 THE INNER PLANETARY SYSTEM DURING TIP-RGB EVOLUTION

After 12 Gyr of slow solar evolution, the final ascent of the RGB will be relatively fast. The solar radius will sweep through the inner planetary system within only five million years, by which time the evolved solar giant will have reached the tip of the RGB and then entered its brief (130 million year) He-burning phase. The giant will first come to exceed the orbital size of Mercury, then Venus. By the time it approaches Earth, the solar mass-loss rate will reach up to $2.5 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ and lead to some orbital expansion (see Section 2). But the extreme proximity of the orbiting planet to the solar photosphere requires the consideration of two effects, which both lead to angular momentum loss and a fatal decrease of the orbital radius of planet Earth.

4.1 Tidal interaction

For the highly evolved giant Sun, we may safely assume (cf. Section 1) that it has essentially ceased to rotate, after nearly two billion years of post-MS magnetic braking acting on the hugely expanded, cool RGB giant. Consequently, any tidal interaction with an orbiting object will result in its suffering a continuous drag by the slightly retarded tidal bulges of the giant solar photosphere.

As shown in Section 2, the orbital radius of planet Earth r_E depends on the angular momentum squared, by the equation

$$r_E = \frac{\Lambda_E^2(t)}{M_E^2 G M_{\text{Sun}}(t)}. \quad (3)$$

Hence, the terrestrial orbit reacts quite sensitively to any loss of angular momentum, by shrinking.

The retardation of the tidal bulges of the solar photosphere will be caused by tidal friction in the outer convective envelope of the RGB Sun. This physical process was analysed, solved and applied by Zahn (1977, 1989, and other work referred to therein), and successfully tested with the synchronization and circularization of binary star

orbits by Verbunt & Phinney (1995). In a convective envelope, the main contribution to tidal friction comes from the retardation of the equilibrium tide by interaction with convective motions. For a circular orbit, the resulting torque Γ exerted on planet Earth by the retarded solar tidal bulges is given by (Zahn 1977, 1989, equation 11)

$$\Gamma = 6 \frac{\lambda_2}{t_f} q^2 M_{\text{Sun}} R_{\text{Sun}}^2 \left(\frac{R_{\text{Sun}}}{r_E} \right)^6 (\Omega - \omega). \quad (4)$$

Here, the angular velocity of the solar rotation is supposed to be $\Omega = 0$, while that of the orbiting Earth, $\omega(t) = 2\pi/P_E(t) = \Lambda^{-3}(t) M_E^3 (GM_{\text{Sun}}(t))^2$, will vary both with the decreasing angular momentum $\Lambda(t) (= 2.67 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$ at present) and with the solar mass in the final solar RGB stages. The exerted torque scales with the square of the (slowly increasing) mass ratio $q(t) = M_E/M_{\text{Sun}}(t)$ ($= 3.005 \times 10^{-6}$ at present) because q determines the magnitude of the tidal bulges. $t_f(t) = (M_{\text{Sun}}(t) R_{\text{Sun}}^2(t)/L_{\text{Sun}}(t))^{1/3} \approx O(1 \text{ yr})$ is the convective friction time (Zahn 1989, equation 7), and the coefficient λ_2 depends on the properties of the convective envelope. For a fully convective envelope (Zahn 1989, equation 15), with a tidal period $\approx O(1 \text{ yr})$, comparable to $2t_f$, we may use $\lambda_2 \approx 0.019 \alpha^{4/3} \approx 0.038$ (with a convection parameter of our tip-RGB solar model of $\alpha \approx 1.7$). This coefficient appears to be the main source of uncertainty (see Section 4.3), because it is related to the simplifications of the MLT.

