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Meteor stream identification: a new approach – III. The limitations of statistics

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ABSTRACT

The new criterion $D_{\rm N}$ for meteoroid stream identification, based on variables directly linked to observations, is applied to four sets of photographic meteors, three of which have been obtained by independent teams, and the fourth obtained by combining the first three. We find that if one of the data sets has statistical properties significantly different from the others, the use of a statistical criterion to determine thresholds for the stream search does not lead to satisfactory results.

Key words: meteors, meteoroids.

1 INTRODUCTION

The identification of meteoroid streams is a multistep process in which several basic ingredients are necessary: a distance function, i.e. a measure of the orbital similarity of meteors, a linking technique, i.e. an algorithm to group similar meteors together, and a criterion to determine whether the obtained groups are 'real'. Statistical criteria have often been used for the last task.

Valsecchi, Jopek & Froeschlé (1999) introduced a new distance function $D_{\rm N}$, based on geocentric quantities directly derived from observations. Then, using the single neighbour linking technique (Lindblad 1971) and values for meteor association thresholds obtained according to the prescriptions of Jopek & Froeschlé (1997), Jopek, Valsecchi & Froeschlé (1999) compared the results obtained with $D_{\rm N}$ to those obtained using the classical distance function $D_{\rm SH}$ of Southworth & Hawkins (1963), on a set of 865 precise photographic meteor radiants. The comparison has shown that $D_{\rm N}$ and $D_{\rm SH}$ give similar results on most streams, but give significantly different ones in the case of ecliptical streams.

A logical step forward from Jopek et al. (1999) would be to apply $D_{\rm N}$ to a larger sample of photographic orbits, in order to look for less conspicuous streams.

Two good-quality data sets are available for this purpose. The first is that presented in Betlem et al. (1997) (359 rad and orbits) or its enlarged version (722 meteors) made available by Betlem (private communication) at the ftp site strw.leidenUniv.nl in the pub/betlem/orbits directory. These meteors were observed in the course of the Photographic Meteor Survey by members of the Dutch Meteor Society (DMS).

The second data set contains the radiant coordinates of 259 bolides observed by the Canadian Meteor Observation and Recovery Project (MORP; Halliday, Griffin & Blackwell 1996).

We thus decided to do a new search for streams on these two data sets, as well as on the 865 Harvard meteors (HD) published by Whipple (1954), Jacchia & Whipple (1961), Hawkins & Southworth (1958, 1961), already used in Jopek et al. (1999).

However, in performing our study we have found that the use of thresholds for the stream search based on the statistical properties of the data sets can lead to serious difficulties if one of the data sets shows certain peculiarities, and that this undesirable property does not go away if we try to mitigate it by immersing the biased data set into a larger one.

Below we present the results of four stream classifications. Due to the limitations just mentioned, we have first made stream searches on the MORP, DMS and HD data sets separately, and then made a stream search in the data set obtained from the combination of MORP, DMS and HD. In the following we refer to this joint sample as MDH.

2 PREPARING THE METEOR INPUT DATA

Before being used for the classification, the available meteor data were examined to check their internal consistency. The test comprised the following steps.

(i) From the geocentric parameters $T_{\rm Obs}$,$\alpha_{\rm G}$, $\delta_{\rm G}$ and $V_{\rm g}$ given in the computer source catalogues, the orbital elements q, e, ω, Ω and i of the meteors have been recalculated.

(ii) Taking the orbital elements from the computer source catalogue, the minimum distance between the orbits of the Earth and of the meteor $D_{\rm min}$ as well the radiant parameters $T_{\rm Obs}$, $\alpha_{\rm G}$, $\delta_{\rm G}$ and $V_{\rm g}$ were determined.

(iii) For each parameter, the difference $({\rm O}-{\rm C})_0$ has been found and compared with the uncertainty given by the authors of the catalogues; if the uncertainties were not available, the set of deviations listed in Table 1 was applied.

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Table 1. Critical values of the deviations of meteor parameters applied in the internal consistency test.

Deviation		Parameter
0.05	day	Apparition time
0.05		Eccentricity
0.01	au	Perihelion distance
0.5	deg	Argument of perihelion
0.2	deg	Longitude of ascending node
0.5	deg	Inclination
0.5	deg	Right ascension
0.5	deg	Declination
0.5	km s^{-1}	Geocentric velocity
0.005	au	Minimum distance between the Earth and the meteor orbit

(iv) If all $(\mathrm{O}-\mathrm{C})_0$ values were less than the corresponding uncertainties or deviations, the radiant and the orbital data of the meteor were accepted as internally consistent.

(v) If even one of the $(\mathrm{O}-\mathrm{C})_0$ values was greater than the limit, for the corresponding meteor steps (i) and (ii) were repeated 100 times, after having introduced uniform noise into the input catalogue data. The noise was taken from intervals (a, b), where a and b are the maximum and minimum value of the parameter, for the appropriate uncertainty or deviation. The influence of the noise on the calculated orbital elements and the radiant parameters was tested relative to the recalculated values free from the noise. We took as 'O' the values of the meteor parameters recalculated from the source catalogues data, and as 'C' the values of the parameters calculated from the noisy data. For each parameter, the maximum $(\mathrm{O}-\mathrm{C})_\mathrm{M}$ was found and compared with $(\mathrm{O}-\mathrm{C})_0$ obtained as the result of steps (i), (ii) and (iii). If for all parameters the $(\mathrm{O}-\mathrm{C})_\mathrm{M}$ values were greater than the corresponding $(\mathrm{O}-\mathrm{C})_0$ values, the data of the meteor were considered to be consistent. Otherwise, the meteor was rejected from the catalogue and placed in the outlier list.

