The maximum possible duration of a total solar eclipse

Jean Meeus

The theoretical longest possible duration of a total solar eclipse for a point on the Earth's surface slowly varies with time. Its value has been calculated from 2000 BC to AD 7000.

It is well-known that the longest possible duration of the phase of totality of a solar eclipse, for a point on the surface of the Earth, is close to seven minutes and a half, and that this occurs for a point close to the equator. The value generally quoted for this theoretical maximum is 7 minutes 31 seconds. This figure comes from an oral paper by Isabel Martin Lewis (US Naval Observatory) which she delivered at the 42nd meeting of the American Astronomical Society at Ottawa in 1929. Unfortunately the paper was never published but there is an abstract for it. Lewis says:[1]

'Calculations were made to test out various combinations of circumstances with a view to obtaining the maximum duration. It appears that the most favorable combination of circumstances possible, at least for some centuries to come, will occur early in the month of July; when the sun is at or near apogee; when the moon is at perigee and at its ascending node and its latitude 24 minutes South; and when the observer is on the equator. A computation for such a combination gave a value of 7m 31.1s with the formulae and constants employed in calculating the duration of total eclipse given in the *American Ephemeris*.'

To maximise the duration, obviously we want the Moon close and the Sun far. Consequently, the longest duration of totality will happen when the Moon is at or close to perigee, and the Earth close to aphelion (the Sun close to apogee). To increase still further the Moon's apparent diameter, the Moon should be at the observer's zenith, because this is the point of the Earth's surface that is closest to the Moon. Moreover, the observer should be at the equator, because there the eastward motion of the Earth's surface is greatest, so the lunar shadow needs a longer time to overtake the observer.

However, these two last conditions are not compatible. Nowadays, the Earth reaches aphelion on or close to July 5, when the declination of the Sun is +23°. Therefore, an observer having the Sun at the zenith in early July cannot be on the equator, but has to be at 23° north latitude, where the rotational speed of the surface is smaller.

Presently, hence, the best conditions are for places somewhere between the equator and 23° north latitude. Danjon[2] writes that, starting from latitude +23° and going southwards without altering the other data, one finds that the duration of totality begins to increase and that it reaches a maximum value at latitude +5°: the diameter of the shadow decreases because the eclipse no longer occurs at the zenith, but initially its speed with respect to the surface decreases more rapidly.

Another condition is that the velocity vector of the site and that of the lunar shadow be almost parallel.

That is still not the end of the difficulties. The characteristics of the Earth's orbit vary with time. Presently, the eccentricity of this orbit is 0.0167 and decreasing; it will reach a minimum of 0.0023 about the year 29,500.[3] For this reason, the aphelion distance of the Earth is gradually decreasing, which increases the diameter of the solar disk at apogee and, generally speaking, decreases the maximum possible duration of a total solar eclipse.

The longitude of the Earth's perihelion, too, varies with time and this also affects the maximum possible duration, as we shall see. Finally, the obliquity of the ecliptic, too, is variable. It was equal to 24°00' in 2800 BC, and will be 23°00' in AD 5600. This variation, too, may affect somewhat the longest possible duration of a total eclipse.

So the problem is rather complicated, and we had to find a method to handle it.

Method of calculation

The best way to tackle the problem was to calculate a large number of *fictitious* eclipses for a given epoch, and to find a method that converges to the longest possible duration of totality.

Firstly, it was evident that the gravitational actions of the planets on the motions of the Moon and the Earth might be neglected:

Actions of the planets on the Moon

The planetary term in the longitude of the Moon with the largest amplitude is the so-called great Venus term; its coefficient is 14.25 arcseconds but its period is 273 *years*, so its variation in one day is negligible: it hardly affects the *speed* of the Moon. In distance, the greatest planetary term has a coefficient of 1.06 km, resulting in a change of only 0.006 arcsecond in the Moon's apparent diameter, which again is negligible.

Actions of the planets on the Earth

The planetary term with the largest amplitude in the longitude of the Earth is due to Jupiter and has a coefficient of 7.2 arcseconds. However its period is 399 days (the synodic period of Jupiter), so the value of this periodic term varies slowly, by at most 0.11 arcsecond per day.

In the radius vector, the total actions of the planets amount to at most 0.00006 AU, resulting in a change of less than 0".12 in the Sun's angular diameter, or at most one third of a second in the duration of a total eclipse. So, for reason of safety, we increased the radius vector of the Earth systematically by +0.00005 AU.

