necessarily be eclipses at every conjunction, and even if there *were* they would in general not occur at the computed times.

With a putative linear separation of about 1·4 AU and a distance getting on for 1 kpc, the components of HD 214974 can be expected to show a maximum angular separation, according to the model above, of hardly 0″·002; it will of course occur at the nodes of the orbit.

Acknowledgement

It is pleasure to thank an anonymous referee for curbing a number of overstatements that existed in the original version of this paper.

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MEASUREMENTS OF THE SOLAR DIAMETER IN KEPLER'S TIME

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We present five measurements of the solar diameter made by Tycho (1591) and Kepler (1601–2) with pinhole instruments, reduced and published by Kepler in 1604[1]. We reproduce their experimental procedure, recovering and discussing their systematic errors.

Introduction

The measurements obtained by projection onto a screen of the solar image generated by a pinhole, made by Tycho and Kepler[1], are the first direct and detailed measurements of the diameter of the solar disc available in the literature, after those made by Aristarchus[2,3] (*circa* 310–230 B.C.), Archimedes[2] (287–212 B.C.), and Ptolemy[4] (*circa* 87–151 A.D.) using dioptræ of the same type as those described by Hipparchus (second century B.C.) and reported by Pappus (fourth century A.D.) in his *Commentary on the Almagest*[4]. Those dioptræ allow one to compare the apparent diameter of the Sun or the Moon with a target disc moving on a rigid rod. The Sun was observed directly with the naked eye at sunrise or sunset in order to prevent eye damage[2]. Allowing for the finite diameter of the pupil, Archimedes gave a range of values for $\vartheta_\odot = [33' - 27']$ for the actual solar diameter. Archimedes quoted also a measurement of $\vartheta_\odot = 30'$ made by Aristarchus. Ptolemy found a systematic discrepancy between the direct measurement with the dioptra and the angular dimensions from calculations involving lunar eclipses. In fact, he judged the solar diameter equal to the lunar one at apogee and calculated (indirect measurement) the latter from lunar eclipses at apogee.

Ptolemy found the solar diameter to be $\vartheta_\odot = 31' 20''$ with no variation worthy of mention during the year[4]. Albategnius (Al-Battani, *circa* 868–929) criticized that affirmation, basing his proof on his observations of two solar eclipses[5] (indirect measurement), while Copernicus quoted him to prove his theory, although he used the measurement of Ptolemy increased by $10''$ (no new measurements) because it fitted his data better[6]. Riccioli[7] in 1656 reviewed the measures of the solar diameter ranging from a minimum of $30' 30''$ given by Kepler[1] to a maximum of $32' 44''$ given by Copernicus[8].

The reasons for the large differences between the data expected from the ephemerides calculated with the actual mean solar diameter of $1919''\cdot 26$ ($\sim 32'$) and Tycho's and Kepler's actual measurements are to be found in their data-analysis procedure that we call 'geometrical correction'. In fact, Kepler's and Tycho's values are systematically ~ 1 arcminute smaller than the actual mean value (*i.e.*, the value for an Earth–Sun distance of 1 AU). We have reproduced those historical experiments with an equivalent device to calculate the uncertainties and explain the above-mentioned systematic error. We then re-reduced the original data, recovering values consistent with the actual mean solar diameter.

Methods of measurement and expected accuracy

The effect of the pinhole projection was already described by Aristotle[9] (384–322 B.C.); it was exploited by Tycho and Kepler at the turn of the sixteenth century for measuring the solar diameter. Kepler also explained its principles.

In 1591 Tycho performed eleven measurements of the solar angular diameter with pinhole instruments[10]. Kepler selected and reduced two observations made by Tycho on 1591 April 12 and December 5 (Julian Calendar dates)[10]. Tycho used a closed tube 2-m long mounted on a side of a wooden *quadrans*[10]. At the upper end of the tube was a square pinhole opening of 2 cm (Aristotle presented among his *Problemata* a section on the way a square pinhole produces round images of the Sun[9]). Tycho measured the diameter of the circular image produced on the lower screen[10].

