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Meteoroid stream identification: a new approach – II. Application to 865 photographic meteor orbits

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ABSTRACT

The new criterion D_N for meteoroid stream identification, based on variables directly linked to observations, is applied to a set of 865 photographic meteor orbits and the results are compared with those obtained using the well-known and widely used D_{SH} criterion of Southworth & Hawkins, using thresholds for meteor association computed according to Jopek & Froeschlé. For 15 streams, including the Lyrids, the α Capricornids, the Perseids, the Taurids, the Quadrantids, the Geminids, the Leonids, the Orionids and the southern δ Aquarids, the two searches are in very good agreement, with D_N often adding a few more members to some streams, at the same level of statistical significance as used for D_{SH}. The Andromedids and the σ Leonids, identified as populous streams using D_{SH}, are not recognized using D_N; on the other hand, using D_N five additional streams are identified, namely the ε Geminids, the Monocerotids, the northern δ Aquarids and the rather populous α Virginids and ε Piscids. In general, D_N gives results essentially equivalent to those obtained with D_{SH} in many circumstances, the most notable exception being for near-ecliptical streams, where the results differ markedly. We show also that the reduced distance function D_R can be successfully applied to search for meteor showers resulting from the same meteoroid stream; in our study the Quadrantids and the southern and northern δ Aquarid showers have been identified as possibly being originated by a single meteoroid stream.

Key words: meteors, meteoroids.

1 INTRODUCTION

Valsecchi, Jopek & Froeschlé (1999, Paper I in this issue) have re-examined the problem of defining an appropriate distance function for meteor orbits, with the purpose of improving the existing methods of meteoroid stream identification, and have introduced a new distance function D_N.

A preliminary version of D_N was described by Valsecchi, Jopek & Froeschlé (1996), and an application of it was performed by Jopek, Valsecchi & Froeschlé (1996). The new distance function successfully detected many previously known streams of three or more members; the results were also compared with a search made using the D criterion D_{SH} of Southworth & Hawkins (1963). As our goal was to ascertain the properties of the new distance function, we applied D_N and D_{SH} to the same meteor data sample, using the same cluster analysis algorithm and the same rules for the meteor association threshold as used by Lindblad (1971a,b). We found that both D_N and D_{SH} gave essentially equivalent classifications for streams of moderate and high inclination, while for low-inclination streams the memberships disagreed significantly.

There are several possible reasons for the observed disagreement, and among them a very important one: the choice of the meteor association threshold.

In this paper we present the results of a comparison between two meteor stream searches, one performed by applying the new distance function D_N proposed by Valsecchi et al. (1999), and the other using the D criterion introduced by Southworth & Hawkins (1963). In the present work the meteor association threshold has been determined by the method described in Jopek & Froeschlé (1997); this approach ensures that the results correspond to the same probability of chance identification of a stream of M members, something that makes our comparison more objective.

Additionally, we present the results of the first application of the new reduced distance function D_R proposed by Valsecchi et al. (1999), which facilitates the search for evolved meteoroid streams that give origin to several meteor showers, with different Keplerian orbits, as described by Babadzanov & Obrubov (1991, 1992a,b, 1993).

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Table 1. The orbital similarity thresholds for 865 precise meteors. The values of $D_{c,M}$ for each stream membership M correspond to the reliability level $W_M = 99$ per cent.

	$D_{c,M}$		
M	(D_{SH})	(D_N)	(D_R)
2	0.015	0.020	0.0002
3	0.054	0.080	0.0023
4	0.079	0.113	0.0059
5	0.098	0.134	0.0089
6	0.110	0.160	0.0116
7	0.117	0.178	0.0148
8	0.123	0.188	0.0158
9	0.132		0.0173
10	0.135		0.0196
11	0.145		0.0207

2 STREAM CLASSIFICATION METHOD

2.1 The catalogue of 865 precise meteor orbits

We used the geocentric and orbital data obtained from the IAU Meteor Data Center (Lindblad (1987, 1991) in the form of computer files `prcorb90.dat` and `prcgeo90.dat`. From these files we extracted a subset containing the 865 most precise orbits; this subset comprises 139 small-camera meteors published by Whipple (1954), 413 Super Schmidt meteors reduced by Jacchia & Whipple (1961), and 313 Super Schmidt meteors listed by Hawkins & Southworth (1958, 1961).

We have corrected the perihelion distances of two meteors because they do not fulfil the Earth-crossing condition:

(i) for meteor number 08348, $q = 0.105$ has been changed into $q = 0.933$;
(ii) for meteor number 08528, $q = 0.040$ has been changed into $q = 0.433$;

both the corrected values were taken from Southworth & Hawkins (1961).

2.2 The distance functions

Let every meteor be considered as a point in the space of orbital elements and geocentric parameters. As a quantitative measure of the distance between two meteor points we have used two distance functions: the first one is the so called D criterion D_{SH} proposed by Southworth & Hawkins (1963), and the second one is our new D_N, introduced in Valsecchi et al. (1999).

