Dual-conjugate AO Performance for ELTs. Mirror Dynamical Range and Actuator Pitch

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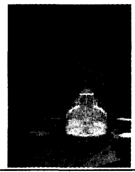
Beyond Conventional Adaptive Optics



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ABSTRACT

Modeling the performance of a dual-conjugate AO system, we use an analytical corrective algorithm given in a recent paper for devising appropriate correction of atmospheric turbulence for ELTs. According to this algorithm the atmospheric correction is achieved based on a minimization of the sum of the residual power spectra of the phase fluctuations seen by the guide stars after correction. Five guide stars are considered to be infinitely distant and distributed in a regular cross on the sky. We present predictions for performance of the Swedish 50 m ELT with Dual-conjugate AO in the K band using a standard seven layer atmospheric model. The average Strehl ratio is used as a quality measure and estimated for different values of actuator pitch and DM2 conjugation altitudes. Influence of outer scale on the mirror RMS strokes is presented. A strategy for choosing the actuator pitch of the DMs is outlined and the performance is investigated for different telescope pointing angles. It is believed that the conclusions drawn here regarding DM stroke and actuator pitch will also be valid for laser guide stars at a finite distance.

1. INTRODUCTION

In a classical adaptive optics (AO) system the atmospheric turbulence is corrected by a single corrective element such as a deformable mirror (DM) normally conjugated to the telescope pupil. Since the atmospheric turbulence has a three-dimensional continuous structure, one DM will be able to compensate only the turbulence in the direct path between the reference source and the telescope. The field angle over which this correction is sufficiently good is limited by anisoplanatism and the isoplanatic angle does not exceed 10-15 arc seconds in the K band. Employing more DMs conjugated to different turbulent layers in the atmosphere will extend the corrected field of view. Such multi-conjugate adaptive systems (MCAO) (Beckers, 1988 and 1989) are very promising for future extremely large telescopes (ELTs).

There are two principle issues related to the concept of MCAO, the first one is the probing of the three-dimensional distributed turbulence, where it is assumed that the continuous atmosphere is divided into several thin layers, and the second issue is the optimal conjugation and control of DMs. Even if the strength of the turbulent layers is known, there is still a problem of optimal atmospheric correction with a number of DMs smaller than the number of layers. An analytical algorithm has been proposed (Owner-Petersen & Gontcharov, in press) for deriving the optimal correction on DMs and modeling the performance of MCAO.

The key idea of the algorithm is to solve the problem of optimal correction in spatial Fourier space. Assuming that the number of guide stars is q = 1...Q and the number of DMs is m =1...M, the residual wavefront error at the telescope pupil can be expressed as

$$\varepsilon_q(\mathbf{r}) = \phi_q(\mathbf{r}) + \sum_{m=1}^{M} \phi_m(\mathbf{r} + L_m \alpha_q), \tag{55}$$

where $\mathbf{r}$ is the two-dimensional vector position in the pupil, $\phi_q(\mathbf{r})$ is the pupil wavefront error for star q, $\phi_m(\mathbf{r})$ is the phase-deformation for mirror m, L_m is the altitude of mirror m and $\alpha_q$is the two-dimensional angular vector position on the sky, see Fig. 1. According to the "Big pupil" approximation (Gontcharov & Owner-Petersen, 2000) the DMs act as additional layers at the corresponding conjugate altitudes in the atmosphere. In two-dimensional spatial Fourier space the residual error is expressed as

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$$\mathrm{E}_q(\mathbf{f}) = \Phi_q(\mathbf{f}) + \sum_{m=1}^{M} \Phi_m(\mathbf{f}) exp(2\pi i L_m \mathbf{f} \cdot \alpha_q) \tag{56}$$

The optimal atmospheric correction is achieved based on a minimization of the sum S(f) of the residual power spectra for the phase fluctuations seen by the guide stars after correction:

$$S(\mathbf{f}) = \sum_{q=1}^{Q} |\mathrm{E}_q(\mathbf{f})|^2 \tag{57}$$

Minimization of the sum $S(\mathbf{f})$ with respect to DM phase-deformation spectra $\Phi_m(\mathbf{f})$ will lead to a system of M linear equations. The solution will give the DM phase-deformation spectra $\Phi_m(\mathbf{f})$, which are transformed back to provide the needed DM phase deformations $\phi_m(\mathbf{r})$.