With the properties of the tip-RGB Sun, a typical value of the tidal drag acting on planet Earth is $\Gamma = d\Lambda/dt = -3.3 \times 10^{26} \text{ kg m}^2 \text{ s}^{-2}$, which gives a typical orbital angular momentum decay time of $\tau = |\Lambda/\Gamma| = 2.6 \times 10^6 \text{ yr}$. This is comparable to the time spent by the Sun near the tip-RGB; since a loss of only ≈ 10 per cent of the angular momentum will be sufficient to reduce the orbital radius (by 20 per cent) to lower it into the solar giant photosphere, this order-of-magnitude calculation clearly illustrates that tidal interaction is crucial. Its full consideration requires a time-step-by-time-step computation of the loss of orbital angular momentum; at each time-step of the solar evolution calculation, we use equation (4), together with the radii and masses of our solar evolution model, to compute the change in angular momentum, and then use equation (3) to compute the change in the orbital radius, and hence the new orbital period of the Earth. Section 4.3 presents the result, which also takes into account the relatively small additional angular momentum losses by dynamical drag, as discussed in the next section.

4.2 Dynamical friction in the lower chromosphere

A further source of angular momentum loss by drag is dynamical friction, from which any object suffers in a fairly close orbit, by its supersonic motion through the gas of the then very extended, cool solar giant chromosphere. In a different context, dynamical drag exerted by a giant atmosphere has already been considered by Livio & Soker (1984). But the specific problem here is to find an adequate description of the density structure of the future cool solar giant. Fortunately, as it turns out (see below), dynamical drag will play only a minor role, very near the solar giant photosphere, and the total angular momentum loss is dominated by the tidal interaction described above. An approximate treatment of the drag is therefore adequate, and we use the recent study by Ostriker (1999).

In the case of supersonic motion (with a Mach number¹ of the order of 2 to 3) in a gaseous medium, dynamical friction consists

¹ Note that $v_E \propto M_{\text{Sun}}(t)$, and so the Mach number, is somewhat lower than would be expected from the present orbital velocity of the Earth of about 30 km s^{-1} .

of about equal shares of the collisionless, gravitational interaction with its wake and of the friction itself. In her study, Ostriker (1999, fig. 3) finds that the drag force exerted on the object in motion is

$$F_d = \lambda_d 4\pi\rho (GM_E/c_s)^2 \quad (5)$$

where λ_d is of the order of 1 to 3. The numerical simulations made by Sánchez-Salcedo & Brandenburg (2001) are in general agreement with the results of Ostriker (1999). Here, c_s is the speed of sound, which in a stellar chromosphere is about 8 km s^{-1} , and ρ is the gas density (SI units). The latter quantity is the largest source of uncertainty, as we can only make guesses (see below) as to what the gas density in the lower giant solar chromosphere will be. The angular momentum loss resulting from this drag is simply

$$d\Lambda/dt = -F_d r_E, \quad (6)$$

and the corresponding lifetime of the orbital angular momentum is $\tau = \Lambda/|d\Lambda/dt|$, as above.

For the lower chromosphere of the K supergiant ζ Aur, employing an analysis of the additional line absorption in the spectrum of a hot companion in chromospheric eclipse, Schröder, Griffin & Griffin (1990) found an average hydrogen particle density of $7 \times 10^{11} \text{ cm}^{-3}$ at a height of $2 \times 10^6 \text{ km}$. Alternatively, we may simply assume that the density of the lower solar chromosphere scales with gravity g , which will be lower by 4.7 orders of magnitude on the tip-RGB, while the density scaleheight scales with g^{-1} (as observations of cool giant chromospheres seem to indicate; see Schröder 1990). The chromospheric models of both Lemaire et al. (1981) and Maltby et al. (1986) suggest particle densities of the order of 10^{17} cm^{-3} at a height of 100 km, and a scaleheight of that order for the present, low solar chromosphere. Scaled to tip-RGB gravity, that would correspond to a particle density of $2 \times 10^{12} \text{ cm}^{-3}$, or $\rho \approx 4 \times 10^{-9} \text{ kg m}^{-3}$, at a height of $5 \times 10^6 \text{ km}$ (0.03 au), and a density scaleheight of that same value.

