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Letter to the Editor

Co-orbital objects in the main asteroid belt

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Abstract. We have investigated the possibility of main belt asteroids being trapped in the 1:1 resonance with other asteroids. Using contemporary mass estimates for the large asteroids (1) Ceres, (2) Pallas and (4) Vesta, it is found that four asteroids are temporarily trapped in such resonances with Ceres and Vesta during numerical integrations spanning 2×10^6 yr. In particular, the asteroid (1372) Haremari is likely to be co-orbiting with Ceres at the present time. The corresponding libration period is found to be $\lesssim 10^5$ yr. The implications of our results for the general dynamical evolution of minor bodies and these asteroids in particular is discussed.

Key words: celestial mechanics, stellar dynamics – minor planets, asteroids

1. Introduction

Since the discovery of the first Jupiter Trojan by Max Wolf in 1906, co-orbital motion has been observed in a variety of locales in the solar system. These include: the saturnian satellites Janus and Epimetheus (Dollfus 1967; Fountain & Larson 1977, 1978) which move in a stable horseshoe configuration with respect to each other (Harrington & Seidelmann 1981; Dermott & Murray 1981; Yoder et al. 1983) the Mars Trojan (5261) Eureka (Bowell et al. 1990; Mikkola et al. 1994) and, more recently, the Earth co-orbital (3753) Cruithne (Wiegert et al. 1997). Cruithne's dynamical behaviour is markedly different from what has been observed in the other cases mainly due to the large eccentricity and inclination of its orbit (Wiegert et al. 1998; Namouni 1999). This type of resonant behaviour is shared by comets (Marsden 1970; Benest et al. 1980) as well as other near-Earth asteroids (Namouni et al. 1999; Christou 2000).

Recently, Yu & Tremaine (1999) and Nesvorný et al. (2000) demonstrated Pluto's ability to trap other Edgeworth-Kuiper belt objects (EKOs) in co-orbital motion with itself. These objects are members of the *Plutino* subgroup of EKOs (Jewitt & Luu 1996) sharing Pluto's motion inside the 3:2 interior mean motion resonance with Neptune (Cohen & Hubbard 1965).

Motivated by the above, we have carried out a numerical search for known objects which are able to or currently are co-orbiting with large main-belt asteroids, and in particular (1) Ceres, (2) Pallas and (4) Vesta.

We have identified four such objects, two for Ceres and two for Vesta, for which we observe numerous occasions of temporary capture into the co-orbital resonance over a span of 2×10^6 yr. One of those objects, (1372) Haremari, is probably transiting from tadpole to horseshoe libration with respect to Ceres at present.

2. Large asteroids as perturbing bodies

The largest asteroids are massive enough to perturb each other, passing asteroids and even planets. These perturbations have in fact been used for the purpose of mass determination, for example, in the cases of (4) Vesta (Hertz 1968), (1) Ceres (Schubart 1971) and (2) Pallas (Schubart 1975).

Standish & Hellings (1989) improved upon these earlier determinations by using Viking lander radio science data to measure the effect of these three asteroids on the motion of Mars.

Hilton (1999, hereafter H99) re-determined the masses of these three asteroids. He showed that the slow change in the relative mean longitude between Ceres and Pallas render the mass determination procedure for these two asteroids dependent on each other.

In the context of co-orbital motion, the above result is not surprising. The radius of Ceres' Hill sphere $\epsilon = (\mu/3)^{1/3}$ (μ being the asteroid-Sun mass ratio) and the difference in their respective osculating semi-major axes are within one order of magnitude of each other. At the very least, this suggests that our search should be carried out using a model that contains *both* these objects. Hereafter, we shall refer to these large asteroids as secondaries, to distinguish them from the co-orbital candidates which we regard as massless particles.