Estimating the corrective shapes of the DMs, the algorithm relies only on the measured atmospheric wavefronts $\phi_q(\mathbf{r})$ referred to the telescope pupil. Hence, no a priory assumption of the atmospheric statistics is invoked. In the minimization procedure the images of the guide stars are sharpened up in the best possible way leading to a solution, which is similar to the one obtained with other corrective algorithms based on a least squares principle. Perhaps the main advantage of this algorithm is the simplicity of the analytical evaluation of the MCAO performance for ELTs under given atmospheric conditions and given guide stars and mirror configurations. It is used here for performance evaluation of the proposed Swedish 50m ELT with an integrated AO system having two DMs (dual-conjugate AO, Gontcharov & Owner-Petersen, 2000; Andersen et al, 1999; Andersen et al, *this conference*; Gontcharov et al, *submitted*).

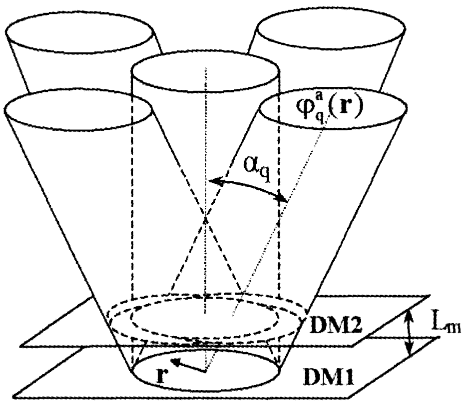

Figure 1. Geometry for phase projections onto the telescope pupil. For simplicity DM1 is shown as being conjugated to the telescope pupil.

Modeling the performance of a dual-conjugate AO system relies on following assumptions. No intrinsic optical aberrations in the telescope and no wavefront sensor measurement errors are included. Five infinitely distant guide stars (NGSs) of infinite brightness are used. They form a regular cross on the sky. In accordance with the "Big pupil" approximation, the common overlap of the five wavefronts at the upper turbulent layer is assumed to have a size close to the size of the telescope pupil. The standard Cerro-Pachon atmospheric model (Vernin et al, 2000) with seven thin layers is implemented for analysis. It is assumed that the layer phase fluctuations have von Karman statistics with zero inner scale.

The optical design of the Swedish 50m ELT consists of a two–mirror Ritchey-Chrétien (RC) system (Wilson, 1996) with adaptively controlled secondary (DM1), which compensates for low altitude turbulence, and a four-mirror Offner-like relay system containing a flat DM2 for high altitude turbulence compensation. Referring the ground level to the primary, the secondary (DM1) is conjugated to – 0.6 km altitude. This altitude is imposed by the RC system geometry and cannot be changed. Hence, the conjugation altitude of DM2 is the only free parameter in the mirror configuration. It is also assumed that the DMs have finite actuator pitch (the distance between adjacent actuators) allowing wavefront compensation up to a maximum frequency limit in two-dimensional spatial Fourier space. Therefore, our modeling takes into account wavefront fitting error (Hudgin, 1977) and angular anisoplanatism (Stone et al, 1994).