There is a further increase of +0.000029 AU to take into account the 'action' of the Moon. Indeed, it is not the centre of the Earth but the barycentre of the Earth–Moon system that describes an elliptical orbit around the Sun. At New Moon, when the Moon is closer to the Sun, the Earth is a little farther than is the barycentre. We thus increased the Earth–Sun distance systematically by a total amount of +0.000079 AU.

So we could neglect all planetary terms in our calculations. Although these terms do affect the times of beginning and end of a particular eclipse at a given place by many seconds, they do not sensibly affect the maximum *duration* of an eclipse on the Earth's surface.

Therefore, for the motion of the Earth we could consider a purely unperturbed elliptical orbit, but taking into account the secular variations of the eccentricity and of the longitude of perihelion. The positions of the Moon were calculated by using only the 'solar' periodic terms of Chapront's lunar theory ELP. These solar terms depend on the following four arguments only:

D, the *mean* elongation of the Moon to the Sun, that is, the difference between the *mean* longitudes of Moon and Sun; its period is 29.53 days, which is the length of the lunation, or the synodic period of revolution of the Moon;

M, the mean anomaly of the Earth (Sun), the longitude difference between the perihelion and the mean Earth; its period is 365.26 days, the anomalistic period of the Earth;

M', the mean anomaly of the Moon; it is equal to 0° at perigee and 180° at apogee; the period is 27.55 days, the anomalistic period of revolution of the Moon;

F, the argument of latitude of the Moon, or the difference between the mean longitude of the Moon and the longitude of the Moon's mean ascending node; it is equal to 0° at the passage of the mean Moon at the ascending node, and to 180° at the descending node; the period is 27.21 days, the Moon's draconic month.

Because D is the *mean* elongation of the Moon, it is not exactly zero at the instant of the *true* New Moon, the instant when the true Sun and the true Moon are in conjunction in celestial longitude for a geocentric observer. The time difference between mean and true New Moon can be as large as 14 hours. However, at a total solar eclipse of very long duration the Moon is close to perigee and hence M' is close to 0°, the Sun is near apogee and hence M does not differ much from 180°, and the Moon is close to a node and hence F is close to either 0° or 180°. Under such circumstances, the time difference between mean and true New Moon is rather small.

All calculations were performed for eclipses near the Moon's ascending and descending nodes separately, because the circumstances differ from one node to the other, as we shall see further on.

For each (fictitious) eclipse we took as reference time the instant of the corresponding mean New Moon. By definition, D is zero at that instant, so there remained three independent variables:

— the value F_0 of the argument F at the instant of mean New Moon; that value should be chosen close to either 0° (for an eclipse near the ascending node) or 180° (near the descending node);

— the value M_0 of the argument M at mean New Moon, to be chosen near 180° (Earth near aphelion);

— the value M'_0 of the argument M' at mean New Moon, to be chosen near 0° (Moon near perigee).

Of course D_0, the value of D at mean New Moon, is 0° by definition. We have to choose the values of F_0, M_0 and M'_0 in such a way as to make the duration of totality to be a maximum for the epoch considered.

To illustrate the way we performed the calculation, suppose we want to find out the greatest possible duration of totality for the year 3000 at eclipses near the ascending node of the lunar orbit. For that epoch, we calculate the eccentricity and the longitude of perihelion of the Earth's orbit, and the obliquity of the ecliptic, from well-known formulae. Then, in order to avoid handling three independent variables, we do the calculation for several fixed values of M_0. Let us start, for instance, with $M_0 = 180°$.

Then, choosing $M'_0 = 0°$ and $F_0 = 0°$, we calculate the so-called Besselian elements of a first eclipse, 'A'. These elements, named after the German mathematician and astronomer F. W. Bessel (1784–1846), characterise the geometric position of the Moon's shadow relative to the Earth; they allow us to perform many calculations about the given eclipse, such as local circumstances or points on the central line. It would be outside the scope of this paper to explain how to calculate and to use these Besselian elements (for a definition of Besselian elements see, for instance, references 12, 13 & 14).

It appears that, for this eclipse, the maximum duration of totality along the central line is 431.645 seconds.

Keeping M_0 unchanged, we calculate the Besselian elements of another eclipse, 'B', by using, say, $M'_0 = 2°$ and $F_0 = 0°$. This yields a maximum duration of 431.532 seconds.

Then, with $M'_0 = 2°$ and $F_0 = 0°.4$, we find the maximum duration of a third eclipse, 'C', to be 427.500 seconds.