(vi) All meteors from the outlier list were carefully analysed with interactive software allowing us to calculate the $(\mathrm{O}-\mathrm{C})$ differences after a change of value of every single parameter. This allowed us to recognize the nature of the high $(\mathrm{O}-\mathrm{C})$ values and sometimes to find typing mistakes made when the computer meteor data files were created.

2.1 The Harvard precise meteor catalogue

We used a subset of 865 precise meteors extracted from the computer files PRCORB90.DAT and PRCGEO90.DAT obtained from the IAU Meteor Data Center (Lindblad 1987, 1991).

The internal consistency test failed 12 times, and all of these cases were analysed separately. As a result, we have corrected the data of the following three meteors:

(i) for meteor 07169, $\delta_\mathrm{G} = 48^\circ\!.8$ has been changed into $\delta_\mathrm{G} = 49^\circ\!.8$;

(ii) for meteor 08348, $q = 0.105$ au has been changed into $q = 0.933$ au;

(iii) for meteor 08528, $q = 0.040$ au has been changed into $q = 0.433$ au.

These corrections were taken from Jacchia & Whipple (1961), and from Hawkins & Southworth (1961).

Because the $(\mathrm{O}-\mathrm{C})_0$ values of the remaining nine cases were only slightly greater than their corresponding $(\mathrm{O}-\mathrm{C})_\mathrm{M}$ values (among them were four members of the Cyclids stream), we decided to consider the data relative to these meteors as internally consistent.

2.2 The MORP meteor catalogue

Detailed data for 259 fireballs observed during 1974–1985 by the Canadian camera network were recently published by Halliday et al. (1996). A subset of these data, consisting of just the orbital elements, was available earlier in computer form in the IAU Meteor Data Center. However, both the orbital data and the radiants of the 259 fireballs were kindly made available to us in computer form by the Editorial Office of Meteoritics and Planetary Science.

For this catalogue, our internal consistency test failed in 16 cases. A detailed analysis revealed the following.

(i) For meteor 750167, the values of the eccentricity and the perihelion distance are misplaced. To correct, we put $e = 0.748$ and $q = 0.646$ au.

(ii) For 12 meteors, the failure can be explained by poor numerical conditioning of the calculations: very small inclination of the orbit, high sensitivity to the eccentricity changes. All these meteors were included in our final data sample.

(iii) For meteors 760229, 800592 and 760268, we were not able to explain their high $(\mathrm{O}-\mathrm{C})_0$ values by poor numerical conditioning, therefore we excluded them from the sample.

As a result, we used the radiant parameters of 256 fireballs.

In comparison with HD meteors, the precision of MORP data is significantly lower, because the exact time of appearance of MORP meteors is in general not known, causing an uncertainty in the right ascension of the radiant amounting, on average, to $1^\circ\!.8$.

2.3 The DMS meteor catalogue

Betlem (private communication) kindly made available to us the orbital and radiant data of 722 meteors observed during 1972–96 by members of the DMS (see the ftp site strw.leidenUniv.nl in the pub/betlem/orbits directory).

The internal consistency test carried out on this data set resulted in singling out 27 meteors. Using our interactive software:

(i) we accepted as consistent the data for 14 meteors, either because the differences between $(\mathrm{O}-\mathrm{C})_\mathrm{M}$ and $(\mathrm{O}-\mathrm{C})_0$ values were only slightly greater than the assumed limits, or because of the poor numerical conditioning of the orbit calculation;

(ii) we rejected the remaining 13 meteors, although possibly eight of them can be improved because their inconsistencies seem to be due to reasons identifiable by the authors of the catalogue.

2.4 The input meteor data used in this study

After passing the internal consistency tests, radiant coordinates of meteors from the MORP and HD catalogues have been transformed to the J2000 reference frame. The MORP data set we use contains 256 meteors, the HD set includes 865 meteors and the DMS set includes 709 items; by joining them, we obtain a set of 1830 radiants of meteors observed in 1936–96.

Fig. 1 shows that the statistical properties of DMS meteors differ significantly from those of the MORP and HD data, for reasons discussed in the next section.