2.3 The cluster analysis method

The set of 865 meteors was tested for stream membership by a cluster analysis program implementing the same algorithm as used by Lindblad (1971a,b). The algorithm is based on a single neighbour linking technique and has the advantage of not requiring any a priori orbital information on the meteor stream; it has, on the other hand, the well-known disadvantage of possibly creating meteor associations composed of long chains of particles with extrema separated by distances much larger than one would a priori consider physically acceptable.

The meteor data were analysed with our software using the distance functions D_{SH}, D_N and D_R (for their formulations, see Valsecchi et al. 1999). For each function the threshold of the orbital similarity was found by the method described in Jopek & Froeschlé (1997); in short, many random samples having the same marginal distributions of the variables as the real sample are generated and these are searched for groupings. If we want a reliability level of 99 per cent, we accept as threshold for finding groupings of M members the value with which we do *not* find such groupings in 99 per cent of the random samples. If after increasing M the threshold does not increase appreciably for the reliability level chosen, this means that we have reached a practical limit on M, beyond which large groupings would be found for very small

Table 2. Meteor streams detected in two searches. The first column gives the stream name, the second its code, the third and fifth ones the number of members M_{SH} and M_N identified by, respectively, D_{SH} and D_N, for the threshold values $D_{c,M}$ given, respectively, in the fourth and sixth columns; finally, the last column gives the number of common members M_C. The flags S and N sometimes added to the stream code denote that the latter refers to the searches made with D_{SH} and D_N, respectively. The absence of the flag means that the code is the same for both distance functions.

Name	Code	M_{SH}	$D_{c,M}$	M_N	$D_{c,M}$	M_C
Lyrids	1N	5	0.054	6	0.080	5
α Capricornids	13	20	0.145	21	0.178	20
Perseids	16	30	0.123	33	0.178	30
κ Cygnids	32	4	0.079	4	0.080	4
Taurids	47	53	0.145	58	0.188	53
Quadrantids	48	14	0.054	14	0.080	14
Geminids	70	51	0.054	51	0.080	51
χ Orionids	72	4	0.079	11	0.188	4
Leonids	94	5	0.079	6	0.080	5
σ Hydrids	107	3	0.054	5	0.134	3
Orionids, η Aquarids	119	20	0.110	19	0.080	19
Southern δ Aquarids	347	11	0.098	11	0.134	11
Draconids	467	2	0.015	2	0.020	2
Cyclids	477	2	0.015	2	0.020	2
α Pegasids	817	2	0.015	3	0.080	2

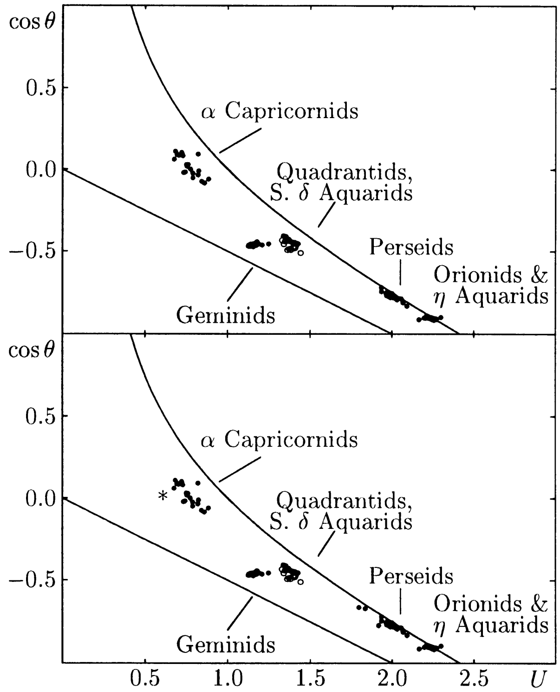

Figure 1. U–$\cos\theta$ diagrams of the streams with more than 10 members detected by both D_{SH} and D_N distance functions (the top and the bottom panels, respectively). Individual meteors belonging to the streams indicated on the plot are represented as full dots; to avoid confusion with the Quadrantids, the southern δ Aquarids are represented by open circles. The asterisk in the lower plot denotes the single southern α Capricornid detected by D_N (see text).