Table 1. The maximum possible duration of a total solar eclipse

Epoch	*Near the ascending node* *min. sec.*	*Near the descending node* *min. sec.*
−2000	7 07.3	7 29.6
−1000	7 18.9	7 34.5
0	7 27.3	7 35.9
+1000	7 31.7	7 33.4
2000	7 32.1	7 27.0
3000	7 28.7	7 16.9
4000	7 21.9	7 03.8
5000	7 12.7	6 48.6
6000	7 03.1	6 32.4
7000	7 01.7	6 32.6

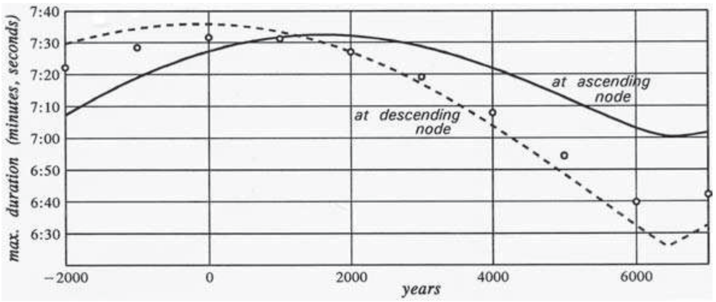

Figure 1. The theoretically longest possible duration of a total solar eclipse in the course of ninety centuries, from 2000 BC to AD 7000, separately at the ascending node of the Moon's orbit (solid line) and at the descending node (dashed line). The small circles are the values for eclipses at the descending node if we suppose that the eccentricity of the Earth's orbit remains constant at its value for the year 2000.

So at this point we have a 'triangle' of three eclipses in the M'_0–F_0 'plane':

eclipse A	$M'_0 = 0°$	$F_0 = 0°$	431.645 sec.
eclipse B	$M'_0 = 2°$	$F_0 = 0°$	431.532 sec.
eclipse C	$M'_0 = 2°$	$F_0 = 0.4$	427.500 sec.

Note that we keep an unrealistically high accuracy of one thousandth of a second here, for the purpose of allowing convergence to a correct, final value.

To converge to an eclipse with the greatest duration of totality, we use the so-called *Simplex* method.[4,5] The triangle ABC is moved 'uphill' in the M'_0–F_0 plane, accelerating, slowing and changing shape as needed. For instance, in the example above, the 'worst' vertex of the triangle is eclipse C, as it has the lowest duration of totality. That worst vertex is rejected and another one is substituted for it, according to certain rules. When programmed properly, the calculation automatically converges to the longest possible totality for the chosen value of M_0. In the present case, we find 447.687 seconds.

Then the whole calculation is repeated for other values of M_0. So we obtain, always for the epoch 3000 and for the ascending node:

M_0	*duration (sec.)*
168°	448.101
172°	448.656
176°	448.508
180°	447.687
184°	446.232

By interpolation, we find that the maximum is 448.684 seconds, or 7 minutes 28.7 seconds, for $M_0 = 173.1°$, that is, seven days before the Earth reaches aphelion.

With this method, hence, for any given epoch the longest possible duration of a total eclipse is *not* found by searching the best geographical latitude at which this longest event occurs, but by changing M'_0 and F_0 (by the Simplex method), then by changing the value of M_0. We didn't even care about the best geographical latitude; the latter is implicitly achieved by our searching method.

The whole calculation is then repeated for the other node, then for other epochs. The results are given in Table 1 and illustrated in Figure 1.

There is one curve for each node, and for any given epoch the longest possible duration of totality is, of course, the largest of the two values. The two curves cross in the year 1246, at the epoch when the longitude of the Earth's perihelion, referred to the mean equinox of the date, was exactly 90°. So in AD 1246 there was a change of node: before 1246 the longest possible duration of totality was for eclipses occurring at the descending node of the lunar orbit; since 1246, it happens at the ascending node.

Nowadays, the longest possible duration is 7 minutes 32 seconds, just one second longer than the value found by Isabel Lewis. Maybe this is due to the fact that Lewis made the calculation for a place exactly on the equator.

Between the years −2000 and +7000, the longest possible duration of totality is always larger than 7 minutes, the maximum being 7 minutes 35.9 seconds, which happened about the year −120. The smallest value for the maximum duration will be 7 minutes 00.4 sec., about AD 6500.

The longitude of the Earth's perihelion will reach the value 180° in AD 6429. From then on, the maximum possible duration will increase again, at both nodes, as seen at the extreme right in Figure 1.