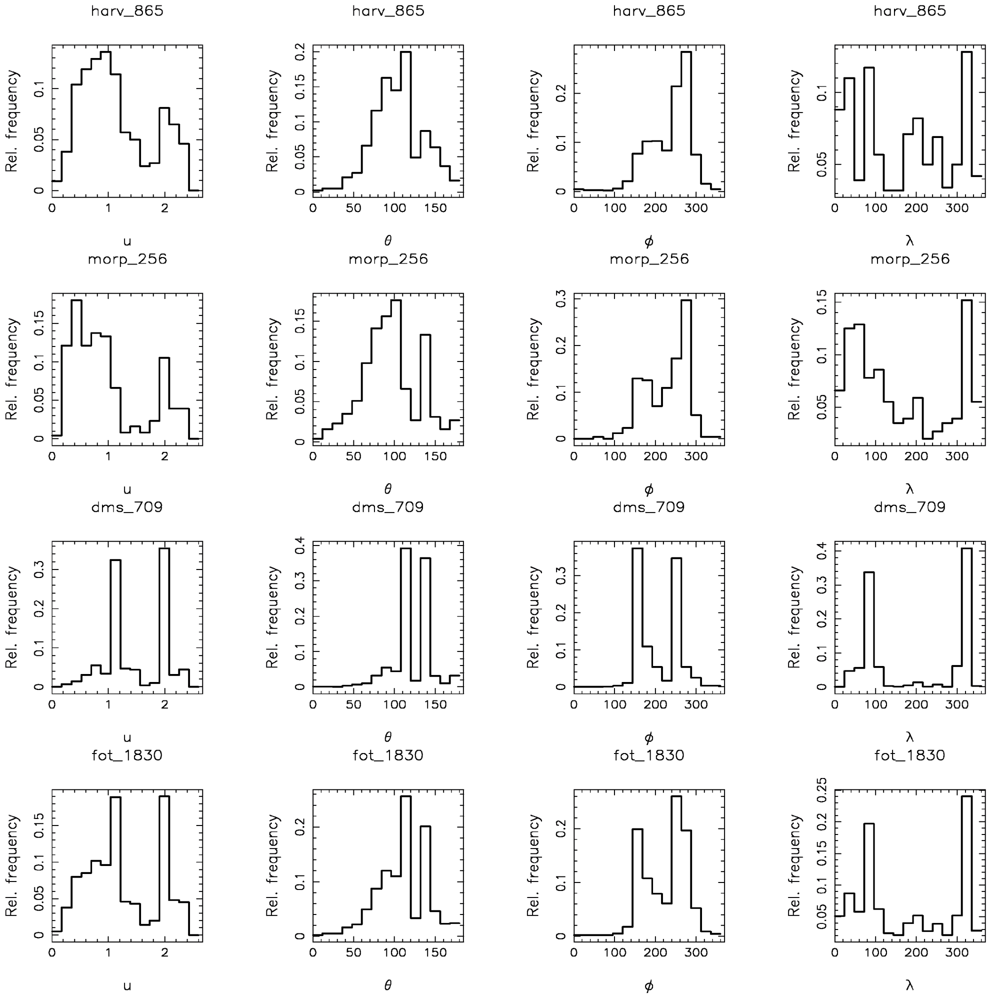

Figure 1. Distributions of U, θ, ϕ and λ for 865 HD meteors, 256 MORP fireballs, 709 DMS meteors and for the 1830 meteors of the combined sample.

3 THE DISTANCE FUNCTION AND THE CLUSTER ANALYSIS METHOD

As a quantitative measure of the difference between two meteor orbits we have used the D_{N} distance function introduced in Valsecchi et al. (1999).

As in Jopek et al. (1999), after having computed all the mutual distances, we processed them with a cluster analysis computer program implementing an algorithm based on the single neighbour linking technique (Lindblad 1971); the values of the meteor association thresholds were estimated with the method described in Jopek & Froeschlé (1997). The only modification with respect to Jopek et al. (1999) has been that, instead of determining the thresholds with a single numerical experiment, 10 of these were carried out; this resulted in obtaining the average thresholds and their standard deviations listed in Table 2.

It turns out that, for a given number of stream members M, the threshold values do not decrease as expected with the increase of sample size. In fact, because the function D_{N} depends on four variables, we would expect the threshold values to decrease proportionally to the fourth power of the sample size. In a plot where the logarithm of the threshold $D_{\mathrm{c},M}$ is given against the logarithm of the sample size N, the points of Table 2 should lie along straight lines of slope equal to -0.25. However, as Fig. 2 shows, the values of thresholds for the DMS sample are clearly too low.

This appears to be due to observational selection, and is strongly reflected in the statistical properties of meteor parameters from this sample. In fact, the histograms for the DMS data set (see Fig. 1) show strong groupings around two values for each variable due to the fact that almost 2/3 of the DMS meteors were observed in August and in December, the months of the Perseids and the Geminids.[1]

In addition, the method we use to find the association thresholds (Jopek & Froeschlé 1997) can be applied to the combined data sample only if the individual samples are statistically similar. In our case this does not happen, as Fig. 2 shows, because the point corresponding to the combined sample lies below the reference line.

We therefore performed separate stream searches in the three data sets, MORP, DMS and HD. A fourth search, in the combined set, was performed just to confirm the results obtained earlier and to give us the possibility to find streams composed of meteors belonging to the different samples, not identified in the separate searches.

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[1] Such a distribution of observations is typical for any non-professional group; they cannot observe meteors systematically over the whole year.

Table 2. The values of thresholds $D_{c,M}$ applied in the meteor association tests. They correspond to the reliability level $W_M = 99$ per cent and are given for each stream population M and for each data sample: 256 meteors for the MORP set, 709 meteors for the DMS set and 865 for the HD set. The last column list the thresholds for the combined sample of 1830 meteors.