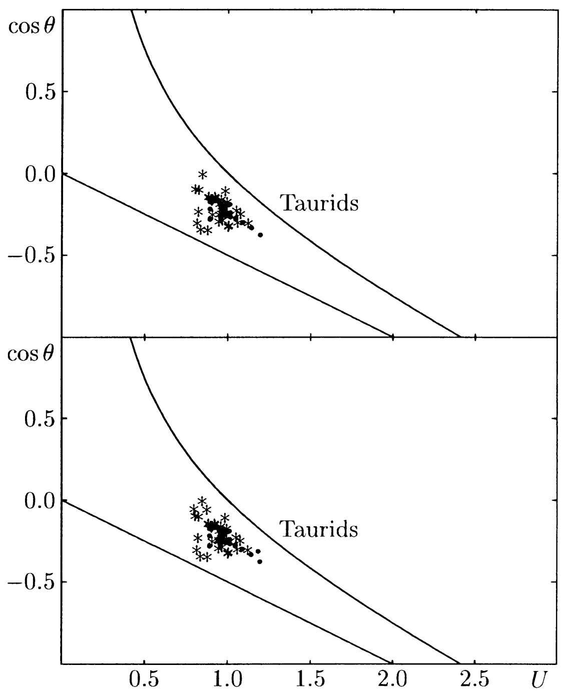

Figure 2. Same as Fig. 1 for the Taurids. Full dots denote northern Taurids, asterisks denote southern Taurids.

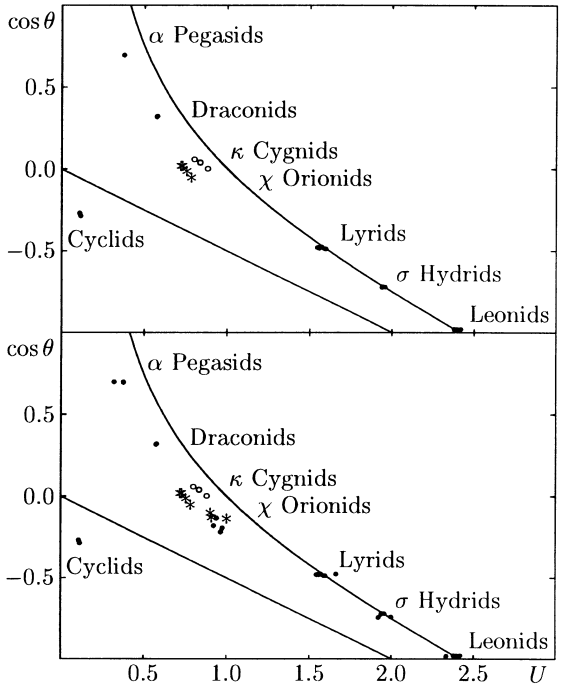

Figure 3. Same as Fig. 1 for the streams with less than 10 members. To avoid confusion, the κ Cygnids are represented by open circles; northern χ Orionids are represented with full dots, southern χ Orionids with asterisks.

increases of the threshold. In this case, we keep the same value of M, and the associated threshold, when looking for larger groups.

We decided to accept all the streams detected with thresholds $D_{c,M}$ such that the reliability level $W_M = 99$ per cent (see Table 1). Several runs were made; starting from $M = 2$ members, all the pairs of meteors (k, l) with distance $D_{kl} < D_{c,2}$ have been found. In the following run (with the new threshold $D_{c,3}$) all the streams of $M = 3$ members have been found, which possibly included some sub-streams already found as two-member streams, and so on.

3 RESULTS OF THE CLASSIFICATIONS OBTAINED WITH D_{SH} AND D_N

Using the function D_{SH} we detected 17 streams, combining 31 per cent of the 865 meteor orbits; in the search made using the D_N, 20 streams were detected, and the stream component included 35 per cent of the meteor sample.

All the streams were compared on the basis of the detailed lists of stream members; as a result, two groups of streams have been sorted out: those recognized in both searches, and those recognized in only one search.

3.1 Streams identified in both searches

Table 2 lists the first group, composed of the 15 streams that were identified in both searches, and Figs 1, 2 and 3 show these streams in the plane U–$\cos\theta$; Fig. 1 contains the streams with more than 10 members, except the Taurids which, to avoid clutter, are plotted in Fig. 2, and Fig. 3 those with less than 10 members. Noticeable in Fig. 1 is the coincidence, in the U–$\cos\theta$ plane, of the Quadrantids and the southern δ Aquarids, already commented upon by Valsecchi et al. (1999).

