

THE ASTROLABE AND SPHERICAL TRIGONOMETRY IN MEDIEVAL INDIA

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1. SPHERICS AND THE INDIAN ASTRONOMICAL TRADITION

The Ptolemaic model of the universe relies on the techniques of Greek spherical trigonometry to derive quantitative predictions from its complex mechanism of uniform circular motions and nested spheres. Classical Indian astronomy, on the other hand, while employing many of the same spheres and circles to describe celestial motions, appears to have remained unaware of the corresponding mathematical methods for the exact solution of spherical triangles. Their function is for the most part amply fulfilled in Sanskrit astronomical texts by the use of plane trigonometry in orthographic projections or analemmas,¹ in formulae apparently derived by means of the relations between plane triangles inside the sphere,² and in approximations assuming small spherical triangles to be plane.

The Islamic descendants of the Ptolemaic model inherited its spherical trigonometry, which was further extended by the mathematicians of the medieval Islamic world. Some of the rules thus developed were eventually incorporated into Indian works dealing with western astronomy, particularly with that popular astronomical device, the Islamic astrolabe.³ But the earliest such work, which was also perhaps the most influential, appears to misunderstand several of the rules in question, blending the resulting incorrect formulae with various approximation devices with an inclusive resourcefulness very characteristic of Sanskrit mathematics. The consequent inconsistencies in the rules, their explanations, and the data indicate that the elements of this new trigonometry upon the surface of the sphere were not fully grasped by their Indian recipients.

2. THE YANTRARĀJA

In A.D. 1370 a Jaina astronomer at the Delhi court of Fīrūz Shāh Tughluq, Mahendra Sūri, wrote (in metrical Sanskrit stanzas, as was customary for medieval Indian scientific works) the *Yantrarāja*, a treatise on the astrolabe (Skt. “yantrarāja”, “king of instruments”), for which his student Malayendu Sūri later provided tables and a commentary.⁴ This appears to be the first Indian work to concern itself explicitly with the methods of medieval Islamic astronomy. Mahendra claims to have written the *Yantrarāja* to provide an orderly and concise summary of the knowledge concerning the theory and construction of astrolabes that was available to him in a profusion of Islamic texts (although it does not seem likely, for reasons that will become clear, that he himself possessed any great mastery of Arabic or Persian).

In his opening verses, he states:⁵

apāre saṃsāre katikati babhūvur na caturāḥ
 paraṃ tair durbodhaṃ gaṇitam araci prauḍhamatibhiḥ ||
 tataḥ svalpaṃ sāraṃ viśadam idam atyantasugamaṃ
 vitanve 'haṃ śāstraṃ suhrdayahrḍānandakṛtaye ||2||

klṛpās tathā bahuvidhā yavanaiḥ svavāṇyā
 yantrāgamā nijanijapratibhāviśeṣāt ||
 tāt vāridhīn iva vilōḍya mayā sudhāvat
 tatsārabhūtam akhilam praṇigadyate 'tra ||3||

In endless transmigration, some have not [yet] become clever; and [even] for those with developed minds, mathematics is difficult to understand. Therefore I rendered as a small epitome this brilliant, entirely complete treatise, to gladden the hearts of friends.

*There are very many treatises on instruments produced in their own language by Westerners, each from the peculiarity of his own intelligence. So having stirred them together like the oceans, I now proclaim the best part, as it were the nectar, of that in its entirety.*⁶

Unfortunately, however, the nectar of knowledge that Mahendra was able to churn from this ocean of learning remained somewhat contaminated by the poison of ignorance; and the resulting procedures reveal a curious blend of misinterpretation and re-interpretation according to approximative methods. Furthermore, Mahendra’s partial comprehension was evidently transmitted to his student and commentator Malayendu Sūri, and in the use of this widely popular text during the subsequent centuries, so far as is known, these inconsistencies were never remarked.

3. THE CALCULATION OF EQUATORIAL COORDINATES

The particular application of spherics encountered in the *Yantrarāja* involves conversion from one system of celestial coordinates into another. To fix a star-pointer in the astrolabe’s rete, which contains a stereographic projection of the celestial sphere onto the plane of the equator centred at the celestial pole(s), it is necessary to calculate from the standard set of ecliptic coordinates (celestial longitude and latitude, or λ and β) two distances computed with respect to the pole of the equator: the ‘true’ declination δ and (instead of the orthogonal right ascension α) the polar longitude λ_* (see Figure 1). Then the cosine of the star’s declination δ determines its radial distance from the centre of the rete (the segment OS in Figure 2), and the angular separation between this radius and the east–west line is $\alpha(\lambda_*)$, the right ascension corresponding to the polar longitude.

Mahendra does not name any of the “very many” foreign sources for the coordinate transformations in which the *Yantrarāja*’s rules originated, and I

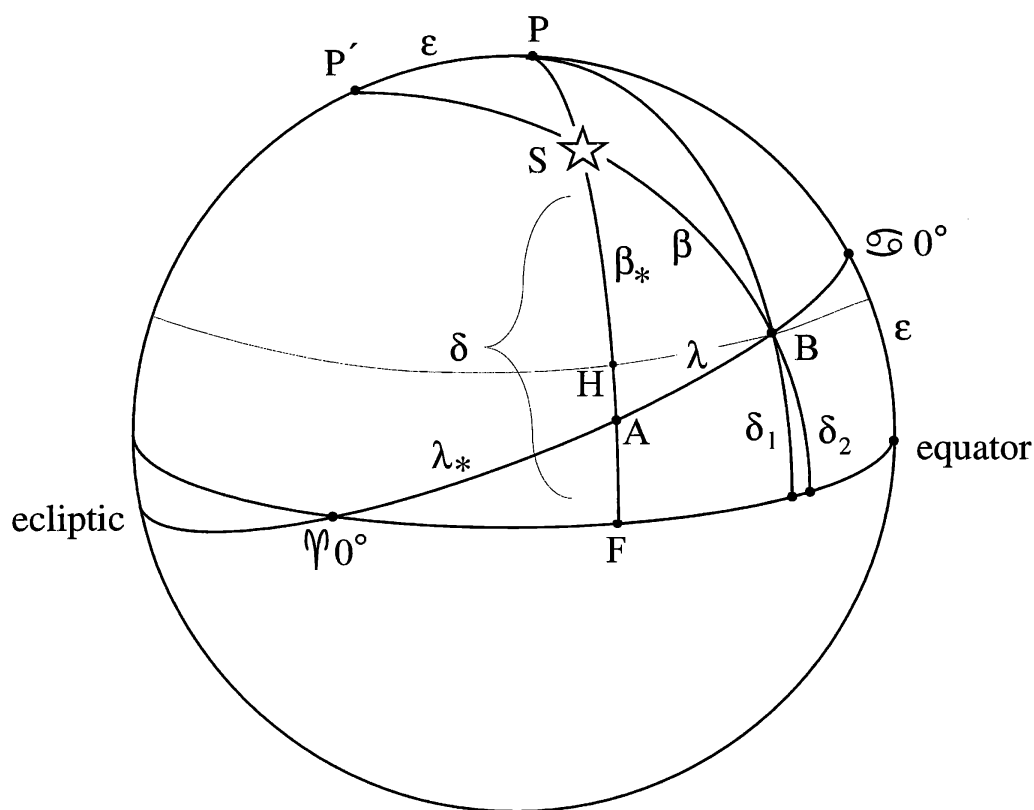


FIG. 1.

have not been able to identify any Islamic text dealing specifically with astrolabe construction that seems to be a likely candidate.⁷ But the quantities δ and λ_* are employed in various contexts in zijes, and the formulae for computing them are frequently stated and sometimes proved.