	$D_{c,M}$							
M	256		709		865		1830	
3	0.1210	±0.0066	0.0305	±0.0016	0.0743	±0.0044	0.0385	±0.0032
4	0.1694	±0.0040	0.0449	±0.0016	0.1086	±0.0025	0.0553	±0.0026
5	0.2051	±0.0041	0.0560	±0.0018	0.1352	±0.0035	0.0694	±0.0028
6	0.2309	±0.0035	0.0644	±0.0016	0.1555	±0.0035	0.0811	±0.0030
7	0.2494	±0.0042	0.0708	±0.0015	0.1708	±0.0033	0.0909	±0.0028
8	0.2634	±0.0060	0.0757	±0.0015	0.1822	±0.0035	0.0990	±0.0024
9	–	–	–	–	–	–	0.1057	±0.0019
10	–	–	–	–	–	–	0.1113	±0.0017
11	–	–	–	–	–	–	0.1161	±0.0018
12	–	–	–	–	–	–	0.1205	±0.0024

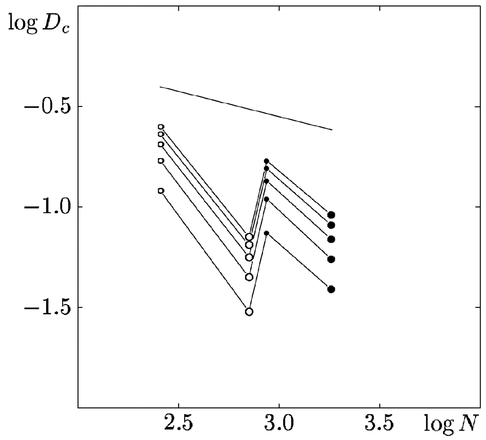

Figure 2. Log–log plot of the thresholds $D_{c,M}$ versus the total number of meteors in the data set N. The segment on the top has an inclination corresponding to the power −1/4, to guide the eye; the five broken lines join, from top to bottom, the values of the thresholds for $M = 3, 4, 5, 6$ and 7 for the MORP (small circles), DMS (large circles), HD (small dots) and combined (large dots) data sets.

Table 2 gives the deviations of the estimated thresholds, that are of the order of $\sim 5 \times 10^{-3}$. To check the influence of these uncertainties, for each data sample we made two searches, one with the thresholds equal to $D_{c,M} - \sigma_{D_{c,M}}$ and the second one with the thresholds equal to $D_{c,M} + \sigma_{D_{c,M}}$.

4 RESULTS OF THE CLASSIFICATION

The main results of all searches are summarized in Table 3. In the last two columns, for the MORP and DMS data sets the values were taken directly from the source catalogues, while those for the HD sample are from Jopek et al. (1999).

The values given in the third and fourth columns were found in this study. The first and second rows of these columns list the results obtained with the upper and lower boundaries of the threshold intervals, respectively (see Table 2).

The authors of the MORP and DMS catalogues singled out streams that have only one and two members in their data sets. We found six and five of such cases, respectively. In Jopek et al. (1999), two streams of two members are listed. In the present study we looked for streams of three or more members. We therefore subtracted six, five and two from the values given in the fifth column of Table 3; the results of this computation are given, in brackets, in the same column.

Table 3. General results of the eight meteor stream searches: N is the sample size, S and P_S are the number of streams and the fraction of stream component detected in this study, S_0 and P_{S_0} are the number of streams and the total fraction of stream members classified by other researches among the same meteors. Our results were obtained using thresholds equal to $D_{c,M} - \sigma_{D_{c,M}}$ and to $D_{c,M} + \sigma_{D_{c,M}}$, first and second rows respectively.

Catalogue	N	S	P_S (per cent)	S_0	P_{S_0} (per cent)
MORP	256	5	28.9	13, (7)	30.5
		5	29.3		
DMS	709	10	79.8	17, (12)	86.3
		10	80.1		
HD	865	15	30.6	20, (18)	33.2
		16	31.4		
MDH	1830	12	47.0	–	–
		13	47.9		

In general, in both searches made in each data sample we obtained very similar numbers of identified streams and corresponding percentages of stream component. Moreover, in general the present searches gave a lower number of stream members, and smaller stream components, when compared with those of Halliday et al. (1996), Betlem (private communication) and Jopek et al. (1999).

4.1 The Harvard streams

For the Harvard sample, the results of the present study (see Table 4) and the results of our earlier paper (Jopek et al. 1999) agree very well. This is not surprising, given that in both studies we have applied very similar approaches. The only difference consists in the more accurate determination of the thresholds $D_{c,M}$ of the present work. In Jopek et al. (1999) the thresholds were found as the output of a single numerical experiment in which 200 artificial samples were analysed for grouping, whereas here we performed 10 such experiments in order to obtain the mean values of the thresholds together with their standard deviations (Table 2).

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Table 4. Meteor streams in the HD data set. The first column gives the stream name, the second its code, the third and fifth columns the number of members M_N and M_J identified, respectively, in the present search for the threshold values $D_{c,M}$ and by Jopek et al. (1999) for the thresholds D_{c,M_J}.

Name	Code	M_N	$D_{c,M}$	M_J	D_{c,M_J}
Lyrids	1	6	0.0787	6	0.080
α Capricornids	13	21	0.1741	21	0.178
Perseids	16	33	0.1741	33	0.178
κ Cygnids	32	4	0.0787	4	0.080
Taurids	47	58	0.1857	58	0.188
Quadrantids	48	14	0.0787	14	0.080
Geminids	70	51	0.0787	51	0.080
χ Orionids	72	11	0.1857	11	0.188
Monocerotids	90	3	0.0787	3	0.080
Leonids	94	6	0.0787	6	0.080
σ Hydrids	107	5	0.1387	5	0.134
Orionids	119	19	0.0787	19	0.080
Virginids	217	12	0.1857	12	0.188
S. δ Aquarids	347	11	0.1387	11	0.134
ε Piscids	431	15	0.1857	16	0.188
α Pegasids	817	3	0.0787	3	0.080

Using the more accurately determined threshold values, in the present search we detected all but two of the streams found by Jopek et al. (1999); we did not find the Northern δ Aquarid and the ε Geminid streams, that had six and four members respectively in Jopek et al. (1999). In Table 2 we can see that the thresholds for six and four members are in the intervals (0.1520, 0.1590) and (0.1061, 0.1111), respectively. Both the corresponding values used in Jopek et al. (1999), i.e. 0.160 and 0.113, are outside the interval allowed, although by a small quantity.

