DECLINE, THEN RECOVERY: AN OVERVIEW OF ACTIVITY IN THE HISTORY OF MATHEMATICS DURING THE TWENTIETH CENTURY

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To the affectionate memory of Roy Porter (1946–2002)

1. INTRODUCTION

A graph of the level of activity in the history of mathematics during the twentieth century resembles a U-shaped curve. Around 1900 and until the Great War the field was very active, building upon a considerable body of work from the 1870s onwards. Since the first half of the 1970s it has been also in a pretty lively state. But in between there was a substantial drop in level, though some figures kept alive a modest flame of learning.

This unusual story is the subject of this article. The first Part (Sections 2–11) is a largely factual summary of activity from 1900 to around the 1990s. The second Part (Sections 12–17) reflects upon the story in various ways: imbalances in the measures of study of the various branches and aspects of the history of mathematics, changes in interpretations of some topics and developments, causes of the decline, relationships with mathematics education, and attitudes positive and negative to our subject among non-practitioners. The account is usually divided by countries, since differences between them are marked; however, work in each one does not necessarily focus upon its own developments, for many historians have not displayed any national preference in their choice of studies. The treatment of work is deliberately selective, especially in recent decades: many of the items cited exemplify some particular category or unusual feature, and silence on the others reflects only the desire to avoid just recording a long list of doubtless excellent achievements.

Recently, some historians of mathematics became interested in the history of their subject to the extent that they published a large collective book on it in 2002.[1] I refer to this book for more ample accounts of all aspects of this article, and also for the history of the history of mathematics before 1900; some features of the book are considered in Section 16.

PART ONE: REPORTAGE

2. THE ENTERPRISES OF FELIX KLEIN

Let us start in Germany, whose language was by far the most important one for our subject around 1900. Then in his early fifties, the mathematician Felix Klein

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(1849–1925) at Göttingen University was heavily involved in various large-scale projects concerning the development of mathematics and mathematical education; the Leipzig publishing house of Teubner was the chief outlet.[2] Of those projects that bore upon historical work, the most important was the *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* (hereafter *EMW*), which had been launched in 1894 as a detailed account of all the main areas of mathematics at the time. The founder President was Klein's former student, the algebraist Franz Meyer (1856–1934).

The *EMW* was divided into six Parts, each with its own editor(s) and publishing schedules:
(1): arithmetic and algebra (editor Franz Meyer), 1250 pages, 1898–1904;
(2): analysis (co-editor Heinrich Burkhardt), 4020 pages, 1900–27;
(3): geometry (co-editor Franz Meyer), 5200 pages, 1903–35;
(4): mechanics (co-editor Felix Klein), 3000 pages, 1901–35;
(5): physics (editor Arnold Sommerfeld), 3280 pages, 1903–26;
(6): section 1: geodesy and geophysics (co-editor Philipp Furtwängler), 970 pages, 1905–22; section 2: astronomy (co-editor Karl Schwarzschild), 2300 pages, 1905–34.

In total it filled around 20,000 pages. Many articles were the first of their kind on their topic, and several are still the last or best. Some of them have excellent information on the deeper historical background. This is especially true of articles on applied mathematics, including engineering, which was stressed in its title.

The German Mathematicians' Union, which itself had been founded in 1890, was closely involved. It also published several important survey-historical articles in its *Jahresbericht.* An outstanding historical example was the 1,804-page review of mathematical methods in analysis by Heinrich Burkhardt (1861–1914), an editor of the second Part of the *EMW*; he placed there a 540-page snippet of this longer piece.[3]

The edition had been planned to include a Part 7 on history, philosophy and education; unfortunately not a line was to appear, but Klein's other major activity lay in education, for at the International Congress of Mathematicians in Rome in 1908 he helped launch an International Commission for Mathematics Education. Up to 1920 nearly two hundred books and pamphlets were produced, and over three hundred reports. They included not only many national reports but also writings on broader topics that are still of interest today, such as teaching mathematics to girls and the use of the history of mathematics in mathematics education.[4] Wilhelm Lorey (1873–1955) was a distinguished author here and elsewhere, especially on institutional history.[5] For its "organ" of regular communication to the outside world the Commission appointed the Swiss journal *L'enseignement mathématique*, which had been founded in 1899.[6]

To be more precise about these projects, they were not just German but very Göttingen, because of Klein's strong emphases on applications and on education. The mathematicians at Berlin, the other main mathematical pole in Germany and a citadel for pure mathematics, were not invited to collaborate on the *EMW*, and are reputed to

have sneered at it. But Klein's commitment to the diffusion of mathematics extended to giving pioneering lecture courses on the history of nineteenth-century mathematics, and on 'elementary mathematics from an advanced standpoint' viewed with a deft use of both history and heuristics; both works have been reprinted or translated.[7]

3. OTHER WORK LARGELY IN GERMAN

The *Vorlesungen über die Geschichte der Mathematik* of Moritz Cantor (1829–1920) was the major general history of mathematics of the time. Underlying its massive scope was his strong belief in the unity of knowledge. Appearing from Teubner from 1880 onwards, the first three volumes all received second editions in the early 1900s, about 2800 pages in total. A fourth and last volume, covering 1759–99, appeared in 1908; by then Cantor was in his late seventies, so that it was a collective 1100-page effort by nine colleagues, which included a short survey of the history of history of mathematics by Siegmund Günther (1848–1923). The article on trigonometry was by Anton von Braunmühl (1853–1908), whose own two-volume *Vorlesungen über die Geschichte der Trigonometrie* (1902–3, also Teubner) imitated Cantor not only in title but also in thoroughness; it remains uneclipsed.[8] Cantor also continued to edit until 1913 a series of *Abhandlungen* on history that he had founded with Teubner in 1877; many valuable pieces appeared there, some of book length.

Denmark had been contributing importantly to the history of ancient mathematics since the late nineteenth century, sometimes to Cantor's discomfort. The mathematician Hieronymus Zeuthen (1839–1920) wrote extensively on Greek mathematics, though in a rather modernizing manner (Section 13) that led him into disputes with Cantor. His compatriot J. L. Heiberg (1854–1928) made the most sensational discovery of the time; in connection with a new edition of Archimedes, he identified in 1906 an important manuscript that had been found some years earlier.[9] Axel Bjørnbo (1874–1911) also wrote on ancient mathematics, but focused attention upon the transmission of Greek and Arabic mathematics into Europe in the Middle Ages, especially in manuscripts. This latter interest was also pursued by the German scholar Heinrich Suter (1848–1922).

The most important journal was the *Bibliotheca mathematica*, which Gustaf Eneström (1852–1923) had founded in 1884 in his native Sweden, initially as an appendix to the mathematical journal *Acta mathematica*. It reached a peak with its third series (1900–15) of 14 volumes, published by Teubner; for example, Heiberg's edition of the Archimedes text appeared there in 1907. However, Eneström somewhat spoilt the enterprise with long lists of criticisms of Cantor's *Vorlesungen*, of greatly varying quality. (As a critic of Cantor he followed Zeuthen.) Among other sources, the mathematical abstracting journal of the time, the *Jahrbuch über die Fortschritte der Mathematik* (1867–1942), maintained a reasonable level of commentary on historical writings until its closure, though its production faltered much in its last decade.[10]

4. MAJOR EDITIONS

Several editions of collected or selected works of mathematicians were prepared or at least started during the late nineteenth century, and very many more during the twentieth century:[11] the total number seems to be *far* greater than those for the other types of scientist. The majority have been just (photo)reprints of the mathematician's writings, some requiring many volumes, but with no special editorial involvement. However, some editions made much greater demands, with introductions, annotations, bibliography, translations and/or indexes. For example, the edition of C. F. Gauss (1777–1855), published and unpublished, had begun to appear already in 1865, but was far from completion by 1900: Klein not only was involved in continuing the edition but also helped to inspire colleagues to produce substantial historical essays on aspects of Gauss's work, towards an "intellectual biography". Several of the essays appeared as supplements to the edition.[12]

Another major edition with a long and interesting history is that for Christiaan Huygens (1629–95), prepared in the Netherlands by an anonymous committee of the Dutch Society of Sciences. Most unusually, they started with the correspondence (10 volumes, 1882–1905) before tackling the publications and manuscripts (12 volumes, 1908–50). Each volume was meticulously indexed, and its sheets are watermarked alternately "Christiaan" and "Huygens".

The edition that has made perhaps the greatest impact upon historians of mathematics is the one for Leonhard Euler (1707–83). A commission was formed under the auspices of the Swiss Scientific Society in 1907, the bicentenary of Euler's birth. Eneström produced a bibliography, published by Teubner for the German Mathematicians' Union,[13] who were also closely involved. From the list Paul Stäckel (1862–1919) sketched out the edition in three series:[14] the final plan proposed 18 volumes for the first series (pure mathematics), 16 for the second (mechanics and astronomy), and 11 for the last (physics, the philosophical *Lettres à une princesse d'Allemagne* (1768–72), and unpublished correspondence).[15] Work started in 1910, and the first two volumes were published by Teubner in the following year; ten more appeared before the Great War, and two during it.

A valuable edition project of a different kind was the *Ostwalds Klassiker der exakten Wissenschaften* (1889–), where many major scientific texts were reprinted (or, where necessary, translated into German) and furnished with notes. Mathematical writings were prominent from the start.