3.1. The Use of δ and λ_* in the Pre-Islamic Indian Tradition

In classical Indian astronomy, on the other hand, star coordinates are much less rigorously determined;⁸ and the methods routinely used for calculating them often depend upon approximations. The true-declination calculations in general use in Indian texts (for such purposes as the computation of conjunctions of stars and planets) usually involve taking the sum or difference of some version of ecliptic declination and latitude. Where the polar coordinates λ_* and β_* are known (for example, in some star-lists), δ is correctly computed⁹ from them and from the ecliptic declination δ_1 of λ_* by

$$\delta = \delta_1(\lambda_*) \pm \beta_*. \quad (1)$$

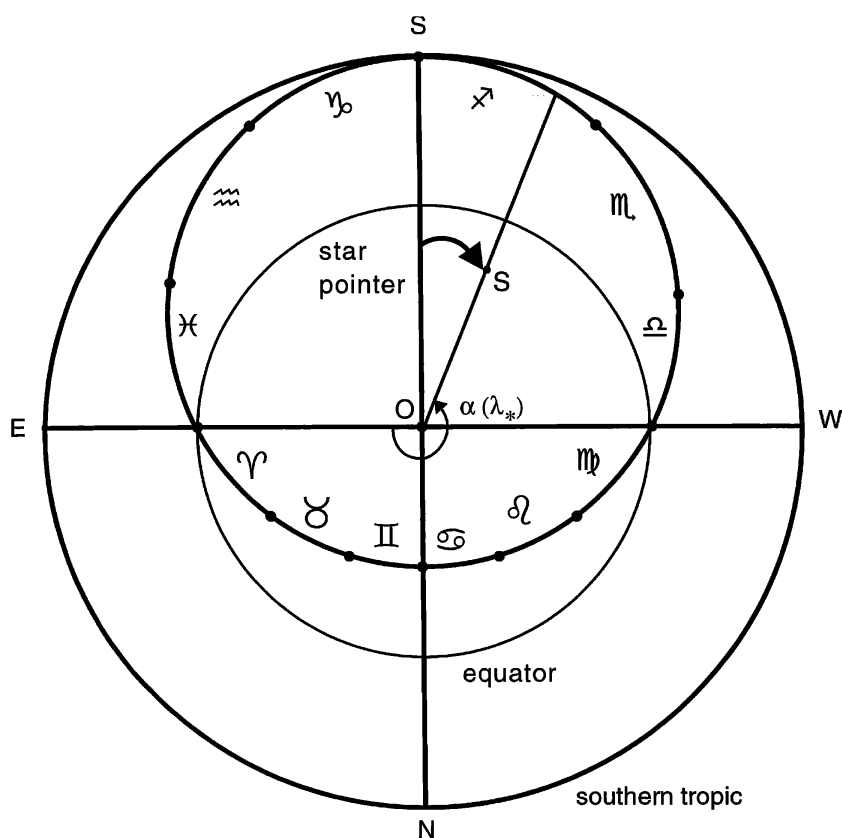


FIG. 2.

However, the polar coordinates themselves are generally produced by approximate formulae. Occasionally, they are even used interchangeably with ecliptic coordinates.¹⁰ Somewhat more correctly, λ_* can be derived from the so-called “ayanadr̥kkarma” or visibility correction due to the deviation of the ecliptic: namely, the determination of the ecliptic arc λ_* , the longitudinal difference between the ecliptic and polar longitudes.¹¹ A typical formula for this quantity¹² is

$$\lambda_* \approx [(\text{Vers } (\lambda + 90)) \cdot (\text{Sin } \varepsilon) \cdot \beta] / R^2. \quad (2)$$

This approximation can be explained with reference to Figure 1. If a small circle is drawn through B parallel to the equator, it will intersect arc SF in some point H , and the arc BH is taken approximately equal to $BA = \lambda_*$. Then if the right triangle SHB is considered to be plane, $BH/\beta = \text{Sin } \angle HSB/R$. It is clear that HSB is zero when $\lambda = 90$, and it is assumed that it is equal to the ecliptic obliquity ε when $\lambda = 0$ (true only if $\beta = 0$). The Versine function is then selected to give interpolated values between these extrema: $\text{Vers } (\lambda + 90)/R \approx (\text{Sin } \angle HSB)/(\text{Sin } \varepsilon)$, from which Equation (2) follows. From this, β_* can be derived by assuming that $\beta_* = SA \approx SH$, and solving for SH by the Pythagorean theorem in the plane triangle SHB ; and finally, Equation (1) can be applied to find a correspondingly approximate value of

δ. In other situations, it is even less rigorously assumed¹³ that

$$\delta \approx \delta_1(\lambda) \pm \beta.$$

3.2. The Yantrarāja's Approach to True Declination (δ)

Mahendra's discussion of these coordinate transforms in his chapter on constructing an astrolabe begins with the determination of the stellar declination δ, for which two rules are given. The first of these (Yantrarāja 1, 41–45) is as follows:

meṣādāv uttaro golas tulādaḥ dakṣiṇaḥ smṛtaḥ ||
trirāśiyuktabāhujaḥ bāṇakoṭijayā hatā ||41||

bhājyāṃtyayā tataś cāpe navateḥ śodhite punaḥ ||
guṇena bhaktā bāṇajyā prāptaṁ bhāgādikaṁ tataḥ ||42||

syād aṃtaraṁ parākṛāṃtis tena yuktā tathonitā ||
gole bhabāṇayor aikye bhede sādhyas tato guṇaḥ ||43||

hataḥ pūrvadhanurmaurvyā bhaktaś cāṃtyajyayā tataḥ ||
labdhāc cāpaṁ yutaṁ hīnaṁ kartavyaṁ ca śareṇa tat ||44||

gole bhapattriṇoraikye bhede syād apamaḥ sphuṭaḥ ||
yāmyasaumyavibhāgena madhyarekhāsamāśritaḥ ||45||

The northern hemisphere is understood [to be] in [the six signs] beginning with Aries, the southern in [the six signs] beginning with Libra. The Sine of the arc [of longitude] increased by 90 degrees is multiplied by the Cosine of the latitude, and divided by the last [Sine, i.e. R]. When the arc from that is subtracted from 90, the Sine of the latitude is divided by the Sine [of that arc]. The result from that is in degrees, etc.; it should be the “difference” [i.e., an additive correction]. The maximum declination [ecliptic obliquity, ε] is increased or diminished by that when [the longitude of] the star and the latitude are in the same or different hemispheres [respectively]. The Sine produced from that is multiplied by the Sine of the previous arc and divided by the Radius. Then the arc from the quotient is to be increased or diminished by the latitude when [the longitude of] the star and the latitude are in the same or different hemispheres. [That] should be the correct declination, either south or north with respect to the equator.

