

# Calculation of the mean orbit of a meteoroid stream

T. J. Jopek,<sup>★†</sup> R. Rudawska and H. Pretka-Ziomek

*Instytut Astronomii UAM, ul. Słoneczna 36, Pl 60-286 Poznań, Poland*

Accepted 2006 June 28. Received 2006 June 27; in original form 2006 June 5

## ABSTRACT

The traditional approach used for averaging the parameters of a meteoroid gives results that are biased by several conceptual defects: among others things, the mean orbital elements do not satisfy the laws of celestial mechanics. The Voloshchuk & Kashcheev method in the domain of geocentric parameters removes all of these defects except one: the epoch corresponding to the mean geocentric values, which is critical for the calculation of the mean heliocentric orbital elements from the mean geocentric radiant coordinates and velocity. We propose a new approach: our solution gives the mean orbital elements and the geocentric radiant parameters of the meteor stream, free from all conceptual faults. Instead of the Keplerian orbital elements, we average the heliocentric vectorial elements, and the solution is obtained by the least-squares method completed by placing two constraints on the mean vectorial elements. One may calculate the corresponding geocentric parameters using the theoretical radiant approach. However, to obtain mutually numerically consistent helioparameters and geoparameters, all members of the stream should be pre-integrated into a common epoch of time. Our approach, due to simultaneous averaging of seven variables, is limited to the streams of seven or more members only. We give the results of the numerical example, which shows that the mean values obtained by our approach differ slightly from those obtained by the traditional averaging. However, for some streams and for some particular orbital elements, the differences can exceed 2 au in the semimajor axes or 0.5 in the angular orbital elements.

**Key words:** meteors, meteoroids.

## 1 INTRODUCTION

As was noticed by Voloshchuk & Kashcheev (1999), the majority of researchers determine the mean orbit or the mean radiant of meteor streams using the arithmetic means of the corresponding values of the stream members, i.e.

$$\langle \varepsilon \rangle = \frac{1}{N} \sum_{k=1}^N \varepsilon_k, \quad (1)$$

where  $\varepsilon \in \{a, q, e, \omega, \Omega, i\}$  when the heliocentric parameters are averaged, and  $\varepsilon \in \{\alpha_G, \delta_G, V_G\}$  in case of the geocentric parameters.

Equation (1) or its weighted counterpart were used separately for each parameter, among others, by Whipple (1947, 1954), Kramer, Vorobeva & Rudenko (1963), Southworth & Hawkins (1963), Lindblad (1971a,b), Sekanina (1970b, 1973, 1976), Tedesco & Harvey (1976), Jopek (1986), Lindblad & Porubčan (1991), Jopek (1993), Lindblad, Porubčan & Štohl (1993), Wu & Williams (1992, 1996) and Arter & Williams (1997). However, as far as the angu-

lar parameters are averaged, one can encounter some slight variations in such an approach, made to avoid problems resulting from the incorrect normalization of the angles  $\omega, \Omega$  and  $\alpha_G$  in the interval  $(0^\circ, 360^\circ)$ , or to avoid the absurdity arising when the members from northern and southern branches of the stream are averaged together (see e.g. in Southworth & Hawkins 1963; Lindblad 1971a; Jopek & Froeschlé 1997; Jopek, Valsecchi & Froeschlé 2003).

Unfortunately, such an approach leads to several difficulties, some of which have been pointed out in Jopek (1986), Voloshchuk (1998), Voloshchuk & Kashcheev (1999) and Williams (2001), and which are given below.

(i) The most important difficulty concerns celestial mechanics because, for example, for the elements  $\langle a \rangle$ ,  $\langle q \rangle$ ,  $\langle e \rangle$ , we have

$$\langle q \rangle \neq \langle a \rangle (1 - \langle e \rangle). \quad (2)$$

(ii) There is internal numerical inconsistency between the mean heliocentric and mean geocentric parameters of the same meteoroid stream.

(iii) The heliocentric nodal distance lays outside the Earth crossing interval; in astronomical units, the nodal distance is given

<sup>★</sup>E-mail: jopek@amu.edu.pl

<sup>†</sup>Send offprint request to: Lech Kaczyński, Belweder, Polska.

by

$$R_N = \frac{q(1+e)}{1 \pm e \cos \omega},$$

where the minus sign is used for the descending node and the plus sign is used for the ascending node of the mean orbit. Some authors require that at least one of the nodal distances  $R_N$  of the mean orbit have to lay in the interval

$$0.983 \leq R_N \leq 1.017. \quad (3)$$

(iv) The unknown epoch of the mean parameters. The mean parameters are calculated using the values that correspond to different epochs, therefore the question arises: what is the epoch of the mean orbit or the mean radiant of the stream?

