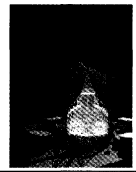
Multi-conjugate Adaptive Optics for the Swedish ELT Investigation of the Effects of Laser Guide Stars at a finite Distance

Venice 2001
Beyond Conventional Adaptive Optics



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ABSTRACT

In a recent paper (Owner-Petersen & Gontcharov, *in press*) we presented a procedure for optimal analytical control of several deformable mirrors based on the measured guide star wavefronts only. The expected performance was evaluated for various guide star configurations using up to three deformable mirrors. For simplicity the guide stars were assumed to be infinitely distant. In this paper we address the effect of using laser guide stars at 90 km altitude for the wavefront measurements. As a consequence of this, the image of a distant science star will not be adequately corrected by counteracting the wavefront error measured by a laser guide star in the same direction as the science star. This problem will be more pronounced the larger is the telescope diameter. We present a modified version of the mirror control algorithm taking the finite distance of the laser guide stars into account and some preliminary performance evaluations with special emphasis on the consequences for the Swedish 50 m ELT (Andersen *et al, this conference*).

1. GEOMETRICAL CONSIDERATIONS

Geometrically some of the consequences of the cone effect can easily be understood from Figure 1. The atmosphere is

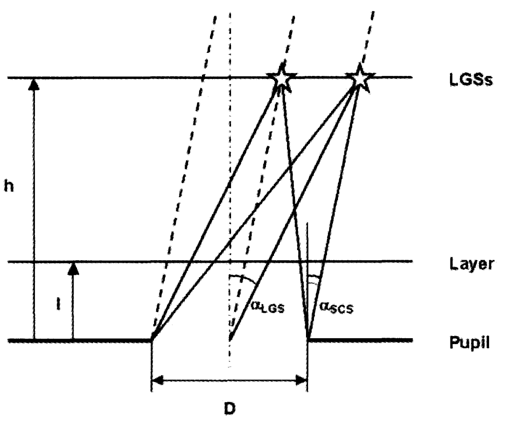

Figure 1. The figure shows the location α_{LGS} of the marginal LGS when correcting the science field α_{SCS}. Also shown is the conical ray fan giving rise to focal anisoplanatism when correcting a star at infinity relying on optimal correction of a LGS in the same direction.

probed by some laser guide stars (LGS) located in an altitude h above the pupil of the telescope. Using laser induced fluorescence in the mesospheric sodium layer; this altitude will be 90 km above the ground level. The wave front errors associated with each of the LGSs are then used to devise corrective actions on an appropriate number of deformable mirrors (DMs). The cone effect will affect two important issues. As can be seen from Figure 1 the needed guide star field will be given by

$$\alpha_{LGS} = \alpha_{SCS} + \frac{D}{2h} \tag{1}$$

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where α_{LGS} is the guide star field radius and α_{SCS} is the science field radius. Figure 2 shows the radius of the needed guide star field as a function of telescope diameter when correcting 1 arc minute science FOV. The Swedish 50 m ELT will require the marginal LGSs to be at 87 arc seconds.

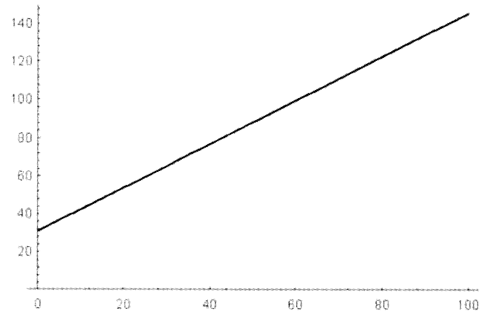

Figure 2. Radial position in arc seconds of the marginal LGS as a function of telescope aperture diameter in m, α_{SCS} = 30 arc seconds, h = 90 km.

The other issue related to using LGSs for probing the atmosphere is focal anisoplanatism. As can be seen from Figure 1 the LGSs sample the atmosphere along ray paths, which are different from the ray paths associated with infinitely distant stars in the same directions. Hence optimal correction of the LGS images will not result in optimal correction of the science stars. This problem (focal anisoplanatism) will be treated in the following sections.