Let us now briefly discuss each stream in turn, also noting when

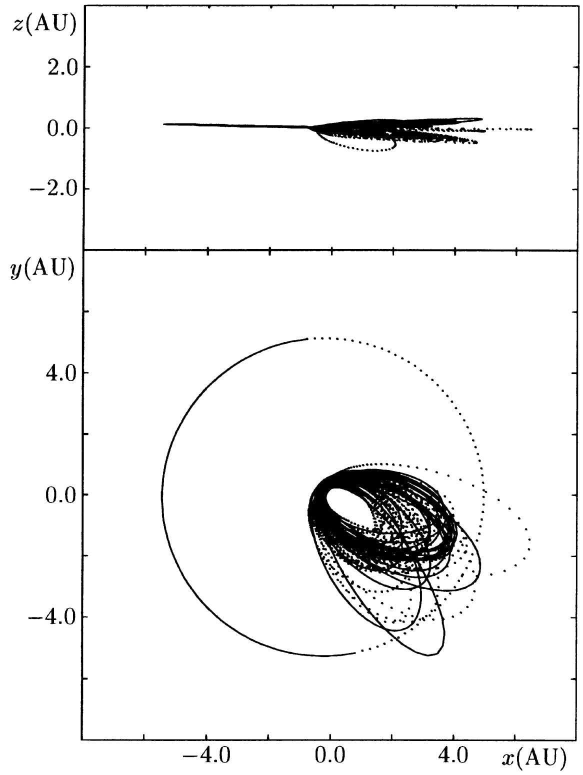

Figure 4. The orbits of the Taurid meteor stream identified with the new D_N distance function. The bottom panel shows the projections of the orbits on the plane of the ecliptic, that is our x–y plane, with the positive x-axis in the direction of the vernal equinox. In the top panel the orbits are projected on the x–z plane. The orbit of Jupiter is shown for reference; the part below the ecliptic of each orbit is dotted. One can easily distinguish the southern and northern branches of the Taurids.

our findings with D_{SH} differ from those of Lindblad (1971a), presumably because of the different threshold we chose.

3.1.1 *Lyrids*

With D_{SH} we found five Lyrids, and with D_N the same five plus an additional one; in both cases, the stream is compact, as the threshold used is $D_{c,3}$ (see the fourth and sixth columns of Table 2, and Table 1). The additional Lyrid found with D_N is numbered 1 in our data set; this is why we attach the label N to it in Table 1. For all other streams found in both searches, the number of the first meteor found is the same, so that no label needs to be attached. Lindblad (1971a) found four Lyrids, using a larger value of D_c ($D_c = 0.1$); we do not know the reason for this discrepancy.

3.1.2 *α Capricornids*

With D_{SH} we found 20 α Capricornids using $D_{c,11}$, the largest threshold giving significant results, and with D_N the same 20, plus an additional one, using $D_{c,7}$, not the largest value allowed; thus, this stream seems to be somewhat more compact in the space of the new variables. Interestingly, the isolated meteor identified by D_N as a member of the α Capricornids belongs to a *southern* branch, not detected at all by D_{SH}.

3.1.3 *Perseids*

With D_{SH} we found 30 Perseids, using $D_{c,8}$, and with D_N the same 30 plus three additional ones, using $D_{c,7}$. Lindblad (1971a) found 27 Perseids, using a slightly larger value of D_c ($D_c = 0.15$); why he found less members using a larger threshold is not clear to us, although this might be partly a result of the fact that one of the meteors, the q of which needed to be corrected, is in fact a Perseid, and also a possibly different number of significant digits in the data available to Lindblad.

3.1.4 *κ Cygnids*

With D_{SH} we found four κ Cygnids, using $D_{c,4}$, and with D_N the same four, using $D_{c,3}$; this stream is apparently somewhat more compact in the space of the new variables. Lindblad (1971a) found nine κ Cygnids, using his rather large threshold $D_c = 0.15$; for $D_c = 0.10$ he found that the stream split into two smaller ones, one of two members and the other of four members.

3.1.5 *Taurids*

With D_{SH} we found 53 Taurids, of which 20 belonged to the northern branch and 33 to the southern one; with D_N we found the same 53, plus five additional ones, with 22 meteors in the northern branch and 36 in the southern one. In both searches we used the largest thresholds allowed; Lindblad (1971a) found 54 Taurids, and the discrepancy should be caused by the slightly larger threshold used by him. The orbits of the 58 Taurids found using D_N are shown in Fig. 4.

3.1.6 *Quadrantids*

Both with D_{SH} and D_N we found 14 Quadrantids, in both cases using $D_{c,3}$, something that indicates a very compact stream.

3.1.7 *Geminids*

Both with D_{SH} and D_N we found 51 Geminids, in both cases using $D_{c,3}$, indicating a very compact stream. Lindblad (1971a) found 50 Geminids, using his much larger value of D_c ($D_c = 0.15$); as for the Perseids, it is not clear to us why Lindblad found fewer members using a larger threshold, although in this case only the different number of significant digits may have played a role, as the were no corrections to be made for the Geminid members.

3.1.8 *χ Orionids*

With D_{SH} we found four χ Orionids, all belonging to the southern branch, using $D_{c,4}$, and with D_N the same four plus seven additional ones, using $D_{c,8}$, the largest one giving significant results; of the 11 χ Orionids identified by D_N, seven belong to the southern branch and four to the northern one, completely missed by D_{SH}. Lindblad (1971a) found 10 χ Orionids, and the discrepancy should be caused by the larger threshold used by him ($D_c = 0.15$); using $D_c = 0.10$ he found only three χ Orionids.

3.1.9 *Leonids*

With D_{SH} we found five Leonids, using $D_{c,4}$, and with D_N the same five plus an additional one, using $D_{c,3}$; this stream is apparently somewhat more compact in the space of the new variables.