This four-step procedure defines two new quantities here termed λ' and β', which may be expressed in terms of the arcs shown in Figure 1 as follows:

$$\cos \lambda' = [(\cos \lambda) \cdot (\cos \beta)]/R, \quad (3)$$

$$\beta' = (\sin \beta)/(\sin \lambda'). \quad (4)$$

Then it uses these quantities to obtain the declination by means of an additive

correction to the latitude:

$$\sin \Delta\delta = [(\sin (\varepsilon \pm \beta')) \cdot (\sin \lambda')]/R, \tag{5}$$

$$\delta = \beta \pm \Delta\delta. \tag{6}$$

Malayendu’s commentary provides a worked example of the procedure which confirms this interpretation.¹⁴

Mahendra’s second rule for δ follows immediately in verses 46–48. With slight emendations to the sandhi of the printed text, it reads:

athavānīya bhakrāṁtiṁ nakṣatradhruvakāt tataḥ ||
tathā śarasya saṁyogo viyogo ’traikyabhedataḥ ||46||

tasmāj jīvā hatotkrṣṭakrāṁtikotijyayā tathā ||
uktabhakarāṁtikotijyā bhaktā prāptād dhanur bhavet ||47||

etāvat krāṁtibhāgāḥ syuḥ saumyā yāmyā yathā vidhi ||
krāṁtisāyakayor yoge viyoge śeṣataś ca dik ||48||

Or else, when one has found the declination of the star from its longitude, then the sum or difference of the latitude and that [is made], depending on the sameness or difference [of their directions]. Then the Sine of that is multiplied by the Cosine of the maximum declination and divided by the Cosine of the declination of the given star. The arc should be [computed] from the quotient. Those are the degrees of declination [δ], north or south according to the direction of the result of the sum or difference of the declination and the latitude.

Malayendu’s remarks and worked example accompanying these verses make it clear that these instructions are equivalent to the expression

$$\sin \delta = [(\sin (\beta \pm \delta_1(\lambda))) \cdot (\cos \varepsilon)]/(\cos \delta_1(\lambda)). \tag{7}$$

3.3. The Relation of the δ -rules to Islamic Counterparts

The *Yantrarāja*, although it claims to be an exposition of “Yavana” or Western astrolabe theory, actually intermingles to some extent Islamic and traditional Indian approaches, not always intentionally and not entirely successfully. The first example of this is provided by the rule in Equations (3)–(6), which bears an obvious similarity to the following procedure given by Ibn Yūnis in his *Hakimī Zij* of 1003:¹⁵

$$\cos \lambda' = [(\cos \lambda) \cdot (\cos \beta)]/R, \tag{8}$$

$$\sin \beta' = [(\sin \beta) \cdot R]/(\sin \lambda'), \tag{9}$$

$$\sin \delta = [(\sin (\varepsilon \pm \beta')) \cdot (\sin \lambda')]/R. \tag{10}$$

Mahendra’s presentation accurately reproduces the first step from Equation (8), but distorts the following ones. Evidently, the initial source of the corruption of the

Islamic rule was the loss of a factor of R from Equation (9). (Such computations in *zīj*es employing the standard value $R = 60$ often omit to specify explicitly multiplication or division by R , merely lowering or raising the sexagesimal place of the answer instead; and some such technique in Mahendra's ultimate source may have assisted the misunderstanding.) The smallness of the result $\sin \beta / \sin \lambda'$ then led Mahendra, or perhaps some earlier recipient of the corrupted formula, to interpret it as an arc rather than a Sine. Similarly, the consequent smallness of the result of Equation (5) suggested that it should be taken not as the desired final result δ but as an additive correction to β . (The confusion may have been increased by the deceptively similar use of the sum of β and the *second* declination δ_2 in the rule described below; also by the combining of latitude and ecliptic declination δ_1 in the traditional Indian approximation discussed above.) Having more or less saved the phenomena in this fashion, the tinkerer left it at that, and it was this version that was eventually preserved and studied by Malayendu.

The formula in Equation (7) is also identifiable with, but not identical to, a "Yavana" counterpart. The version presented by Ibn Yūnis¹⁶ and later by Tūṣī in the *Zīj-i Ilkhānī*¹⁷ states:

$$\sin \delta = [(\sin (\beta \pm \delta_2(\lambda))) \cdot (\cos \varepsilon)] / (\cos \delta_2(\lambda)), \quad (11)$$

where $\delta_2(\lambda)$ is the so-called "second declination" or arc of the secondary to the ecliptic passing through λ between the ecliptic and the equator. The only difference in Mahendra's version is the substitution of δ_1 for δ_2 as the "declination of the star" (an interpretation confirmed not only by Malayendu but by the fact that the verses nowhere give a rule for computing δ_2). Did Mahendra Sūri (or whoever was originally responsible for its formulation) simply mix up the two declinations, or did he deliberately replace the second by the first to simplify the computation? The modified formula has the advantage of not requiring the calculation of δ_2 , which in any case is usually close in value to the ecliptic declination δ_1 . Hence the originator of the rule in its present form made either a convenient approximation or a very natural mistake, and it is probably not possible to say with certainty which it was.

Given these discrepancies between the original formulations of the above δ -rules and their representations in the *Yantrarāja*, it seems unlikely that Mahendra was himself entirely competent to read and understand such sources as the abovementioned *zīj*es. Rather, he probably depended at least partially on the assistance of translations, Muslim colleagues, and/or interpreters,¹⁸ one or more of whom inadvertently misled him about some details of the procedures. In short, these spherical trigonometry formulae were adopted, via any number of intermediary interpretations, by astronomers whose own tradition supplied little or no background in the underlying theory, and some corruption of the rules was a natural result.

Possibly adding to the confusion was the fact that in many cases, the results produced by the corrupted versions are not glaringly wrong. The rule in Equation

(7) is of course exact at the extrema $\lambda = 0$ and $\lambda = 90$ (since there $\delta_1 = \delta_2$), and usually within 1° of the true value of δ elsewhere. The first approximate formula, from Equations (3)–(6), is exact only for $\beta = 0$, when $\delta = \Delta\delta = \delta_1$. As β increases, the accuracy of the rule deteriorates rapidly for $\lambda \ll 90$. These discrepancies, though uncorrected, did not go entirely unnoticed: Malayendu’s commentary compares the results of the *Yantrarāja*’s two formulae in the case of the star Rohiṇī (α Tauri) and acknowledges the small difference between them, but adds that “because of its smallness, it is not wrong”. (This is by no means always the case; Malayendu has been fortunate (or perhaps diligent) in selecting an example where the agreement between the rules is good.) This remark seems to indicate that one if not both of the formulae is expected to be imprecise. The value of δ in the worked example illustrating the method of Equation (6) also appears in Malayendu’s star table; therefore, at least some of these numbers were worked out by one of Mahendra’s methods rather than borrowed from a foreign table (as is the case with the ecliptic coordinates).¹⁹ A recomputation of all the coordinates is required to determine exactly how the calculations were made; but in any case it is clear that whatever imprecision was noted by Mahendra or his pupil, this was not considered bad enough to inspire a stricter examination of the rules’ theoretical soundness.