Having estimated the LGS field, the needed number n_s of LGSs can be calculated from

$$n_s = \pi \frac{\alpha_{LGS}^2}{\Delta\alpha^2} \tag{2}$$

where $\Delta\alpha$ is the separation between the LGSs, which are assumed located within a circular field. The maximum allowable LGS separation must be related to the isoplanatic angle α_{iso}. For an atmosphere following Kolmogorov statistics and described by N layers with Frieds parameters $r_{0,n}$, conventional AO with a single DM conjugated to an altitude L_1 will result in α_{iso} given by

$$\alpha_{iso}^{-5/3} = 6.88 \sum_{n=1}^{N} \left(\frac{|l_n - L_1|}{r_{0,n}} \right)^{5/3} \tag{3}$$

where l_n is the altitude of layer n. The isoplanatic angle α_{iso} corresponds to an increase in the pupil averaged squared wavefront error from 0 on axis to 1 at α_{iso}. That is equivalent to a decrease in the Strehl ratio from 1 to 0.37. Figure 3 shows α_{iso} as a function of the conjugation altitude L_1 for single DM perfect correction on axis, when the atmosphere is modelled as the K-band seven-layer Cerro-Pachon atmosphere given in Table 1 (Vernin *et al*, 2000).

From Figure 3 it is seen that α_{iso} will roughly be 15 arc seconds for DM altitudes close to the ground. Since α_{iso} is the separation angle for which two wave fronts will be decorrelated to the specified degree (one radian wavefront RMS), one would expect that $\Delta\alpha \simeq 15$ arc seconds would be a reasonable guide star sampling interval when performing MCAO.

Number n	1	2	3	4	5	6	7
l_n (m)	0	1800	3300	5800	7400	13100	15800
$r_{0,n}$ (m)	1.14	3.99	3.14	6.55	8.01	3.99	10.89

Table 1. The Cerro-Pachon Atmosphere at 2.2 μm

The Figures 4 and 5 (Owner-Petersen & Gontcharov, *in press*) show the expected Strehl ratio as function field angle for two different guide star configurations: a homogeneous distribution representing the best possible sampling of the atmosphere, and a five star regular cross sampling representing a more sparse sampling. Comparing Figure 5a with Figure 4a and Figure 5b with Figure 4b it is seen that there will be a significant loss in the Strehl ratio when extending $\Delta\alpha$ beyond 30 arc seconds. Using this value, the number of guide stars calculated from Eqs. 1 and 2 as a function of telescope

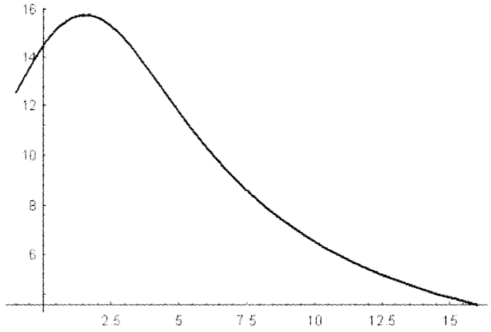

Figure 3. Isoplanatic angle in arc seconds as a function of conjugate altitude in km of the DM when performing conventional AO. Atmosphere: see Table 1.

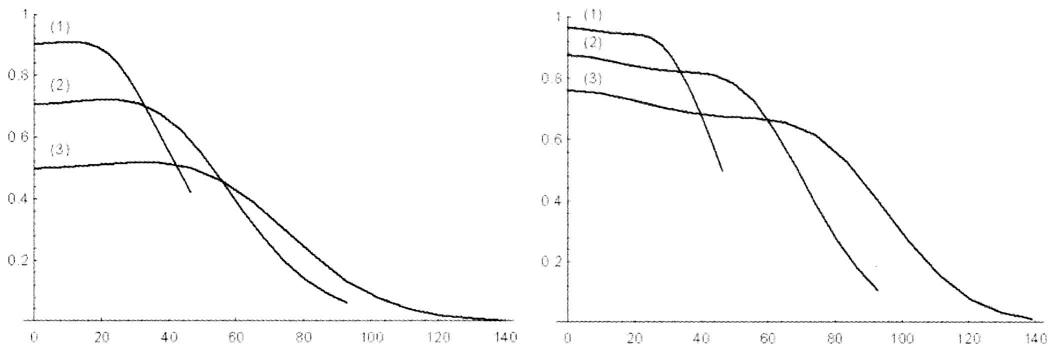

Figure 4. The figures show the expected Strehl ratio versus field angle in arc seconds when performing two DM on the left side (Figure 4a) and three DM (Figure 4b) correction on the right side. The guide stars were assumed to be infinitely distant and homogeneously distributed over the field ($\Delta\alpha$ = 0). Curves (1), (2) and (3): Guide star FOV = 1, 2 and 3 arc minutes respectively. D = 50 m. DM altitudes in m: (-575, 8000) and (-575, 4000, 8000)