3.1.10 *σ Hydrids*

With D_{SH} we found three σ Hydrids, using $D_{c,3}$, and with D_N the same three plus two additional ones, using $D_{c,5}$. Lindblad (1971a) found five σ Hydrids, and the discrepancy should be caused by the larger threshold used by him; using $D_c = 0.10$ he found four σ Hydrids.

3.1.11 Orionids, η Aquarids

With D_{SH} we found 19 Orionids and one η Aquarid, using $D_{c,6}$, while with D_N we found the same 19 Orionids but failed to recognize as belonging to the same stream the single η Aquarid, using $D_{c,3}$. Evidently, the Orionids are very compact in the space of the new variables, and the single η Aquarid is too peripheric. Lindblad (1971a) found only 18 Orionids; as for the Geminids, it is not clear to us why Lindblad found fewer members using a larger threshold.

3.1.12 Southern δ Aquarids

Both with D_{SH} and D_N we found 11 Southern δ Aquarids, in both cases using $D_{c,5}$, something that indicates a very compact stream.

3.1.13 Draconids

Both with D_{SH} and D_N we found two Draconids, in both cases using $D_{c,2}$.

3.1.14 Cyclids

Both with D_{SH} and D_N we found 2 Cyclids, in both cases using $D_{c,2}$. Lindblad (1971a) found six Cyclids, and the discrepancy should be caused by the larger threshold used by him.

3.1.15 α Pegasids

With D_{SH} we found two α Pegasids, using $D_{c,2}$, and with D_N the same two plus an additional one, using $D_{c,3}$. Lindblad (1971a) found four α Pegasids, and in this case also, the discrepancy should be caused by the large threshold used by him.

3.2 Streams identified in only one search

Tables 3 and 4 list the streams that have been identified by only one of the two distance functions, and Fig. 8 shows these streams in the plane U–$\cos\theta$.

We now briefly discuss each of these streams stream in turn, again noting when our findings with D_{SH} differ from those of Lindblad (1971a).

Table 3. Meteor streams detected only by D_{SH}. The first column gives the stream name, the second its code, the third the number of members M_{SH} identified by D_{SH}, for the threshold value $D_{c,M}$ given in the fourth column, and the last column gives the number of members in common with a stream found only by D_N. The flags S and N added to the stream code denote that the latter refers to the searches made with D_{SH} and D_N, respectively.

Name	Code	M_{SH}	$D_{c,M}$	
σ Leonids	66S	26	0.145	
Andromedids	130S	12	0.145	6 in 431N

Table 4. Meteor streams detected only by D_N. The first column gives the stream name, the second its code, the third the number of members M_N identified by D_N, for the threshold value $D_{c,M}$ given in the fourth column, and the last column gives the number of members in common with a stream found only by D_{SH}. The flags S and N added to the stream code denote that the latter refers to the searches made with D_{SH} and D_N, respectively.

Name	Code	M_N	$D_{c,M}$	
ε Geminids	44N	4	0.113	
Monocerotids	90N	3	0.080	
α Virginids	217N	12	0.188	
Northern δ Aquarids	371N	6	0.160	
ε Piscids	431N	16	0.188	6 in 130S

3.2.1 σ Leonids

Using D_{SH} we found a very populous σ Leonid stream, including 26 meteors, using $D_{c,11}$, the highest allowed threshold; 15 meteors belong to the northern branch, and 11 to the southern one. All these meteors were attributed to the sporadic background by D_N. Lindblad (1971a) found 27 σ Leonids, using a slightly larger threshold. As can be seen in the top frame of Fig. 5, where the members of this stream are represented by small open circles, there is considerable dispersion in the U–$\cos\theta$ plane.

3.2.2 Andromedids

Using D_{SH} we identified 12 members of the Andromedids stream, with $D_{c,11}$, the highest allowed threshold; three meteors belong to

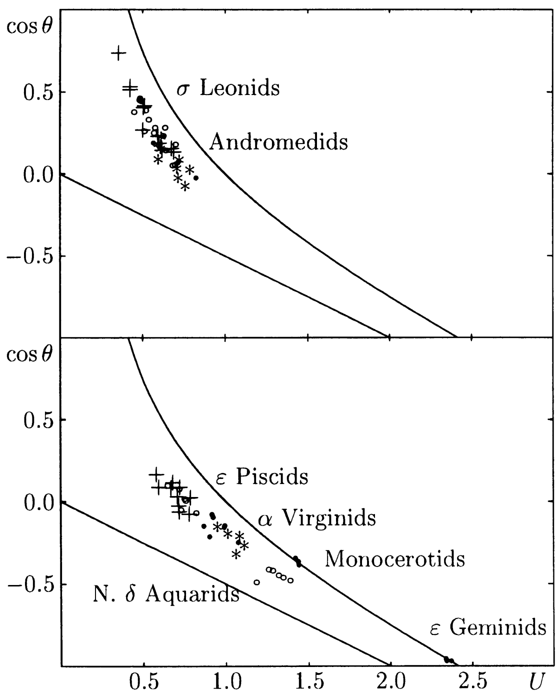

Figure 5. Same as Fig. 1 for the streams detected by only one search. In the top frame northern σ Leonids are indicated by open circles, southern σ Leonids by crosses, northern Andromedids by full dots and southern Andromedids by asterisks. In the bottom frame full dots indicate northern α Virginids, Monocerotids and ε Geminids, open circles indicate northern δ Aquarids and northern ε Piscids, asterisks indicate southern α Virginids and crosses indicate southern ε Piscids.