In view of the statistical character of our approach, the failure to find the Northern δ Aquarids and the ε Geminids does not appear to be serious. We then conclude that the confidence level at which these two streams are identifiable is just slightly lower than the 99 per cent. Also the slightly smaller number of ε Piscids found in the present search has the same explanation.

4.2 The Canadian fireball streams

The search in the MORP data set detected five streams; one of them, the δ Arietid stream, was not recognized by the authors of MORP data sample (Table 5). On the other hand, three other streams with $M_H > 2$ were not detected in the present search, namely the Orionids (four members), the χ Orionids (three members) and the α Capricornids (three members).

Table 5. Meteor streams amongst the MORP data set. The first column gives the stream name, the second its code, the third and fifth columns the number of members M_N and M_H identified, respectively, in the present search for the threshold values $D_{c,M}$ and by Halliday et al. (1996). The last column gives the number of common members M_C.

Name	Code	M_N	$D_{c,M}$	M_H	M_C
Taurids	3	28	0.2344	23	23
Perseids	10	27	0.1276	27	27
Leonids	15	8	0.2694	9	7
δ Arietids	16	7	0.2344	0	0
Geminids	26	5	0.1276	5	5

Table 6. Meteor streams in the DMS data set. The first column gives the stream name, the second its code, the third and fifth columns the number of members M_N and M_A identified, respectively, in the present search for the threshold values $D_{c,M}$ and by the authors of the source catalogue. The last column gives the number of common members M_C.

Name	Code	M_N	$D_{c,M}$	M_A	M_C
α Capricornids (N)	6	5	0.0560	8	5
Perseids	8	243	0.0772	247	243
Geminids	26	223	0.0757	223	223
Lyrids	29	5	0.0560	5	5
Orionids	32	12	0.0449	12	12
κ Cygnids	40	10	0.0449	18	10
Taurids	106	7	0.0644	18	7
Quadrantids	508	38	0.0644	38	38
Leonids	582	21	0.0560	23	21
α Monocerotids	614	4	0.0246	4	4

We did not find the six streams with $M_H < 3$ found by Halliday et al. (1996), namely the δ Draconids, μ Virginids, σ Hydrids, δ Cancrids and δ Leonids, something that can be understood given that we were looking for streams with three or more members. On the other hand, we found all the 23 Taurids, the 27 Perseids and the five Geminids found by Halliday et al. (1996). In the case of the Taurids, five more members were detected, one of which had been assigned, in the source catalogue, to the Northern χ Orionids. For the Leonids, we found only seven of the nine members of Halliday et al. (1996), plus an additional, previously undetected, member.

4.3 The DMS streams

Our search in the DMS sample resulted in 10 streams of three or more members. As shown in Table 6, some of our results agree well with the classification done by the authors of the source catalogue; we detected the same Geminids, Lyrids, Orionids, Quadrantids and α Monocerotids. In the case of the Perseids and of the Leonids, in our search these streams are just slightly less numerous.

There are, however, some more serious discrepancies. With the assumed reliability level of 99 per cent, we did not identify six members of the Monocerotid stream and eight members of the δ Aquarid stream. Of the 18 Taurids in the source catalogue we have confirmed only seven members, all belonging to the Southern branch.

Concerning the Virginids, the υ Pegasids, the ι Aquarids, the σ Hydrids and the Leo Minorids, indicated as one or two member streams by the authors of the DMS catalogue, for the reasons already explained we could not confirm their existence in this data sample.

4.4 Streams in the combined data set

In Table 7 we list the results of the search in the combined meteor sample. Given the larger size of this data set, we would expect to find at least the same number of streams and of their members found in the searches done on the separate data sets. Moreover, we would expect that the fraction of the stream component should be the same or higher than in the case of the separate data sets.

Therefore, we should have found more than 18 streams, including at least 50 per cent of the sample. In fact, we detected only 13 streams, whose members amount to 48 per cent of the sample. In

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Table 7. Meteor streams found in the MDH data set. The first column gives the stream name, the second its code, the third the number of members M_{MDH} detected in the present search for the threshold values $D_{\mathrm{c},M}$ given in the fourth column. In the column labelled $M_{\mathrm{HD}+}$ we give the number of members detected in the separate search in the HD sample, and in the column labelled $M_{\mathrm{HD}-}$ the number of members not detected in the separate search. The meaning of the labels $M_{\mathrm{MORP}+}$, $M_{\mathrm{MORP}-}$, $M_{\mathrm{DMS}+}$, $M_{\mathrm{DMS}-}$ of the remaining columns is analogous.