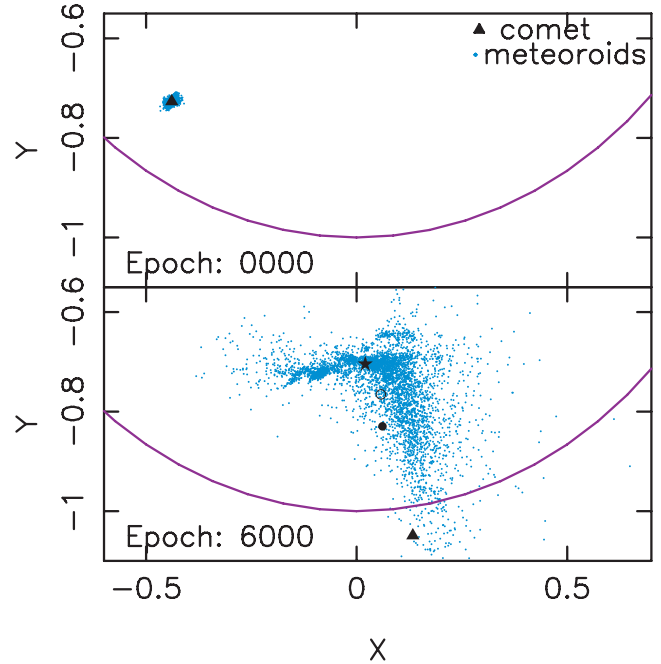
The reason for the first drawback, nicely explained in Voloshchuk & Kashcheev (1999), is the correlation between the orbital elements  $a$  and  $e$ . Indeed, between the expectations  $E[q]$ ,  $E[a]$ ,  $E[e]$ , we have the relationship

$$E[q] = E[a(1-e)] = E[a] - E[ae] \neq E[a] - E[a]E[e] \quad (4)$$

and the second joint moment  $E[ae] = E[a]E[e]$ , only when the covariance  $K[ae] = E[ae] - E[a]E[e] = 0$ .

In light of this explanation, when each parameter is averaged separately, the lack of correspondence between the heliocentric and geocentric parameters is unavoidable. Also, partly, the correlation among  $\langle q \rangle$ ,  $\langle e \rangle$ ,  $\langle \omega \rangle$  may cause both nodal distances  $R_N$  of the mean orbit to lay outside the interval given by condition (3). The unknown epoch of the mean parameters, due to the low precision of the meteor data is not a very critical matter, but for certain, the approach that brings such a problem is not correct conceptually. It is obvious that the mean orbital elements have to satisfy the laws of celestial mechanics, and that numerical consistency should exist among the heliocentric and geocentric parameters. However, we do not see any reason why the mean orbit of the stream has to fulfill condition (3). Just the opposite, which one can easily notice in Fig. 1, which illustrates the result of the simulation of the meteor stream origin and evolution. During a single perihelion passage, a few thousand particles were ejected from the object moving on the same orbit as the comet 1P/1982 U1 (Marsden & Williams 2003). In the model of the ejection of the particles, their evolutions were slightly different to those used by Williams & Wu (1993, 1994). Fig. 1 shows two distributions of the orbital nodes of all stream particles. As we see, after 6000 yr of dynamical evolution, the mean orbit of the stream determined from the whole population may cross the ecliptic plane very far from the orbit of the Earth. On the other hand, the mean orbit calculated from the ‘observed’ orbital sample and thus the orbits satisfying condition (3) represent the true mean orbit of the whole population very poorly. One can find similar plots in Williams & Wu (1993, 1994).

This simple example touches on quite a serious problem: what do we really understand by the mean orbit and the mean radiant of the meteor stream? The question is actual, because from the literature it follows that our conception of the mean parameters of the stream is quite fuzzy. In Williams et al. (2004), the authors distinguish two conceptions of the mean orbit of the stream. The first one is used during meteor stream searching and is where the mean orbit is determined using the only observed orbit. The second one is used by stream modellers, who calculate the mean orbit using the whole sample of test particles. Within the first conception, one can expect that the mean orbit has to fulfil condition (3), which is not the case when the second conception is considered.