the Northern branch, and nine to the southern one. The search with D_N attributed six of these meteors to the sporadic background, whereas the other six were put in the ε Piscids stream. Also Lindblad (1971a) found 12 Andromedids, using a slightly larger threshold; however, with $D_c = 0.10$ he only found two Andromedids, thus indicating a very loose stream.

3.2.3 *ε Geminids*

Using D_N we found four members of the ε Geminids stream, using $D_{c,4}$. All these meteors were attributed to the sporadic background by D_{SH}.

3.2.4 *Monocerotids*

Using D_N we found three members of the Monocerotids stream, using $D_{c,3}$. All these meteors were attributed to the sporadic background by D_{SH}.

3.2.5 *α Virginids*

Using D_N we found a rather populous α Virginids stream, composed of 12 members, using $D_{c,8}$, the highest allowed threshold; of these 12 α Virginids, seven belong to the Northern branch, and five to the Southern one. All these meteors were attributed to the sporadic background by D_{SH}.

3.2.6 *Northern δ Aquarids*

With D_N we found six northern δ Aquarids, using $D_{c,6}$. All these meteors were attributed to the sporadic background by D_{SH}.

3.2.7 *ε Piscids*

Using D_N we found a rather populous ε Piscids stream, composed of 16 members, using $D_{c,8}$, the highest allowed threshold; of these meteors, six belong to the northern branch, and 10 to the southern one. The search done with D_{SH} assigned 10 meteors to the sporadic background, while the other six were incorporated into the Andromedids stream; of the latter, one belongs to the Northern branch of the ε Piscids identified by D_N, and the remaining five to the southern branch.

3.3 Summary of the streams detected with D_N

Approximate geocentric and orbital data concerning the 15 streams identified with D_N are given in Table 5. The dates given there in the second column bracket those of the meteors of our sample assigned to each stream, and the other quantities are simply the averages over those of the members of each stream.

For completeness, we also reported the same data for the two streams detected only with D_{SH} in Table 6.

4 STREAMS OBTAINED WITH THE REDUCED DISTANCE FUNCTION D_R

The reduced distance functions D_R (for the formulation, see Valsecchi et al. 1999) and the thresholds given in Table 1 were used in an additional search amongst the 865 precise meteor orbits. Fourteen groups were detected which included 52 per cent of the meteor sample.

As already noted, D_R can identify members of the same stream that are on orbits of different e, i, ω and Ω, although its results must

Table 5. Approximate geocentric and orbital data for the streams found by D_N amongst 865 precise photographic meteor orbits at the reliability level $W_M = 99$ per cent; α_G, δ_G, i, ω and Ω are given for B1950.0. For streams identified as single ones, but possessing both a northern and a southern branch, the data are given separately for each branch.

Stream name	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
Lyrids	21/4	23/4	272	33	0.92	0.99	80	214	32	47	119	198
α Capricornids (N)	24/7	22/8	314	−9	0.58	0.78	6	268	134	23	89	262
α Capricornids (S)	22/8		333	−18	0.63	0.62	4	89	329	18	89	276
Perseids	4/8	21/8	47	58	0.95	0.95	113	150	139	59	139	164
κ Cygnids	19/8	22/8	289	56	0.98	0.76	39	202	147	25	88	194
ε Geminids	11/10	22/10	100	26	0.81	0.96	174	231	203	70	166	258
Taurids (N)	16/9	15/11	44	19	0.32	0.85	3	299	213	30	104	267
Taurids (S)	17/9	21/11	40	10	0.34	0.82	6	118	27	28	104	275
Quadrantids	2/1	4/1	230	49	0.98	0.68	72	171	282	41	116	176
Geminids	7/12	14/12	112	32	0.14	0.90	24	324	261	35	117	258
χ Orionids (N)	9/12	30/12	97	26	0.38	0.83	3	291	265	28	101	267
χ Orionids (S)	4/12	14/12	80	17	0.51	0.79	5	96	77	25	93	276
Monocerotids	10/12	15/12	103	8	0.18	1.00	37	129	80	43	112	286
Leonids	15/11	20/11	153	22	0.98	0.92	162	173	235	71	170	168
σ Hydrids	4/12	15/12	127	2	0.24	0.98	126	122	78	58	137	295
Orionids	19/10	24/10	96	16	0.57	0.97	165	83	29	66	155	288
α Virginids (N)	4/4	12/5	222	−8	0.41	0.83	8	288	34	28	99	263
α Virginids (S)	2/4	24/4	208	−17	0.32	0.87	7	118	198	31	103	275
Southern δ Aquarids	21/7	8/8	340	−16	0.08	0.97	27	151	308	41	118	278
Northern δ Aquarids	5/8	25/8	347	2	0.10	0.95	21	328	142	38	117	262
ε Piscids (N)	19/9	13/10	6	10	0.58	0.76	5	268	190	22	89	263
ε Piscids (S)	14/9	9/10	4	−5	0.61	0.73	4	85	5	21	88	276
Draconids	9/10		271	47	1.00	0.70	25	177	196	17	71	175
Cyclids	19/10		50	−7	0.83	0.12	3	119	26	3	108	296
α Pegasids	12/11		342	22	0.97	0.68	7	200	230	11	42	226