2. LAYER TRANSFER FUNCTIONS AND CRITICAL FREQUENCY

Analytical solutions to the system of linear equations for the DM phase-deformations implicate division with a determinant, the value of which depends on mirror and guide star configuration. For certain frequencies the determinant is zero and the solutions are ill conditioned. This can be illustrated with the use of so-called layer transfer functions (Beckers, 1988). A single turbulent layer with the phase disturbance spectrum Φ^{l}_{n}(f) at the altitude l $_n$ will contribute to the wavefront error in the pupil as

$$\Phi^a_q(\mathbf{f}) = \Phi^\ell_n(\mathbf{f}) exp(2\pi i l_n \mathbf{f} \cdot \alpha_q) \tag{58}$$

The layer transfer functions $T_{n,m}(\mathbf{f})$ describe how correction of a single layer is distributed among the DMs.

$$\Phi_m(\mathbf{f}) = -T_{n,m}(\mathbf{f})\Phi^\ell_n(\mathbf{f}) \tag{59}$$

$T_{n,m}(\mathbf{f})$ can be found solving the system of M linear equations for the mirror phase deformations. Performing correction for 5 NGSs positioned in a regular cross with two DMs conjugated to –0.6 km and 8 km altitudes, the layer transfer functions (for layers at 0, 2, 4, 6, 8, 10, 12, 14 and 16 km altitudes) are plotted in Fig. 2.

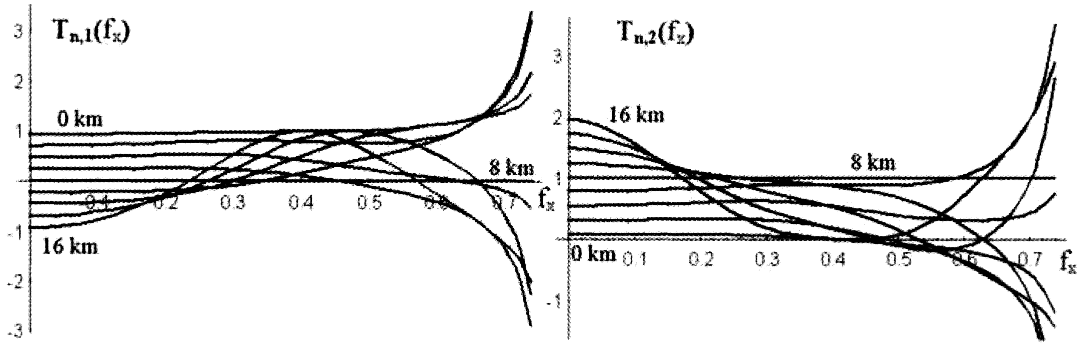

Figure 2. Layer transfer functions for DM1 (left), and DM2 (right). Here f is given in m^{-1} and the cross arm is 30 arc seconds.

It is worth noticing that DM2 corrects all phase disturbance coming from the 8 km layer, whereas DM1 corrects most of the phase disturbance at 0 km layer and does not affect the 8 km layer. The layers which are between the conjugation altitudes of DMs (DM span) are corrected by DMs in co-action ($T_{n,1}(\mathbf{f})$ and $T_{n,2}(\mathbf{f})$ have the same sign) and the layers, which are outside the DM span are corrected in counter action. When the frequency approaches the critical values corresponding to ill-behaved solutions, the amplitude of the layer transfer functions grows to infinity. To assure stable corrections on the DMs one may correct only below the critical frequencies, where the layer transfer functions have finite values. The lowest critical frequency is given by f_0=1/ (L_1t L_2)α_0 , where L_1 and $L_2$are the conjugation altitudes of DMs and α_0 is the angular separation between neighboring guide stars (the cross arm, see Fig. 3). It means that depending on guide star separations and DM conjugation altitude difference there will be critical frequencies, which must be left uncorrected.

A geometrical interpretation of the reason for the critical frequency is depicted in Fig. 3. A single periodic phase disturbance with frequency $f_0$projected onto the DMs along the direction of an NGS will be seen as an identical pattern on both DMs, regardless of the layer altitude. On the other hand, in terms of layer transfer functions, corrective action of DMs depends on layer altitude, which cannot be identified under such conditions. Therefore, there is no information for the algorithm how to distribute corrections among DMs. Avoiding the critical frequency effects, one could correct only up to the maximum working frequency evaluated as f_{max}= 0.95f_0. Alternatively the critical frequencies f_0 ,2f_0 ...can be filtered out from DM actions so that one can use even smaller actuator pitch to reduce fitting errors. This would be necessary in the case of large angular separations of the guide stars.