For the computation of the orbital angular momentum loss of the Earth, presented below (see Figs 2 and 3), we apply the latter, rather higher values of the future chromospheric gas density, together with the (also more pessimistic) assumption of $\lambda_d = 3$ (using $c_s = 8 \text{ km s}^{-1}$). The typical angular momentum decay-time by dynamical friction in the low ($h \approx 0.03 \text{ au}$) chromosphere of

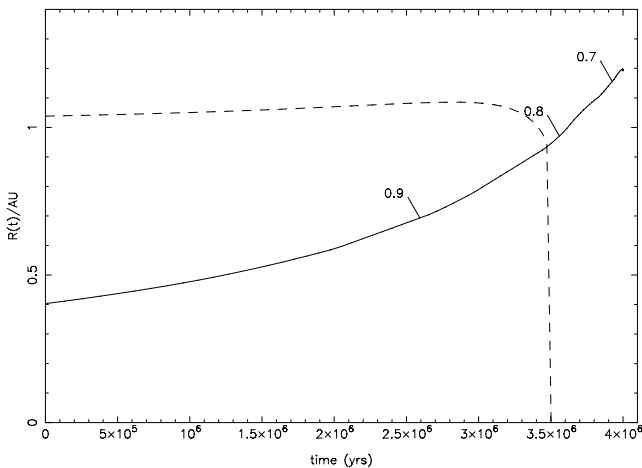


Figure 2. The final four million years of solar evolution before the tip-RGB, showing the radii of the Sun and of the orbit of planet Earth (dashed curve) – taking account of angular momentum losses by tidal interaction and supersonic drag. The labels on the solar radius track give values of $M_{\text{Sun}}(t)/M_{\odot}$, as in Fig. 1.

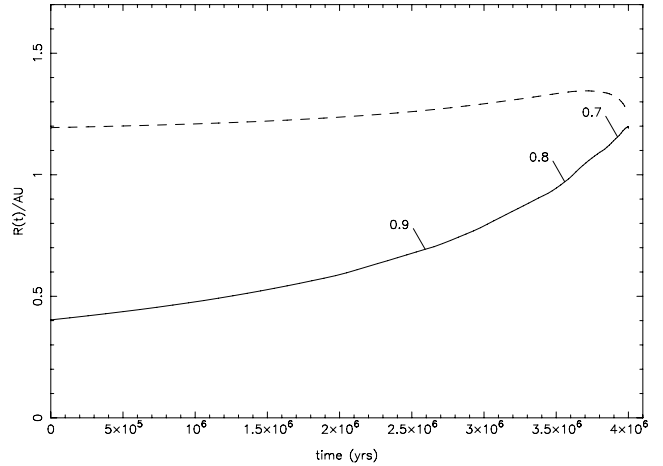


Figure 3. As Fig. 2, but for a planet with a present orbital radius of 1.15 au.

the tip-RGB solar giant is 14 million years – significantly longer than that for tidal interaction. Hence, this illustrates that dynamical friction is of interest only in the lowest chromospheric layers, adding there just a little to the drag exerted by tidal interaction. None the less, we include it, using equations (5) and (6), to calculate the additional angular momentum change to be included in equation (3).

4.3 ‘Doomsday’ confirmed

As explained in the previous two sections, we use equations (3) to (6) to compute, at each time-step of our evolutionary calculation, a detailed description of the orbital evolution for planet Earth in the critical tip-RGB phase of the Sun under the influence of tidal interaction and dynamical drag. The resulting evolution both of the orbital radius of the Earth and of the radius of the solar giant is shown in Fig. 2. This shows that, despite the reduced gravity from a less massive tip-RGB Sun, the orbit of the Earth will hardly ever come to exceed 1 au by a significant amount. The potential orbital growth given by the reduced solar mass is mostly balanced and, eventually, overcome by the effects of tidal interaction. Near the very end, supersonic drag also becomes a significant source of angular momentum loss.

As shown in Fig. 2, engulfment and loss of planet Earth will take place just before the Sun reaches the tip of the RGB, 7.59 Gyr ($\pm 0.05 \text{ Gyr}$) from now. According to our calculation, it occurs when the RGB Sun has still another 0.25 au to grow, about 500 000 yr before the tip-RGB. Of course, Mercury and Venus will already have suffered the same fate as Earth some time before – respectively, 3.8 and 1.0 million years earlier.