As with every scientist of that time, Kepler used to make his own instruments for different experiments. The one used by Kepler for the measurement of the solar diameter is shown in Fig. 1; it is 4-m long and has an altazimuth mount. The 6-mm pinhole is located in the mask, M, and the light beam ends on the screen, S, situated in a dark camera. During the measurement ($\sim$ 1 s), the motion of the solar disc on the screen (and on the sky) is $\sim$ 15 arcseconds due to the diurnal motion of the Sun. Therefore Kepler's method could achieve no better accuracy than about 15 arcseconds. To minimize his errors, Kepler prepared in advance a set of different discs to be placed on the screen S, that could be quickly compared with the Sun's image. Additional errors arose from the measurements of the pinhole's diameter, d_p, in the mask M, and of the focal length $f = |\vec{MS}|$ (see Fig. 1).

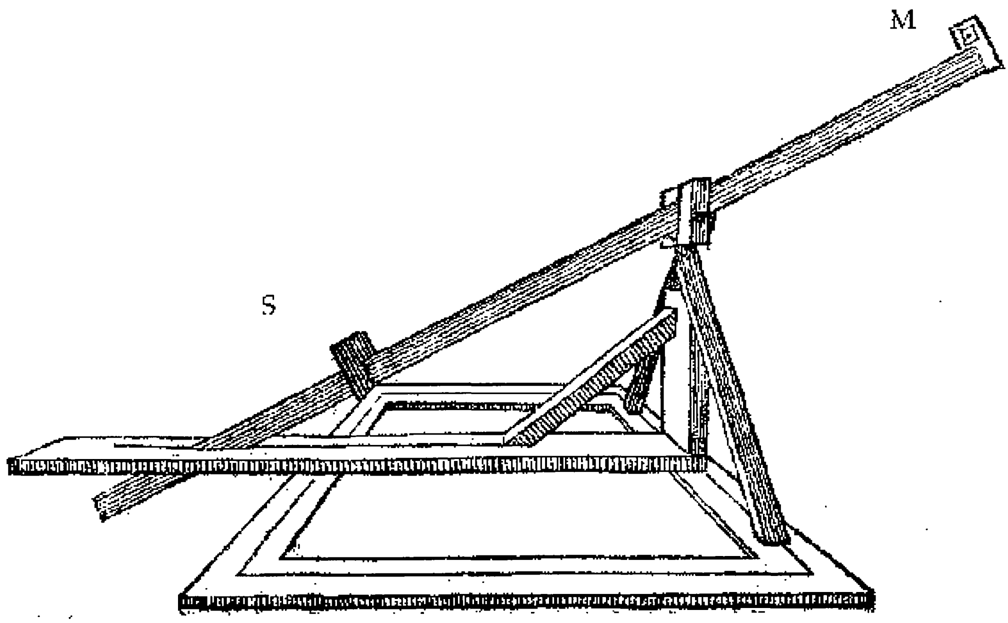

FIG. 1
Kepler's instrument for measuring the solar diameter[1].

Correction of the data

The 'geometrical correction' is the algorithm used both by Tycho and Kepler and consists firstly of subtracting the diameter of the pinhole, d_p, from the measured diameter, d_m, of the solar disc upon the screen:

$$d_{corrected} = d_m - d_p, \qquad (1)$$

and then computing the angular diameter of the Sun, $\vartheta_\odot$, from the formula:

$$\vartheta_\odot = \arctan\,(d_{corrected}/f). \qquad (2)$$

This operation suggests immediately the advantages of performing new measurements with a larger focal distance in order to reduce the relative error: while the diameter of the image, d_m, rises linearly with the focal distance, the diameter of the pinhole becomes relatively smaller. Kepler's problem was to build stable instruments larger than 4 m; Tycho's instrument was even smaller