5. WORK IN OTHER LANGUAGES

French mathematicians soon began to prepare their own translation and elaboration of the *EMW*, as the *Encyclopédie des sciences mathématiques pures et appliquées*, with Teubner working with Gauthier-Villars. All six Parts were started, and some of the revisions were very remarkable: in particular, the historical content of several articles was increased, thanks especially to additional notes and material furnished by Eneström and Tannery. Some of the articles on set theory and functions were

so good that their German colleagues re-translated them back in the mid-1920s as additional pieces for their own second Part. But much greater enthusiasm was shown for translating articles on pure than applied mathematics; Klein's philosophy did not fully diffuse into France. Then the death in 1914 of the general editor of the *Encyclopédie*, Jules Molk, and the general circumstances of the War, led to Gauthier-Villars's withdrawal from the project in 1916; some articles stop in mid-sentence at the end of a 32-page signature.[16]

Efforts were made to launch Part 7, on the history, philosophy and education of mathematics. In particular, "The First Congress on Mathematical Philosophy" was held in Paris in April 1914, closely following one on mathematics education held there under the auspices of the International Commission for Mathematics Education.[17] But Heinrich Timerding (1873–1945), who was responsible for the Part, recorded at the 1914 Congress unexplained "very great difficulties" in its preparation, and after the War it seems to have been abandoned, in both the French and the German versions.[18]

Other French work in our subject was characterized by that concern with history mixed with philosophy that has long marked their scholarship. A striking example is Louis Couturat (1868–1914), for in advocating (the history of) symbolic logics he isolated himself from his compatriot mathematicians; his historical studies of logic in G. W. Leibniz (1646–1716) were better received. Other figures include Gaston Milhaud (1858–1918) and Abel Rey (1874–1940) on Greek science and philosophy; the mathematician Pierre Duhem (1861–1916), especially on medieval mechanics and the philosophy of science; and Leon Brunschvicg (1869–1944) and Pierre Boutroux (1880–1922) on progress in mathematical thought, and also as important contributors to an edition of the works of Pascal (14 volumes, 1904–14). The editions of the works of René Descartes (13 volumes, 1897–1913) and Pierre de Fermat (5 volumes, 1897–1912, 1922) excited admiration for the level of detailed scholarship deployed; Paul Tannery (1843–1910) was prominent in their preparation, in addition to his own major studies of Greek mathematics.

In Italy Gino Loria (1862–1954) launched in 1898 a *Bollettino di bibliografia e di storia delle scienze mathematiche*, imitating the title of a remarkable *Bullettino* that had been edited by Baldassare Boncompagni (1821–1894) between 1868 and 1887. Loria's journal also lasted for twenty years, and then continued as an appendix to the *Bollettino di matematica*; but it never reached the same level of importance of its predecessor. However, he also published the first "guide" to our subject,[19] where he reflected on the practice of history and its own history as well as providing bibliographies. His main research speciality lay in the history of geometry, an interest shared by Roberto Bonola (1874–1911), whose book on *Non-Euclidean geometry* (English translation 1912) is still well known.

Ettore Bortolotti (1866–1947) specialized in Italian mathematics during the Renaissance and afterwards, following a tradition from Boncompagni. Bonola's doctoral supervisor Federigo Enriques (1871–1956) stands out among his countrymen as not only an excellent mathematician but also a French-style *storico-filosofo*, viewing a mathematical theory as dynamically intertwined with its history. As well as writing for

Part 3 of the *EMW*, he tried to promote Part 7 on history, philosophy and education. He also applied his approach to (the history of) logic; a school in Turin around Giuseppe Peano (1858–1932) concentrated on the formalization of logic and of mathematical theories, but also adorned their accounts with extensive historical notes.[20]

The most important Italian edition of the period was that for Galileo Galilei (1564–1642), prepared by Antonio Favaro (1847–1922) in 20 volumes (1890–1909). However, although officially "national", it met with little response at first, and has become known mainly through a reprint with additions (1929–39).

While a few British and American authors had contributed to the *EMW*, no English edition was prepared, and efforts to start one in Britain met with apathy. The main historians of the time were Sir Thomas Heath (1861–1940), with his various editions and commentaries on Greek mathematics; Sir Thomas Muir (1844–1934), with his mammoth four-volume *The theory of determinants in the historical order of development* (1906–23); and Sir Edmund Whittaker (1873–1956), with several very fine obituaries and articles on applied mathematics (including one in the *EMW*), and also a well-known *History of the theories of aether and electricity* (1911, second edition in two volumes 1951–53). These men received knighthoods for other activities. Philip Jourdain (1879–1919) is also worth recalling, for some fine pioneering work in the history of set theory and symbolic logics.[21] At Cambridge W. W. Rouse Ball (1850–1925) wrote extensively on the history of the local mathematicians, and produced a remarkable and unique history of *Mathematical recreations and problems of past and present times* (1892), which took its seventh edition in 1917 and is still in print in its 13th edition (1987).[22]

In addition, a separate tradition developed in Britain, concerning probability and mathematical statistics. A principal modern founder of mathematical statistics was Karl Pearson (1857–1936); he also took an active interest in its own history, especially from the 1920s onwards, when historical pieces by him and others appeared in the journal *Biometrika* that he had started in 1901.[23]

Many valuable long survey-historical articles appeared in the 10th (1902–3) and especially the 11th (1910–11) editions of the *Encyclopaedia Britannica.* These vast and glorious quartos contain the equivalent of little books on many topics, mathematics included; for example, to my knowledge the 120-page pair on "ships" and "shipbuilding" surpasses anything of that time on recent developments, the *EMW* included.[24] Earlier editions of the *Encyclopaedia Britannica*, and some contemporary encyclopaedias in other languages, also contain valuable but neglected articles; historians should be more aware of such sources.

The USA was emerging as a significant mathematical country around 1900, partly under the influence of Klein.[25] Some benefits for historical work flowed, especially the relationship between history and mathematics education. The Swiss-born Florian Cajori (1859–1930) had already reported on *The teaching of the history of mathematics in the United States* in 1890. At the Teachers' College in Columbia University in New York from 1901, D. E. Smith (1860–1944) was pursuing the same line; indeed, it was a suggestion by him in *L'enseignement mathématique* in 1905 that led to the

creation of the International Commission for Mathematics Education (Section 2). G. B. Halsted (1853–1922) was a pioneer advocate of non-Euclidean geometry in the USA, to the extent of translating some of the original major texts.

For publishing, one unusual venue was the Open Court Publishing Company, which had started in the late 1880s under the direction of the German mathematician and philosopher Paul Carus (1852–1919) as a channel for importing German culture, especially philosophy, into the USA. Carus's journal *The monist* (1890–) was an important site for papers (including translations) on logic and on the philosophy of mathematics and of science. As well as publishing new books, Open Court also issued translations, including reprinting some of Halsted's. Among several German authors, Ernst Mach (1838–1916) was quite a hero, and others included the mathematicians Richard Dedekind (1831–1916) and Georg Cantor (1845–1918). The latter was handled by Jourdain in 1915, soon after he became the Company's European Editor: he succeeded Carus as General Editor in 1919, but died himself later that year.[26]

Our subject was pursued also in the Far East, though the West may have known only of two books in English by the Japanese historian Yoshio Mikami (1875–1940), one of them written with Smith. The respective publishers were Teubner (in Cantor's *Abhandlungen* series) and Open Court.[27]

6. A FAIRLY QUIET TIME: TO THE SECOND WORLD WAR

After 1920 the level of activity fell off quite noticeably. One continuing line was that of the French *historien-philosophe*: new figures of this genre include the Russian-born Alexandre Koyré (1892–1964), who deployed his religiosity and Platonism to convey a rather purist vision of the development of astronomy and mechanics from Copernicus to Galileo; Gaston Bachelard (1868–1944), who wrote on some aspects of the history of applied mathematics and of physics; and Jean Cavaillès (1903–44), who specialized in the history of foundational questions in mathematics[28] and contributed to the 1930s editions of the works of Dedekind and Cantor. Cavaillès's death during the Second World War is no accident of chronology, for he sacrificed himself for the cause of the French Resistance, and is now buried in the chapel of the Sorbonne in Paris, near to Descartes.

In the Netherlands E. J. Dijksterhuis (1892–1965) produced some notable work on ancient mathematics; for example, studies of mechanics, and an excellent partial translation of Euclid's *Elements* (1929–30). This edition launched a book series under his co-editorship to encourage the links between history and education among school-teachers.[29]

In Switzerland (and elsewhere) the Euler edition continued in various hands, especially those of Otto Spiess (1878–1966) and also Andreas Speiser (1885–1970): fifteen more volumes were published between 1920 and 1944, though none in the second series for mechanics. From 1933 Teubner shared the publication with the Zurich house Orell Füssli. Spiess also began planning an edition of the works of Euler's friends and mentors, the Bernoulli family.

Several articles in most Parts of the *EMW* continued to appear. Those for geometry

and mechanics were the last to be completed, in 1935, when the President was Constantin Carathéodory (1873–1950), a major mathematician with considerable historical interests. A second edition of the *EMW* was being worked out, with foundations of mathematics, algebra and number theory as the initial topics; but the Second World War took it as a victim, and it seemed to stop around 1950 after only a few articles had been published.

A notable German figure was Heinrich Wieleitner (1874–1931). He continued the effort to update Cantor's *Vorlesungen*, and wrote on the history of various mathematical topics, especially during the Middle Ages. Contemporary with him was Johannes Tropfke (1866–1939), whose work was more closely linked to educational purposes. His *Geschichte der Elementarmathematik in systematischer Darstellung* (1902–3) was greatly enriched in its multi-volume second (1921–24) and third (1930–37) editions by studies of original texts. A further posthumous volume was produced in 1940 by Kurt Vogel (1888–1985); forty years later he co-edited a large new edition of the first three volumes as one big book, covering arithmetic and algebra.[30] Vogel's own research lay initially in ancient mathematics, and later with the medieval European period, including the transmission of Greek and Arabic mathematics there. Several compatriots, such as Carl Schoy (1877–1925) and the philologist Julius Ruska (1867–1949), specialized in Arabic mathematics, including astronomy. Some mathematicians formed seminars on our subject in their universities, especially at Kiel, Frankfurt (led by Lorey), and Bonn.

Vogel's interest in ancient mathematics was shared by the most internationally significant German-born figure of this period. Otto Neugebauer (1899–1990) specialized in ancient mathematics, in ways interestingly different from those of the French philologist François Thureau-Dangin (1872–1944).[31] For most of the 1930s he also ran a series of *Quellen und Studien zur Geschichte der Mathematik*, a successor to Cantor's series.