As mentioned in the Introduction, a similar calculation was already carried out in the context of extra-solar planets by Rasio et al. (1996), who basically came to the same conclusions; their fig. 2 is reminiscent of ours. They also employed the orbital decay rate predicted by Zahn’s theory, but their solar evolution model used the old Reimers mass-loss relation, and they did not make any adjustments to match the effective temperatures found empirically at the tip of the giant branches (see Section 2).

Do the remaining uncertainties allow the possibility for Earth to escape the ‘doomsday’ scenario? As far as the mass-loss alone is concerned, this seems unlikely: according to the study of HB stars in globular clusters by Schröder & Cuntz (2005), η is remarkably well constrained and cannot exceed $9 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$, or the

total RGB mass loss would become so large that the tip-*RGB* star would miss He-ignition and not reach the *HB* at all. The full width of the *HB* towards lower T_{eff} is achieved already with an η of $7 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$. Furthermore, the benefit of larger orbits with a reduced solar mass is to some extent compensated for by a larger solar giant.

Dynamical drag does not become important until the planet is already very near the photosphere, i.e. *after* tidal drag has already lowered the orbit. Hence, the most significant uncertainty here comes from the scaling of the tidal friction coefficient λ_2 (of Zahn 1989). For this reason, we computed several alternative cases, and from these we find the following.

(1) With the mass-loss rate unchanged, the value of λ_2 would have to be significantly smaller for an escape from the ‘doomsday’ scenario, i.e. less than 0.013, instead of our adopted value of 0.038. But Zahn’s scaling of λ_2 has been empirically confirmed within a factor of 2, if not better (see Verbunt & Phinney 1995). Very recently, realistic 3D simulations of the solar convection have also resulted in an effective viscosity which matches that of Zahn’s prescription surprisingly well (Penev et al. 2007). Rybicki & Denis (2001), by comparison, used a value ($K_2 = 0.05$ in the notation of their very similar calculation of tidal angular momentum loss) which is entirely consistent with Zahn’s scaling of λ_2 for a convection parameter of $\alpha = 2$.

(2) We then considered solar evolution models with a reasonably larger mass-loss rate ($\eta = 9 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$) in combination with tidal friction coefficients of 1/1, 2/3 and 1/2 of the one given by Zahn. In each of these cases, planet Earth would not be able to escape doomsday but would face a delayed engulfment by the supergiant Sun – 470 000, 230 000 and 80 000 yr before the tip-*RGB* is reached, respectively.

(3) Finally, we checked the outcome for a reasonably lower mass-loss rate ($\eta = 7 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$) in combination with the same tidal friction coefficients as above. The engulfment would then happen rather earlier than with more mass loss – 540 000, 380 000 and 270 000 yr before the tip-*AGB* is reached.

These computations confirm that reducing the solar mass enlarges the planetary orbit more than the tip-*RGB* solar radius, so that the best way to avoid the doomsday scenario would be to have as high a mass-loss rate as possible. However, we believe that the value of η in Case 2 is already as high as it can be without violating agreement of evolved models with observations, and that the smallest value used there for the tidal friction coefficient is also at the limits of what is allowed by observational constraints. The only possible escape would be if our solar giant models were too cool (by over 100 K in Case 2), and therefore larger than the real Sun will be. Hence, to avoid engulfment by the tip-*RGB* Sun would require that all three parameters (η , λ_2 and T_{eff}) were at one edge of their uncertainty range, which seems improbable. Rather, our computations confirm, with reasonable certainty, the classical ‘doomsday’ scenario.

4.4 ‘Doomsday’ avoidable?

Even though this is an academic question, given the hostile conditions on the surface of a planet just missing this ‘doomsday’ scenario, we may ask: what is the minimum initial orbital radius of a planet in order for it to ‘survive’? Fig. 3 shows, by the same computation as carried out for Fig. 2, that an initial orbital radius of 1.15 au is sufficient for any planet to pass the tip-*RGB* of a star with $M_i = 1.0 M_{\odot}$. Since, as shown in Section 5, the tip-*AGB* Sun will not

reach any similarly large extent again, such a planet will eventually be orbiting a white dwarf.