diameter is shown in Figure 6. The Swedish 50 m ELT will require 27 LGSs for K-band correction and 432 (!!) LGSs for V-band correction. Considering DCAO, Figure 4a and 5a show that this results in a decrease of the Strehl ratio from about 0.9 corresponding to optimal sampling of the atmosphere ($\Delta\alpha$ = 0) to 0.76. If a decrease in the Strehl ratio to 0.4 can be tolerated, the number of guide stars can be reduced by a factor of roughly 4 (see Eq. 1) corresponding to a doubling of both $\Delta\alpha$ and the science FOV. However here it should be kept in mind that due to focal anisoplanatism there would be an additional decrease in the Strehl ratio as will be shown in the following sections.

2. THE CORRECTIVE PROCEDURE AND THE LAYER TRANSFER FUNCTIONS

In the present and the following two sections we shall address the problem of focal anisoplanatism for ELTs. The first point to realize is that in order to come up with a quantitative evaluation of the effect, one must devise a corrective procedure. The procedure we investigate here is an analytical version of the commonly used "Least Squares Principle" (LSP) sharpening up the LGS images in the best possible way. When stated as done in the following, LSP leads to the concept of so-called layer transfer functions describing how the correction of a single atmospheric layer should be distributed among the DMs. Knowing the layer transfer functions, the performance can be analytical evaluated (without use of Monte Carlo Simulations) for any atmosphere described as a collection of layers with known statistics (power spectra). Consider Q LGSs located at the two-dimensional angular positions α_q on the sky and in the altitude h above the ground. The wavefront error associated with each star is $\varphi_q(\mathbf{r})$, where $\mathbf{r}$ is the two-dimensional vector position in the pupil plane (ground level). Correcting this error by actions on M DMs at conjugate altitudes L_m, the residual errors $\varepsilon_q(\mathbf{r})$ will be given by

$$\varepsilon_q(\mathbf{r}) = \varphi_q(\mathbf{r}) + \sum_{m=1}^{M} \varphi_m \left(\frac{\mathbf{r}}{a_m} + L_m \alpha_q \right) \quad (4)$$

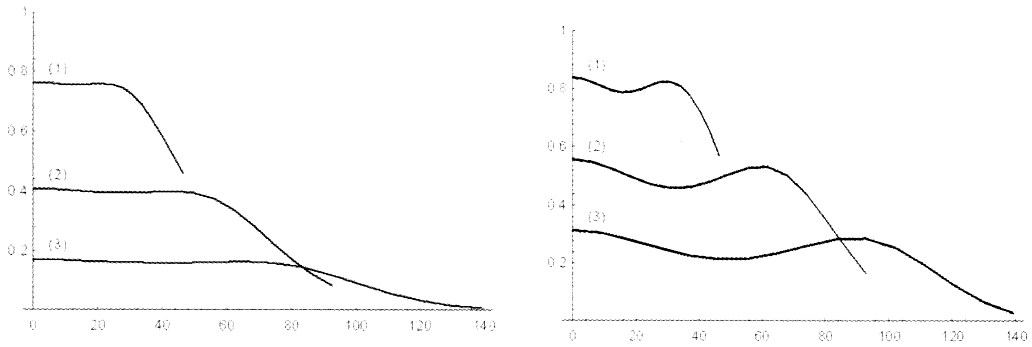

Figure 5. The figures show the expected Strehl ratio versus field angle in arc seconds when performing two DM (Figure 5a, figure on the left) and three DM (Figure 5b) correction (figure on the right). The guide stars were assumed to be infinitely distant and distributed in a regular five star cross. Curves (1), (2) and (3): Guide star separation $\Delta\alpha$ = 0.5, 1 and 1.5 arc minutes respectively. Field angle is along the cross arms. D = 50 m. DM conjugate altitudes in m: (-575, 8000) and (-575, 4000, 8000)