In 1934 Neugebauer moved from Göttingen University to Copenhagen University; then in 1939 he decided to emigrate to the USA. There the interest in history and education had continued. Smith produced an influential *History of mathematics* in 1923 and *A source book in mathematics* six years later. The journal *Scripta mathematica* was launched in 1932 by Jekuthiel Ginsburg (1889–1957) partly to pursue this cause, and also to treat educational and philosophical topics in mathematics (after his death this policy was quickly dropped). The practice of teaching history courses spread to many educational institutions in the USA, probably far more than in any other country.

Among activities related to research, in 1934 Smith and Ginsburg put out *A history of mathematics in America before 1900*; the publisher was the Mathematical Association of America, in a series funded by a legacy from the widow of Carus, the founder-editor of *The monist.* Smith also bought many mathematical books and masses of manuscripts, collections of major importance now housed in Columbia University. Cajori published with Open Court an important *History of mathematical notations* (1928–29); isolated from sources in Colorado, he used to pay scholars to

transcribe texts from holdings in major libraries.[32] In 1934 he put out a revised edition of Andrew Motte's 1729 translation of Newton's *Principia mathematica*, which has become very well known.

Perhaps the best American research historian was R. C. Archibald (1875–1955); he wrote on several topics in nineteenth-century mathematics, which then was still much unstudied. It is a pity that he produced no substantial tome: he lacked the prolificacy of the Belgian-born George Sarton (1884–1956), who had moved to the USA in 1915. Best known as the founder-editor of the history of science journals *Isis* in 1912 and *Osiris* in 1936, Sarton is one of very few historians of science who has taken history of mathematics seriously. He produced in 1936 a valuable handbook on *The study of the history of mathematics*, as a companion to a contemporary review of *The study of the history of science*.[33]

The best-known American figure is the Scottish-born E. T. Bell (1883–1960). He produced books such as *Men of mathematics* (1937) and *The development of mathematics* (1940) — unfortunately, for the first book especially is a hit-and-miss attempt to chronicle the lives or supposed lives of major mathematicians, with no references given. Apparently the style of work fitted the man himself, for he also wrote science fiction.[34] His historical books are still reprinted quite regularly.

Special factors attended the development of the subject in the Soviet Union. Before the Revolution of 1917 work was modest, though V. V. Bobynin (1849–1919) and A. V. Vasiliev (1853–1929) were known internationally for their work on Russian mathematics, and I. Y. Timchenko (1863–1929) in Odessa for excellent contributions to the history of mathematical analysis. After the political change the subject was encouraged among both historians and leading mathematicians, as part of the general conception of history of knowledge as a Good Thing; but of course it had to be presented in a Good Way, and on Good Topics. Thus the history of the calculus, the one branch of mathematics that Karl Marx had studied intensively, became a preferred branch; and writers on all topics frequently cited the Marx–Engels correspondence in their prefaces, though often giving only factual accounts thereafter. In the same vein, history courses were given in several universities. Leading figures include Sof'ya Yanovskaya (1896–1966), who had a special concern for the history of logic and of foundational topics in mathematics.

In the Far East, activity seems to have fallen away: Mikami wrote a great deal about Japan and China but published relatively little. But India came into the picture. B. Datta (1888–1958) and A. N. Singh (1901–54) produced *Hindu mathematics: A source book* in English (1935–38); and some Western achievements were well captured by G. Prasad (1876–1935), especially in his survey of fourteen figures entitled *Some great mathematicians of the nineteenth century: Their lives and their works* (1933–34).

7. A FAIRLY QUIET TIME: THE POST-WAR PERIOD UP TO THE 1970S

In Germany, Wieleitner's most important student was J. E. Hofmann (1900–73). Fascinated by genius in mathematics, he gave much attention to major figures of the

early modern period, normally up to 1800.[35] In 1954 he founded a seminar for the history of mathematics at the Mathematical Research Centre at Oberwolfach, a series of week-long meetings that has become of major importance for our subject (Section 8). Among his contemporaries, Helmut Gericke (1909–) was the first historian of mathematics to gain a full chair in the history of science in Germany (Munich, 1963); his main contributions lay in the history of algebra and of foundational questions, and he wrote some more general histories. The most prominent of Hofmann's students is Christoph Scriba (1929–), who gained a chair in Berlin in 1969 and then in Hamburg in 1975. In research he has focused especially on geometry, number theory, the early development of the calculus, and historiographical questions.

Of all historical figures who fell under Hofmann's scrutiny, the most significant for him was Leibniz. So he worked on the *Sämtliche Briefe und Schriften* of Leibniz, which is possibly the most massive edition of all for a single historical figure. Conceived in seven series under the auspices of the German Academy of Sciences in Berlin, volumes had begun to appear in 1923, but slowly: the first for the mathematical ones (letters in the third series, publications and manuscripts in the seventh) came out only in 1976 when Hofmann completed the first volume of letters. The complications of the edition involve not only its sheer scale (for example, the Paris period 1672–76 alone will need eight volumes) and the extreme difficulty of transcribing many manuscripts, but also the post-War fact that the direction lay with the Academy of Sciences of the German Democratic Republic while the documents and many of the scholars were in Hanover and elsewhere in the Federal Republic.

Another major German edition was a *Gesamtausgabe* of publications and manuscripts for Johannes Kepler (1571–1630). It had begun to appear in 1937 under the Bavarian Academy of Sciences, and volumes were published steadily even during the War and until 1959, in the hands of von Dyck and Franz Hammar and especially Max Caspar (1880–1956). Then there was a long pause, until further volumes started to come out again in 1983, under the leadership of Volker Bialas; most of the edition is now done.

The Gauss edition had been completed in 1933, but continued interest in his life and work and its context led to the founding in the 1960s of a *Gauss-Gesellschaft*, which publishes a journal of *Mitteilungen*. Some biographies appeared, but none yet matches the massive range of his achievements; so Klein's hope of an "intellectual biography" (Section 4) remains unfulfilled.

This last judgement can be made also for Euler. He was the third subject in a series of useful booklet-length biographies of major mathematicians that was published from 1947 to 1980 by the Swiss journal *Elemente der Mathematik*. But the Euler edition continued well. The original scheme of three series for his publications and correspondence in 18 + 16 + 11 volumes had become 29 + 31 + 12 volumes for the publications alone,[36] and 33 more volumes appeared from Orell Füssli between 1945 and 1970.

As usual, most activity in France was conducted in Paris. In addition to more *historiens-philosophes* — most notably Georges Canguilhem (1904–95), and also

François Rostand with original but much neglected meditations on inexactness in mathematics[37] — two distinct *équipes* were formed. One was led by René Taton (1915–) and Pierre Costabel (1912–89), with the *Revue d'histoire des sciences* (1947–) as a main outlet; the other operated under Gaston Bachelard's daughter Suzanne (1919–). Much work of both groups focused on French mathematics from the time of the founding of the Académie des Sciences in the 1660s up to the early twentieth century. The school-teacher Jean Itard (1902–79) became involved in the interface between history and education as well as researching on Greek mathematics and the seventeenth century.

Separate from both groups was the Bourbaki *collectif* of prominent French mathematicians, who presented central parts of mathematics for mathematicians in a highly formalized manner in their multi-volume treatise *Eléments des mathématiques*. But they included short historical essays on various topics, which they collected together as a volume of *Eléments d'histoire des mathématiques* (1969 and later editions); it is one of the historical sources best known to mathematicians for relatively modern developments, and has received several translations.[38]

Some of the Italian work was driven by the powerful academic place there of the history of philosophy, and so focused upon foundational questions. An important example is Ludovico Geymonat (1908–91), especially after the Second World War when the demise of the Fascists allowed him to take university chairs, at Turin and then at Milan. Ettore Carruccio (1908–80) wrote on questions of method and proof in general and also on Italian mathematics; Attilio Frajese (1902–86) specialized in Greek mathematics, including Italian translations; and Ugo Cassina (1897–1964) analysed the achievements of his old master Peano and his followers, and also prepared an edition of Peano's works (3 volumes, 1957–61).

In Britain the principal efforts centred upon Isaac Newton (1642–1727). A seven-volume edition of his correspondence was produced between 1959 and 1977, under various editorial hands. More directly significant for the subject was the edition of his numerous unpublished *Mathematical papers,* which were edited by D. T. Whiteside (1932–) with the assistance of Michael Hoskin (1930–) and Adolf Prag (1906–2004), and appeared between 1967 and 1981 as eight large quartos from Cambridge University Press. The medieval and early modern periods attracted some fans, including Alistair Crombie (1915–96), who from 1967 chaired a Thomas Harriot Seminar to study the achievements of that remarkable but shadowy polymath.

In the Soviet Union the leading historian was Adolf Pavlovich Yushkevich (1906–93). His extensive studies of the medieval period, the development of the calculus and of the work of Euler were complemented by a massive study of the development of mathematics in Russia prior to the Revolution, one of the first modern histories of the mathematics of a country.[39] Among many other contributions, in 1948 he co-founded an annual for the subject, entitled *Istoriko-matematicheskogo issledovaniya* (*Historico-mathematical writings*); it was the first new journal (or equivalent) for decades. Of his colleagues, Isabella Bashmakova (1921–) stands out for her studies of ancient and medieval mathematics, and of algebraic number theory. Collective

Soviet productions included several good editions of the works of Russian mathematicians: for example, those of N. I. Lobachevsky and P. L. Chebyshev are far more complete and scholarly than the earlier French-language ones.[40] Several biographies of mathematicians, Russian/Soviet or otherwise, were written, and given print-runs in the tens of thousands.

Among the Soviet colonies the German Democratic Republic became the most active, thanks especially to Hans Wussing (1927–), who built up an important Institute for the History of Science at Leipzig University. His own main speciality was the history of group theory, but he and many colleagues also prepared a wide range of more general works, especially a valuable volume of short biographies of many mathematicians of which there is no counterpart in any other language.[41] In Berlin K.-R. Biermann (1919–2002) became a leading authority on institutional history and on Gauss (and also on Alexander von Humboldt). In a typical post-war German situation *Ostwalds Klassiker der exakten Wissenschaften* (Section 4) continued in both countries, in West Germany with a "new series".