Secular variations

Because the eccentricity of the Earth's orbit is presently decreasing, the aphelion distance of the Earth decreases, and the smallest annual value of the Sun's apparent diameter increases. By 2000 BC, the Sun's diameter at apogee was 1885.0 arcseconds; presently it is 1887.7 arcseconds, and by AD 6000 the Sun's diameter will not decrease below 1891".2. This results in a general decrease of the longest possible duration of a total solar eclipse.

To better show this effect due to the decreasing eccentricity, we repeated the calculation for eclipses at the descending node, for several epochs, but by keeping the eccentricity as constant, equal to its value in AD 2000, namely 0.016 7086. The resulting values are shown by the small open circles in Figure 1. We notice that the values thus found are smaller than the actual values before AD 2000, and larger thereafter.

However, superposed on this general trend of decrease, there are periodic variations due to the varying longitude of the perihelion of the Earth's orbit. These long-period variations, well seen in Figure 1, are explained in Figure 2.

We must bear in mind that, in order to maximise the duration of totality, the motion of the place of observation and that of the Moon's shadow should be as nearly parallel as possible. Consequently, the Moon's umbra should move as nearly perpendicular to the North–South direction as possible, as seen in Figure 3.

Let us now return to Figure 2. In the year 1246 (upper drawing), the Sun's apogee A coincided with the summer solstitial point, at longitude 90°. A solar eclipse taking place at the very date of the summer solstice would have occurred exactly at apogee, but then the Moon's motion would not have been exactly in the west–east direction. To have a longer duration

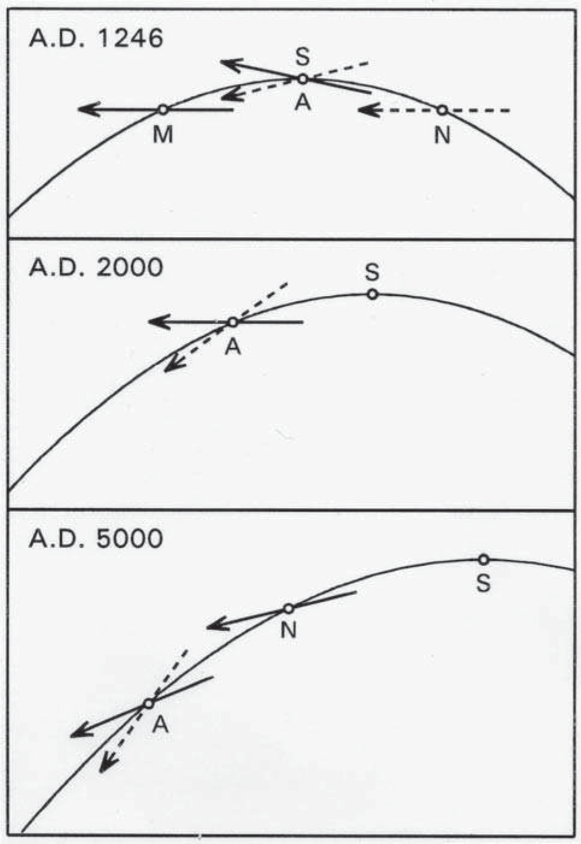

Figure 2. Changing conditions due to the variable longitude of the perihelion of the orbit of the Earth. Each drawing represents a part of the sky, and the lower border is parallel to the celestial equator. The curved line is a part of the ecliptic. *S* is the summer solstitial point, the position reached by the Sun about June 21. *A* is the position of the Sun's apogee (Earth at aphelion). Celestial longitudes increase from the right to the left. The arrows are the paths of the Moon at a central solar eclipse at the ascending node (solid line) and at the descending node (dashed).

of totality, the eclipse should have taken place either about 8 days after apogee, at the ascending node, M, or about 8 days before apogee, at N, the descending node.

In AD 2000 (second drawing of Figure 2), the apogee of the Sun has moved to longitude 103°, or 13 degrees past the solstitial point S. If an apogee eclipse occurred at the Moon's descending node, the path would make a rather large angle with the west–east direction; see the dashed arrow. But an eclipse at the ascending node would be quite right. This is the reason why presently the longest duration of totality takes place at apogee eclipses at the ascending node. For AD 2000, the maximum possible duration is 7 minutes 32 seconds at the ascending node, but 'only' 7m 27s at the descending node.