Name	Code	M_{MDH}	$D_{\mathrm{c},M}$	$M_{\mathrm{HD}+}$	$M_{\mathrm{HD}-}$	$M_{\mathrm{MORP}+}$	$M_{\mathrm{MORP}-}$	$M_{\mathrm{DMS}+}$	$M_{\mathrm{DMS}-}$
Lyrids	1	11	0.0871	6	0	–	–	5	0
α Capricornids (N)	13	22	0.0990	11	0	0	3	5	3
Perseids	16	307	0.1205	33	0	27	0	243	4
κ Cygnids	18	23	0.1205	4	3	0	1	10	5
Taurids (N)	47	33	0.1205	18	0	11	0	–	4
Taurids (S)	47	50	0.1205	22	0	15	0	7	6
Quadrantids	48	52	0.0694	14	0	–	–	38	0
Geminids	70	279	0.0811	51	0	5	0	223	0
Monocerotids	90	5	0.0553	3	0	–	–	0	2
Leonids	94	33	0.0990	6	0	5	0	21	1
σ Hydrids	107	4	0.0553	4	0	0	0	0	0
Orionids	119	32	0.0811	19	0	0	1	12	0
S. δ Aquarids	347	16	0.0909	11	0	–	–	–	5
α Monocerotids	1735	4	0.0385	–	–	–	–	4	0

addition, we did not discover any streams not already found in the separate searches. The reason for this has already been mentioned in Section 3 – the thresholds used to test associations amongst MDH meteors are too small, due to the strong observational selection of the DMS meteors.

As already remarked earlier, two thirds of this sample is composed of meteors observed in August or in December (see Fig. 1). This means that more than half of the DMS data have a strong concentration with respect to one of the variables, λ, the longitude of the Earth at the time of the meteor apparition. This is the reason why in Table 2 we can see such low values of the thresholds $D_{\mathrm{c},M}$; these are correct for the meteors observed in August and December, but for the other months they are definitely too small.

This bias exists also in the combined sample. Therefore, as we can see in Table 7, the results we obtained for meteors observed in August and December are in agreement with the above-mentioned expectations. In fact, we detected in this data set more α Capricornids, Perseids, κ Cygnids, Monocerotids and Southern δ Aquarids than we found in the separate searches. Moreover, we detected the same amount of Geminids and one member less of the σ Hydrids. This last case is a further example of the influence of the above-mentioned bias. In the MDH sample, to find four members of the σ Hydrids we needed $D_{\mathrm{c},4} = 0.0553 \pm 0.0026$ whereas, in the HD data set, we were allowed for the same group to use the value 0.1086 ± 0.0025 (see Table 2).

Due to the artificially low values of the thresholds, the reliability of the remaining streams detected in the MDH data set is higher. Despite this, we detected for many streams the same number of meteors as the sum of the results of all the separate searches. In particular, this is true for very compact groups such as the Lyrids, the Quadrantids and the α Monocerotids, detected in the separate searches at low thresholds (see Tables 4 , 6 and 7).

On the other hand, we did not find all the separately identified Taurids and Leonids. In the latter case, 33 members were found with $D_{\mathrm{c},M} = 0.0990$, and they include three Leonids from the MORP sample, in which five meteors were classified as Leonids at $D_{\mathrm{c},M} = 0.02344$ (see Tables 5 and 7). Of the 83 Taurids detected in the MDH data set, only 73 were also detected in the separate searches. In the MDH sample, the association threshold for this stream was equal to 0.1205, which is almost two times more than in case of the DMS sample and about one third smaller than in the case of the HD sample. It is therefore not surprising that in the MDH sample we detected 10 additional Taurids from the DMS catalogue, but failed to identify as belonging to this stream 18 Harvard meteors. The above-mentioned arguments explain the small number of Taurids found in our search in the DMS data set. In fact, we found only seven of the 18 meteors singled out by Betlem (private communication) as Taurids members.

It might be tempting to ask why we did not increase the thresholds 'by hand', so as to be able to identify all the members of all the meteor streams possibly present in the DMS data. The answer is that, by manually adjusting the thresholds in specific cases, we would introduce in our procedure an element of arbitrariness that we want to avoid as much as possible. This was the very reason to introduce thresholds automatically determined by objective statistical tests in Jopek & Froeschlé (1997).

It turns out that the DMS data set is quite peculiar, and with it a statistical approach such as ours is in fact not effective, as it leads to artificially low thresholds.

4.5 Mean parameters of the meteor streams found

In Tables 8–11 we give the mean values of the geocentric and heliocentric parameter of the streams.

For the orbital elements q and e, and for the geocentric velocity V_{G}, we give the arithmetic means. The angular orbital elements i, ω and Ω and the radiant spherical coordinates α_{G} and δ_{G} were first converted into components of the unit vector pointing towards the pole of the orbit, the perihelion point, the ascending node of the orbit, and the radiant point. They were then averaged, and the averages transformed back to the angular representation. The averaged values of θ and ϕ were calculated in a similar way; we averaged the versors of the individual vectors $\boldsymbol{U}$, and the resulting components were used to find the mean angles θ and ϕ (this procedure is equivalent to the calculation of the arithmetic means of the components of the vector).

5 DISCUSSION AND CONCLUSIONS

Neslusaň, Svoreň & Porubcaň (1995) and Neslusaň & Welch (2002) used another approach to the similarity threshold determination. To find the threshold, the so-called break point on the plot – cumulative

Table 8. Geocentric and orbital data of the streams found in the HD sample at the reliability level 99 per cent; α_G, δ_G , i, ω and Ω are given for J2000.0. For streams identified as single ones, but possessing both a Northern and a Southern branch, the data are given separately for each branch.