3. Initial search strategy

We have used JPL's HORIZONS ephemeris tool (Giorgini et al. 1996) to select those numbered asteroids with semi-major axes within one Hill parameter (ϵ) distance of the secondary at

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Table 1. Dynamical and physical parameters of the asteroids of interest in this paper. The respective secondaries are shown in bold. Columns 2–7 give osculating elements on November 18, 1999 (JD2451500.5) as given by the Asteroid Orbital Elements Database (Bowell et al. 1994) available electronically at *ftp://ftp.lowell.edu/pub/elgb/astorb.html*. Column 8 refers to the current formal uncertainty in the objects' position. Columns 9 & 10 provide secondary mass estimates as reported by H99 and Standish & Hellings (1989) with 1-σ errors given as superscripts. The last column gives the value of Hill's parameter derived using H99's mass estimates.

Object	a (AU)	e	I (°)	ω (°)	Ω (°)	M (°)	1-σ unc. ($''$)	μ (H99) ($\times 10^{10}$)	μ (SH89) ($\times 10^{10}$)	ϵ ($\times 10^{4}$)
(1) **Ceres**	2.76632	0.0783	10.58	73.92	80.50	356.65	0.01	$4.39^{\pm 0.04}$	$5.0^{\pm 0.2}$	5.27
(1372) Haremari	2.76586	0.1494	16.43	87.81	327.78	292.87	0.12	–	–	–
(8877) Rentaro	2.76604	0.0924	2.58	140.95	331.17	188.69	0.42	–	–	–
(2) **Pallas**	2.77235	0.2296	34.85	310.26	173.20	343.46	0.02	$1.59^{\pm 0.05}$	$1.4^{\pm 0.2}$	3.76
(4) **Vesta**	2.36153	0.0900	7.13	149.60	103.95	328.93	0.01	$1.69^{\pm 0.11}$	$1.5^{\pm 0.3}$	3.83
(855) Newcombia	2.36118	0.1802	10.91	232.78	17.47	336.21	0.13	–	–	–
(4608) 1988 BW3	2.36210	0.2193	7.45	202.38	203.41	138.64	0.08	–	–	–

the epoch JD2451500.5. This procedure yielded 23 asteroids for Ceres, 6 for Pallas and 17 for Vesta. The candidate asteroids were then integrated backwards and forwards, progressively increasing the timespan of the integrations from 10^4 to 10^5 yr guided by the expected conjunction period $T_{\rm conj} \geq 4\pi/\left(3n\epsilon\right)$ with the secondary (n is the mean motion).

The integration method employed was a 12th order Runge–Kutta–Nystrom (RKN) (Dormand et al. 1987) with an output step of 40 yr.

4. Results

4.1. Co-orbiting asteroids

Progressive elimination of candidates reduced the list size to four (Table 1). One of these asteroids, (1372) Haremari, exhibits horseshoe libration with Ceres in the immediate future with a period of $\sim 10^5$ yr. For the remainder of the sample, namely (855) Newcombia, (4608) 1988 BW3 and (8877) Rentaro, the relative mean longitude $\Delta\lambda = \lambda_{\rm asteroid} - \lambda_{\rm secondary}$ circulated with a period of the same order. Extended integrations of these asteroids for $\pm 10^6$ yr showed that they all undergo periods where $\Delta\lambda$ is involved in horseshoe or tadpole libration.

In order to assert the reality of this result we have carried out a large number of these extended integrations using a variety of physical models and also two different integration schemes, the RKN method already mentioned and the SWIFT_RMVS3 symplectic integration code available in the public domain (Levison & Duncan 1994). The model of the Solar System that we employed included all planets from Venus to Neptune.

We have integrated the motion of the Ceres candidates

- using the two different mass estimates for Ceres reported in Standish & Hellings (1989) and H99 given in Table 1.
- with and without perturbations by Pallas using the mass estimate reported in H99.

For the integrations involving the Vesta candidates we have included Ceres and Pallas with masses from H99. Due to the large uncertainty in the mass of Vesta we have carried out tests using both the formal mass estimate and a reduced Vesta mass which results from subtracting three standard deviations from the former. In all of our 10^6 yr integrations we find the energy constant to be preserved down to 1 part in 10^{10}. This implies that the motion of the large asteroids – crucial for the purposes of this study – does not suffer from significant numerical dissipation.