3.4. *The Yantrarāja’s Procedures for Polar Longitude (λ_*), and Their Relation to Islamic Astronomy*

The next step in the coordinate conversions of the *Yantrarāja* is the determination of the polar longitude λ_* , traditionally accomplished in Indian astronomy by the “ayanadr̥kkarma” procedure described above. Again, Mahendra gives two different rules for this computation: the first is an accurate rendition of a solution from spherics, while the second is a standard Indian formula that, however, shows evidence of Islamic influence. The first (*Yantrarāja* 1, 49–52) is explained as follows:

dr̥kkarmaṇy ayanam matvā mrgādyam atha karkajam ||
satribhajyām parakrāṁtimauryā hatvā vibhājitaḥ ||49||

uktodukrāṁtikot̥yākhyamauryā prāptam kalādi yat ||
tena hatveṣukot̥ijyām vibhājyāṁtyajyayā tataḥ ||50||

labdhāt kodaṁdakot̥ijyām utpādyā bhaśarād guṇaḥ ||
saṁtādyoktaphalenoktadhanuḥkot̥ijyayoddhṛtaḥ ||51||

prāptāc cāpaṁ nu dr̥kkarmaphalam pūrvoktayuktitaḥ ||

When one has determined the ayana beginning with Capricorn or Cancer [i.e., whether $270 < \lambda \leq 90$ or $90 < \lambda \leq 270$], [and] when one has multiplied the Sine [of λ] plus three signs by the Sine of the maximum declination, [the product] is divided by the Cosine of the declination of the aforesaid star. Whatever the

quotient is in minutes etc., when one has multiplied the Cosine of the latitude by that and divided by R, and then produced the Cosine of the arc from the result, Sin β is multiplied by the previously stated result and divided by the Cosine of the aforesaid arc. The arc from the quotient is the *drkkarma* correction. According to the aforesaid rule, it is negative when the [ayana of the] star and the latitude are in the same direction, positive when different.

This rather complicated but correct procedure directs the reader to make use of two new quantities here called x and y , in a calculation equivalent to the following sequence of equations, where the symbols refer to arcs in Figure 3.

$$\sin x = [(\cos \lambda) \cdot (\sin \varepsilon)]/(\cos \delta), \quad (12)$$

$$\sin y = [(\sin x) \cdot (\cos \beta)]/R, \quad (13)$$

$$\sin \lambda\lambda_* = [(\sin \beta) \cdot (\sin x)]/(\cos y), \quad (14)$$

$$\lambda_* = \lambda \pm \lambda\lambda_*. \quad (15)$$

The quantity λ_* is well known in the *zīj* literature as the “culminating degree”, or the point on the ecliptic that crosses the meridian simultaneously with the given star, but there it seems to be more usually computed by means of its right ascension;²⁰ I have not yet encountered this particular formula in an Islamic text. Its validity can

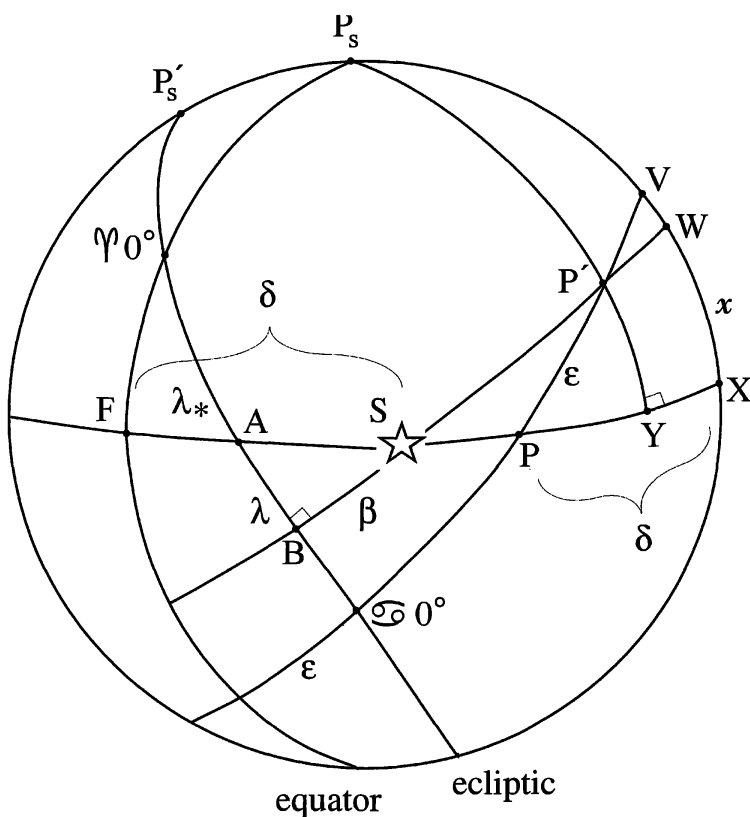


FIG. 3.

be confirmed in the context of medieval spherics by consideration of the spherical triangles in Figure 3 centred on the star S (drawn so that the arcs SW , SX , and AY all equal 90°). The two acute angles of the right triangle SBA containing the arc $\lambda\lambda_*$ are given by $\angle ASB = x$ and $\angle SAB = y$. In the spherical quadrilateral $SXVP'$, the Menelaos Theorem²¹ implies that the product of the sines of three arc segments with no endpoint in common equals the product of the sines of three other arcs without a common endpoint, e.g., that

$$(\sin x)/(\sin VW) = [(\sin \varepsilon)/(\sin VP')] [(R/(\cos \delta))].$$

The “rule of four quantities” (stating that for two right spherical triangles having equality of one acute angle, the ratio of the triangles’ hypotenuses equals the ratio of their sides opposite that angle) implies that in the two right triangles $VP'W$ and BP' (Cancer 0), whose acute angles at the vertex P' are equal,

$$(\sin VP')/(\sin VW) = R/(\cos \lambda);$$

so by substitution

$$\sin x = [(\cos \lambda) (\sin \varepsilon)]/(\cos \delta).$$

By the same rule applied to other pairs of triangles,

$$\begin{aligned} (\sin y)/(\sin x) &= (\sin P'S)/(\sin SW) = (\cos \beta)/R, \\ (\sin \beta)/(\cos y) &= (\sin SA)/R, \text{ and} \\ (\sin \lambda\lambda_*)/(\sin x) &= (\sin SA)/R; \end{aligned}$$

these are easily combined to produce Equation (14). A somewhat similar approach to computing a difference arc of the ecliptic from another set of coordinates is given in the *zīj* of Ḥabash al-Ḥāsib.²²

The second rule for this quantity appears in Mahendra’s two subsequent verses:

pūrvavat satirāśījyā tacchareṇa hatā hṛtā ||
kṛtābhranaṃdavasubhir labdhaṃ bhāgādikaṃ tataḥ ||53||

hataṃ nirakṣalagnena taddhiṣṇyasthitarāśinaḥ ||
hṛtaṃ khakhāgnibhir bhāgādikaṃ dṛkkarma pūrvavat ||54||

As before, the Sine [of λ] plus three signs is multiplied by its latitude and divided by 8904. The quotient is in degrees etc. Then it is multiplied by the rising time of the sign in which that [star] stands, and divided by 300. The dṛkkarma is in degrees etc., as before.

In other words,

$$\lambda\lambda_* = [((\cos \lambda) \cdot \beta)/8904] \cdot [\alpha/300], \tag{16}$$

where α is the right ascension in *vinādīs* (1/3600 of a day; thus one sign or 30° corresponds to 300 *vinādīs*) of the sign in which λ stands. This rule is one of many variants of the traditional “ayanadṛkkarma” procedure illustrated by Equation (2)

above.²³ Mahendra (or his source) has substituted the constant $1/8904$ for the factor $\sin \varepsilon/R^2$, and multiplied the whole by $\alpha/300$. This second factor appears to be intended to adjust the arc $\lambda\lambda_*$ slightly by stretching it near the solstices and shrinking it near the equinoxes, in accordance with the size of the right ascension of the corresponding sign. Other authors use instead $1800/\alpha$, where α is in asus (sixths of a vinādī);²⁴ it is not clear why Mahendra has inverted the factor.