In the USA the links with education or at least with popularization were continued by new figures such as Carl Boyer (1906–76), with a general history (1968) and histories of some specific branches; Philip Jones and Howard Eves, very active in the Mathematical Association of America; and Morris Kline (1908–92), whose *Mathematics in Western culture* (1953) brought mathematics to a remarkably wide audience. His 1200-page *Mathematical thought from ancient to modern times* (1972) was then novel in the large proportion of space given to the nineteenth and early twentieth centuries, as was Edna Kramer's consideration of *The nature and growth of modern mathematics* (1970). The Dutch-born Dirk Struik (1894–2000), resident in the USA since the mid-1920s, pioneered efforts from the 1930s to bring social factors to bear upon the history of mathematics — in his case, influenced by Marxism.[42] His *A concise history of mathematics* was soon recognized as the best of its compass upon its first appearance in 1948, and has been translated into eighteen languages. Among many other activities, he was heavily involved in an excellent edition of "the principal works" of Simon Stevin (1548–1620), including English translations, that was initiated in the Netherlands by Dijksterhuis and others (5 volumes, 1955–66).

The mathematical component of the work of Bernard Cohen (1914–2003) included the history of economics and of computing, but it was dominated by studies of Newton. In particular, in 1972 he was the co-editor of a new edition of the third edition (1726) of Newton's *Principia mathematica*, and in 1999 of a new English translation of the work that should come to eclipse the Motte-Cajori translation of 1934 noted in Section 6. Koyré was the other editor of the 1972 edition just mentioned; he came to enjoy a considerable influence in the USA because of his periods of academic residence there, especially at the Institute for Advanced Study at Princeton, where Marshall Clagett (1916–) pursued the history of medieval mathematics, and then Egyptian science.

Quite separate from these workers, during the mid-1950s C. Truesdell III (1919–2000) began to make notable inroads into the neglected histories of elasticity theory and hydrodynamics; in work undertaken largely in connection with the

Euler edition, he elevated Euler to a very high place.[43] In 1960 he also launched with Springer the *Archive for history of exact sciences*, where history of mathematics features frequently.

A department for the history of mathematics was established under Neugebauer at Brown University in 1948. But it was a Marxist self-exilist from the USA who was to have the greatest impact on the practice of the history of mathematics, as we shall now see.

8. REVIVAL FROM THE 1970S: THE ROLE OF THE INTERNATIONAL COMMISSION ON THE HISTORY OF MATHEMATICS

One of the main interests in mathematics for the American Kenneth O. May (1916–77) was information retrieval, and this led him to study its history seriously. In 1973 he published with the University of Toronto Press (his university) an 800-page bibliography for the subject that, while worryingly dense with errors of reference, provides a comprehensive classification of historical literature.[44] The year following, the Canadian Society for the History and Philosophy of Mathematics was formed.

At that time May, with support from Taton and Yushkevich, also set up the International Commission on the History of Mathematics within the framework of the International Union for the History and Philosophy of Science. Under his Chairmanship, and that of his successors (initially Scriba and then Joseph Dauben (1944–)), it has been one of the most active Commissions, with national representatives right across the world. In 1985 it was also recognized by the International Mathematical Union. Since 1989, and forthwith at each International Congress for the History of Science, the Commission has awarded May medals, in memory of its founder, to historians who have enriched the subject not only by their scholarship but also by developing the community of historians. The first awards went to Struik and Yushkevich, followed in 1993 by Scriba and Wussing and in 1997 by Taton.

One of the main tasks of the Commission is to administer *Historia mathematica*, a journal created by May in 1974. Originally published by University of Toronto Press, it was transferred three years later to Academic Press. Intended as more than a repository of articles and reviews, it included departments on projects and on archival sources, abstracts of relevant publications, news of meetings and appointments, and other news. One of May's main decisions was to incorporate an abstracts section: at first about 250 items per year were listed, by the mid-1980s the number had grown to 300–400, and for some years now at least 700 have appeared. Under May's successor editors, initially Dauben (1977–85) and Eberhard Knobloch (1943–) (1985–93), it has played an important role in the growth in activity.

A very significant stimulus to the subject was the series of week-long meetings at Oberwolfach that was founded by Hofmann (Section 7). After his death in 1973, the series was taken over by Scriba, and until 2000 it met annually or biennially. It allowed fifty historians of mathematics from around the world not only to hold a meeting but also to plan other events: in particular, the Commission met there. Without this opportunity some of its and others' initiatives might well not have occurred, or

at least would have been much harder to effect.

One of Dauben's main achievements as Chairman was to edit a substantial annotated bibliography of historical literature in our subject, which appeared in 1985. Smaller in scale than May's, it was more focused in organization and information. An extended edition was prepared under the direction of Albert Lewis (1943–) and published in 2000 as a CD-ROM by the American Mathematical Society.[45]

9. REVIVAL FROM THE 1970S: BY COUNTRY OR REGION

Only a few features of the last thirty years in the most active countries can be recorded here; some preference is given to collective rather than individual enterprises.

Italy became particularly active;[46] the depth of work is well exemplified by the volume of 1987 on Italian mathematics between the two World Wars.[47] The Italian Mathematical Society has sponsored since 1980 the *Bollettino di storia delle scienze mathematiche*, edited by Enrico Giusti (1940–). In addition, a general magazine on mathematics and its history, *Lettera pristem* (the latter word is the acronym of a study group) started in 1991, partly modelled upon the American general journal *Mathematical intelligencer.* The history of mathematics also appears sometimes in the similar publication *Informazione filosofiche*. In 1994 the *Circolo Matematico di Palermo* added to its renowned *Rendiconti* a biennial historical supplement, edited by U. Bottazzini (1947–). A national Society was formed in 2001, with Silvia Roero (1952–) as founder President.

A long-term project that continues an Italian tradition from Bortolotti and others is a Centre for Medieval Mathematics, founded in 1981 at Siena University by Laura Toti Rigatelli (1941–) and Raffaella Franci (1940–). Two years later it launched a series of editions of original Italian sources with a bibliography of the many articles on Renaissance mathematics written by Gino Arrighi (1906–2001) and often published in obscure locations.[48]

Another major centre is Paris, where various *équipes* function, with much work on the various important French educational institutions and societies. In 1980 the *Cahiers du séminaire d'histoire des mathématiques* was launched, with some speciality for transcribing manuscripts and correspondence. Conferences take place occasionally at Luminy (Marseilles) in the mathematics centre of the French Mathematical Society, which also supported in 1995 the founding of the journal *Revue d'histoire des mathématiques* to succeed the *Cahiers.*

The German Mathematicians' Union launched in 1985 a book series of German documentary sources and editions. It also supports a History section, inspired largely by its absorption in the early 1990s of the sister society of the former German Democratic Republic. A book series with Vandenhoeck & Ruprecht contained some excellent doctoral theses. A project led by Menso Folkerts (1943–) tracks down medieval manuscripts, including those of the German *Rechenbuch* tradition of early algebra. In 1988 he also launched a book series "Algorismus" for the history of science (often mathematics) and became the editor of another one, "Boethius" (1963–).

Soviet work continued in the same style and quantity as previously, especially in Moscow and Leningrad. Yushkevich edited a three-volume general history of mathematics, up to the nineteenth century, followed by a trio edited with A. N. Kolmogorov (1903–87) on the nineteenth century itself; the latter has been translated into English.[49] The change of the Soviet Union into Russia does not seem to have had major effects on the work done there. With the death of Yushkevich in 1993, his mantle has fallen upon Sergei Demidov (1942–), who maintains the annual *Istoriko-matematicheskogo issledovaniya*.

Czechoslovakia has produced a respectable body of work, led by Lubos Novy (1929–) and Jaroslav Folta (1933–); some of it was concerned with the Czech speciality of geometry. Rather less has come out of Poland, but in compensation the history of logic is active there, especially concerning the country's great importance in that subject since the 1920s. Thanks to the organizer Christa Binder (Vienna), week-long meetings take place in Austria, serving as a valuable focus for historians especially in Central Europe and Germany.

A British Society for the History of Mathematics was formed in 1971, in order to provide a forum for adherents to the subject. Since the mid-1980s the Society has substantially increased both its membership and range of meetings, and developed a tri-annual *Newsletter* from 3–4 pages of typescript in 1986 to 60 or so pages of print nowadays; it was elevated to a *Bulletin* in 2004. Its founder President was Gerald Whitrow (1912–2000), whose historical work focused upon cosmology and theories of time, an area of interest shared by C. W. Kilmister and J. D. North (1936–). The latter has also worked much on the medieval and early modern periods, including in the Thomas Harriot Seminar (Section 5), which has continued since the early 1980s under the direction of Gordon Batho. Interest grew in the 1970s within the Faculty of Mathematics of the Open University (where distance learning is the main strategy), in connection with a special course for our subject.

In the USA large special sessions take place regularly at meetings of the American Mathematical Society and the Mathematical Association of America, and are well attended. In 1988 the Society started a book series, which is now co-sponsored by the London Mathematical Society. Among large-scale projects, the *Dictionary of scientific biography* gave appropriate attention to mathematics, both in its original 16 volumes (1970–80) and the two supplementaries (1990); the editor Charles Gillispie (1918–) had Boyer and Clagett on his editorial board. Many researchers are to be found in the USA, with the links to education particularly strong.

In Mexico City a group around Alejandro Garciadiego (1953–) started in 1985 the journal *Mathesis*, which has been especially concerned with the history of foundational subjects. Interest has grown substantially in Latin America: in 1992 an Association for History, Philosophy and Pedagogy of Mathematics was established, and nine years later there was founded a national Society in Brazil, together with a journal. Part of the stimulus has come from policies in the Iberian Peninsula to encourage the history of science as an academic discipline. Spain has become quite active, especially with a group under Mariano Hormigon (1946–) at Zaragoza; in

Portugal a seminar for our subject was formed in the early 1990s, with an especial concern for meetings suitable for students.