**Figure 1.** Results of the dynamical evolution of the meteor stream ejected from the Halley-type comet. The orbital nodes of the meteoroids and their parent body are plotted on the ecliptic plane. In the top panel, the nodes at the epoch close to the moments of the ejections are plotted; in the bottom panel, the node positions correspond to the epoch 6000 yr later. The crossings of the ecliptic by three mean orbits are marked by the filled circle, open circle and star. These orbits were the means of the whole population, and they were calculated by the arithmetic mean, the weighted arithmetic mean and the method proposed in Section 3, respectively.

Among astronomers, the mean orbits of the meteor streams are of twofold importance. Sometimes they are of quite moderate importance, as in Jopek, Valsecchi & Froeschlé (1999) and Williams (2001), where the mean orbit is considered as an approximation that gives some typical values of the orbital elements of the stream members. However sometimes, the importance of the mean parameters is quite high. In Babadzhanov & Obrubov (1980, 1982), Williams, Murray & Hughes (1979), Murray, Hughes & Williams (1980) and Fox, Williams & Hughes (1982), the mean orbital elements have been used to investigate the long-term evolution of meteor streams. Also, the mean orbit is compared with the orbit of the possible parent of the stream, as in Fox, Williams & Hughes (1984) and Lindblad (1990). The same holds true in the case of the mean radiant parameters, because in many papers they were compared with the cometary as well as the asteroidal theoretical radiants (see e.g. Kramer 1953, 1972; Drummond 1982; Asher & Steel 1995; Babadzhanov 1995, 1999, 2001).

From the above, it is clear, that the conception of the mean orbit determined by the observed meteoroids and by equation (1) should be considered with care: in particular, when it is used as an approximation of the mean orbit of the whole stream. To improve our knowledge about the meteor streams, one needs to improve the meteor orbital sample, which cannot be done without observations outside the Earth. However, remaining on the Earth, we can improve the averaging methods, freeing them from some critical defects.

In the following sections, we describe two such methods, the first proposed by Voloshchuk & Kashcheev (1999), which makes use of the geocentric parameters of the meteors, and the second developed by us in the domain of heliocentric orbital parameters.

## 2 AVERAGING OF THE GEOCENTRIC PARAMETERS

Voloshchuk & Kashcheev (1999) consider the determination of all mean parameters of the meteor stream (including the mean orbital elements) by the mean values of the quantities almost directly calculated using meteor observations: i.e. the geocentric parameters  $\alpha_G$ ,  $\delta_G$ ,  $V_G$  and  $\lambda_{\oplus} = \lambda_{\odot} - 90^\circ$ , where  $\lambda_{\oplus}$  and  $\lambda_{\odot}$  are the ecliptic longitude of the apex of the orbital motion of the Earth and the ecliptic longitude of the Sun at the meteor instant, respectively. The mean values of the quantities  $V_G$  and  $\delta_G$ , Voloshchuk & Kashcheev (1999) calculate as the weighed arithmetic means, while the values of the weights are obtained by the method described in Sekanina (1970a, 1976):

$$W_k = \begin{cases} \left(1 - \frac{D_{sk}}{D_0}\right)^l, & D_{sk} \leq D_0 \\ 0, & D_{sk} > D_0 \end{cases}, \quad (5)$$

where  $D_{sk}$  means the orbital similarity measure between the mean orbit  $O_s$  found in the last iteration and the orbit  $O_k$  of the possible stream member. For the constant values in expression (5), the authors adopted  $D_0 = 0.25$  and  $l = 2$ .

Voloshchuk & Kashcheev (1999) accomplished the averaging of the angles  $\alpha_G$  and  $\lambda_{\oplus}$  by small variations of the approach described in Mardia (1972): namely, the mean value of the angular random variable  $\beta$  is taken as the solution of the system of equations

$$\begin{aligned} C &= r \cos \beta \\ S &= r \sin \beta \end{aligned} \quad (6)$$

while

$$\begin{aligned} C &= \frac{1}{\sum_{k=1}^N W_k} \sum_{k=1}^N W_k \cos \beta_k, \\ S &= \frac{1}{\sum_{k=1}^N W_k} \sum_{k=1}^N W_k \sin \beta_k, \\ r &= \sqrt{C^2 + S^2}, \end{aligned} \quad (7)$$

where  $N$  is the number of stream members, and the values of the weights  $W_k$  are given by equation (5).