4.2. General comments

In this and the following section we concern ourselves with a phenomenological study of the asteroids' motion. A detailed analysis of the underlying dynamics is currently under progress and will be reported in a forthcoming paper. The two Ceres co-orbitals execute multiple librations of the horseshoe or tadpole type while the objects associated with Vesta less so, presumably due to Vesta's smaller mass. In all cases, these temporary librations appear in $\sim 90\%$ of our 10^6 yr integrations up to this point.

It is observed that the variations introduced in our dynamical model as described in the previous section do not affect the overall efficiency of resonance capture; the time spent in the resonance as a fraction of the integration timespan does not change appreciably. However, the addition of Pallas in particular causes small changes in the time series of $\Delta\lambda$ for the Ceres co-orbitals which propagate with time and become noticeable typically $\sim 2 \times 10^5$ yr after the beginning of each integration. These result in the history of resonant libration (horseshoe or tadpole) to be different between the two models from that point onward until the end of the integration. The introduced variations in the mass of Vesta have a similar effect on the motion of the co-orbitals of that asteroid.

Regarding the dynamical studies of the Plutinos by Yu & Tremaine (1999) and Nervorný et al. (2000) we note two significant differences between the respective dynamical regimes.

In the case of the Plutinos the main external forcing is due to the 3:2 mean motion resonance with Neptune and the Kozai resonance. Here the leading perturbation is the planetary secular forcing. Moreover, in our case the co-orbital and forcing periods are not well separated. This last feature of the dynamics may present problems for secular theories which assume adiabatic invariance as, for example, Morais (1999).

4.3. The motion of (1372) Haremari

The integrations corresponding to the nominal orbit of one of the asteroids, (1372) Haremari, exhibited libration into the 1:1 resonance with Ceres at the present epoch. The relative mean longitude of the asteroid at present is close to $180°$.

It is of interest to determine the sensitivity of the present dynamical state of Haremari on the observational uncertainties of its orbit. To this end, we have generated 6 dynamical "clones" of the asteroid (denoted by $\pm X$, $\pm Y$, $\pm Z$) by adding and subtracting to its initial spatial co-ordinates a quantity equal to 3 times its $1-\sigma$ ephemeris uncertainty given in Table 1. These clones were then integrated with the RKN code for $\pm 10^6$ yr using a dynamical model which included all 8 major planets as well as Ceres, Pallas and Vesta. A similar method was used by Michel (1997) in his integrations of the near-Earth asteroid (4660) Nereus.

It was found that out of the 7 integrated clones (6 plus the nominal orbit) only one did not librate in the immediate past while all clones are co-orbiting with Ceres in the immediate future. In all cases, the asteroid resides in the resonance for at least 50% of the integrated timespan. Thus it appears highly probable that this asteroid is presently a Ceres co-orbital.

The general characteristics of the resonant motion may be demonstrated by an example. Fig 1 shows the evolution of the relative semimajor axis $a_{\rm r} = \left(a_{1372} - a_{\rm Ceres}\right)/a_{\rm Ceres}$ and the relative mean longitude corresponding to clone -X as a function of time.

The clone transits from tadpole (libration of $\Delta\lambda$ near -60°)to horseshoe motion ($\Delta\lambda$ avoiding the secondary at 0°). The residence interval in each libration mode is several times 10^5 yr while the libration period varies between 6×10^4 and 9×10^4 yr. A further two of the clones undergo the same transition near $t = 0$ leading us to suspect that this may be the most likely evolution for the actual asteroid.

Note that the oscillation of $a_{\rm r}$ that accompanies this type of motion is *not* symmetric with respect to $a_{\rm r} = 0$. Instead, the axis of symmetry appears to be displaced upwards by 2×10^{-5}. We have also noted a smaller negative displacement in the case of (8877) Rentaro. No such asymmetry is visible in our integrations of the Vesta co-orbitals.