Another region to develop its own interests is Africa. Under the inspiration of Paulus Gerdes (Mozambique), a Commission on the History of Mathematics in Africa was established in 1987. A good deal of revisionist history is emerging from its meetings and *Newsletter*, especially concerning the content of African mathematics and developments after the ancient glory of Egypt, which hitherto were usually ignored.[50] Partly in this connection, considerable interest has developed in ethnomathematics; an astonishing range of theories created worldwide in all sorts of contexts has been exposed. In addition to Gerdes, leading practitioners include Ubiratan d'Ambrosio (1926–) in Brazil (a May medallist in 2001) and Marcia Ascher (1935–) in the USA.[51]

Further north on that continent, Arabic and Islamic sciences have gained a new level of interest and subsidized support, with mathematics featuring prominently. An Institute has been established at Aleppo, and meetings occur regularly. For mathematics, one main task is to locate unknown manuscripts; in addition to many indigenous sources, important versions of lost works by Apollonios and by Diophantos have been found. Several editions of the works of important figures have appeared, some with translations into a Western language. (Some texts have been edited more than once, with some disturbance of the scholarly atmosphere.[52]) Several manuscripts are in India, whose own rich history has received a considerable impulse: a national Society was formed in 1978, and their *Bulletin* was launched the following year under the title *Ganita-Bharati*. The annual meetings of the Society have become more international recently, and proceedings have appeared for the 2001 gathering.[53]

Similarly, the history of mathematics in the Far East has increased both in the countries involved and elsewhere — in China after a period of isolation reinforced by the "cultural revolution". An important international example is a project between Dauben and colleagues in China, Taiwan and Singapore to produce an English edition of "Ten classics of ancient mathematics", in enhanced echo of the collaboration decades earlier between Smith and Mikami (Section 5). In the same international spirit, several historians in Far Eastern countries study aspects of Western mathematics, even effecting some translations of primary sources.[54]

One major general task is to convey to non-readers of Chinese the *sense* as well as the content of the mathematics involved; for six characters may have been rendered as (say) fifteen words in English, and in another order of expression. To cope with this problem Jock Hoe introduced a semi-symbolic language corresponding as closely as possible to the order of the characters, thereby bringing the reader closer to Chinese thought.[55] His proposal has not been adopted to the extent that it deserves.

10. REVIVAL FROM THE 1970S: OLD AND NEW EDITIONS

Among the editions mentioned earlier,[56] the mathematical series of the Leibniz edition (Section 7) are now supervised by Knobloch. Reorganized during the 1980s with a new eighth series to cover some science and technology, the scale of the enterprise

has forced planning of future volumes into long periods,[57] and editions of individual manuscripts or groups of them are appearing elsewhere. Since its founding in 1979 the journal *Studia Leibnitiana* has been keeping the reader abreast.

In the Euler edition, the first and third series were completed in 1956 and 2003 respectively, and two astronomy volumes in the second one are now left to be published.[58] However, the fourth series, comprising two sub-series of his correspondence and his notebooks, is far from complete; only four of the ten thick volumes of letters are out, and none for the notebooks. From 1952 Orell Füssli was the sole publisher of the edition; this responsibility was taken over in 1981 by Birkhäuser (Basel), who are also issuing the edition of the Bernoulli family conceived by Spiess, helped by funds that he left. Currently under the general editorship of P. de Radelet-Grave, it includes the contributions of the Bernoullis' close colleague Jacob Hermann (1678–1733).[59] Birkhäuser also started in 1989 (initially with a house in the German Democratic Republic) the book series "Science networks" for the history of mathematics and the physical sciences, parallel to one for both "Studies and sources" already launched by their parent company Springer in the 1980s as a successor to Neugebauer's series of the 1930s (Section 6).

Among newer editions, some large ones involve historical figures in the interaction between mathematics, logic and philosophy. The *Gesamtausgabe* for the Bohemian polymath Bernard Bolzano (1781–1848) (1969– , Frommann) began with a beautiful biography by the Austrian historian Eduard Winter (1896–1982).[60] Now run largely in German-based hands based around Jan Berg, the edition will contain in its more than fifty volumes not only Bolzano's many publications but also transcriptions of his mathematical notebooks and unpublished essays, and correspondence. It succeeds some partial editions produced earlier in the century in Bohemia and in the Czechoslovak Republic.

For scholarly editions of more recent figures, there is the 30-volume chronological edition of the *Writings* of C. S. Peirce (1839–1914) (1982– , Indiana University Press) in the tradition of algebraic logic (among his many accomplishments); and one of comparable compass for the papers and manuscripts of Bertrand Russell (1872–1970) (1983– , now Routledge) in the very different line of mathematical logic (and again many other things). The publications and manuscripts of Felix Hausdorff (1868–1942), important in set theory and several other mathematical topics, are in hand (8 volumes, 2003– , Springer); those of Kurt Gödel (1906–78) have recently been completed (5 volumes, 1986–2003, Oxford University Press).

In addition, interest in the history of the allied area of computing grew rapidly during the 1980s, especially with the launch of the *Annals for the history of computing* in 1979 and reprints of several original books and unpublished reports. The mathematician most to benefit from this activity was Charles Babbage (1792–1871), whose collected works appeared in 1989 (11 volumes, Pickering), edited by Martin Campbell-Kelly.

Other major editions in progress include Pierre Crépel directing a full edition of Jean d'Alembert (1717–83), to be published in around forty volumes by the

National Council for Scientific Research; and various hands toiling over the *Collected papers* of Albert Einstein (1879–1955) (1987– , Princeton University Press). Among editions solely of manuscripts, an outstanding example is Clerk Maxwell's (3 volumes, 1990–2003, Cambridge University Press), carried out by Peter Harman; and Scriba is co-editing the correspondence of John Wallis (1616–1703) (2003– , Oxford University Press).

The photoreprint industry has been very busy. Much of the mathematical output reproduces several older editions of works, an extraordinary number of old books, some older books in the history of mathematics (including several cited earlier), many mathematical journals, and at least one historical journal (Boncompagni's). Some volumes even appeared in paperback, and several with new introductions or other editorial material (in particular, by Hofmann for Olms Verlag). Most of this reprinting has happened since the Second World War: one motive was to make good library losses created by bombing, while two others were the creation of new universities and the expansion of old ones, many with empty or rather bare libraries. Historians, the historically minded and librarians have benefited enormously, especially when the item has been made available in paperback.

11. THE *COMPANION ENCYCLOPEDIA* (1994)

It is appropriate to end this chronological review with the *Companion encyclopedia of the history and philosophy of the mathematical sciences*, for it is a comprehensive two-volume book of 1806 pages that appeared under my editorship in 1994 from Routledge. The principal general aim of the book was to convey the *ubiquity and variety* of the various branches and aspects that make up "the mathematical sciences" — pure, applicable, applied, probability and statistics, and cultural and educational aspects. It contains 176 articles, written by 133 authors of 18 nationalities. The normal terminal point for articles was in the 1930s (if the topic in question was still then active).

After a general editorial introduction, the book is divided into the following Parts:

1. Non-Western traditions up to Western superventions
2. Medieval and Renaissance, up to around 1600–1700
3. Calculus and analysis
4. Functions, series and methods in analysis
5. Logics, set theories, foundations of mathematics
6. Algebras and number theory
7. Geometries and topology
8. Mechanics and mechanical engineering
9. Physics and mathematical physics, and electrical engineering
10. Probability and statistics, and the social sciences
11. Higher education and institutions
12. Mathematics and culture
13. Reference and information

The issue of generality was prime. The book naturally has articles on central historical topics such as the Greeks, early algebra, number theory, Newtonian mechanics, and Euclidean and non-Euclidean geometry; but the many other topics include: Korean mathematics; Ancient methods of doing fractions; Medieval optics and instruments; Prehistory of fractals; Nomography; Algebraic and mathematical logics; Polish logics; Calculating machines; Lie groups; Linear programming; Operational research; Graph theory and combinatorics; Servomechanisms; Methods of navigation; Meteorology; Capillarity; Biomathematics; Actuarial mathematics; Probability and statistics in medicine; Probability and statistics in engineering and technology; Mathematical economics; Tilings; Mathematics in prose literature; Mathematics in poetry; and History of history of mathematics.

The invitation from Routledge in 1988 to produce the book was very timely; for I doubt if it would have been possible even five years previously to find authors to cover all the topics needed. The reception has been positive enough for a paperback reprint to appear in 2003, from the Johns Hopkins University Press.

PART TWO: APPRAISALS

12. CHANGES IN APPROACH AND IN BALANCE

> We are in the midst, it would appear, of the springtime of the history of nineteenth-century mathematics.
>
> M. Bernkpof, 1971[61]

The growth in activity since the 1970s has led to changes not only in quantity; different (im)balances between branches, and some new approaches, are also evident. The examples about to be noted are all significant, but not exhaustively so.

The largest change in time period concerns the attention now given to the nineteenth and early twentieth centuries. When I started working on that period nearly forty years ago, it had become largely deserted: most topics had received no detailed histories at all; general histories (of which there had been quite a few) rarely went beyond familiar stories about the discovery of non-Euclidean geometries and a few bits on group and set theory (all mistaken or over-simplified accounts, historical research was to reveal); and the Oberwolfach meetings had few lectures on it. Now it seems to gain more attention than any other — not surprisingly, for once entered, an incomparably vast panorama is made visible. As an adjunct to *Historia mathematica*, an occasional book series of articles on the "history of modern mathematics" deals exclusively with developments since 1800.[62]

One aspect of mathematics since 1800 to benefit is institutional history, where the origins of national differences in modern mathematics can often be traced. After the exciting work of Klein's Commission early in the century (Section 2), institutional history suffered an especially severe drop. However, we now have, for example, the admirable examinations by Biermann of mathematics in the Berlin Academy,[63]

and by Hélène Gispert of the French Mathematical Society.[64] The remarkable rise of the USA in mathematics from the late nineteenth century onwards, late but then fast, has also been well chronicled.[65] Scientific institutions running before 1800 and involving mathematics have been newly studied. The social history of mathematics has also gained attention, not only from Struik's approach (Section 7) and concern with institutional history and education but also using approaches pursued in the history of other sciences.[66]

Another important change concerns probability and mathematical statistics. It achieved professionalization only in the twentieth century, and so constituted a third grouping, alongside but rather separate from pure and applied mathematics. However, in imitation of its past, historians of mathematics often ignored probability and mathematical statistics, even the historical work available such as that fostered by Pearson in Britain (Section 5). For example, the treatment in Part 1 of the *EMW* was very modest. But the situation has now changed considerably, for much historical work has appeared since 1980. Of especial note is the two-volume study of *The probabilistic revolution* (1987) edited by Lorenz Krüger and others, the principal result of a large research project run at the University of Bielefeld in Germany in the early 1980s. Several other substantial monographs, and many papers, have appeared from individual authors, and one fervently hopes that the tradition of historians' ignoring the history of probability and statistics has ceased.