3. CHOICE OF PITCH SIZE AND IMPACT OF OUTER SACLE ON DM STROKES

Modeling the performance of the Swedish 50 m ELT with Dual-conjugate AO, it is assumed that corrective actions of the DMs have a maximum frequency f_{max}= 0.95f_0, which defines the minimum allowable actuator pitch on both DMs.

$$\text{pitch}_{\text{min}} = \frac{1}{2f_{max}} \tag{60}$$

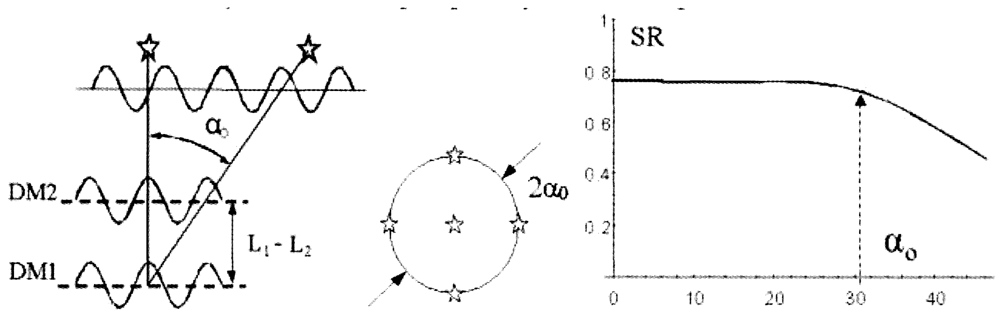

Figure 3. Left: Interpretation of the critical frequency effect. Middle: 5 NGS in a regular cross within $2\alpha_0$field of view. Right: Strehl ratio as a function of field angle along cross arm in arc seconds, $2\alpha_0$= 1 arc minutes.

Performance evaluations are carried out for two fields of view $2\alpha_0$ = 1 arc minutes and $2\alpha_0$ = 2 arc minutes. The Strehl ratio averaged over the 5 NGS is used as quality measure (see Fig. 3). The size of the outer scale L_0 is 50 m. The conjugation of DM1 is fixed to – 0.6 km altitude, whereas the conjugation of DM2 is chosen as 10, 8, 6 and 4 km. These conjugate altitudes correspond to the following minimum actuator pitch values (in units of r_0 imaged on the DM) of 0.95, 0.77, 0.59, 0.41 for $2\alpha_0$ = 1 arc minutes and 1.9, 1.54, 1.18, 0.82 for $2\alpha_0$ = 2 arc minutes respectively. The averaged Strehl ratio is shown in Fig. 4 as a function of actuator pitch, which is identical for both DMs. It is seen that 4 km conjugation gives the best performance, since it has the smallest pitch and consequently the largest r_0/pitch (2.44 and 1.22) and the lowest fitting error. For the Cerro-Pachon atmospheric model the optimal conjugation altitude is approximately 8 km (but very shallow), if one filters the critical frequency and correct the atmospheric phase disturbance spectrum also between $f_0$and $2f_0$. Correction at higher frequencies is not so effective, since in that part the power of the spectrum decreases fast.