Stream name	M	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
Lyrids	6	Apr 21	Apr 23	272.0	33.3	0.919	0.989	79.8	214.2	32.4	47.1	118.6	197.8
α Capricornids (N)	20	Jul 25	Aug 23	314.2	−8.6	0.584	0.780	6.1	268.2	135.0	22.8	89.1	261.9
α Capricornids (S)	1	Aug 23		333.4	−17.8	0.630	0.623	3.8	89.0	329.8	18.1	89.2	276.4
Perseids	33	Aug 4	Aug 22	47.3	58.2	0.948	0.951	112.7	150.3	139.4	59.0	139.4	163.9
κ Cygnids	4	Aug 19	Aug 22	289.5	55.6	0.980	0.763	38.5	201.9	147.6	24.9	88.1	194.2
Taurids (N)	22	Sep 17	Nov 15	44.7	19.8	0.317	0.853	3.4	298.8	214.1	29.6	103.8	267.3
Taurids (S)	36	Sep 18	Nov 22	40.6	10.3	0.340	0.820	6.0	117.9	27.6	27.8	103.8	275.2
Quadrantids	14	Jan 2	Jan 4	230.2	49.2	0.978	0.676	71.9	171.1	283.2	41.2	116.3	176.2
Geminids	51	Dec 7	Dec 16	112.9	32.3	0.141	0.898	23.5	324.2	261.6	34.6	117.4	258.2
χ Orionids (N)	4	Dec 9	Dec 30	97.3	25.9	0.376	0.833	3.0	291.1	266.0	28.2	100.6	267.4
χ Orionids (S)	7	Dec 4	Dec 14	80.9	17.6	0.507	0.790	4.5	95.6	77.9	24.6	93.4	275.3
Monocerotids	3	Dec 11	Dec 15	103.3	8.0	0.182	0.998	36.8	129.3	81.2	42.5	111.6	286.0
Leonids	6	Nov 16	Nov 20	153.6	21.8	0.985	0.918	162.4	173.2	235.4	70.8	169.6	168.0
σ Hydrids	5	Dec 4	Dec 15	127.1	1.8	0.237	0.978	126.5	122.5	79.1	58.0	137.2	295.0
Orionids	19	Oct 19	Oct 25	96.3	16.1	0.565	0.963	164.6	83.2	29.8	66.4	155.2	287.4
Virginids (N)	7	Apr 4	May 12	223.1	−8.8	0.411	0.831	7.6	287.9	34.5	28.3	98.8	262.9
Virginids (S)	5	Apr 2	Apr 24	208.3	−17.1	0.322	0.868	7.3	117.6	199.2	31.0	103.0	275.4
S. δ Aquarids	11	Jul 22	Aug 8	341.0	−15.8	0.078	0.972	26.9	151.1	308.2	40.8	118.0	278.1
ε Piscids (N)	6	Sep 20	Oct 14	6.4	10.5	0.583	0.755	4.9	268.3	190.7	21.9	89.3	263.1
ε Piscids (S)	9	Sep 15	Oct 9	4.3	−5.1	0.608	0.740	4.5	85.6	4.2	21.0	87.8	276.4
α Pegasids	3	Nov 13	Nov 13	342.3	22.4	0.965	0.680	7.3	200.1	230.4	10.6	41.7	226.2

Table 9. Same as Table 8 for the 256 fireballs of the MORP data set.

Stream name	M	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
Taurids (N)	12	Oct 18	Nov 30	53.8	22.5	0.360	0.827	3.5	294.1	225.0	28.0	101.8	266.9
Taurids (S)	16	Oct 11	Nov 11	51.8	14.4	0.351	0.841	5.1	114.8	40.9	28.6	102.2	274.3
Perseids	27	Jul 31	Aug 14	42.6	57.4	0.956	0.948	112.6	152.1	136.9	58.9	139.5	165.1
Leonids	8	Nov 8	Nov 23	154.4	22.8	0.973	0.834	160.1	169.8	234.8	69.5	168.0	164.7
δ Arietids (N)	1	Dec 6		53.8	21.7	0.743	0.962	1.4	240.0	253.9	21.3	72.8	267.5
δ Arietids (S)	6	Dec 11	Jan 1	67.3	15.6	0.817	0.658	2.5	55.1	87.9	14.7	70.4	276.4
Geminids	5	Dec 13	Dec 15	113.7	32.5	0.143	0.894	23.8	324.3	262.1	34.3	117.6	257.9

Table 10. Same as Table 8 for the 709 photographic meteors of the DMS data set.