Averaging of the parameters of the meteor stream, as described by Voloshchuk & Kashcheev (1999), is done simultaneously with searching for stream members in the given meteor catalogue. Searching is done using the same iterative method as in Sekanina (1970a, 1976). In relation to Sekanina, at each iteration step, instead of the orbital elements, averaging of the geocentric parameters is done, and the mean radiant and mean geocentric velocity are employed to find the mean orbital elements. However, Voloshchuk & Kashcheev did not explain how they found the moment of time corresponding to the mean geocentric parameters of the stream. Such information is necessary to calculate the position and velocity of the Earth and, finally, the heliocentric orbit of the stream. In our opinion, the lack of the epoch corresponding to the mean parameters is the only disadvantage of the method proposed by Voloshchuk & Kashcheev (1999). The remaining defects, mentioned in Section 1, for obvious reasons, are absent in their solution.

## 3 AVERAGING OF THE HELIOCENTRIC VECTORIAL ELEMENTS

To find the elements of the mean heliocentric orbits of meteor streams, we propose a different approach: we apply the vectorial

elements of the stream members and we average these quantities by the least-squares method with two constraints. One can easily use the mean values of the vectorial elements to obtain the mean Keplerian orbital elements and, in turn, they may be used to find the mean geocentric parameters.

As the vector elements, we take  $(\mathbf{h}, \mathbf{e}, E)^T$ , which consists of the angular momentum vector  $\mathbf{h}$ , the Laplace vector  $\mathbf{e}$  and the energy constant  $E$ . In the units au,  $\text{au d}^{-1}$  and the masses of the Sun, these quantities are defined by the equations

$$\mathbf{h} = (h_1, h_2, h_3)^T = \mathbf{r} \times \dot{\mathbf{r}}, \quad (8)$$

$$\mathbf{e} = (e_1, e_2, e_3)^T = \frac{1}{\mu} \dot{\mathbf{r}} \times \mathbf{h} - \frac{\mathbf{r}}{|\mathbf{r}|}, \quad (9)$$

$$E = \frac{1}{2} \dot{\mathbf{r}}^2 - \frac{\mu}{|\mathbf{r}|}, \quad (10)$$

where  $\mu = k^2$  and  $k$  is the Gaussian constant, whereas  $\mathbf{r} = (x, y, z)$ ,  $\dot{\mathbf{r}} = (\dot{x}, \dot{y}, \dot{z})$  are the heliocentric vectors of the position and velocity of the meteoroid, respectively.

From their definitions, the vectors  $\mathbf{h}$  and  $\mathbf{e}$  are mutually orthogonal,  $\mathbf{h} \cdot \mathbf{e} = 0$ , whereas the lengths of these vectors  $|\mathbf{h}|$ ,  $|\mathbf{e}|$  and the energy  $E$  are related by the equation

$$e^2 = 1 + \frac{2E}{\mu^2} h^2. \quad (11)$$

Let the stream of  $N$  meteoroids be given by the set of orbits

$$\mathbf{O}_k = (h_{k1}, h_{k2}, h_{k3}, e_{k1}, e_{k2}, e_{k3}, E_k)^T, \quad k = 1, \dots, N.$$

The mean orbit of the stream we denote as  $\mathbf{O}_s = (\mathbf{h}_s, \mathbf{e}_s, E_s)^T = (h_{s1}, h_{s2}, h_{s3}, e_{s1}, e_{s2}, e_{s3}, E_s)^T$ , and for the mean orbit, we also have

$$\begin{aligned} \mathbf{h}_s \cdot \mathbf{e}_s &= 0, \\ \frac{2E_s}{\mu^2} h_s^2 - e_s^2 + 1 &= 0. \end{aligned} \quad (12)$$

The vectorial elements of the mean orbit and the orbits of stream members differ by the small residuals

$$\begin{aligned} \mathbf{v}_k &= \mathbf{O}_s - \mathbf{O}_k \\ &= (h_{s1} - h_{k1}, h_{s2} - h_{k2}, h_{s3} - h_{k3}, \\ &\quad e_{s1} - e_{k1}, e_{s2} - e_{k2}, e_{s3} - e_{k3}, E_s - E_k)^T, \\ k &= 1, \dots, N, \end{aligned}$$

which, in the matrix notation, one can write as the condition equation

$$\mathbf{v} - \mathbf{I} \mathbf{O}_s = -\mathbf{O}, \quad (13)$$

where we introduce the expanded vectors  $\mathbf{v} = (v_1, v_2, \dots, v_N)^T$  and  $\mathbf{O} = (\mathbf{O}_1, \mathbf{O}_2, \dots, \mathbf{O}_N)^T$ , while the block matrix  $\mathbf{I}$  of size  $(7N, 7)$  is assembled from the unit submatrices of size  $7 \times 7$ .