About AD 5000, the Sun's apogee is at still larger longitudes, closer to the autumnal equinoctial point. That part of the ecliptic is rather highly inclined to the west–east direction; think of the ecliptic in eastern Leo. Even at an apogee eclipse at the Moon's ascending node, the Moon's path would be too much inclined to yield a very long duration of totality, and an eclipse at the descending node would be worse still. Compare this with the solar eclipses in early October of 1959 and 2005 (Figure 4). To yield a longer duration of totality, we have to go back to a smaller celestial longitude, at N, where the Moon's path at the ascending node makes a smaller angle with the west-east direction, resulting in a longer duration, although this occurs at some distance from apogee. In all, the situation is less favourable than around AD 2000.

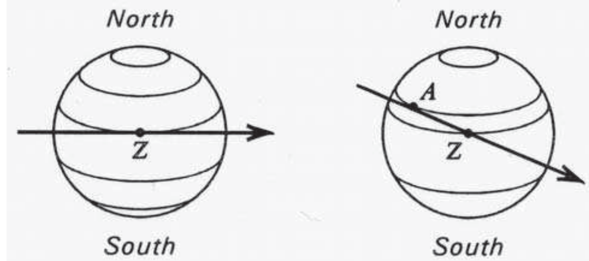

Figure 3. Two views of the Earth as seen from the Sun at the time of a central solar eclipse. The arrows indicate the motion of the centre of the Moon's shadow. *Z* is the centre of the Earth's 'disk'. For an observer at *Z*, the Moon and the Sun are at the zenith at the time of central eclipse, so it is there that the diameter of the lunar umbra at the Earth's surface is a maximum.
In the drawing at left, the shadow moves exactly in the west-east direction, and at *Z* its motion and that of the observer are exactly parallel, so the duration of totality is maximised. In the case illustrated at right, the two motions at *Z* are no longer parallel. They are parallel at *A* (where the motion of the umbra is tangent to a circle of latitude), at a large distance from *Z*. But in *A* the breadth of the shadow is smaller than at *Z*.

Such considerations explain why the longest possible duration of a total solar eclipse generally does not occur exactly when the Sun is at apogee.

Remarks

1. In August 1982, the IAU General Assembly adopted the value $k = 0.272\,508$ for the ratio of the Moon's radius to the equatorial radius of the Earth. However, this value corresponds to the *mean* radius of the lunar globe.

Since a solar eclipse is not regarded as total as long as rays from the Sun shine through the valleys of the Moon, a smaller value than the mean value of k is to be used for the calculation of the umbral cone (total and annular phases). In our calculations, therefore, we adopted the value $k = 0.272\,274$ as recommended by the *Explanatory Supplement* of 1961,[6] although a somewhat larger value has been adopted later. From 1969 to 1980, the *Astronomical Ephemeris* used $k = 0.272\,281$ for computing the radius of the umbra.

If, instead of $k = 0.272\,274$, we used the IAU value 0.272 508 of the *mean* radius of the Moon, the maximum possible duration of totality at epoch 2000 would change from 7 minutes 32.1 seconds to 7 minutes 37.3 seconds. But in that case the first and last 2 or 3 seconds of 'totality' would not really yield a total event.

2. We have calculated the value of the theoretically longest possible duration of a total solar eclipse in the course of 90 centuries. But what about the *real* eclipses with long duration? Table 2 lists these eclipses with duration of 7 minutes 20 seconds or longer, between 2000 BC and AD

Table 2. Solar eclipses with long duration of totality, years –2000 to +4000

Date	Maximum duration min. sec
–761 June 05	7 25
–743 June 15	7 27
+363 June 27	7 24
381 July 08	7 22
1062 June 09	7 20
2168 July 05	7 26
2186 July 16	7 29
2204 July 27	7 22

4000. Julian calendar is used before AD 1583. Notice the presence of the Saros period, for instance the group 2168–2186–2204.

3. The record eclipse of 2186 July 16 will occur seven days after the passage of the Earth at aphelion. The Moon will reach perigee only 54 minutes before mid-eclipse, and the distance between the centres of Earth and Moon will reach a minimum of 357,345 kilometres.

However, this is not the least possible distance between the centres of Earth and Moon. During the period AD 1500–2500, the least distance is 356,371 km, on 2257 January 1.[7] This is 974km closer than the perigee distance of 2186 July 16. One might think that bringing the Moon closer to the Earth by 974km would increase the diameter of the Moon's shadow at the Earth's surface by 8.9 kilometres, which would increase the maximum duration of totality by about 11 additional seconds (15 seconds increase due to the shadow growth, but 4 seconds loss from faster lunar angular velocity); this would yield a duration of about 7 minutes 29 seconds + 11 seconds = 7 minutes 40 seconds.