We have systematically eliminated a number of possible causes for this asymmetry such as the effects of Pallas, a numerical artifact due to the sparse sampling or indirect pertubations by Jupiter. We believe the explanation lies with the differential precession of the longitude of pericentre ϖ between the asteroids and their respective secondaries. A similar situation arises in the presence of an oblate central body (Greenberg 1981).

The respective periods of circulation are 35, 21 and 24×10^3 yr for (1372) Haremari, (8877) Rentaro and (1) Ceres while those for (855) Newcombia, (4608) 1988 BW3 and (4) Vesta are $37, 36$ and 35×10^3 yr respectively. Application of Kepler's third law yields differences in a_r of 4×10^{-5} for (1372), -2×10^{-5} for (8877), 3×10^{-6} for (855) and 6×10^{-7} for (4608) which agrees in relative size and sign with the observed asymmetries but only within a factor of two. Thus, although this line of investigation appears promising, more work is clearly needed to ascertain the mechanism giving rise to this feature.

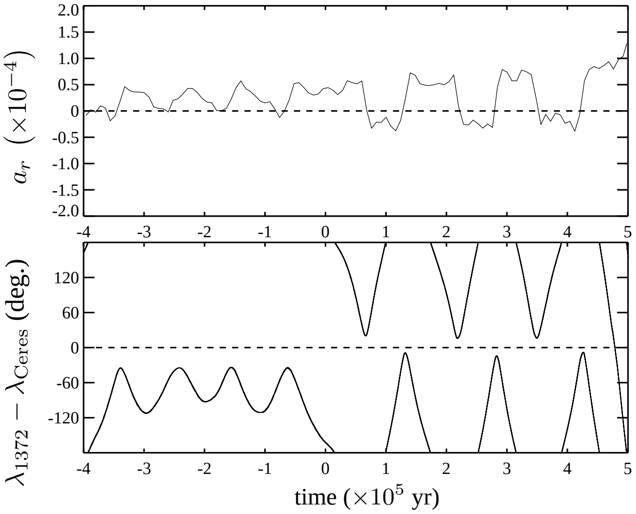

Fig. 1. Dynamical evolution of clone -X corresponding to asteroid (1372) Haremari over a timespan of 9×10^5 yr. Displayed quantities are the semi-major exis (top panel) and mean longitude (bottom panel) relative to the secondary.

5. Discussion

This work has demonstrated that large asteroids such as (1) Ceres or (4) Vesta can maintain smaller asteroids in the co-orbital resonance at least temporarily.

One may wonder whether such capture is possible or even significant for interplanetary matter in a broader size spectrum. In view of the many unknowns of this problem – to be clarified by more detailed work – the following should be regarded as possibilities to be investigated rather than definitive statements.

Collisionally generated dust grains can be temporarily captured in mean motion resonances with planets (Dermott et al. 1994; Liou & Zook 1995). However, the narrow width of the resonant regions combined with the long libration periods for the cases examined here would argue against resonant capture.

On the other hand, the Yarkovsky effect operating on metre-sized fragments (e.g. Bottke et al. 1998) should make capture possible for a non-negligible fraction of the existing population.

For the asteroids in the size range investigated here, migration towards the co-orbital resonance may be induced by slow diffusive processes known to operate in the main belt such as three-body resonances (Nesvorný & Morbidelli 1998; Morbidelli & Nesvorný 1999). For example, in the inner planet region, an observed slow migration of the semi-major axis of asteroids leads to their temporary capture into the 1:1 resonance with Earth and Venus (Christou 2000).

Thus, the present dynamical state of those main belt asteroids as revealed in this work does not necessarily support a genetic relationship with their secondaries. The same conclusion is reached by inspection of their respective proper elements as computed by Milani & Knežević (1994); these four asteroids are not related to the Ceres and Vesta groupings of Zappalà et al. (1995). It is unclear whether the interaction with such a low-mass secondary over a significant fraction of the age of the Solar System can expel the asteroid from its original location in proper element space.

Therefore, we tentatively conclude that these asteroids are physically unrelated to Ceres or Vesta. Further work in this direction, and in particular the long-term dynamics of such objects, should provide a clearer picture of their origin.

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