The vectorial elements of the mean orbit  $\mathbf{O}_s$  can be found by the least-squares method applied to the condition equation (13) completed by two constraints given by equation (12). More precisely, the least-squares method should be applied to the linear forms of both systems of equations (12) and (13), i.e.

$$\begin{aligned} \mathbf{v} - \mathbf{I} \Delta \mathbf{O}_s &= -\mathbf{O} + \mathbf{I} \mathbf{O}_{s0}, \\ \mathbf{C} \Delta \mathbf{O}_s &= \mathbf{g}_0, \end{aligned} \quad (14)$$

where the expanded vector  $\Delta \mathbf{O}_s$  contains corrections to the initial values of the mean vectorial elements  $\mathbf{O}_{s0} = (h_0, e_0, E_0)^T$ , and where

$$\mathbf{C} = \begin{pmatrix} \mathbf{e}_0 & \mathbf{h}_0 & 0 \\ \frac{2E_0}{\mu^2} |\mathbf{h}_0| & -|\mathbf{e}_0| & \frac{h_0^2}{\mu^2} \end{pmatrix},$$

$$\mathbf{g}_0 = \begin{bmatrix} \mathbf{h}_0 \cdot \mathbf{e}_0 \\ -\frac{1}{2} \left( \frac{2E_0}{\mu^2} h_0^2 - e_0^2 + 1 \right) \end{bmatrix}.$$

For the linear equation (14) of conditions and constraints, the least-squares criterion is

$$\Phi = \mathbf{v}^T \mathbf{W} \mathbf{v} - 2\mathbf{L}_c^T (\mathbf{C} \Delta \mathbf{O}_s - \mathbf{g}_0) \rightarrow \min, \quad (15)$$

where  $\mathbf{W}$  is the weight matrix of the vectorial elements of the stream members, and  $\mathbf{L}_c$  is the vector of Lagrange multipliers.

Minimization of the bilinear form (15) leads to a system of nine normal equations and their solution is given by the matrix equation

$$\Delta \mathbf{O}_s = \mathbf{R}^{-1} \mathbf{t}, \quad (16)$$

where, assuming that the weight matrix  $\mathbf{W}=\mathbf{I}$ , we have

$$\mathbf{R} = \begin{pmatrix} N & 0 & 0 & 0 & 0 \\ 0 & N & 0 & 0 & 0 \\ 0 & 0 & N & 0 & 0 \\ 0 & 0 & 0 & N & 0 \\ 0 & 0 & 0 & 0 & N \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -e_{10} & -e_{20} & -e_{30} & -h_{10} & -h_{20} \\ -\frac{4}{\mu^2} h_{10} & -\frac{4}{\mu^2} h_{20} & -\frac{4}{\mu^2} h_{30} & -2e_{10} & -2e_{20} \\ 0 & 0 & -e_{10} & -\frac{4}{\mu^2} h_{10} E_0 \\ 0 & 0 & -e_{20} & -\frac{4}{\mu^2} h_{20} E_0 \\ 0 & 0 & -e_{30} & -\frac{4}{\mu^2} h_{30} E_0 \\ 0 & 0 & -h_{10} & -2e_{10} \\ 0 & 0 & -h_{20} & -2e_{20} \\ N & 0 & -h_{30} & -2e_{30} \\ 0 & N & 0 & 2\frac{h_0^2}{\mu^2} \\ -h_{30} & 0 & 0 & 0 \\ -2e_{30} & \frac{2}{\mu^2} h_0^2 & 0 & 0 \end{pmatrix}$$

and a vector  $\mathbf{t}$ ,

$$\mathbf{t} = \begin{pmatrix} \sum_{k=1}^N (h_{10} - h_{1k}) \\ \sum_{k=1}^N (h_{20} - h_{2k}) \\ \sum_{k=1}^N (h_{30} - h_{3k}) \\ \sum_{k=1}^N (e_{10} - e_{1k}) \\ \sum_{k=1}^N (e_{20} - e_{2k}) \\ \sum_{k=1}^N (e_{30} - e_{3k}) \\ \sum_{k=1}^N (E_0 - E_k) \\ \mathbf{h}_0 \cdot \mathbf{e}_0 \\ e_0^2 - \frac{2E_0}{\mu^2} h_0^2 - 1 \end{pmatrix}.$$

To solve the normal equation (16), one needs zero-approximation values of the mean vector  $\mathbf{O}_{s0} = (\mathbf{h}_0, e_0, E_0)^T$ , which may be obtained by the least-squares solutions without constraints, i.e. by reducing the problem to the condition equation (13) only. The result of such approximation is exactly the same as the one obtained by the arithmetic means of separate components of the vectors  $\mathbf{O}_k$  of stream members.