This reasoning is incorrect, however. The extreme perigees of the Moon take place only during the period of the year when the Earth is *closest* to the Sun. In other words, the Moon at extreme perigee and the Sun near apogee are incompatible conditions. See more details in Meeus, 2002.[8]

What, then, is the smallest possible Earth–Moon distance at the time the Earth is at aphelion? Or 10 or 20 days earlier or later? We avoided this difficult question by calculating the Besselian elements using the solar terms of Chapront's ELP lunar theory, as we have said.

4. Stranger still is the statement by Reynolds and Sweetsir[9] that 'totality may theoretically not exceed 7 minutes 58 seconds.' How did they obtain such an impossibly large value? Isabel Lewis[1] attributed that very long duration to Du Sejour,[10] who indeed found 7 minutes 58 seconds as maximum.

Du Sejour correctly stated that the eclipse should occur shortly before the Moon reaches the ascending node of its orbit. But probably he made the same error as that mentioned above under (3), as he wrote: 'Nous avons supposé que toutes les circonstances qui concurrent à donner la plus grande durée de la demeure dans l'ombre, ont lieu à la fois.' He adopts for the maximum *polar* parallax of the Moon the value 1°01'17"; using the modern value 6356.76 km for the Earth's polar radius, this yields a distance of 356,607 km.

Moreover, several values may not have been accurately known by Du Sejour. For instance, he gave tables for the ellipticity of the Earth's meridian 'avec rapport des axes comme 177 à 178, 200 à 201, 229 à 230, 299 à 300', so he did not know the exact value of the flattening of the Earth. We now know that the ratio of the axes is 297.3 to 298.3. So it is no surprise that his value for the maximum possible duration of totality is too large.

Isabel Lewis[1] further writes: 'Du Sejour's value for the duration is wrong because he used erroneous values for the semi-diameters and parallaxes of sun and moon and their hourly motions, as well as a value for the compression of the earth that was greatly in error.'

Indeed, using the presently adopted value of 959.63 arcseconds for the Sun's semidiameter at unit distance, we find that at about AD 1775 the Sun's semidiameter at apogee was 943.77 arcseconds. Du Sejour, however, adopted 15'42", or 942", in his calculations. This discrepancy of 1.77 arcseconds with the actual value, by itself, results in a duration that is too large by ten seconds. This fact alone explains a large part of the error of Du Sejour's value.

Camille Flammarion[11] repeated this incorrect value of 7m 58s, but without reference. Flammarion's unquoted source is Arago, from whose *Astronomie Populaire* he took not only the title, but many subjects, without mention. In Vol. III, pp. 550–551, Arago quotes Du Sejour for the data on eclipse duration, without comments, which means he accepted them. It is incomprehensible, however, that some authors still repeated this in 1995.

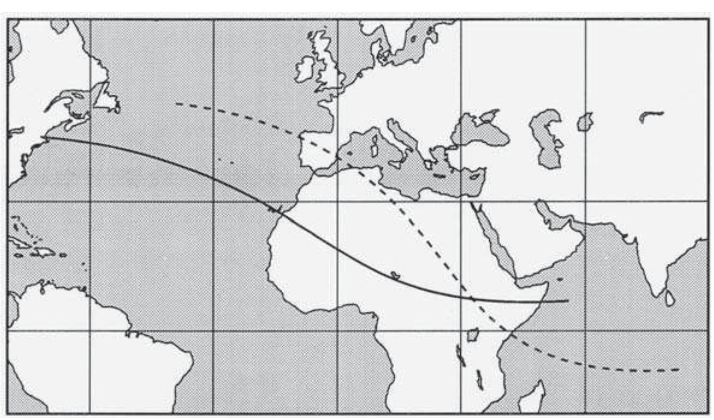

Figure 4. The central lines of the total solar eclipse of 1959 October 2 (solid line) and of the annular eclipse of 2005 October 3 (dashed line). Both eclipses occur near the date of the autumnal equinox. At the eclipse of 1959, the Moon was near the ascending node of its orbit, and therefore the path was less inclined to the west–east direction than at the eclipse of 2005, when the Moon will be at the descending node.

Acknowledgments

The writer thanks Fred Espenak (NASA) and Salvo De Meis (Milan, Italy) for help with references.

Address: Leuvense steenweg 312, box 8, B-3070 Kortenberg, Belgium. [jmeeus@compuserve.com]