Table 6. Approximate geocentric and orbital data for the streams found only by D_{SH} amongst 865 precise photographic meteor orbits at the reliability level $W_M = 99$ per cent; α_G, δ_G, i, ω and Ω are given for B1950.0. The data are given separately for the northern and the southern branch.

Stream name	Dates		α_G (°)	δ_G (°)	q (au)	e	i (°)	ω (°)	Ω (°)	V_G (km s^{-1})	θ (°)	ϕ (°)
σ Leonids (N)	18/3	22/5	191	8	0.79	0.69	6	241	23	17	74	257
σ Leonids (S)	9/4	14/6	204	−15	0.80	0.67	2	59	227	16	71	275
Andromedids (N)	16/9	7/11	6	8	0.63	0.76	4	261	193	21	85	265
Andromedids (S)	14/9	6/11	10	−5	0.64	0.73	5	81	92	20	86	278

then be supported by accurate modelling of stream formation. Actually, the fact that a pair of meteor orbits is characterized by a small value of D_R is a *necessary*, but not a *sufficient* condition to infer that they belong to the same meteoroid stream.

Our search confirmed the above reasoning, in the sense that for only one of the 14 groupings found can we speak of an actual meteoroid stream. Its members are listed in Table 7, where they are sorted according to the known major showers to which they belong, when appropriate: the Quadrantids, the southern δ Aquarids and the northern δ Aquarids.

This finding is in line those obtained a few years ago by Babadzanov & Obrubov (1993), who simulated the evolution of meteoroid particles ejected from comet 96P/Machholz 1. They showed that these particles may produce eight meteor showers – and among others the ones mentioned in Table 7.

The last part of Table 7 lists some interlopers, identified by our stream-searching algorithm as potential members of the meteoroid stream. Such a phenomenon is rather normal in every cluster analysis method, and in this case is reinforced by the fact that we are making our cluster analysis using only two variables in a problem that has a higher dimensionality. In Fig. 6 the orbits of the members of the Quadrantid–Aquarid meteoroid stream are plotted.

Table 7. The members of the Quadrantid–Aquarid meteoroid stream detected amongst 865 precise photographic meteor orbits by the reduced distance function D_R at the reliability level $W_M = 99$ per cent; α_G, δ_G, i, ω and Ω are given for B1950.0.