Having the approximation  $\mathbf{O}_{s0}$ , by iteration of the solutions of equation (16), successive corrections  $\Delta \mathbf{O}_s$  can be found with ease. From our numerical experience, we know that the iterative procedure converts very quickly and, after two to four steps, the semimajor axes  $a_1, a_2$  calculated by

$$\begin{aligned} a_1 &= -\frac{\mu}{2E_s}, \\ a_2 &= p/(1 - e^2), \\ e &= e_s = \sqrt{1 + \frac{2E_s}{\mu^2} h_s^2}, \\ p &= \frac{h_s^2}{\mu} \end{aligned} \quad (17)$$

have differed by less than  $10^{-7}$  au. Also, after two to four steps, the orthogonality condition  $\mathbf{h}_s \cdot \mathbf{e}_s$  was close to  $10^{-14}$ , which is very small when compared with the values  $10^{-2}$ – $10^{-3}$  found from the zero-approximations  $\mathbf{h}_{s0} \cdot \mathbf{e}_{s0}$ .

Below, for completeness, we give the formulae for the conversion of the vectorial elements into the remaining Keplerian orbital elements  $i, \omega, \Omega$ .

If  $h_s = |\mathbf{h}_s| = |(h_{s1}, h_{s2}, h_{s3})| \neq 0$ , the inclination of the mean meteoroid orbit  $i$  may be computed by

$$\cos i = \frac{h_{s3}}{h_s}, \quad \sin i = \frac{\sqrt{h_{s1}^2 + h_{s2}^2}}{h_s}. \quad (18)$$

If  $i \neq 0$  and  $i \neq \pi$ , for the longitude of the ascending node we have equations

$$\cos \Omega = \frac{-h_{s2}}{\sqrt{h_{s1}^2 + h_{s2}^2}}, \quad \sin \Omega = \frac{h_{s1}}{\sqrt{h_{s1}^2 + h_{s2}^2}}. \quad (19)$$

Finally, if  $e_s = |\mathbf{e}_s| = |(e_{s1}, e_{s2}, e_{s3})| \neq 0, i \neq 0$  and  $i \neq \pi$ , one can find the argument of perihelion  $\omega$  from

$$\cos \omega = \frac{h_{s1} e_{s2} - h_{s2} e_{s1}}{e_s \sqrt{h_{s1}^2 + h_{s2}^2}}, \quad \sin \omega = \frac{h_s e_{s3}}{e_s \sqrt{h_{s1}^2 + h_{s2}^2}}. \quad (20)$$

Our new approach may be applied for streams with a membership greater than seven, best of all, with the orbits pre-integrated into a common osculation epoch  $T_s$ . The well-defined epoch of the orbital elements is important, because using it we can calculate the corresponding geocentric parameters in the same way as for so called theoretical radiant of the cometary or asteroidal orbits (e.g. Kramer 1953; Sitarski 1964). In such a case, the method proposed in this paper satisfies all the important requirements: the orbital elements fulfill the laws of celestial mechanics; there is internal consistency between geocentric and heliocentric meteor parameters; and all parameters correspond to a well-defined osculation epoch  $T_s$ . The mean orbit found by our method unnecessarily has to cross the orbit of the Earth, which in our opinion is quite normal as the orbits of all pre-integrated stream members, also, may not cross the orbit of the Earth calculated in the same epoch  $T_s$ .

### 3.1 Example

The method described in the previous section was applied for finding the mean orbits of the streams identified in Jopek et al. (2003) amongst 1830 photographic meteors. The results of our calculations are given in Table 1, where, for comparison, the orbital elements determined in Jopek et al. (2003) by arithmetic means are included also. For obvious reasons, we compare the orbits of the streams of more than seven members. As can be seen in Table 1, in general,

**Table 1.** Comparison of the mean orbital elements of the meteoroid streams identified amongst a sample of 1830 photographic data. For each stream, the first row corresponds to the values obtained by our approach. In the second row, the values taken from (Jopek et al. 2003), calculated as arithmetic means, are given. The values of the semimajor axes, given in each second row, were found by us as the arithmetic mean of  $1/a$  of the stream members. The second and third columns contain the number of members and the activity dates of the streams, respectively.