Thus far, Equation (16) represents a standard Indian solution to this problem; but the value $1/8904$ reveals Islamic influence. It is not unusual²⁵ to replace the factor $\sin \varepsilon/R^2$ by a numerical constant depending on a specific value of the standard radius R , but any of the typical Indian values²⁶ of R will produce a factor somewhere between $1/8500$ and $1/8000$. It seems likely, therefore, that this modified formula was adapted for use with the Islamic value $R = 3600$, which Mahendra uses in the *Yantrarāja*. Curiously, the value of the factor $1/8904$ is still somewhat inaccurate for Mahendra's value of $\varepsilon = 23;35^\circ$; it should be $1440;15/3600^2 \approx 1/8998$. It is interesting to note that if ε is taken as $23;51$, $\sin \varepsilon/R^2 = 1455;36,36/3600^2 \approx 1/8903$. Using $\sin \varepsilon = 1455;36$ or even $1455\frac{1}{2}$ would give the $1/8904$ of Mahendra's formula. Did Mahendra adopt the rule of some earlier synthesizer who used the Ptolemaic value²⁷ of ε instead of the more common $23;35$, without noticing the discrepancy with the parameters of the *Yantrarāja*? Another roughly contemporary Sanskrit astrolabe text, the first chapter of the *Yantraratnāvalī* of Padmanābha (A.D. 1423), also uses parameters implying a comparable value for ε — in this case, about $23;50$ — but like Mahendra, Padmanābha names no Arabic or Persian source, and seems not to notice that his stated value of ε is at variance with this hidden parameter.²⁸ Again, the *Yantrarāja*'s commentary does not concern itself with tracing or proving the given formulae, confining itself to the usual gloss and worked example. The results of Malayendu's application of each of Mahendra's two rules to the coordinates of Rohiṇī differ slightly, but as before, Malayendu concludes that the discrepancy is small enough to neglect.

4. The *Yantrarāja*'s Rules in an Eighteenth-century Text

Later Sanskrit treatises on the astrolabe, as far as is now known, add little to the exploration of these issues. The majority of such works discuss the mechanical details of making and using the astrolabe rather than the trigonometric formulae underlying its construction.²⁹ These formulae seem to have been of such limited interest that even some copies of the *Yantrarāja* itself omit the chapter containing them.³⁰ One notable exception to this trend is the *Yantrarājakaḥ* (or *Yantrarājaghaṭana*)³¹ written by Mathurānātha Śūkla in 1782. Mathurānātha, like Mahendra, provides rules for determining the star positions,³² and in fact, they are almost precisely those given by Mahendra in Equations (3)–(6), and (12)–(15).

esāṃ spaṣṭarāśyādīn asmāc cakrād avadhārya grantheṣṭavarṣety uktarītyā
yantranirmāṇakālikān vidhāya tataḥ krāntim āniya ḍṛkkarmaśuddhān vidadyāt |

tadarthaṃ ca prathamatas tātkaḷikād iṣṭarkṣāt krāntim ānayet | tadānayanaprakāro
yathā ||

nakṣatrādhruvakāt trirāśisahitād dorjyeṣukoṭijyayā |
niḡhnā trijyakayā hṛtā dhanur ataḥ śodhyaṃ khanandāṃśakāt ||

bhājyas tasya guṇena bheṣujaguṇo labdhena yuktonitā |
syātkrāntiḥ paramaikyabhinnahariti tryādharkṣatadbāṇayoḥ ||1||

tato guṇaḥ pūrvaharajyayā hato hṛt trijyayā labdhaguṇasya yaddhanuḥ ||
hīnānvitaṃ bheṣulavairbhabāṇayorbhinnaikadiktve 'pama īhitodunaḥ ||2||

iti krāntyānayanavidhiḥ || atha dr̥kkarmaśodhanaprakāraḥ ||

tryādharkṣadhruvaśiṅjiniḥ paramakrāntijyayā ghnī hṛtā |
svaspaṣṭāpamakoṭicāpajaguṇairāptena koṭeriṣoḥ ||

jīvāsaṃguṇitā hṛtā paramayā labdhāddhanuḥ syāt tataḥ |
koṭijyā hyanayā purāptahatabheṣujyā hṛtāptāddhanuḥ ||1||

idaṃ bhabāṇāyanadigbhidaikye dhanarṇamuktaṃ bhagaṇadhruveṣu ||
dr̥kkarmaśuddhāni bhavanti bhāni yantrāya yogīni tadākhilāni ||2||

Having determined the [longitudes] of those [stars] beginning with the correct sign with respect to that [ecliptic] circle, [and] having established them for the time of the construction of the instrument by means of the rule beginning “The year given by the text...”,³³ [and] having calculated the declination[s] from that, one should correct them by the dr̥kkarma. For that purpose, one should first calculate the ecliptic declination for the given star at that time. Then the procedure is as follows: The Sine from the longitude of the star increased by three signs is multiplied by the Cosine of the latitude and divided by the Radius. The arc from this is to be subtracted from ninety degrees. The Sine of the star’s latitude is to be divided by the Sine of that; the maximum declination is increased or decreased by the quotient when the directions of [the longitude of] the star plus three signs and its latitude are the same or different, [respectively]. The Sine from that is multiplied by the Sine from the previous divisor and divided by the Radius. The arc of the resulting Sine is diminished or increased by the degrees of latitude when the directions of [the longitude of] the star and its latitude are different or the same. [That] is the declination of the given star. Thus the rule for the calculation of declination.

Now, the method for the dr̥kkarma-correction. The Sine of the star’s longitude increased by three [signs] is multiplied by the Sine of the maximum declination and divided by the Cosine of its own corrected declination [δ]. The Cosine of the latitude is multiplied by the quotient and divided by the greatest [Sine, i.e. R]. The arc from the quotient should be [made]. The Cosine from that

[is computed], and the Sine of latitude, multiplied by the previous quotient, is divided by that. The arc from the quotient [is made]. This is stated [to be applied] positively or negatively to the stellar longitudes, as the latitude and ayana of the star are in different directions or the same direction. [Thus] the stars are corrected by drkkarma; [when] affixed to the instrument [in the rete], they are complete.

The only substantial difference between these rules and those of the *Yantrarāja* is that Mathurānātha (or his intermediary source) has managed to distort Equation (5) even further by making the addition or subtraction of ϵ and β' dependent on the direction not of λ but of $\lambda + 90^\circ$. Moreover, although he omits the rule in Equation (7), he retains the otherwise unnecessary requirement of computing δ_1 . In the worked examples immediately following the above explanation, Mathurānātha also follows Malayendu in choosing the coordinates of Rohiṇī to demonstrate these procedures. Thus, although the *Yantrarājakaḥ* must have had other Islamic or Islamic-derived sources as well,³⁴ it is clear that at least some of it is merely borrowed from the *Yantrarāja*, and borrowed without any very close scrutiny of the material. This is very much in keeping with the pragmatic approach of Mahendra Sūri and his pupil four centuries earlier when confronted with the new problems and methods of astrolabe construction: the necessary rules were appropriated even if not well understood, and any accompanying inconsistencies lost in the blurring of the distinction between approximation and error. The theoretical justification of the rules was a matter of much less concern to the practical astronomers who first transformed them into Sanskrit, and apparently it did not gain significantly in importance during the subsequent four hundred years.