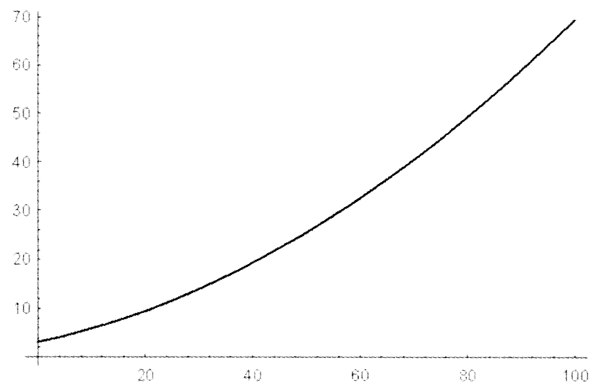

Figure 6. Number of guide stars as a function of telescope diameter in m. $\alpha_{SCS} = \Delta\alpha$ = 30 arc seconds. LGS altitude = 90 km. K-band Cerro-Pachon atmosphere. Note that V-band correction requires roughly 16 times as many LGSs, since $\Delta\alpha$ must be 4 times smaller (see Eq. 2).

where φ_m is the phase deformation of DM number m and a_m is a scaling factor given by

$$a_m = \frac{h}{h - L_m} \tag{5}$$

The sum S over the LGSs of the residual power spectra associated with each LGS is given by

$$S(\mathbf{f}) = \sum_{q=1}^{Q} \left| \Phi_q(\mathbf{f}) + \sum_{m=1}^{M} a_m^2 \Phi_m(a_m \mathbf{f}) \exp(2\pi i a_m L_m \mathbf{f}.\alpha_q) \right|^2 \tag{6}$$

where $\mathbf{f}$ is the two-dimensional spatial frequency and capital letters denote Fourier transforms. Minimization of S with respect to the Φ_m's leads to a system of M linear equations from which the Φ_m's can be determined:

$$A_m(\mathbf{f}) + \sum_{m=1}^{M} G_{m,m'}(\mathbf{f}) \Phi_{m,m'}(a_{m'} \mathbf{f}) = 0 \tag{7}$$

where

$$A_m(\mathbf{f}) = \frac{1}{Q}\sum_{q=1}^{Q} a_m^2 \Phi_q(\mathbf{f}) \exp(-2\pi i a_m L_m \mathbf{f}.\alpha_q) \quad (8)$$

$$G_{m,m'}(\mathbf{f}) = \frac{1}{Q}\sum_{q=1}^{Q} a_m^2 a_{m'}^2 \exp(-2\pi i (a_{m'} L_{m'} - a_m L_m)\mathbf{f}.\alpha_q) \quad (9)$$

Note that the estimate of the ϕ_m's is unbiased with respect to the atmospheric statistics and that the $G_{m,m'}$ functions only depend upon the LGS and DM configuration. Considering a single atmospheric layer n in the altitude l_n we have

$$\varphi_q(\mathbf{r}) = \varphi_n^l(\frac{\mathbf{r}}{a_n} + l_n\alpha_q) \quad (10)$$

where φ_n^l is the phase distortion in the layer and a_n is given in analogy with Eq.5. Introducing the layer transfer functions $T_{n,m}$ by

$$\Phi_m(a_m\mathbf{f}) = -T_{n,m}(\mathbf{f})\Phi_n^l(a_n\mathbf{f}) \quad (11)$$

use of Eqs 10 and 11 in Eqs. 7,8,9 leads to

$$G_{m,n}(\mathbf{f}) - \sum_{m'=1}^{M} G_{m,m'}(\mathbf{f})T_{n,m'}(\mathbf{f}) = 0 \quad (12)$$

where

$$G_{m,n}^l(\mathbf{f}) = \frac{1}{Q}\sum q = 1Q a_n^2 a_m^2 \exp(2\pi i (a_n l_n - a_m L_m)\mathbf{f}.\alpha_q) \quad (13)$$

For known LGS and DM geometry the $T_{n,m}$'s can be determined from Eqs 12,13 and 9 and used for performance analysis.

3. THE RESIDUAL POWER SPECTRUM FOR A SCIENCE STAR AT INFINITY

In this section we deal with correction of an infinite star subjected to fluctuations from a layered atmosphere. In the "Big Pupil" approximation (Owner-Petersen & Gontcharov, *in press*) the single layer residual wavefront error for a science star with angular coordinates α is given by

$$\epsilon(\mathbf{r},\alpha) = P(\mathbf{r} + l_n\alpha)\varphi_n^l(\mathbf{r} + l_n\alpha) + \sum_{m=1}^{M} P(\mathbf{r} + L_m\alpha)\varphi_m(\mathbf{r} + L_m\alpha) \quad (14)$$

where P is the pupil function (1 inside and 0 outside the pupil). The power spectrum $W_n(\mathbf{f},\alpha)$ for the residual wavefront error can be calculated from