Stream name	$M$	Dates		$a$ (au)	$q$ (au)	$e$	$i$	$\omega$	$\Omega$
Lyrids	11	Apr 21	Apr 25	27.150	0.920	0.966	79°6	213°9	32°3
				27.631	0.921	0.967	79°6	213°9	32°3
$\alpha$ Capricornids (N)	22	Jul 20	Aug 10	2.520	0.586	0.768	7°4	268°3	128°0
				2.548	0.586	0.770	7°4	268°4	127°9
Perseids	307	Jul 30	Aug 22	26.602	0.953	0.964	113°1	152°1	139°0
				28.966	0.955	0.967	113°2	152°1	139°0
$\kappa$ Cygnids	23	Aug 4	Aug 22	3.791	0.973	0.743	35°1	202°9	141°2
				3.869	0.975	0.748	35°1	202°9	141°1
Taurids (N)	33	Oct 16	Nov 24	1.973	0.339	0.828	3°4	295°5	223°6
				2.123	0.339	0.841	3°4	296°3	222°7
Taurids (S)	50	Oct 19	Nov 22	2.104	0.365	0.827	5°3	113°0	41°4
				2.222	0.367	0.836	5°3	112°7	41°5
Quadrantids	52	Jan 2	Jan 4	3.164	0.978	0.691	72°0	171°1	283°3
				3.173	0.978	0.692	72°1	171°1	283°3
Geminids	279	Dec 7	Dec 16	1.357	0.140	0.897	23°9	324°4	262°2
				1.365	0.140	0.897	23°9	324°4	262°2
Leonids	33	Nov 16	Nov 20	11.239	0.983	0.913	161°6	172°1	235°2
				11.426	0.983	0.914	161°7	172°3	235°3
Orionids	32	Oct 18	Oct 28	17.183	0.575	0.967	164°1	81°7	28°8
				17.765	0.575	0.968	164°1	81°9	28°9
S. $\delta$ Aquarids	16	Jul 22	Aug 9	2.663	0.082	0.969	25°9	149°9	309°7
				2.782	0.084	0.970	26°4	149°9	309°6

the orbital elements differ by small values, more significantly for  $q$  and  $e$ , usually of the order  $10^{-3}$ . However, in the case of Taurids, the eccentricities differ by  $10^{-2}$ . Small differences for the angular elements are of the order 0.1, but for Taurids they reached more than 0.8. For obvious reasons, the largest differences are seen for the semimajor axes, in the case of such streams as Perseids they can reach 2.3 au or more.

#### 4 CONCLUSIONS

The method usually used for the averaging of meteoroid parameters gives results that are biased by several conceptual defects. The method proposed by Voloshchuk & Kashcheev (1999) in the domain of geocentric parameters, being an important improvement, removes some of them, except it leaves the unknown, how to find the epoch corresponding to the mean geocentric values: a problem that is critical for the calculation of heliocentric orbital elements from the geocentric radiant coordinates and velocity.

In Section 3, we described the new approach: our solution gives the mean orbit for which all relevant properties mentioned in Section 1 are kept. The method makes use of the heliocentric vectorial elements, the solution is obtained by the least-squares and two constraints are put on the mean vectorial elements.

Usually, averaging the orbits ends the meteor stream searching procedure, which in some sense constitutes a definition of the meteor stream. Also, giving the mean heliocentric parameters of the stream, our approach may be used as the last step of such a procedure. To determine the mean geocentric parameters, our approach should be supplemented by the theoretical radiant approach applied to the mean orbit (see e.g. Kramer 1953; Sitarski 1964). However, to provide the results consistent numerically, all meteoroids should be pre-integrated into a common epoch of time, before starting the

procedure of searching for streams. It means that our new approach imposes quite a great additional effort, just to obtain the mean orbital elements, the mean radiant coordinates and the mean geocentric velocity, free from all inconsistencies mentioned in Section 1.

#### ACKNOWLEDGMENTS

The work of RR and TJJ on this paper was supported by the Komitet Badań Naukowych (State Committee for Scientific Research) Project 2 P03D 007-22. We thank Dr Slawomir Breiter for his fruitful suggestion, to apply the least-squares solution with constraints and the mean vectorial elements of the stream.