APPENDIX: MALAYENDU'S COMMENTARY ON THE QUOTED VERSES

Yantrarāja 1, 41–45

When the epoch position of the star at [the time of] the beginning of [this] book [as listed in 1, 22–37] stands in the six signs beginning with Aries, [that] is the northern hemisphere; when standing in the six signs beginning with Libra, [that] is the southern hemisphere. From that, when one has set down the longitude of the star on a tablet, and added three signs [90°] in the signs place, the Sine should be calculated as before. That is the “star-Sine”. And it is called the “Sine of [longitude plus] three” and the “Sine of longitude”. It is multiplied by the Cosine of the latitude and divided by the Radius, 3600. When one has calculated the arc from the result, and subtracted [it] from 90, then the quotient from the Sine of the star's latitude divided by the Sine produced from that remainder is called the “difference”, beginning with degrees. The degrees of the maximum declination, 23;35,0, are increased or diminished by that, the sum [being] when the aforesaid epoch longitude is in the same direction as the latitude standing beneath it, and the difference in the case of difference in their directions, [respectively]. Then after one has computed the Sine of that [and] multiplied it by the Sine computed from the previously calculated arc, it should be divided by the Radius, 3600. Then whatever is the quotient is the Sine of the declination. Then after one has computed the arc from the quotient, that arc is added or subtracted, as before, when there is similarity or dissimilarity of the hemispheres of stars, [i.e., their location in longitude and in latitude] in the part north or south of the equator; [thus] the degrees of declination are corrected.

Now an example: In the year Saṃvat 1427 the longitude of Rohiṇī (in the northern hemisphere on account of its standing in [the six signs] beginning with Aries), corrected for precession, [was] 2,1;33,52,

[its] south latitude 5;10. And when the longitude, 2,1;33,52, is increased by three signs, there results 5,1;33,52. From this, according to [the rule] beginning “Up to three [signs], [an arc] is not to be altered [i.e., is already reduced to the first quadrant]...” [*Yantrarāja* 1, 10], the Sine of the arc is computed. Since the arc [reduced to the first quadrant] is 0,28;26,8, the result found beneath the degrees of arc (28) in the Sine table is 1690;6. The [interpolation-]difference beneath that is 55;13. When the [fractional part] in minutes and seconds (26,8) standing beneath the degrees of arc is multiplied by that difference and divided by 60, and the obtained result 24;<4>3 is added to the previous result 1690[;6], the resulting Sine of the arc is 1714;9. (And it is always [done] in this way.)

Now the Cosine of the latitude: As the south latitude of Rohiṇī is 5;10, this is subtracted from 90; there results 84;50. The Sine is computed from this as before: 3585;25 [error for 3585;18]. The previously-computed Sine of the arc, 1714;9, is multiplied by that, and there results 6145941;58. When it is divided by the last Sine [*R*], there results 1707;12. Here it is assumed that this itself is a Sine. Now the calculation of [its] arc: Since the result standing beneath 28° in the Sine-table is 1690;6, it is subtracted from the previously-computed Sine beginning with minutes (1707;12). The remainder is 17;6. The 28° of arc thus obtained are accompanied by zero [as a fractional part]. After the remainder (17;6) is multiplied by 60, it is divided by the difference standing beneath 28° (55;13). The resulting quotient in minutes etc. is 18;34 [error for 18;35]. This is the resulting arc from the Sine: 28;18,34. (The arc is always to be calculated in this way.) When this arc is subtracted from 90, the remainder beginning in degrees is 61;41,26. From this, as before, the Sine is 3169;20. When the previously-computed Sine (334;10 [misprint for 324;10]) of the star’s latitude (5;1[0]) is divided by this, the difference in degrees etc. is 0;6,8. Then, because of the difference in direction of [the longitude of] the star Rohiṇī (added to <or subtracted from> three signs) and its latitude, there is dissimilarity of its hemispheres. Therefore the degrees of the maximum declination 23;35,0 are decreased by that difference 0;6,8 — there results 23;28,52. From this, as before, the Sine is 1434;22. This is multiplied by the Cosine of the arc [i.e., of the first arc found, the complement of λ'] (3169;20), and there results 4545986;15. It is divided by the final Sine (3600). The quotient is the Sine of the declination, 1262;45 [should be 1262;46], and its arc is calculated as before: 20;33,1 [33 is a misprint for 32; the seconds value should be 5 or 6 depending on which value of the above $\sin \delta$ is used]. The degrees of declination are to be known as [equal to] this much. Now when the latitude 5;10 is subtracted from the arc (because of the difference in direction of [the longitude of] the star Rohiṇī increased by three signs and of its latitude), the remaining degrees of north declination of the star Rohiṇī are correct: 15;22,1. By so many degrees, the star Rohiṇī stands to the north of the equator. The calculation of the declination of all stars is done by this method.

[When the above procedure is repeated using correct values in place of Malayendu’s slightly erroneous ones, and following his example in rounding all arc-calculations to two sexagesimal places and all Sine-calculations to one, it yields a final value of 15;22,7° for the declination of Rohiṇī.]

Yantrarāja 1, 46–48

“Or else”: by another method, when one has ascertained the hemisphere as before and has computed the arc from the star’s epoch longitude, the [ecliptic] declination is computed from that. The sum of that declination with the latitude, when they are in the same direction, or their difference when in different directions, is made. And when one has computed the Sine as before from that sum or difference in degrees, and multiplied that by the Cosine of the maximum declination, and found the Cosine of the previously computed declination, this [product] is divided by that [Cosine]. Then whatever the quotient is, its arc is obtained as before. That arc is just the degrees of declination of the star in question, [which] are north or south. How is their direction [determined]? It is said: When the declination and the latitude have the same direction, the direction of the declination of the star is to be known from the sum. When the latitude and declination have different directions, when the lesser is taken from the greater, the direction of the declination of the given star is from the remainder.

Now an example: The epoch longitude of the star Rohiṇī, corrected by precession (in the northern hemisphere, because of its being in [the hemisphere beginning with] Aries), is 2,1;33,52. The latitude is southern, 5;10. Now from its being less [than 90°], this longitude is just the arc; the degrees of the arc are 61;33,52. Their declination is computed as [follows]: The result obtained from what is beneath the degrees of arc (61) in the tables of <Sine and> declination-results is 1228;57. The difference beneath

that is 12;14. When the [amount] in minutes and seconds standing beneath the degrees of arc, 33;52, is multiplied by that difference and divided by 60, the quotient in minutes etc. is 6;54. When [that] is added to the previous result, 1228;57, there results the declination in minutes, 1235;51. (The declination is always to be found in this way.) When these are divided by 60, the quotient is the northern declination, beginning with degrees: 20;35,51. The southern latitude, in degrees etc., of the star Rohiṇī is 5;10. Now when the difference of these two is taken (because of their difference in direction), the latitude being subtracted from the larger declination, the “declination-remainder” is northern: 15;25,51. From this the Sine is found as before: 957;50. This is multiplied by the Sine (3299;12) of the complement (66;25) of the maximum declination (23;35), by making a “gomūtrika” [multiplication procedure]: 3160083;44 [is the result]. When it is divided by the Cosine (3359;54 [error for 3369;45]) of the declination found for that (20;35,51), the quotient in two [sexagesimal] places is 937;52. The arc from that, as before, is 15;5,51. This arc is just the degrees of declination; and they are to be understood as northern, because the previously-computed declination-remainder was northern. In the procedure for declination, there is some difference in minutes and seconds between the method previously stated and this one; because of its smallness, it is not wrong.

[Malayendu’s last two calculations are rather puzzling; if the erroneous Cosine 3359;54 is used to calculate $\sin \delta$, the final answer ought to be 15;8,42. If the entire procedure is performed accurately, the resulting value of δ is 15;5,59.]