Similar judgements can be passed for some other topics. The history of mathematical economics is largely studied by historians of economics, but its presence is gradually being noticed by historians of mathematics.[67] The same kind of situation holds for links between mathematics and the arts (including architecture);[68] and for scientific instruments of all kinds, whether mathematical as such (such as compasses or calculators) or scientific/technological with a significant mathematical element (such as telescopes and winches). But relationships between mathematics and religions, which go *both* ways in influence, have never been deeply studied.[69]

The rise of feminist history in general has made due impact, internationally. Biographies of several eminent mathematicians have been written recently, as well as collective biographical volumes.[70] A fair proportion of historians today are female, whereas before the 1960s there were hardly any.

Biographies of men also still attract attention, but the form of treatment has changed from the normal style of factual reportage of a century ago. A remarkable pioneer was Pearson, for his three-volume *Life and labours of Francis Galton* (1914–30) even applied Galton's own biometric methods! Nowadays, the personal and the technical are often mixed together in a particularly attractive kind of biography, corresponding to the way that the historical figure lived his life; for example among several, Thomas Hankins on W. R. Hamilton (1980), Andrew Hodges on Alan Turing (1983), C. C. Gillispie on P. S. Laplace (1997), or Dauben on Georg Cantor (1979).[71]

Cantor's work lay largely in set theory, which formed a major part of foundational studies in mathematics around 1900. Such research lies at the interface of mathematics with logics and with philosophy — perfect circumstances for a ghetto (as evidenced,

for example, by the absence of logic from the first edition of the *EMW*). I tried to improve the situation by founding in 1980 the journal *History and philosophy of logic*, to cover all periods. Also available is *Philosophia mathematica* (1964–), which came under the auspices of the Canadian Society for the History and Philosophy of Mathematics in 1991. Large editions for three figures in this area were described in Section 10. It is becoming better recognized that the old story about formalism versus logicism versus intuitionism is only part of the rich picture of foundational studies pursued between 1870 and the 1930s.

After these various positive signs, let us consider a few negative ones. The dichotomy between pure and applied mathematics, with pure as superior, is itself an historical development, occurring largely from the mid-nineteenth century onwards and tied in fairly closely with the increase in professionalization of mathematics. Along with it grew a rather snobbish mentality of fortress mathematics, presented as the queen of the sciences and with a hierarchy of preference among its topics: pure first, led off by arithmetic and number theory, and then analysis, geometries, algebras, topology; applied mathematics (and also numerical methods) second best; and probability and especially mathematical statistics and mathematical economics quite likely out of sight altogether. Much modern historical writing tends to adopt the same ranking. The Bourbakists (Section 7) are a prominent example, especially in their *Eléments d'histoire des mathématiques* (1969 and later editions). In the same spirit one of the fraternity, Jean Dieudonné (1906–92), edited a general history of mathematics during the eighteenth and nineteenth centuries; although probability and logic were included, applications were omitted, and in silence.[72]

This is disappointing because it is unhistorical; many mathematicians did *not* always subscribe to such rankings, especially before the mid-nineteenth century but also for many afterwards. Thus general histories of mathematics written fortress-style are simply *not* general; large areas or periods often disappear partly or totally. (One of the principal aims of the *Companion encyclopedia* was to attack the fortress mentality.) For example, the balance of historical research on Euler is quite out of line with that in his own work. There has been no satisfactory general history of mechanics since the early years of the last century;[73] the histories of engineering mathematics, and also of military mathematics, are especially neglected.[74] Similarly, mathematical physics is not granted the importance comparable to its status from the early nineteenth century onwards; historians of physics have written much of the best work. Many of the accounts of applied mathematics written early in the century, such as in the *EMW*, have no modern counterparts.

Finally, apart from recreational mathematics, much less attention has been paid to the history of amateur mathematics. Developments in school-level mathematics are also rather unfashionable, in contrast to Klein's Commission (Section 2). Another overly neglected topic is mathematical activity in society. The best studies to date are the bibliographical and prosopographical investigations by Peter Wallis (1918–92) and his wife Ruth of British "philomaths",[75] subscription lists and publications (such as Newtoniana) in the eighteenth century; other countries please imitate.

Many of these changes discussed above involve innovations and novelties, and revisions of historical understanding. One important example of the latter concerns Greek mathematics. From the 1880s Tannery and Zeuthen proposed to read much of it as "geometric(al) algebra": their phrase, which denoted the common algebra of constants and variables, roughly like Descartes's. In this view the more simple Books of Euclid's *Elements* consist of identities, usually of quadratic or bilinear forms, and many of the later constructions correspond to the extraction of roots from equations. This interpretation soon became standard; one influential example among many is Heath's editions and histories mentioned in Section 5. In the 1920s it was adopted by figures such as Neugebauer, who then looked for the algebraic origins in Babylonian mathematics; and by his follower B. L. van der Waerden (1903–96), whose articles and especially the book *Science awakening* (1954) have been influential sources.

However, Dijksterhuis avoided geometric algebra in his Dutch translation of Euclid of 1929–30 noted in Section 5; and during the last thirty years, substantial reservations have been voiced by Sabatei Unguru, Ian Mueller and others. No direct historical evidence backs up this algebraization of the geometry; and if it had happened, why was it never made explicit in the centuries after Euclid? Further, algebra with symbols is quite different from geometry with diagrams; and common algebra is the wrong algebra anyway, for Euclid never multiplied geometrical magnitudes together.[76] Other historians also avoided the tradition of geometric algebra. For example, Wilbur Knorr (1944–97) saw *The ancient tradition of geometric problems* (1986) as much preoccupied by geometric problem-solving, relating its repertoire of curves to objects of nature such as ivy leaves; his account of *The evolution of Euclid's Elements* (1975), and David Fowler's reading of *The mathematics of Plato's Academy* (1987, 1999), have opened up questions about the Euclidean algorithm and its mathematical richness, and also about papyrology. Similarly, Jens Høyrup has been rethinking the context and content of their Babylonian and Mesopotamian predecessors.[77] From suggestions such as these, ancient mathematics should have an interesting and different future.

This rejection of a standard historical interpretation in a particular area exemplifies a *general* methodological change; historians now see that they should adopt a more interventionist attitude. Prior to the 1960s (and long before 1900) most writing was empirical: that is, the facts were assembled and maybe classified, with the accounts largely confined to technical details. General questions of historical method were not normally raised (for example, not in the *EMW*; maybe Part 7 would have considered them). In addition, the notations and terms were often imported from current mathematics or at least from a later time than that supposedly under study. One sense of the phrase "Whig history" applies to such approaches. They are deeply *deterministic* in character, and thereby unhistorical: to continue with the above example, it asserts that since Euclid certainly influenced the development of algebra in the West (and earlier among the Arabs), then he *must* have been trying to create what *they* produced partly in inspiration from him.

By contrast, to some extent under influence from the history of science, such methodologies are now seriously in doubt. The admission of indeterministic approaches has also led historians to doubt or even reject the view that mathematical knowledge grows cumulatively; the possibility of major changes, even revolutions in mathematics, has been discussed,[78] partly in connection with Thomas Kuhn's well-known ideas on *The structure of scientific revolutions* (1962). This increased sense of historical period also encourages (some) historians to distinguish Descartes from the Greeks, and also not to shower Euler Truesdell-style with vectors and matrices, Lagrange on algebra with abstract group theory, or all sorts of people with set theory and axioms. Then historical readings of a text can be more clearly distinguished from modern analyses of it (which itself is a legitimate form of mathematical research, of course).

One central issue here is the distinction between the *history of* some mathematical notion, its creation and development at some period in past time; and the *heritage from* it during later periods and maybe even up to now.[79] While the distinction is quite general, it is particularly pressing in the history of mathematics because many mathematical ideas have a duration of relevance and re-interpretation that does not apply in other sciences: for example, Euclid can be modernized from first line to last, which cannot possibly be done with, say, Ptolemy and the history of astronomy or Galen and the history of medicine. In my view the measure to which this distinction is recognized could have a notable bearing upon future developments of our subject. Or maybe it will just deteriorate into another orthodoxy, to be rejected by a later generation!

14. EDUCATIONAL MOTIVES, POSITIVE AND NEGATIVE

In 1977 I published a bibliographical survey article on our subject;[80] now it looks very quaint. Why should our subject have grown so much in the last thirty years or so? Of the various motives to be found, the largest single one is *negative reaction to practices in mathematical education*. This usually occurs at one of two career stages: (1) as a reaction against being bottled-fed as an undergraduate loads of clever theories which however show no connection with human action, have no motivation whatsoever, and so lead from nowhere to nothing except more complicated versions of themselves; or (2) as a reaction to bottle-feeding undergraduates as a teacher with loads of clever theories which however show no connection with human action, have no motivation whatsoever, and so seem to lead only to more complicated versions of themselves.

