

## Image Fidelity

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### Abstract.

We analyse model images of Cas A and other complex sources to determine how mosaicing and  $uv$  sampling affect the image fidelity. These results suggest that it is quite difficult to achieve an image fidelity much better than 1%. We analyse three models: multiple compact sources, Cas A, and eye charts. In all three models, the image fidelity improves as the  $uv$  sampling is increased. More complex images require better  $uv$  sampling. MFS synthesis is an effective way to increase the  $uv$  sampling for continuum observations. Undersampling the pointing for mosaiced images degrades the image fidelity. A heterogeneous array, which samples more short  $uv$  spacings with the interferometer, gives better image fidelity than a homogeneous array with the same number of antennas and collecting area.

### 1. Introduction

Many astronomical studies require a comparison of source images with different instruments and sampling. The *image fidelity* — how closely the image represents the real source distribution — depends not only on random noise but also on errors in the data, sampling, and imaging artifacts. In a recent study of the supernova remnant Cas A, we compared images from 1.5 to 87 GHz to look for spectral index variations across Cas A (Wright *et al.* 1999). We found that the image fidelity was critically dependent on adequate  $uv$  sampling. By combining data from 75 to 87 GHz using multifrequency synthesis and multiple configurations of the BIMA array we obtained an image fidelity of 1% to 2% of the peak intensity. Analysis of the imaging errors shows that residual amplitude and phase errors, primary beam, and pointing errors each contribute about 1% to the image errors.

This paper presents an empirical study of the image fidelity obtained as a function of the  $uv$  sampling, the pointing sampling, and the image complexity. We image three representative models: multiple compact sources, Cas A, and eye charts. We compare the results obtained with homogeneous and heterogeneous arrays such as the combined BIMA and OVRO arrays. These results are discussed more fully in BIMA memo 73.

## 2. Random $uv$ Sampling

Randomly sampled  $uv$  data create a standard against which array configurations can be measured if the uniformity or efficiency of  $uv$  coverage is the criteria of success. For simple sources, well sampled  $uv$  tracks with large gaps between them may be well suited to measure model parameters. However, large gaps can hide any source structure whose visibility has not been measured, so for complex sources, any gap in the sampled  $uv$  plane can detract from the image fidelity. Complex images require better  $uv$  sampling since deconvolution is not perfect in the presence of noise. For simple models it may be possible to improve the image fidelity by first subtracting well determined sources from the  $uv$  data.

We imaged a model composed of eight compact sources with flux densities from 1 to 0.01 Jy with randomly sampled  $uv$  data. The image noise was 1 mJy/beam. The images were deconvolved using the CLEAN algorithm, which is well matched to deconvolving a field of compact sources. With only 1000  $uv$  samples there were many spurious sources at the 1% level and a 20% error in the measured flux density of the 50 mJy source. With 2000  $uv$  samples there were several spurious sources at the 0.3% level. With 4000  $uv$  samples there were no spurious sources at the 0.3% level, although several appear at the 0.2% level and there was 20% error in the measured flux density of the 10 mJy source. In all cases, the RMS noise was close to the theoretical noise level of 1 mJy/beam, and the dynamic range was 1000:1. The level at which spurious sources appear, or the maximum difference between the image and model are useful measures of the image fidelity, but these are hard to use if the real source distribution is unknown. Empirical estimates of the image fidelity may be obtained by varying the imaging parameters and measuring the RMS differences between the sources on the ensemble of acceptable images.

We also imaged several eye chart models with randomly sampled  $uv$  data. The eye charts contain structures on a range of angular scales from 0.5" to 90", and it is easy to visually assess the image defects. The eye chart letters fill about 8% of the pixels. The image fidelity of the eye chart degrades gracefully as the  $uv$  sampling is reduced from the Nyquist rate. With  $uv$  sampling at 40% of the Nyquist rate, the eye chart is degraded, but readable; at 4% all the letters are poorly defined.

## 3. Various Array Configurations

The eye chart models were sampled using various array configurations. Single field images were used to compare the effect of  $uv$  sampling with the different arrays. The arrays were scaled to the same resolution using 6, 16, and 27 antennas. Narrow bandwidth observations were compared with multifrequency synthesis spanning an 8% bandwidth. Again we see a steady improvement in image fidelity as the  $uv$  sampling is increased.

We compared mosaiced images with the current 10 antenna BIMA array and a 15 antenna heterogeneous array with nine 6.1 m antennas and six 10.4 m antennas using 19 pointings at 100 GHz. The three primary beam patterns were modelled as Gaussians in the joint deconvolution. The RMS on the residual images is 3 times better with the heterogeneous array.

## 4. Heterogeneous Arrays and Observing Strategies

The smaller antennas are well suited for mapping large source structure and are best placed at short baselines in order to sample  $uv$  spacings down to the diameter of the smallest antenna. Mosaicing algorithms can recover visibilities about half a dish diameter shorter than the shortest measured spacing (*e.g.* Cornwell, 1988). A direct Fourier transform of the  $uv$  data with respect to pointing center gives similar results.