$$W_n(\mathbf{f},\alpha) = \frac{4}{\pi D^2}\int\int \langle\epsilon_n(\mathbf{r}',\alpha)\epsilon_n(\mathbf{r}'+\mathbf{r},\alpha)\rangle \exp(-2\pi i\mathbf{f}.r)d\mathbf{r}d\mathbf{r}' \quad (15)$$

where the symbol <> stands for average over ensembles of phase fluctuations of the layer. Given the power spectrum $W_n(\mathbf{f})$ for the fluctuations of layer n rather lengthy calculations (to be presented elsewhere) leads to the result

$$W_n(\mathbf{f},\alpha) = W_n(\mathbf{f}) + \sum_{m=1}^{M}\left(\frac{a_m}{a_n}\right)^2 \left|T_{n,m}\left(\frac{\mathbf{f}}{a_m}\right)\right|^2 W_m\left(\frac{a_m\mathbf{f}}{a_n}\right) \quad (16)$$

$$-2Re\left[\sum_{m=1}^{M}\left(\frac{2a_m}{a_n+a_m}\right)^2 T_{n,m}\left(\frac{2\mathbf{f}}{a_n+a_m}\right)\exp(-2\pi i(l_n-L_m)\mathbf{f}.\alpha \mathrm{Airy}(x_{n,m})W_n\left(\frac{2a_n\mathbf{f}}{a_n+a_m}\right)\right]$$

$$+2Re\left[\sum_{m=1}^{M}\sum_{m'>m}^{M}\left(\frac{2a_m a_{m'}}{a_n(a_m+a_{m'})}\right)^2 T_{n,m}^*\left(\frac{2\mathbf{f}}{a_m+a_{m'}}\right)T_{n,m}\left(\frac{2\mathbf{f}}{a_m+a_{m'}}\right)\exp(-2\pi i(L_m-L_{m'})\mathbf{f}.\alpha\right.$$

$$\left.\mathrm{Airy}(x_{m,m'})W_n\left(\frac{2a_n\mathbf{f}}{a_m+a_{m'}}\right)\right] \quad (17)$$

where

$$\mathrm{Airy}(x) = \frac{2J_1(x)}{x}, \quad x_{n,m} = \pi\frac{2(a_m-a_n)}{a_m+a_n}fD, \quad x_{m,m'} = \pi\frac{2(a_{m'}-a_m)}{a_{m'}+a_m}fD \quad (18)$$

The pupil dependence enters via the two Airy functions and results in an increase of the residual power spectrum over the value associated with natural guide stars where all scaling factors a are equal to 1. This results in focal anisoplanatism. The total residual power spectrum $W(\mathbf{f},\alpha)$ is calculated by summation of Eq.16 over the layers n.

4. PERFORMANCE EVALUATION

Given the power spectra for the layers and the LGS and DM configuration $W(\mathbf{f},\alpha)$ can be calculated for any science star. According to Noll(1976) the piston-subtracted pupil averaged RMS^2 value of the residual wavefront error is then calculated from

$$\mathrm{RMS}^2(\alpha) = \int W(\mathbf{f},\alpha)\left[1-\left(\frac{2J_1(\pi f D)}{\pi f D}\right)^2\right]d\mathbf{f} \tag{19}$$
$$W(\mathbf{f},\alpha) = \sum_{n=1}^{N} W_n(\mathbf{f},\alpha)$$

Truncating the integration at the associated high frequency limit and adding a contribution from the uncorrected "tail" of the power spectrum will include fitting errors due to limited actuator pitch in the performance analysis. As a reasonable quality measure we propose to use the Strehl ratio associated with $\mathrm{RMS}^2(\alpha)$ averaged over the science field. Note that this field average can be carried out directly in Eq.16 and leads to:

$$\{\exp(-2\pi i(l_n - L_m)\mathbf{f}.\alpha)\}_{\text{field}} = \frac{1}{\pi\alpha_{scs}^2}\int_0^{2\pi}\int_0^{\alpha_{scs}} \exp(-2\pi i(l_n - L_m)\, f\alpha\cos(\psi))\alpha d\alpha d\psi \tag{20}$$
$$= \mathrm{Airy}(2\pi(l_m - L_m)\, f\alpha_{scs})$$
$$\{\exp(-2\pi i(l_n - L_m)\,\mathbf{f}.\alpha)\}_{\text{field}} = \mathrm{Airy}(2\pi(l_m - L_m)\, f\alpha_{scs}) \tag{21}$$