This reaction has highlighted the question of using history for educational purposes, both for the teaching of pupils and students and the training of teachers. Two non-exclusive uses are possible, for all clients: (1) *informative*, giving instruction, such as *special* courses, in the history of mathematics as part of an educational programme in(volving) mathematics; and (2) *integrated*, incorporating historical material *into* the design and execution of mathematics instruction, such as mathematics courses. The activity early in the twentieth century seems usually to have followed the informative kind. Moritz Cantor's *Vorlesungen* are a prominent case:

they are typical historical books in that they give accounts about the past, in this case concerning mathematics, but their educational utility is left to the teacher-reader. Presumably his teaching (at Heidelberg University) of some parts of those large volumes carried the same aim.[81]

This informative use is also common today; but the integrated use has now also come into play, especially at school level. Between them they form the largest single concern of members of (at least) the British and Brazilian Societies for the History of Mathematics. A French group known by the acronym "IREM" takes history as one of its concerns, and has won admiration at home and abroad, especially for the integrated use.[82] Various publications of the Mathematical Association of America give this use some prominence,[83] whereas in the days of Cajori and Smith early in the twentieth century the informative use seems to have been more favoured. In Denmark in 1987 the government required that historical material be included in the mathematics curriculum in secondary schools, and later made it mandatory also in the training of school teachers.[84]

When history is deployed at university level, it seems to be used informatively, in special courses for undergraduates. Thanks to an initiative taken by H. J. M. Bos (1940–) at Utrecht, between 1995 and 1997 the Socrates programme of the European Union supported three three-week summer schools in our subject for (prospective) doctoral students that were held in European universities. Courses of this type, or for school-teachers, are given in various countries.

Several of the dozens of journals in mathematical education are aware of the bearing of both uses of history, and feature historical material; for example, in the *Zentralblatt für Didaktik der Mathematik*, which also runs an abstracting department for the whole field. The strength of this interest was substantial enough to inspire the founding in the 1980s under UNESCO of an International Study Group on the Relations between History and Pedagogy of Mathematics, which organizes meetings and distributes a *Newsletter*; one substantial volume of proceedings has appeared.[85]

The principal educational benefit from historical immersion is this obvious but profound truth: *mathematics is there because somebody thought it up, and for a reason — and moreover his, not ours*. Further, the interaction between research and education seems to have been richer in mathematics than in other sciences: sometimes, especially in analysis and mechanics, a mathematician's research was stimulated by teaching experience. But *how* does one get from the old text to the modern classroom? I have proposed the name "history-satire" for the idea that the teacher learns the history to some extent and then imitates the story (ponder upon this word) in the classroom but without getting bogged down in the complicated historical details. Many advocates of history in education follow a similar line, which is sometimes aired in the journals for mathematics education.

An important consequence is that one may teach a theory in different historico-satirical versions over the years; for example, some basic theorems of the calculus in the ancient Greek and medieval techniques of exhaustion, the pre-calculus forms of the early to mid-seventeenth century, and the very different versions of Newton,

Leibniz and Lagrange before the more rigorous but less intuitive ones inspired by A. L. Cauchy and Karl Weierstrass in the nineteenth century. The German mathematician Otto Toeplitz (1881–1940) explored this kind of approach already in the late 1920s in the Bonn seminar (Section 6).[86]

15. COTTAGE INDUSTRY OR GHETTO?

> The history of mathematics is essentially different from the history of other sciences in its relationship with the history of science, because it was never an integral part of the latter ... mathematics being far more esoteric than the other sciences, its history can only be told to a select group of initiates....
>
> It is a pity that this should be so, for the history of mathematics should really be the kernel of the history of culture.
>
> George Sarton, 1936[87]

This quandary captures well the professional situation for historians of mathematics. On the positive side, the two abstracting journals, *Zentralblatt für Mathematik* and *Mathematical reviews* (launched in 1931 and 1940 respectively, much under the influence of Neugebauer, incidentally), greatly improved their departments for history during the mid-1970s. Some of the more popular mathematical journals imposed or improved their refereeing procedures for historical articles. Further, as part of the heritage from Sarton, the historical bibliographies of *Isis* have always given mathematics normal treatment.

But the sample of professional mathematicians and statisticians who were or are generally sympathetic to history must be small among the total population of tens of thousands (as in other sciences, no doubt). For most of the rest our subject is far too historical: it counts only as a side-show, where the accuracy of an historical text is of little importance as long as it is Bell-style "fun".[88] One may even have to be "revolutionary" in declaring one's interest in history: a friend confessed to me that he "proved his silly little theorems" in his chosen branch of mathematical research in order to gain tenure, and *then* went open about his tendencies.

Similarly, the value of history for education is still not *widely* recognized among mathematical educators. For example, a workshop on the topic was held at the University of Toronto in 1983, in memory of May. Money was available to publish the proceedings; but they were rejected by thirteen publishers, one for our failure to satisfy the standards normally found in mathematical education but by the others for fear of poor sales. (However, one referee wanted to keep such an "interesting" manuscript.) The proceedings eventually appeared, thanks to an initiative taken by the French Society for the History of Science.[89]

A converse predicament arises relative to the historians of science; to them our subject is too mathematical.[90] While several of the journals in that field will take mathematical articles, it is well-nigh impossible for historians of mathematics to lecture before historians of science and expect their material to be discussed with the seriousness granted to the history of chemistry, say, or medicine; indeed, in my

experience the reception has sometimes been negative. As for professional historians in general, contact normally obtains only for ancient and some medieval and Renaissance mathematics, where the mathematics as such may be elementary (although not always; and the history can well be very tricky). Historians will recognize from (ironically, mathematical) words like "1753" and "1876" that history is at hand; but the strength of their disinclination to study it is, shall we say, impressive.

Reasons for distaste here are simple to determine: mathsphobia is widespread among the population, and historians furnish no exception, even when it compromises their professional concerns. But then history suffers, for the ignorance of historians takes precedence over proper scholarship about the past. Further, a vicious circle arises; for this mathsphobia usually arises from the historian's unpleasant childhood experiences of mathematics, which was governed by policies in mathematical education, which will have ignored history, so that ....

16. "WRITING THE HISTORY OF MATHEMATICS": A LOOK AT THE COMMUNITY

Meta-historical concerns have been very prominent around the International Commission on the History of Mathematics. As well as the Dauben and Lewis bibliographies mentioned in Section 8, it mounted symposia on historiography at International Congresses for the History of Science: the 1989 occasion resulted in proceedings,[91] and the meditations offered in 1993 led to the formation of a working group to edit a book under the editorship of Dauben and Scriba. With the cooperation of dozens of colleagues around the world and some special gatherings of the working group at Oberwolfach, a 700-page book appeared in 2002 from Birkhäuser, under the main title *Writing the history of mathematics*.[92]

The first major decision concerned the manner by which the corpus of historical work should be divided: by branches of mathematics? or by countries in which mathematics was practised? or into periods? Quickly it became clear to the working group that the governing criterion was *the country in which the historian worked*, whatever topic was studied and whether it developed in one's homeland or elsewhere. As has emerged above in some examples, the country has usually had a greater bearing upon the historian than the subject matter that he studied. For example, German historians seem to have shown a philosophical and philological concern with knowledge as such, which is evident also, though in different ways, among French *historiens-philosophes* and some Italian colleagues. British and American historians have rarely exhibited such concern; on the other hand, their interest in connections with education are rather more marked, as is now also the case in Central and Latin America. (Finer analyses could consider the differences in practice in different regions of a country.) Historians seem rarely to have advocated some philosophical stance, though they may have held one privately; some have expressed belief in the unity of mathematics (M. Cantor, perhaps Klein), and some have favoured Marxism (Struik, May, Wussing).

These differences are manifest to some extent also in the posts and careers that historians have (frequently) been forced to adopt. Some French historians had associations with philosophical groups or institutes (for example among those named earlier,

Couturat, Koyré and Bachelard, and also Geymonat in Italy) that seem to be rather rare elsewhere. Some historians came to ancient mathematics from a philological background: Heiberg, Ruska, Thureau-Dangin. Links to departments of the history of science have been rather infrequent, even when departments in that subject were formed: they are most evident in the Germanies (Gericke, Scriba, Wussing, Folkerts), France (Taton, Costabel) and the Soviet Union (Yushkevich, Demidov and several other colleagues not named above). A few historians worked substantially in the histories of other sciences: Sarton, Struik, Cohen, Clagett, Folta.

Connections have always been strongest with mathematics departments in universities, at two levels. Several historians have practised principally as professional, even professorial, mathematicians: for example, Klein, Zeuthen, Enriques, Timchenko, Peano, Burkhardt, Stäckel, Spiess, Prasad, Bell, Carathéodory, Truesdell, Kline, Struik, Dieudonné, van der Waerden. Others held a more minor post, even an honorary one, simultaneously with a paid position as a school-teacher: Suter, Tropfke, Wieleitner, Loria, Dijksterhuis, Vogel, Hofmann, Itard. A few, such as Smith and May, were employed in mathematics education, and Muir was an educational administrator in South Africa. Late in his life Cajori became Professor in the History of Mathematics, at Berkeley. A few historians have survived on special funding: Sarton at Harvard, and Whiteside until 1975 when he gained a post at Cambridge University while working on the penultimate volume of his Newton edition. Some historians made a living by other means: Tannery as a business manager, Eneström and Bjørnbo as librarians, Heath in the British Treasury, Thureau-Dangin in the Louvre Museum in Paris, Whittaker as an astronomer, Jourdain by writing and editing, Mikami in the Japanese Imperial Academy.

The historians of today show roughly the same proportions of professional appointment and connection, with perhaps a somewhat larger percentage of school-teachers. There have been very few institutions for our subject; however, the group at the Open University in Britain was recently re-formed into a Centre under the direction of J. J. Gray (1947–).