Yantrarāja 1, 49–52

In the calculation of the *ḍṛkkarma*, when one has found the ayana — north or south, [depending on the star’s position] in [the semicircle of the ecliptic] beginning with Capricorn or Cancer [respectively] — of the star’s epoch longitude; and, as before, has calculated the Sine of the arc after it is increased by 90°; and multiplied it by the Sine (1440;15) of the maximum declination (23;35); and divided that by the Cosine of the previously-computed [true] declination; and multiplied the Cosine of the latitude by the result obtained; [then the resulting amount] should be divided by the Radius, 3600. Then, when one has computed the arc and its Cosine (for the purpose of [future] division) from the number [thus] obtained, and computed the Sine of [the star’s] latitude and multiplied it by the aforesaid result, it is divided by the Cosine of the previously-computed arc. Then the arc is to be calculated from the quotient, and that is the *ḍṛkkarma* result. As before, it is negative in the case of similarity between the star’s ayana and its latitude (because of [their being] in the same direction), and positive in the case of their dissimilarity (because of difference in direction). When that result is applied negatively or positively, as stated, to the star’s epoch longitude at the commencement of the book in the given year, that star is *ḍṛkkarma*-corrected.

Now, an example: Since the epoch longitude of Rohiṇī, corrected by precession, in the northern ayana (from its beginning with Capricorn) is 2,1;33,52 and its southern latitude 5;10, the Sine of the arc (computed as before) from Rohiṇī’s epoch longitude added to three signs is 1714;9. This, multiplied by the Sine (1440;15) of the maximum declination (23;35), becomes 2468804;32. By dividing by the Sine (3471;1[0]) of the complement (74;37,59) of the previously-computed true declination of Rohiṇī, [one obtains] the quotient 711;15. The Sine (3471;1) of the complement (84;50) of the latitude (5;10) of Rohiṇī is multiplied by that; there results 2550067;36. When it is divided by the final Sine 3600, the quotient is 708;22. The arc from that, as before, beginning with degrees, is 11;21,5. The complement is 78;38,[5]. From this, as before, the Sine is 3529;27. This Cosine of the arc is set down separately for use in a subsequent division.

Now the latitude is 5;10. From this, as before, the Sine is 324;10. This is multiplied by the previous result 711;15, and there results 230563;32. When it is divided by the Cosine of the previously-calculated arc, 3529;27, the quotient is 65;28. The arc from this, as before, is 1;2,21 [error for 1;2,31]. This is just the *ḍṛkkarma*-result. When one has made it positive in view of the difference of the two (on account of the southern latitude and the northern ayana of the star Rohiṇī), it is added to Rohiṇī’s epoch longitude. There results Rohiṇī’s *ḍṛkkarma*-corrected epoch longitude, 2,2;36,23. The *ḍṛkkarma*-calculation of all stars is [done] in this way.

[The arithmetic errors in this example are numerous and inconsistent, but if the procedure is done correctly, the value of $\lambda_\circ$ is 1;2,22].

Yantrarāja 1, 53–54

When one has ascertained the ayana as before, the “Sine [of longitude] increased by three signs” derived from the epoch longitude of the star is multiplied by its latitude and divided by 8904. The quotient from that is in degrees etc. When that is multiplied by the “ascension at zero latitude” (the vinādīs of the risings at Laṅkā of Aries etc. are 278, 299, 323, and so forth), [i.e.,] by the amount of the rising at Laṅkā of [the sign] beginning with Aries associated with the given star, and divided by 300, the quotient in degrees etc. is just the ḍṛkkarma-result. That is applied negatively or positively, as before, to the given epoch longitude.

Now, an example: Since the longitude of Rohiṇī corrected by precession, 2,1;33,52, is in the northern ayana beginning with Capricorn, and [its] latitude is southern (5;10), the Sine of the arc [plus] three signs, as before, is 1714;9. This is multiplied by the latitude, 5;10, and there results 8856;26. This is divided by 8904, and the quotient beginning with degrees is 0;59,40. That is multiplied by the amount of the right ascension of Gemini (because of the star Rohiṇī’s standing in the sign Gemini), 323, and there results 321;12,20. That is divided by 300, and the quotient beginning with degrees is 1;4,14. That is just the ḍṛkkarma-result. As before, it is added to [the longitude of] the star Rohiṇī, because of the difference [in direction] between its latitude and its ayana. The ḍṛkkarma-corrected [longitude of] the star Rohiṇī is 2,2;38,6. Here there is some difference between this and the previously-computed ḍṛkkarma-corrected [longitude], 1 minute and 43 seconds; because of its smallness, it is not wrong.

[The recomputation of Malayendu’s example yields a value for λ_* of 1;4,16.]

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1. The Indian development of this Hipparchan technique is discussed in O. Neugebauer and D. Pingree, *The Pañcasiddhāntikā* (Copenhagen, 1970–71), II, 41–44.
2. Several such formulae, particularly as represented in the work of the Mādhava school in late-medieval Kerala, are discussed by R. C. Gupta in “Solution of the astronomical triangle as found in the *Tantrasaṅgraha* (A.D. 1500)”, *Indian journal of history of science*, ix (1974), 86–99, and in “Mādhava’s rule for finding angle between the ecliptic and the horizon and Āryabhaṭa’s knowledge of it”, in *History of oriental astronomy*, ed. by G. Swarup *et al.* (Cambridge, 1987), 199–202. As Gupta points out, many of these rules are indistinguishable from ones derived via spherics *per se*, and the typical lack of any proof in the authors’ presentation of them renders it impossible to determine exactly how they were worked out by their inventors. But the absence of explicit reference to the solution of spherical triangles in Indian mathematical works makes it most likely that such rules were established from various ingenious manipulations of triangles within the sphere rather than on its surface.
3. Some aspects of the Indian reception of the astrolabe are discussed in S. R. Sarma, “Astronomical instruments in Mughal miniatures”, *Studien zur Indologie und Iranistik*, xvi (1992), 235–76.
4. See D. Pingree, “History of mathematical astronomy in India”, in *Dictionary of scientific biography*, xv, 533–633, p. 626; Sarma, *op. cit.* (ref. 3), 238–9; and Y. Ohashi, “Early history of the astrolabe in India”, *Indian journal of history of science*, xxxi (1997), 199–295.
5. *Yantrarāja* 1, 2–3. The edition used is *The Yantrarāja*, ed. by K. V. Raikva (Bombay, 1936). The work was previously edited by S. Dvivedi (Benares, 1883).
6. The reference is to the Puranic legend of the gods’ churning of the oceans to obtain the divine nectar of immortality; see, e.g., *Viṣṇupurāṇa* IX, 81–98.