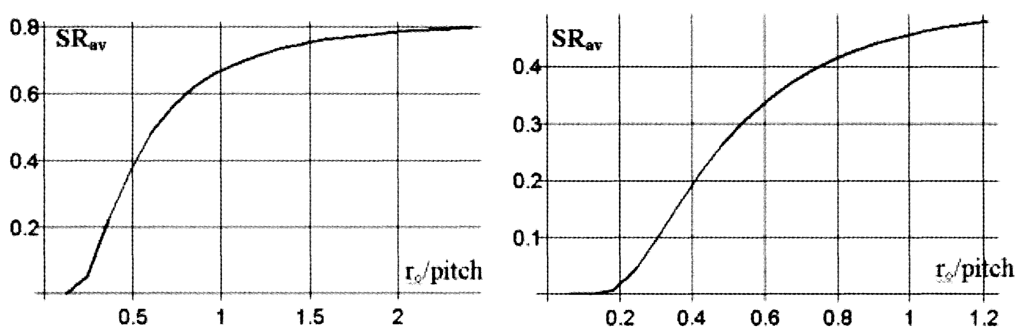

Figure 4. Average Strehl ratio for two fields of view $2\alpha_0$= 1' (left) and $2\alpha_0$=2' (right). DM2 altitudes 4, 6, 8, and 10 km. Note the curves almost coincide.

The optimal choice of actuator pitch depends on the atmospheric turbulence profile and corresponding DM strokes required for its correction. The RMS strokes on DMs are depicted in the Fig. 5 as a function of pitch size for $2\alpha_0$= 1 arc minutes. It is seen that stroke amplitudes remain constant within quite wide range of pitch size, but it grows with decreasing DM2 conjugation altitudes, since many turbulent layers lay outside the conjugation span of DMs. As it will be shown in the next section changing the telescope pointing from zenith increases the DM strokes.

So far the outer scale L_0 was assumed to be equal to the telescope diameter D for all the layers. For larger outer scale the DM strokes will grow reaching their maximum for an infinite outer scale (Kolmogorov model). Fig.6 shows this tendency for a field of view $2\alpha_0$= 1 arc minutes and conjugate altitudes of DM2 at 4,6,8 and 10 km. For the Kolmogorov model the RMS strokes are more than 3 times higher compared to the case of 50 m outer scale. Hence the knowledge of the outer scale for a given astronomical site is of great importance for determining the requirements for the dynamical range of the DMs.

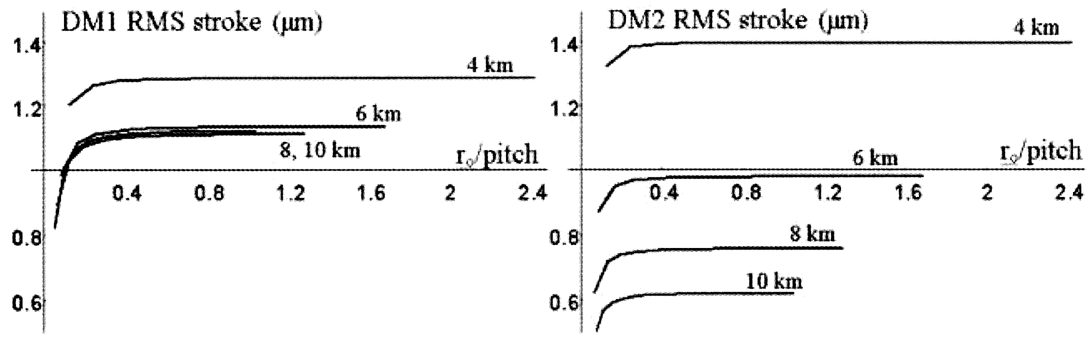

Figure 5. RMS strokes on DMs for different conjugations of DM2, $2\alpha_0$= 1 arc minutes, outer scale 50 m.

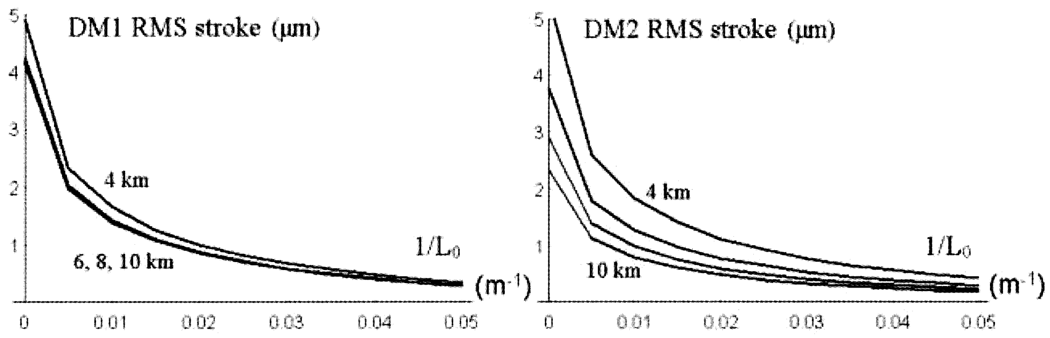

Figure 6. RMS stroke on DMs as a function of reciprocal outer scale size, common for all layers.