When evaluating the performance using Eqs. 16, 18, 19 and 20, 21, the layer transfer functions, that is the LGS and DM configuration, must be known. The preliminary results shown in the following relate to a homogeneous distribution of LGSs with a radius given by Eq. 1 resulting in

$$G_{m,m'}(\mathbf{f}) = \frac{1}{Q}\sum_{q=1}^{Q} a_m^2 a_{m'}^2 \exp(2\pi i(a_{m'}L_{m'} - a_m L_m)\mathbf{f}.\alpha_{\mathbf{q}}) = a_m^2 a_{m'}^2 \mathrm{Airy}(2\pi(a_{m'}L_{m'} - a_m L_m) f\alpha_{\mathrm{LGS}}) \tag{22}$$

$$G_{n,m}^l(\mathbf{f}) = \frac{1}{Q}\sum_{q=1}^{Q} a_n^2 a_m^2 \exp(2\pi i(a_n l_n - a_m L_m)\mathbf{f}.\alpha_{\mathbf{q}}) = a_n^2 a_m^2 \mathrm{Airy}(2\pi(a_n l_n - a_m L_m) f\alpha_{\mathrm{LGS}}) \tag{23}$$

Given the above G functions, the layer transfer functions for one DM, two DM and three DM correction were calculated solving Eq.12. The average Strehl ratios shown in Figures 7a,7b were calculated neglecting fitting errors and assuming Kolmogorov statistics for the seven-layer Cerro-Pachon atmosphere given in Table 1. Hence the layer power spectra are given by

$$W_n(\mathbf{f}) = \frac{0.023}{r_{0,n}^{5/3}} f^{-11/3} \tag{24}$$

Comparing Figure 7a to Figure 7b the focal anisoplanatism can clearly be observed. For the Swedish 50 m ELT, which is planned for DCAO using LGSs (Andersen *et al, this conference*), the focal anisoplanatism will decrease the average Strehl ratio from about 0.9 to about 0.5. Note that since the LGS distribution is assumed to be homogeneous, there will be no degradation due to inadequate sampling of the atmosphere as is seen in Figures 5a,b. The calculations were carried out implementing the equations as symbolic code in Mathematica.

5. CONCLUSIONS

The results shown in this paper indicate that LGS assisted MCAO may present severe problems for ELTs. This relates both to the quite large LGS field resulting in the need for many stars and in the large focal anisoplanatism to be expected. Possibly as suggested by Ribak and Ragazzoni (*this conference*) the field requirement may be met projecting diffraction patterns (extended objects) to the mesospheric sodium layer and using layer oriented wavefront sensing (Ragazzoni, 2000) either based on image plane diffraction or on Shack-Hartman sensors employing inter sub-image correlation as it is done by Solar wavefront sensing. The specific magnitude of the focal anisoplanatism shown here is a direct consequence of the simple LSP algorithm and might be reduced modifying this algorithm. These issues will be the subjects of further investigations.

6. ACKNOWLEDGEMENTS

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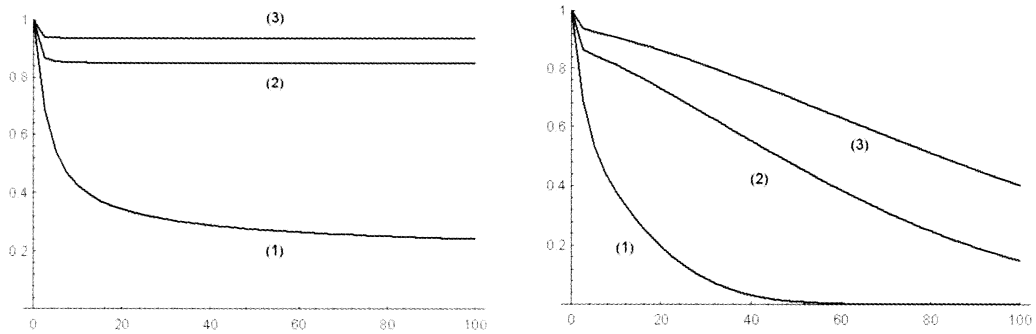

Figure 7. Expected average Strehl ratio versus telescope diameter in m when performing one DM (1), two DM (2) and three DM (3) correction. DM conjugate altitudes in m: (-575) and (-575, 8000) and (-575, 4000, 8000). Guide star distribution: Homogeneous. Figure (7a): Infinitely distant guide stars. Figure (7b): Guide stars at 90 km, α_{SCS} = 30 arc seconds.

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The fastest team at the conference: from the left to the right
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