Short  $uv$  spacings can be obtained from single dish observations with the larger antennas. For many projects it is desirable to have approximately the same single dish sensitivity as the interferometer data. This is important for detecting and mapping large angular size sources, especially transient sources such as comets. This argues for using several of the larger antennas for single dish observations. Using the larger antennas on the longest interferometer baselines provides a more uniform sensitivity in the  $uv$  data and reduces the required  $uv$  data sample rate.

A heterogeneous array provides an overlap in the spatial frequencies derived from direct interferometer observations with the smaller antennas, and from the single dish observations with the larger antennas. This overlap provides additional constraints on the calibration of these data. In a homogeneous array spatial frequencies close to the antenna diameter are critically dependent on the surface accuracy at the edges of the antennas.

## 5. Cas A Models

We made images of Cas A using a VLA image as a model. Cas A is a complex source; the VLA model contains structures on all scales from  $0.4''$  to  $5'$ . The VLA image was sampled at spatial frequencies corresponding to 4 configurations of the 15 antenna array with a minimum  $uv$  spacing of 3 m, to represent the potential gain from adding 2.5 m antennas to the heterogeneous array. The model  $uv$  data, with added noise appropriate for the combined array, was imaged and deconvolved using the Maximum Entropy Algorithm (MEM), the SDI clean algorithm, and a linear mosaic of the separately deconvolved subfields. MEM was most successful in recovering the large scale structure. The linear mosaic was much faster.

### 5.1. $uv$ Sampling

We first studied the effect of  $uv$  sampling with no mosaicing. We made images using multifrequency synthesis (MFS) and deconvolved using the MEM algorithm with 50 iterations. We added model  $uv$  data, one configuration at a time. With 1 configuration and 1 frequency channel at 100 GHz, we recover only 22% of the total flux density and corresponding large scale structure. With 2 configurations and MFS we recover 32%, with 3 configurations and MFS we recover 48%, and with 4 configurations and MFS we recover 65% of the total flux density. The last image is almost indistinguishable by eye from the VLA image convolved to the same resolution. Residual images were formed by subtracting the VLA model image convolved to the same resolution. The residual image for Cas A looks like the surface of the moon; large scale missing flux with error

hills and craters close to bright features where MEM has not done so well. The RMS on the residual image is 3.4 mJy/beam which is 3.5% of the peak on the convolved VLA image. The off-source RMS is 0.5 mJy/beam, and the thermal noise is 30 micro Jy. Clearly neither the thermal noise nor the dynamic range is a good estimate of the image fidelity. The  $\chi^2$  image from MEM provides an estimate.

From this study, we conclude that: i) The image fidelity improves with the density of sampled  $uv$  points. ii) The recovered flux density and large scale structure depends strongly on the shortest  $uv$  spacings. iii) The residual images show the missing large scale structure (corresponding to spacings less than the minimum interferometer spacing) and show deconvolution errors on the strongest peaks, where MEM does not do a good job.

## 5.2. Mosaic Observations

We imagined the Cas A model with 19 pointings in a hexagonal mosaic with 1' spacings and with a 36 s sample interval. With 4 configurations and MFS, we recovered 91% of the total flux density. The residual image, after subtracting the VLA model convolved to the same resolution, has an RMS error 1.4% of the peak on the mosaiced image. The mosaiced image has recovered most of the large scale structure, but there is an error pattern with an angular scale of 1'–2' corresponding to the shortest interferometer spacings sampled. Increasing the mosaic pattern to 37 pointings with 50'' spacings at 100 GHz reduced the RMS residual to 0.4% and increased the total flux density recovered to 94%. Increasing the sample interval to 3 minutes so that the  $uv$  data for each pointing are undersampled increased the RMS residual to 4% and degraded the image significantly. Undersampling the pointing leads to aliasing of the  $uv$  data at spatial frequencies close to the antenna diameter. These are exactly the spatial frequencies which tie to those sampled directly by the shortest interferometer spacings. For simple, discrete source distributions, undersampled pointing may allow us to observe a larger region in a fixed amount of time, but for complex source distributions such as Cas A, the image fidelity is degraded.

The above tests all used a minimum  $uv$  spacing of 3 m. Using a minimum spacing of 6 m recovered only 54% of the total flux density. The single field observations with a minimum interferometer spacing of 3 m reproduced more of the large scale structure than the mosaics with a minimum spacing of 6 m.

We compared images obtained with homogeneous and heterogeneous arrays with the same number of antennas and total collecting area. Although the overall RMS in the residual images is within 7% in both mosaics, the on-source errors are smaller in the heterogeneous mosaic. The heterogeneous array, which samples more short  $uv$  spacings with the interferometer, gives much better image fidelity.

## References

- BIMA memo 73, <http://bima.astro.umd.edu/memo/memo.html>  
Cornwell, T.J., 1988, A&A 202, 316  
Wright, M., Dickel, J., Koralesky, B., & Rudnick, L., 1999, ApJ, 518, 284