7. The concise treatises of Pseudo-Māshā'allāh (in a Latin version; see R. T. Gunther, "Messahalla on the astrolabe", in *Early science in Oxford*, ed. by R. T. Gunther, v (Oxford, 1929), 137–231, and P. Kunitzsch, "On the authenticity of the treatise on the composition and use of the astrolabe ascribed to Messahalla", *Archives internationales d'histoire des sciences*, xxxi (1981), 42–62), ibn 'Īsā (C. Schoy, "'Alī ibn 'Īsā, das Astrolab und sein Gebrauch", *Isis*, ix (1927), 239–54), al-Birūnī (A. Dallal, "al-Birūnī's *Book of pearls concerning the projection of spheres*", *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, iv (1987/88), 81–138), and Tūsī (whose *Bīst bāb*, in the Sanskrit version of Nayanāsukha, is published in *Yantrarājavicāravimśādhyāyī*, ed. by V. Bhaṭṭācārya (Benares, 1979)) confine themselves to practical procedures for making, marking, and manipulating the astrolabe, rather than calculating coordinates. al-Birūnī's longer work, *Isṭī'āb al-wujūh al-mumkinah fī ṣan'al al-aṣṭurlāb*, may prove more comprehensive.
8. See D. Pingree and P. Morrissey, "On the identification of the *yogatārās* of the Indian *nakṣatras*", *Journal for the history of astronomy*, xx (1989), 99–119.
9. Brahmagupta, for instance, prescribes this rule in *Brāhmasphuṭasiddhānta* 10, 15–16 (*Brāhmasphuṭasiddhānta*, ed. by S. Dvivedi (Benares, 1901–2)).
10. Pingree and Morrissey, *op. cit.* (ref. 8), 111.
11. It is generally employed in the calculation of heliacal risings and settings of the planets; Mahendra uses the term to include the λ_* -calculations for stars, whether by Indian or foreign methods.
12. This is the standard form of the rule in the Āryapakṣa or school of Āryabhaṭa — see, for example, *Āryabhaṭīya* 4, 36 (*Āryabhaṭīya*, ed. by K. S. Shukla and K. V. Sarma, i (New Delhi, 1976)). A version frequently seen in the rival school of Brahmagupta, the Brāhmapakṣa, substitutes the Sine function for the Versine in this formula, as in *Brāhmasphuṭasiddhānta* 6, 3.
13. Cf. *Brāhmasphuṭasiddhānta* 7, 5 and Lalla's *Śiṣyadhīvrddhidatantra* 9, 1–2 (*Śiṣyadhīvrddhidatantra of Lalla*, ed. by B. Chatterjee (New Delhi, 1981)).
14. See the Appendix for translations of this and the following glosses and examples by Malayendu, as they are rendered in the published edition; square brackets indicate editorial insertions or comments, and angle brackets deletions.
15. See D. A. King, "The astronomical works of ibn Yunus", PhD. dissertation, Yale University, 1972, 295–6. Although, as was pointed out earlier, the *Yantrarāja* has not been directly linked to this or any other particular Islamic source, it is interesting that both Ibn Yūnis and Malayendu Sūri choose α Tauri (Aldebaran, Skt. Rohiṇī) as an example to demonstrate the δ -calculations.
16. King, *op. cit.* (ref. 15), 293–5.
17. See Javad Hamadani-Zadeh, "Nāṣīr ad-Dīn on determination of the declination function", in *History of oriental astronomy*, ed. by G. Swarup *et al.* (Cambridge, 1987), 185–9.
18. Little is known at present about the specific works used to teach Islamic astronomy in India in this early period; see, however, D. Pingree, "Islamic astronomy in Sanskrit", *Journal for the history of Arabic science*, i (1978), 315–30, and S. R. Sarma, "Yantraprakāra of Sawai Jai Singh", *Studies in history of medicine and science*, x–xi (1986, 1987), (supplement) 1–139.
19. Pingree and Morrissey, *op. cit.* (ref. 8), 115.
20. See, e.g., King, *op. cit.* (ref. 15), 302–5, and M.-Th. Debarnot, *Kitāb maqālīd 'ilm al-hay'a: La trigonométrie sphérique chez les Arabes de l'est à la fin du X^e siècle* (Damascus, 1985), 216–18.
21. A more detailed explanation of the spherical trigonometry used here is given in, e.g., Edward S. Kennedy, "The history of trigonometry", in E. S. Kennedy *et al.*, *Studies in the Islamic exact sciences* (Beirut, 1983), 1–29.
22. Debarnot, *op. cit.* (ref. 20), 52–53.
23. In this form, it more closely resembles Brahmagupta's version (see ref. 12); Mahendra's use of $\sin(\lambda + 90) = \cos \lambda$ would therefore seem to place him among the followers of the Brāhmapakṣa,

- who tend to use the Sine as the interpolating function in this procedure in preference to the Versine favoured in the Āryapākṣa.
24. See, e.g., Śrīpati's *Siddhāntaśekhara*, 9, 6 (*Siddhāntaśekhara*, ed. by B. Misra (Calcutta, 1932, 1947)); Bhāskara's *Siddhāntaśiromaṇi*, Gaṇita 7, 4 (*Siddhāntaśiromaṇi*, ed. by B. D. Sastri (Benares, 1967; repr. as Kashi Sanskrit Series 72, Benares, 1989)). Muñjāla uses an equivalent expression in *Laghumānasa* 52 (*Laghumānasa*, ed. and transl. by N. K. Majumdar (Calcutta, 1951)).
 25. See, e.g., Śiṣyadhivṛddhidatantra 8, 3, and Vateśvara's *Vateśvarasiddhānta*, 6, 10 (*Vateśvarasiddhānta*, ed. and transl. by K. S. Shukla (New Delhi, 1986)).
 26. Most commonly $R = 3438$ (cf. *Āryabhaṭīya* Gītikā 12); other well-known values are 3270 (*Brāhmasphuṭasiddhānta* 2, 5) and 3415 (*Siddhāntaśekhara* 3, 6). If the standard Indian obliquity, $\varepsilon = 24^\circ$, is used in place of the *Yantrarāja*'s smaller value, then the increased amount of the factor $\sin \varepsilon / R^2$ diverges still further from Mahendra's number.
 27. This is the obliquity used in the zij of al-Khwārizmī (c. A.D. 840; see E. S. Kennedy, *A survey of Islamic astronomical tables*, in *Transactions of the American Philosophical Society*, ns., xlvii/2 (1956), 123–77 (p. 148)), but I know of no other zij that uses it as a table parameter, although Ptolemy's value was certainly well-known to Islamic astronomers.
 28. Y. Ohashi, "A note on some Sanskrit manuscripts on astronomical instruments", in *History of oriental astronomy*, ed. by G. Swarup *et al.* (Cambridge, 1987), 191–5; see also Ohashi, *op. cit.* (ref. 4), 225.
 29. For example, the discussions of the astrolabe in the works associated with the court of the eighteenth-century ruler Sawai Jai Singh — the *Yantrarājaracanā* (ed. by K. Jyotirvid (Jaipur, 1953)), the *Yantraprakāra* (Sarma, *op. cit.* (ref. 18)), the *Yantrarājavicāravimśadyāyī* (Bhaṭṭācārya, *op. cit.* (ref. 7)), and the *Yantraprabhā* (published with the *Yantrarājaracanā* in Jyotirvid, *op. cit.*) of Śrīnātha — all neglect these theoretical considerations, as does the earlier *Yantraśiromaṇi* of Viśrāma (published with the *Yantrarāja* in Raikva, *op. cit.* (ref. 5)). It would be interesting to know whether the work of Padmanābha mentioned above gives a fuller treatment of them.
 30. Such abbreviated versions are preserved in the manuscripts Benares Hindu University B.3318 and B.521.
 31. See Y. Ohashi, "Sanskrit texts on astronomical instruments during the Delhi Sultanate and Mughal periods", *Studies in history of medicine and science*, x–xi (1986–87), 165–81, p. 175.
 32. All references to the *Yantrarājakaḷpa* are based on the manuscript Benares 35245; the rules in question appear on ff. 21v–22v.
 33. This refers to the formula for correcting celestial longitudes for precession given earlier by Mathurānātha on f. 13v.
 34. Witness Mathurānātha's use of $\varepsilon = 23;30,17$ (f. 23r) instead of Mahendra's value.