4. EFFECT OF TELESCOPE POINTING ON SYSTEM PERFORMANCE

Changes in telescope pointing impose additional requirements on the dynamical range of the DMs and limits system performance. For a given zenith angle Z the power spectrum of a turbulent layer n is given by von Karman as

$$P_n(f) = \frac{0.023}{r_{o,n}^{5/3}\cos Z}(f^2 + f_{os,n}^2)^{-11/6} \tag{61}$$

where $f_{os,n} = 1/L_{0,n}$ is the outer scale contribution. The layer height relative to the telescope entrance pupil becomes $h = h_0/\cos Z$. The conjugation heights of the DMs remain the same. We choose $L_0 = D$ =50 m for all 7 layers as previously. Fig. 7 shows the growth of DM strokes with increasing telescope pointing angle for a field of view $2\alpha_0$= 1 arc minutes. It is evident from a comparison of DM strokes at Z = 0 degrees to those at Z = 60 degrees that the demands on the mirror dynamical range are doubled. The system performance characterized by the average Strehl ratio (SR_{av}) is presented in Fig. 8, where the zenith angle is in the range of 0...60 degrees. A significant drop in the Strehl ratio occurs for zenith angles exceeding 30 degrees. When pointing the telescope far off zenith, DM2 is lowered with respect to the layers and the atmosphere gets thicker along the line of sight. The latter effect is the main factor degrading the system performance. Adjustment of DM2 conjugation altitude with change of telescope pointing will lead to a new value of the critical frequency, which should be either filtered out or used for determining a new upper frequency limit for the correction. It could be an effective way to retain the correction quality for a certain "dual layered" atmospheric turbulence profile, but for the Cerro Pachon profile it gives no significant improvement.

5. CONCLUSIONS

Modeling of DCAO for the Swedish 50 m ELT has been carried out based on an analytical corrective algorithm. Both fitting errors and angular anisoplanatism is accounted for in the estimations of the system performance. The angular anisoplanatism depends on the field of view (see Figures 4 and 8) and has been illustrated by using two fields $2\alpha_0$ = 1 arc minutes and $2\alpha_0$ = 2 arc minutes. The fitting error is given by the DM pitch size, which is limited by critical frequency,

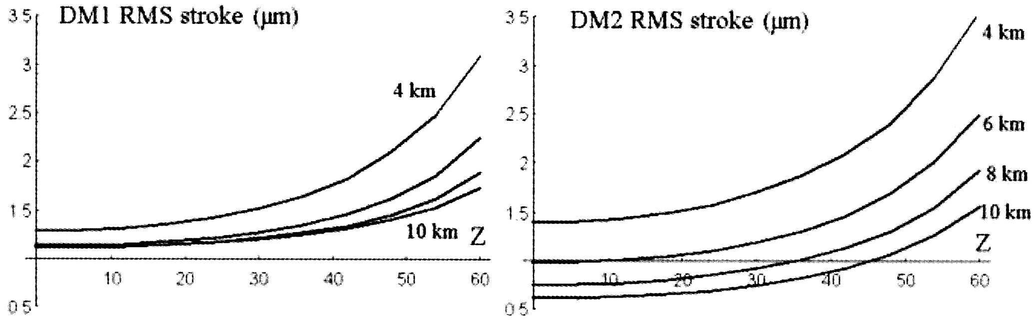

Figure 7. RMS DM strokes as a function of zenith angle Z in degrees. Telescope pointing, $2\alpha_0$= 1 arc minutes.