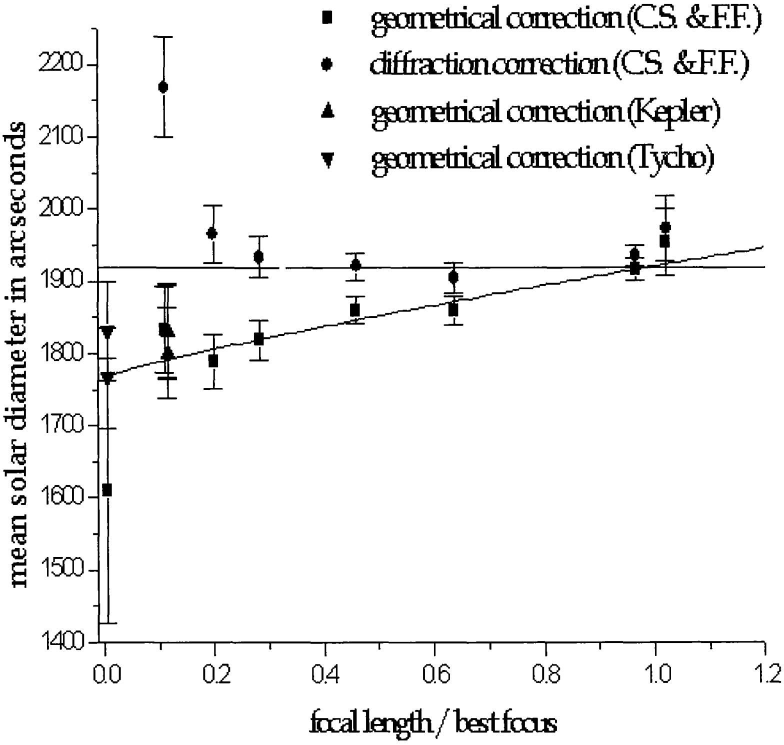

FIG. 2

'Geometrical correction' and 'diffraction correction' in comparison. All the data are rescaled for different Sun–Earth distances (from the ephemerides) and compared with the actual mean solar diameter of 1919·26 arcseconds (horizontal line). The fitting curve $y(x) = 155 \cdot x^{0 \cdot 84} + 1766$, with $x = f/f_b$ (reduced $\chi^2 = 0 \cdot 56$) calculated with our 'geometrical-correction' data has been used for evaluating the systematic error and reducing the historical data in Fig. 3. The original data of Tycho and Kepler with the 'geometrical correction' are also plotted as triangles.

than Kepler's, with larger relative uncertainties. We avoided this problem by locating a plane mirror before the mask M, in order to orientate the light beam horizontally, and thus easily obtained focal lengths $f \geqslant 20$ m. Comparing our data (see Fig. 2) processed with the 'geometrical correction' with the actual mean solar diameter of $1919'' \cdot 26$, we recovered the amount of the systematic scaling factor between the true value and the corrected one for the focal lengths adopted by Tycho and Kepler. Finally we used that scaling factor to recover the true values of the historical data shown in Fig. 3. For evaluating the uncertainties of Tycho and Kepler's measures, we adopt the smallest fraction of the unit of measure that Kepler reported in his text[1].

Diffraction enlarges the thickness of the solar limb and at a given focal length, f, its amount is given by the first zero of the Bessel function:

$$r = f \cdot \frac{1 \cdot 22\lambda}{d_p}; \quad (3)$$

The 'diffraction correction' is obtained by subtracting this from the measured diameter:

$$d_{diffraction} = d_m - r, \quad (4)$$

which produces values of the diameter systematically larger than the expected value.

A best focal length, f_b, has been defined by Joseph Petzval and reported by Rayleigh[11]: $f_b = (d_p)^2/2\lambda$, where $\lambda = 5 \cdot 5 \times 10^{-7}$ m is the wavelength of the light (assumed monochromatic). The existence of an optimal focal length is shown by the following argument. The geometrical distortion becomes significant for short focal lengths, when the dimension of the solar image approaches d_p; therefore the diameter of the whole image has to be $d_m \gg d_p$. Similarly, diffraction affects the resolution of features that on the solar surface are closer than an angular distance $\theta_{Rayleigh} = 1 \cdot 22\lambda/d_p$. It corresponds to a dimension on the image, r, and increasing the focal length and d_m, the same information (in terms of resolution) is spread out over a larger surface, losing contrast. Hence d_m can not be arbitrarily big. Therefore the best focus, f_b, is obtained when $r \sim d_p$, and therefore

$$f_b \sim \frac{1}{\lambda} d_p^2. \quad (5)$$

In Fig. 2 we obtain also an experimental confirmation of the Petzval formula, with no systematic errors on the measurements made around the best focus f_b.