A more general discussion of planetary survival during post-main-sequence evolution has been given by Villaver & Livio (2007), who suggest that an initial distance of at least 3 au is needed for the survival of a terrestrial-size planet when one also takes into account the possible evaporation of the planet by stellar heating. However, they use stellar models and mass-loss rates that have the maximum radius and mass loss occurring on the *AGB*. That has been the expected result for many years, but is quite different from what we find (Section 5 and Table 1) with the improved mass-loss formulation of Schröder & Cuntz (2005, 2007). Hence, Villaver & Livio’s results may be unduly pessimistic.

In any case, it is clear that terrestrial planets can survive if sufficiently far from their parent star. If it were possible to increase the orbital radius from its initial value, then an increase of only 8 per cent of angular momentum should yield the pre-*RGB* orbital size required by planet Earth to escape engulfment. Is that conceivable?

An ingenious scheme for doing so which, in the first place, could increase the time-scale for habitation by intelligent life for the whole of the Sun’s *MS* life-time, was proposed by Korycansky, Laughlin & Adams (2001). They pointed out that a suitable encounter of the Earth every 6000 yr or so with a body of large asteroidal mass could be arranged to move the orbit of the Earth outwards; Kuiper Belt objects might be the most suitable. The energy requirements could be reduced by incorporating additional encounters with Jupiter and/or Saturn. Although still very large by today’s standards, the energy requirements remain small compared to those for interstellar travel.

On the face of it, this scheme seems far-fetched, but Korycansky et al. (2001) show that it is in principle possible, both technically and energetically, although currently somewhat beyond our technical capabilities; however, there is no immediate hurry to implement the scheme, which could await the development of the relevant technology. It would have the advantage of improving conditions for the whole biosphere, whereas any scheme for interplanetary ‘life rafts’ that could move slowly outwards to maintain habitable conditions would, on cost and energy grounds, necessarily be confined to a small fraction of the human population – with all the political problems that would produce – plus perhaps a tiny proportion of other species. None the less, the asteroidal fly-by scheme has its own problems, not least the danger of a benign close approach turning into a catastrophic accidental collision, and possibly also triggering orbital instability – cf. also Debes & Sigurdsson (2002).

5 TIP-AGB SOLAR EVOLUTION

The loss of 1/3 of the solar mass during the rise to the tip of the *RGB* will make a significant impact on the further evolution as an *AGB* star. There is very little shell mass left, into which the two burning shells (H followed by He) can advance (on a radial mass scale). Hence, the *C/O* core cannot grow as much as with a conservative model without mass loss, and the whole core region will not contract as much, either. Consequently, the *AGB* luminosity, determined by the density and temperature in the H-burning shell, will not reach as high levels as in a conservative *AGB* model, and neither will the *AGB* radius of the late future Sun (see Table 1).

According to our evolution model, the regular tip-*AGB* evolution will be ended with a luminosity of only $2090 L_{\odot}$, $T_{\text{eff}} = 3200 \text{ K}$ and $R = 149 R_{\odot}$. The *AGB* mass-loss rate, according to the relation of Schröder & Cuntz (2005), will reach only $2.0 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ (see Fig. 4), since the luminosity will not be sufficient to drive a

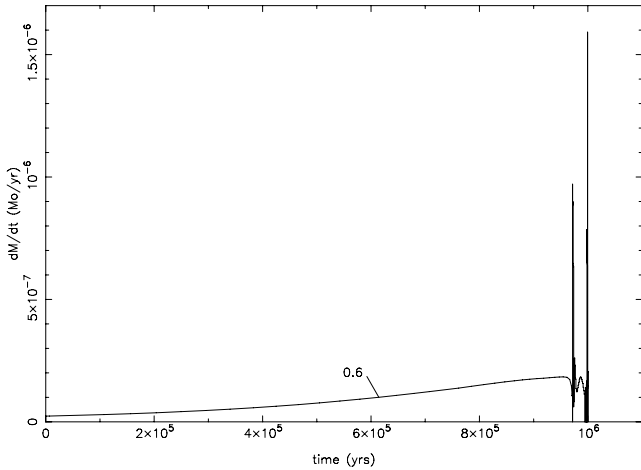


Figure 4. Solar mass loss during the final one million years on the AGB will remain mainly of the order of $2 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ and not provide sufficient CS shell mass to form a regular PN. Only the last two TPs (tip-AGB and post-AGB; see text) are resolved.