Stream name	M	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
α Capricornids (N)	5	Jul 31	Aug 4	306.8	−8.7	0.598	0.796	7.6	265.8	129.0	22.9	88.0	259.8
Perseids	243	Aug 2	Aug 16	46.1	57.8	0.956	0.969	113.3	152.3	139.3	59.4	139.8	164.8
Geminids	223	Dec 13	Dec 16	114.0	32.3	0.140	0.897	24.0	324.5	262.3	34.6	117.6	258.1
Lyrids	5	Apr 21	Apr 25	272.5	33.6	0.924	0.940	79.4	213.6	32.2	46.5	118.7	197.2
Orionids	12	Oct 18	Oct 22	94.1	15.4	0.592	0.975	163.4	79.7	27.0	66.8	155.7	289.7
κ Cygnids	10	Aug 11	Aug 14	285.4	50.6	0.974	0.771	35.5	204.3	139.9	23.6	85.0	197.1
Taurids (S)	7	Nov 4	Nov 6	53.5	14.9	0.345	0.843	5.0	115.2	42.4	28.8	102.4	274.3
Quadrantids	38	Jan 4		230.4	49.3	0.978	0.698	72.1	171.1	283.4	41.5	116.2	176.2
Leonids	21	Nov 17	Nov 19	154.1	22.2	0.983	0.936	161.5	171.8	235.3	70.9	168.9	166.3
α Monocerotids	4	Nov 22		117.3	1.0	0.497	1.016	134.8	89.2	59.3	63.2	144.4	305.5

numbers of the identified meteors versus values of the D criterion – is used. After removing the members of the stream, the reliability of the results is estimated by comparison of the density of the sporadic radiants in the areas surrounding and containing the shower radiants under study (the so-called background-number–density test). To find the stream members the authors applied a different stream definition than the one we have used in the present paper. Namely, they defined a stream as a set of orbits concentrated near the mean orbit of the stream. This approach works well for many streams, but not in all cases. Neslušan & Welch (2002), for example, state that this method did not work for the α Capricornids with our D_N criterion.

Table 11. Same as Table 8 for the 1830 photographic meteors of the combined data set.

Stream name	M	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
Lyrids	11	Apr 21	Apr 25	272.2	33.4	0.921	0.967	79.6	213.9	32.3	46.8	118.6	197.5
α Capricornids (N)	22	Jul 20	Aug 10	307.1	−8.9	0.586	0.770	7.4	268.4	127.9	22.6	89.2	260.2
Perseids	307	Jul 30	Aug 22	45.9	57.8	0.955	0.967	113.2	152.1	139.0	59.3	139.7	164.7
κ Cygnids	23	Aug 4	Aug 22	284.3	51.5	0.975	0.748	35.1	202.9	141.1	23.2	85.0	196.0
Taurids (N)	33	Oct 16	Nov 24	52.4	21.9	0.339	0.841	3.4	296.3	222.7	28.8	102.8	267.1
Taurids (S)	50	Oct 19	Nov 22	51.7	14.0	0.367	0.836	5.3	112.7	41.5	28.2	101.3	274.7
Quadrantids	52	Jan 2	Jan 4	230.4	49.3	0.978	0.692	72.1	171.1	283.3	41.4	116.2	176.2
Geminids	279	Dec 7	Dec 16	113.8	32.3	0.140	0.897	23.9	324.4	262.2	34.6	117.6	258.1
Monocerotids	5	Dec 11	Dec 15	103.3	8.0	0.185	0.997	36.0	128.8	81.5	42.3	111.3	285.9
Leonids	33	Nov 16	Nov 20	154.0	22.1	0.983	0.914	161.7	172.3	235.3	70.6	169.1	166.8
σ Hydrids	4	Dec 9	Dec 15	128.3	1.7	0.227	0.974	125.9	124.1	80.7	57.6	136.9	294.7
Orionids	32	Oct 18	Oct 28	95.6	15.8	0.575	0.968	164.1	81.9	28.9	66.6	155.4	288.3
S. δ Aquarids	16	Jul 22	Aug 9	341.9	−15.6	0.084	0.970	26.4	149.9	309.6	40.6	117.5	278.3
α Monocerotids	4	Nov 22		117.3	1.0	0.497	1.016	134.8	89.2	59.3	63.2	144.4	305.5

Another complication to the problem is to take into account the accuracy of the individual meteor catalogues. Porubcaň (1977, 1978) has shown that the dispersion of the orbital data among various catalogues can differ even by a factor of 2.

The results shown in Section 4 evidence the inadequacy of a purely statistical approach to determine the quantitative thresholds for stream membership in cases in which the data set under study is seriously affected by selection effects. The reason for the failure, in the specific case examined in this paper, is the fact that one of the three data sets examined contains meteor data collected very unevenly over the year, mostly at the time of some major showers.

However, there is a more general issue concerning the statistical determination of membership thresholds. Let us concentrate the discussion on only one of our data sets, namely HD, that is arguably homogeneous, and suppose that, for the sake of the argument, it were much larger than its actual size, still maintaining the same distributions of observed quantities. If we then applied to this much larger hypothetical set the same procedure described in Section 3 to find the thresholds, we would inevitably find that, with respect to those given in Table 2, the new thresholds would be smaller by a factor of $(865/N)^{0.25}$, where N is the size of the hypothetical, larger, HD-like data set.

Now, it is clear that, for large enough N, the thresholds can become arbitrarily small, and this seems to be more a bug than a feature of the method. In fact, except perhaps the case of very sparse data, the acceptable mutual distances between meteors of the same stream in a given data set should be determined by physical, rather than by statistical arguments.

By this, we mean that the thresholds should be the result of an accurate modellization of the time evolution of a stream, taking into account all the phenomena relevant for the meteoroids of the size of interest (i.e., different physical and dynamical processes should be taken into account for photographic meteors and for radar meteors). It is possible that the thresholds determined in this way would turn out to be different in different regions of the parameter space (e.g. ecliptical, low-velocity streams, as opposed to high-velocity, retrograde ones), and it is clear that a statistical technique such as that discussed in this paper may have problems in dealing with such a situation.

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