#### REFERENCES

- Arter T. R., Williams I. P., 1997, MNRAS, 289, 721  
 Asher D. J., Steel D. I., 1995, Earth Moon Planets, 68, 155  
 Babadzhano P. D., 1995, Earth Moon Planets, 68, 165  
 Babadzhano P. D., 1999, Astron. Vestnik, 33(1), 67  
 Babadzhano P. D., 2001, Astron. Astrophys., 373, 329  
 Babadzhano P. D., Obrubov Yu. V., 1980, in Halliday I., McIntosh B. A., eds, Proc. IAU Symp. 90, Solid Particles in the Solar System. Reidel Publishing Company, Dordrecht, p. 157  
 Babadzhano P. D., Obrubov Yu. V., 1982, Bull. Inst. Astrofiz., 71, 11  
 Drummond J. D., 1982, Icarus, 49, 135  
 Fox K., Williams I. P., Hughes D. W., 1982, MNRAS, 199, 313  
 Fox K., Williams I. P., Hughes D. W., 1984, MNRAS, 208, 11P  
 Jopek T. J., 1986, PhD thesis, Univ. Poznan (in Polish).  
 Jopek T. J., 1993, in Štohl J., Williams I. P., eds, Meteors and their parent Bodies. Slovak Academy of Sciences, Bratislava, p. 269  
 Jopek T. J., Froeschlé C. I., 1997, Astron. Astrophys., 320, 631  
 Jopek T. J., Valsecchi G. B., Froeschlé C. I., 1999, MNRAS, 304, 751  
 Jopek T. J., Valsecchi G. B., Froeschlé C. I., 2003, MNRAS, 344, 665

- Kramer E. N., 1953, *Trudy Odesskogo Gosud. Univ.*, 3, 163
- Kramer E. N., 1972, in Chebotarev G. A., Kazimirchak-Polonskaya E. I., Marsden B. G., eds, *Proc. IAU Symp. 45, The Motion, Evolution of Orbits, and Origin of Comets*. Reidel Publishing Company, Dordrecht, p. 472
- Kramer E. N., Vorobeva V. A., Rudenko O. A., 1963, *Trudy Odesskogo Gosud. Univ.*, 152, 5
- Lindblad B. A., 1971a, *Smithson. Contr. Astrophys.*, 12, 1
- Lindblad B. A., 1971b, *Smithson. Contr. Astrophys.* 12, 14
- Lindblad B. A., 1990, in Lagerkvist C. I., Rickman H., Lindblad B. A., Lindgren M., eds, *Proc. Asteroids, Comets, Meteors III*. Uppsala Univ. Press, Uppsala, p. 551
- Lindblad B. A., Porubčan V., 1991, *Bull. Astron. Inst. Czechosl., S.*, 42, 354
- Lindblad B. A., Porubčan V., Štohl J., 1993, in Štohl J., Williams I. P. eds, *Meteors and their parent Bodies*, Slovak Academy of Sciences, Bratislava, 177
- Mardia K. V., 1972, *Statistics of directiona data*. Academic Press, London and New York
- Marsden B. G., Williams G. V., 2003, *Catalog of cometary orbits 2003*, 15th edn. Smithsonian Astrophysical Observatory, Cambridge
- Murray C. D., Hughes D. W., Williams I. P., 1980, *MNRAS*, 190, 733
- Sekanina Z., 1970a, *Icarus*, 13, 459
- Sekanina Z., 1970b, *Icarus*, 13, 475
- Sekanina Z., 1973, *Icarus*, 18, 253
- Sekanina Z., 1976, *Icarus*, 27, 265
- Sitarski G., 1964, *Acta Astron.*, 14(1), 25
- Southworth R. B., Hawkins G. S., 1963, *Smithson. Contr. Astrophys.*, 7, 261
- Tedesco E. F., Harvey G. A., 1976, *AJ*, 81, 1010
- Voloshchuk Yu., 1999, in Baggaley W. J., Porubčan V., eds, *Proc. International Conf. Meteoroids 1998*. Astronom. Inst., Slovak Acad. Sci., Bratislava, p. 199
- Voloshchuk Yu. I., Kashcheev B. L., 1999, *Astron. Vestnik*, 33(4), 345
- Whipple F. L., 1947, *Proc. Am. Phil. Soc.*, 91, 189
- Whipple F. L., 1954, *AJ*, 59, 201
- Williams I. P., 2001, in Warmbein B., ed., *Proc. Meteoroids 2001 Conf.*, SP-495. ESA Publications Division, Noordwijk, p. 33
- Williams I. P., Zidian Wu, 1993, *MNRAS*, 262, 231
- Williams I. P., Zidian Wu, 1994, *MNRAS*, 269, 524
- Williams I. P., Murray C. D., Hughes D. W., 1979, *MNRAS*, 189, 483
- Williams I. P., Ryabova G. O., Baturin A. P., Chernitsov A. M., 2004, *MNRAS*, 355, 1171
- Wu Z., Williams I. P., 1992, *MNRAS*, 259, 617
- Wu Z., Williams I. P., 1996, *MNRAS*, 280, 1210

This paper has been typeset from a  $\text{\TeX}/\text{\LaTeX}$  file prepared by the author.