dust-driven wind (see Schröder et al. 1999). Also, even if it did: only a little shell mass will have been left to lose after the RGB phase, only $0.116 M_{\odot}$.

Hence, for this non-dust-driven AGB solar mass loss, we have adopted the same mass-loss description as equation (1). This mass loss, in combination with our solar evolution model, yields the following prediction: during the final 30 000 yr on the very tip-AGB, which are crucial for any build-up of sufficient circumstellar (CS) material to form a planetary nebula (PN), the solar giant will lose only $0.006 M_{\odot}$. A further $0.0015 M_{\odot}$ will be lost in just 1300 yr right after a final TP on the tip-AGB. That marks the very end of AGB evolution, and it allows the solar supergiant briefly to reach a luminosity of $4170 L_{\odot}$ and $R = 179 R_{\odot}$, with a mass-loss rate of $10^{-6} M_{\odot} \text{ yr}^{-1}$, but with T_{eff} already increased to 3467 K. Again, there will be no involvement of a dust-driven wind. Since common PNe and their dusty CS envelopes reveal a dust-driven mass-loss history of more like 10^{-5} to $10^{-4} M_{\odot} \text{ yr}^{-1}$ during the final 30 000 yr of tip-AGB evolution, we must conclude that the Sun will *not* form such a PN.

Since a CS shell of nearly $0.01 M_{\odot}$ will, nevertheless, be produced by the tip-AGB solar giant, a rather peculiar PN may be created by the emerging hot stellar core – it might be similar to IC 2149. Although most of the peculiar, strongly bi-polar PNe appear to stem from massive stars, this particular object has only a slim total mass of 0.01 to $0.03 M_{\odot}$, lacking a massive envelope (see Vázquez et al. 2002). Hence, these authors argue that this PN appears to be the product of a low-mass star with M_i close to $1 M_{\odot}$.

A final mass of $0.0036 M_{\odot}$ is lost by the post-AGB star, which on its way to become a hot subdwarf undergoes at least one more TP. For the resulting solar white dwarf, our evolution model yields a final mass of $0.5405 M_{\odot}$.

6 CONCLUSIONS

We have applied an improved and well-tested mass-loss relation to RGB and AGB solar evolution models, using a well-tested evolution code. While the HZ in the inner Solar system will already move considerably outwards in the next five billion years of solar MS evolution, marking the end of life on Earth, the most critical and

fatal phase for the inner planetary system is bound to come with the final ascent of the Sun to the tip of the RGB.

Considering in detail the loss of angular momentum by tidal interaction and dynamical drag in the lower chromosphere of the solar giant, we have been able to compare the evolution of the RGB solar radius with that of the orbit of planet Earth. Our computations reveal that planet Earth will be engulfed by the tip-AGB Sun, just half a million years before the Sun will have reached its largest radius of 1.2 au, and 1.0 (3.8) million years after Venus (and Mercury) have suffered the same fate. While solar mass loss alone would allow the orbital radius of planet Earth to grow sufficiently to avoid this ‘doomsday’ scenario, it is mainly tidal interaction of the giant convective envelope with the closely orbiting planet which will lead to a fatal decrease of its orbital size.

The loss of about 1/3 of the solar mass already on the RGB has significant consequences for the solar AGB evolution. The tip-AGB Sun will not qualify for an intense, dust-driven wind and, hence, will not produce a regular PN. Instead, an insubstantial CS shell of just under $1/100 M_{\odot}$ will result, and perhaps a peculiar PN similar to IC 2149.

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