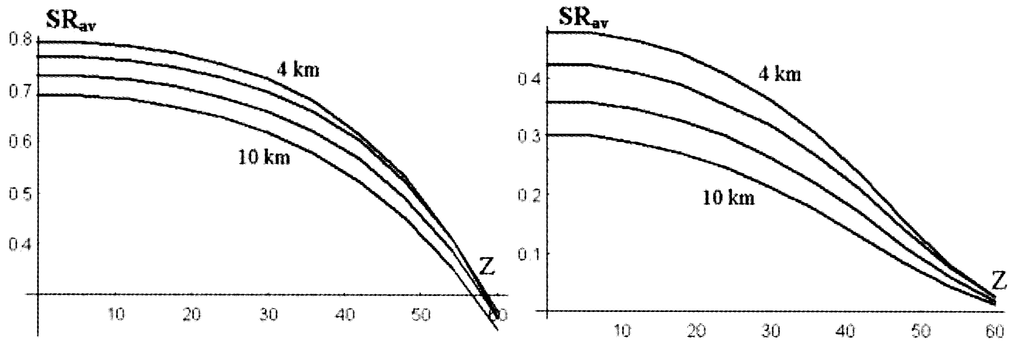

Figure 8. Dependence of average Strehl ratio on telescope pointing angle in degrees, $2\alpha_0$= 1 arc minutes (left) and $2\alpha_0$ = 2 arc minutes (right). Correction is performed up to f_{max}. The degradation with increasing DM2 conjugate altitude is due to the fitting error.

unless the critical frequencies are filtered out. The lowest critical frequency is defined by the angular star separation and the conjugation span of the DMs. Therefore, a low DM2 conjugation altitude allows a smaller actuator pitch and gives larger RMS strokes on the DMs. The optimal DM2 conjugation altitude should be found by taking into account the atmospheric profile and the size of the outer scale, which is very important for determining the required DM strokes.

Modeling the telescope pointing effect has shown that for zenith angles exceeding 30 degrees the system performance deteriorates rapidly and the actuator strokes increase. Changing the conjugation altitude of DM2 with pointing will not significantly improve the system performance for a distributed atmospheric profile, but it could be beneficial in some special cases, where a single strong turbulent layer at a high altitude is present.

We believe that $SR_{av} \sim 0.7$ is optimal in the K band. It can be achieved for a field of view of 1 arc min with 5 NGSs and 8 km conjugated altitude of DM2. The actuator pitch is 0.77 r_0 and the DM RMS strokes are around 1 micron for 50 m outer scale. Using laser guide stars (Owner-Petersen & Gontcharov, *this conference*), one should expect only a decrease in system performance (the Strehl ratio is reduced from 0.75 to about 0.5 for zenith pointing), but the mirror strokes and the actuator pitch are expected to remain nearly the same.

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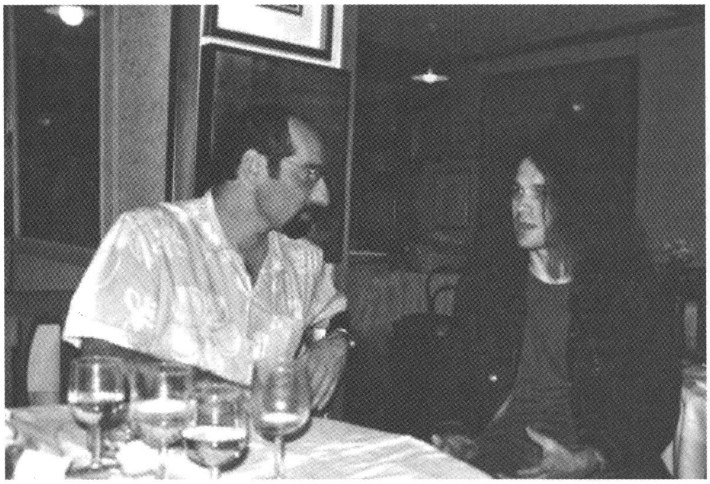

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