On the decline after the Great War, here are some tentative thoughts. The deaths around that time of Eneström, Molk, Burkhardt, Duhem, Stäckel, Jourdain, Zeuthen and Cantor, the retirement of Klein (who died in 1925), and of course economic and social difficulties in general, both reduced activity and harmed some major sources and projects: the loss of *Bibliotheca mathematica* and the abandonment of the French *Encyclopédie* were particularly serious. Despite some of the volumes in the Euler and Gauss editions, the continuation of the series of *Ostwalds Klassiker*, the essays on Gauss, the final articles for the *EMW*, and Neugebauer's series, the reduction of writing in German is quite noticeable. There was also a rise and spread of philosophical positivism that deprecated history (of science) in general, either implicitly or explicitly. The reduced interest in Europe in the links between history and education may have deterred some potential new recruits. On the other hand, the level of activity in the USA increased somewhat, in line with its growing importance as a mathematical country. There must be more processes involved, which would be

worth exploring in the context of the development of the history of other sciences at that time.[93]

After the history of mathematics has been written, it should be published. Some publishers have demonstrated a strong commitment to our subject, including the many editions of the works of mathematicians of which only a few have been mentioned above. The output was and is largely books, but some houses have sustained journals for our subject or friendly to it.[94] In Germany Teubner stands out, especially through its links with Klein: unfortunately, soon after the Great War and seemingly as a consequence of it, Teubner greatly reduced its publication of mathematics in all forms (although it maintained the *EMW* and the Euler edition). This decision left a publishing vacuum that was quickly filled by Julius Springer Verlag, which is the parent of today's Springer. In Switzerland Birkhäuser was and is prominent, in France Gauthier-Villars has a good record, as do the Oxford and Cambridge University Presses in Britain and the USA. Other houses in the USA include several more university presses as well as commercial firms, among which the Open Court Publishing Company occupies a special place; and the American Mathematical Society and the Mathematical Association of America are now significant.

In recent years the means of publishing have extended, especially with the development of the internet. Dozens of sites are now available, some associated with the societies named earlier; others are due to departmental or even individual initiatives and often focus upon biographies, countries or regions, particular theorems, or educational possibilities.[95] As usual, the benefits of access are compromised by greatly variable quality control.

17. CONCLUDING REMARKS

The history of mathematics shows that since Antiquity mathematics has developed as a wide and ever-widening rainbow of ideas and theories.[96] But the rainbow has negative as well as positive aspects. It delights those who look at its many colours; but it is also *distant* from us, and seemingly irrelevant to our cultural lives — including those of most mathematicians, philosophers and historians.

This article was written in order to bring information about an academic field that is little noticed by outsiders; and its own history is not well known to many of its practitioners. The account belongs to meta-history, or the history of history, a part of history about which historians in general are curiously incurious. But all of us who worked on *Writing the history of mathematics* found the experience enlightening and unusual in many ways; we would encourage comparable attention to be paid to the histories of the histories of other sciences.

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12. See especially the ten essays comprising the second parts of Gauss, *Werke*, vols x–xi (Leipzig, 1922–33; repr. Hildesheim, 1973), 1,300 pages in all. Some essays had appeared earlier as papers with the Göttingen Academy.
13. G. Eneström, *Verzeichniss der Schriften Leonhard Eulers* (Leipzig, 1910–13). See also ref. 36.
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66. Two pioneering sources are M. Otte and H. Jahnke (eds), *Epistemological and social problems of the sciences in the early nineteenth century* (Dordrecht, 1980); and H. Mehrtens, H. Bos and I. Schneider (eds), *Social history of nineteenth century mathematics* (Basel, 1981). More recent successors include C. Goldstein and others (eds), *L'Europe mathématique: Mathematical Europe* (Paris, 1996); and U. Bottazzini and A. Dahan (eds), *Changing images in mathematics: From the French Revolution to the new millennium* (London, 2001). An early deployment of postmodernism in our subject was made in H. Mehrtens, *Moderne Sprache Mathematik* (Frankfurt/Main, 1990), in the context of foundational studies in mathematics around 1900.
67. Notable recent contributions include B. Ingrao and G. Israel, *The invisible hand: Economic equilibrium in the history of science* (Cambridge, Mass., 1990); and P. Mirowski (ed.), *Natural images in economic thought* (Cambridge, 1994).
68. Recent notable works include P. Radelet-de Grave and E. Benvenuto (eds), *Entre mécanique et architecture / Between mechanics and architecture* (Basel, 1995); J. V. Field, *The invention of infinity: Mathematics and art in the Renaissance* (Oxford, 1997); and P. Gouk, *Music, science*

and natural magic in seventeenth-century England (New Haven and London, 1999).

69. On Christianity, G. B. Chase and C. Jongsma (eds), *Bibliography of Christianity and mathematics: 1910–1983* (Sioux Center, Iowa Dordt College Press, 1983: available on www.messiah.edu/acdept/-depthome/mathsci.acms/index.html) lists publications on a wide range of themes, including philosophical, logical and educational issues, but rather few specifically on history. In particular, it confirms the lack of literature (as of 1983) on the period after 1750, which is explored in I. Grattan-Guinness, "Christianity and mathematics: Kinds of link, and the rare occurrences after 1750", *Physis*, n.s., xxviii (2001), 467–500. The converse relationship is tentatively explored in I. Grattan-Guinness, "Manifestations of mathematics in and around the Christianities: Some examples and issues", *Historia scientarum*, (2) xi (2001), 48–85; also in *Revista Brasiliera de historia da matematica*, i (2001), 21–56.
70. See, for example, L. S. Grinstein and P. J. Campbell (eds), *Women of mathematics: A biobibliographical sourcebook* (New York, 1987).
71. T. L. Hankins, *Sir William Rowan Hamilton* (Baltimore, 1980); A. Hodges, *Alan Turing: The enigma* (London, 1983); C. C. Gillispie (main author), *Pierre Simon Laplace: A life in exact science* (Princeton, 1997); and J. W. Dauben, *Georg Cantor* (Cambridge, Mass., 1979).
72. J. Dieudonné (ed.), *Abrégé d'histoire des mathématiques* (2 vols, Paris, 1978). The great difference between the mathematics profession in the two centuries is also passed over. But several articles are excellent.
73. On the historiography of mechanics see I. Grattan-Guinness, "The varieties of mechanics by 1800", *Historia mathematica*, xvii (1990), 313–38.
74. A recent pioneering volume on military mathematics across the two World Wars is B. Booß-Bavnbek and J. Høyrup (eds), *Mathematics and war* (Basel, 2003).
75. See especially P. J. and R. Wallis, *Biobibliography of British mathematics and its applications*, Part 2: *1701–1760* (Newcastle-upon-Tyne, 1986).
76. For a non-specialist presentation of these changes of interpretation, together with references to the main recent historical literature, see I. Grattan-Guinness, "Numbers, magnitudes, ratios and proportions in Euclid's *Elements*: How did he handle them?", *Historia mathematica*, xxiii (1996), 355–75 (with printing correction, xxiv (1997), 213).
77. J. Høyrup, *Lengths, widths, surfaces: A portrait of old Babylonian algebra and its kin* (New York, 2002).
78. D. Gillies (ed.), *Revolutions in mathematics* (Oxford, 1992).
79. This issue is developed in some detail in I. Grattan-Guinness, "History or heritage? An important distinction in mathematics and for mathematics education", *American mathematical monthly*, cxi (2004), 1–12; and "The mathematics of the past: Distinguishing its history from our heritage", *Historia mathematica*, to appear.
80. I. Grattan-Guinness, "History of mathematics", in A. Dorling (ed.), *Use of mathematics literature* (London, 1977), 60–77.
81. The same point about selective teaching from published "Vorlesungen" applies not only to Cantor but also possibly to von Braunmühl's large history of trigonometry (Section 3). The publisher for both authors was Teubner, who regularly published substantial "Vorlesungen" on mathematical subjects; hence there was some tradition in that form of title.
82. See, for example, J. Fauvel (ed.), *History in the mathematics classroom: The IREM papers* (Leicester, 1990).
83. See, for example, R. Calinger (ed.), *Vita mathematica: Historical research and integration with teaching* (Washington, D.C., 1996).
84. I am indebted for this information to Kirsti Andersen.
85. J. Fauvel and J. van Maanen (eds), *History in mathematics education: The ICMI study* (Dordrecht,

2000).

86. Toeplitz's treatment of the calculus was published posthumously as *Die Entwicklung der Infinitesimalrechnung* (Darmstadt, 1949); English translation, *The calculus: A genetic approach* (Chicago, 1963).
87. G. Sarton, *The study of the history of mathematics* (Cambridge, Mass., 1936; repr. New York, 1957), 4.
88. A selection of examples, in connection with the predicament of the British Society for the History of Mathematics, is given in I. Grattan-Guinness, "A residual category: Some reflections on the history of mathematics and its status", *Mathematical intelligencer*, xv/4 (1993), 4–6.
89. I. Grattan-Guinness (ed.), *History in mathematics education* (Paris, 1987). This old tome is not mentioned in the recent volume of the same title (ref. 85); such is history. A moderate amount of historical material, mostly for informative use, appears in L. S. Grinstein and S. I. Lipsey (eds), *Encyclopedia of mathematics education* (New York and London, 2001).
90. I. Grattan-Guinness, "Does History of Science treat of the history of science? The case of mathematics", *History of science*, xxvii (1990), 149–73.
91. S. S. Demidov and M. Folkerts (eds), "History and the historiography of mathematics", *Archives internationales d'histoire des sciences*, xlii (1992), 5–144.
92. See ref. 1.
93. The critical bibliographies in *Isis* for the period would provide many basic data, using the appropriate parts of the cumulative version for 1913–65 edited by Magda Whitrow (5 vols, London, 1971–82). Also useful are the lists of various kinds furnished in G. Sarton, *Horus: A guide to the history of science* (New York, 1952); and in Loria, *op. cit.* (ref. 19).
94. J. W. Dauben, "*Historia mathematicae*: Journals of the history of mathematics", in M. Beretta and others (eds), *Journals and the history of science* (Florence, 1998), 1–30; and "*Historia mathematica*: 25 years/context and content", *Historia mathematica*, xxvi (1999), 1–28.
95. M. Mohan, "Useful web links on history of mathematical sciences", *Ganita-Bharati*, xxv (2003), 29–44.
96. This rainbow metaphor formed the sub-title of my own general history, *The Fontana history of the mathematical sciences* (London, 1997), which also trades as *The Norton history of the mathematical sciences* (New York, 1998). I also rejected the normal chronological balance mentioned in Section 10: 1800 is reached only halfway through.