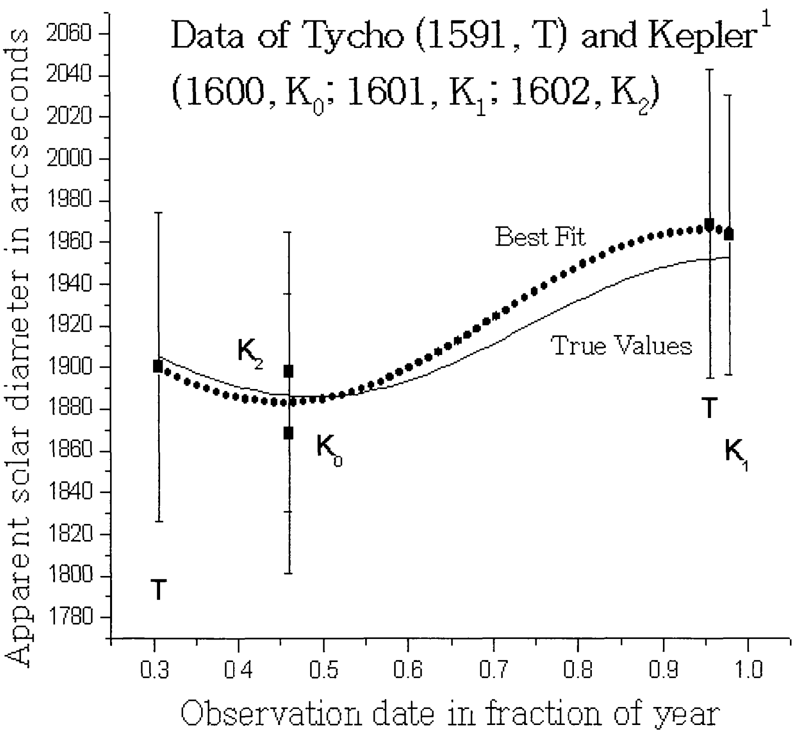

FIG. 3

The data taken by Tycho and Kepler starting in 1591 April 12, Julian Calendar, corrected for the systematic error due to their 'geometrical correction' and plotted in Gregorian dates. The seasonal variation of the apparent solar diameter is recovered fitting the data with a sinusoid of period one year.

Conclusions

The diffraction of light was discovered by Grimaldi[12] in 1665, decades after Kepler's measurements, although some effects of light interference were noticed in 1646 by two Bohemian scholars, Balthasar M. Haněl and Balthasar Conrad[13], repeating the experiments of Christofor Scheiner for measuring the solar diameter in a 'camera obscura'[14]. Therefore Tycho and Kepler could not apply the diffraction correction to their data. Applying the modern error analysis to their data we recover the seasonal variation of the apparent solar diameter with a mean solar diameter of 1924″ ± 35″ at 95% confidence level, consistent with the actual mean solar diameter (see Fig. 3).

The conclusion of Kepler "*Sed res certa est et cuilibet obvia exploratu, diametrum Solis in apogæo 30′ in perigæo 31′ esse,*" that the solar diameter at aphelion is 30′ and at perihelion 31′, is consistent with all the measurements made with pinholes that he examined, and also with the actual values of 31·5′ and 32·5′ once the systematic 'geometrical correction' is taken into account.

Acknowledgments

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CORRESPONDENCE

To the Editors of 'The Observatory'

Nevertheless, Galileo... Galilee... same thing!

As an avid reader of your witty *Here & There* column, I certainly did not miss your 2001 June entry regarding the *Financial Times* misprint, which converted 'Galileo' into 'Galilee'. But the fun doesn't end there (nor here)! I have been intrigued by the possible meaning of Galileo Galilei's name, both on account of the similarity between his first and last names, and on account of the similarity of each to the famous name 'Galilee'. At first I did take a peep into an Italian-English dictionary, but for definitive results I eventually managed to contact a *bona fide* Italian native, who answered my questions: Mr. Marco Berni, of the Institute and Museum of History of Science in Florence, Italy, to whom