Date yr m d	e	q (AU)	ω (°)	Ω (°)	i (°)	α_R (°)	δ_R (°)	V_G (km/s)	θ (°)	ϕ (°)	Name
52 07 25	0.978	0.063	154	302	27	336	−17	43.0	119	277	S. δ Aquarids
52 07 27	0.977	0.069	152	304	26	337	−17	43.2	118	277	
52 07 28	0.973	0.064	154	305	30	340	−16	42.7	120	278	
52 07 28	0.979	0.064	153	305	26	338	−16	43.4	118	277	
52 07 29	0.969	0.067	154	306	28	341	−16	41.8	120	278	
53 08 05	0.967	0.098	147	313	24	343	−15	41.6	116	278	
53 08 06	0.961	0.109	145	313	25	344	−16	41.3	116	279	
53 08 06	0.972	0.109	144	314	19	342	−14	41.7	114	277	
53 08 08	0.963	0.092	149	316	27	348	−14	41.2	117	279	
52 07 29	0.970	0.071	153	306	29	340	−17	42.2	119	279	
53 08 05	0.976	0.065	333	132	24	340	−2	43.0	119	263	N. δ Aquarids
52 08 18	0.970	0.075	332	145	17	352	2	41.6	118	265	
53 08 10	0.960	0.097	328	137	22	343	0	40.5	117	262	
51 01 03	0.715	0.974	168	282	74	230	48	44.1	117	175	Quadrantids
54 01 03	0.682	0.970	165	282	69	235	49	41.5	114	174	
54 01 03	0.664	0.978	170	282	73	229	49	43.1	117	176	
54 01 03	0.685	0.983	180	282	73	225	52	43.2	117	180	
54 01 03	0.674	0.978	170	282	72	230	49	43.1	117	176	
54 01 03	0.675	0.977	170	282	72	230	49	43.1	117	176	
54 01 04	0.683	0.975	168	282	71	232	49	42.4	115	175	
54 01 04	0.701	0.980	173	283	73	228	49	43.7	117	177	
54 01 04	0.692	0.975	169	284	72	231	48	43.3	117	175	
54 01 02	0.659	0.983	177	281	72	226	51	42.8	117	179	
54 01 03	0.679	0.979	172	282	70	231	50	42.1	115	176	
54 01 03	0.662	0.977	170	282	72	230	49	42.7	116	176	
54 01 03	0.640	0.981	174	282	71	229	50	42.2	116	178	
54 01 04	0.670	0.977	170	282	73	230	48	43.4	117	176	
53 01 16	0.934	0.216	308	296	47	153	33	42.1	115	247	?
53 01 21	0.961	0.088	149	121	19	149	7	41.4	117	277	?
53 03 13	0.793	0.445	284	352	62	218	24	41.0	116	228	?
53 06 05	0.717	0.979	204	74	72	301	45	42.4	115	191	?
49 08 21	0.959	0.094	149	328	24	359	−9	40.6	117	278	?
52 10 21	0.729	0.952	206	208	71	127	85	42.4	115	191	Ursids ?
53 11 09	0.795	0.559	89	46	65	85	−20	41.4	116	320	?

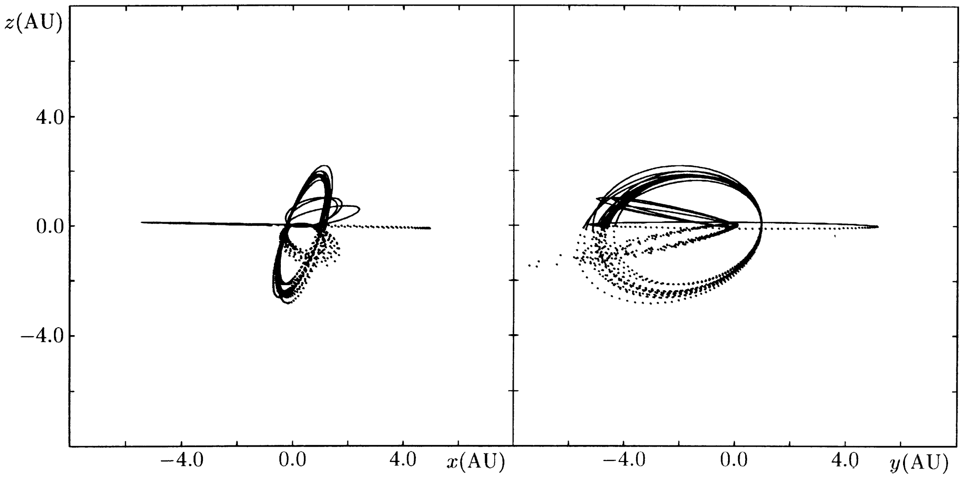

Figure 6. The Keplerian orbits of the Quadrantid–Aquarid meteoroid stream detected by D_R, in the x–z plane (left panel), and in the y–z plane (right panel, axes defined as in Fig. 4). The Aquarid component consists of the northern and southern branches, characterized by high eccentricities, very small perihelion distances and small inclinations. The Quadrantids have orbits with moderate eccentricities, high inclinations and large perihelion distances.

5 CONCLUSIONS

The contents of Tables 2, 3 and 4 allow us to draw the following conclusions.

(i) 15 major and minor streams (Table 2) have been identified with both distance functions.

(ii) For six streams, the κ Cygnids, the Quadrantids, the Geminids, the southern δ Aquarids, the Draconids and the Cyclids, the membership was identical in the two searches.

(iii) For the Orionids the membership was also identical, but D_N failed to identify as belonging to the same stream an η Aquarid meteor.

(iv) For seven streams, the Lyrids, the α Capricornids, the Perseids, the Taurids, the Leonids, the σ Hydrids and the α Pegasids, D_N was able to add a few more members to those identified by D_{SH}.

(v) For the χ Orionids D_N was able to add many more members to those identified by D_{SH}.

(vi) Two streams were identified with D_{SH} and not with D_N, the σ Leonids and the Andromedids; these two streams, however, especially the first, are rather loose in the U–$\cos\theta$ plane, where the spread should be significantly less than for the conventional orbital elements used in D_{SH}, because U and $\cos\theta$ are secular near-invariants (Valsecchi et al. 1999).

(vii) Five streams, the ε Geminids, the Monocerotids, the α Virginids, the northern δ Aquarids and the ε Piscids, were identified with D_N and not with D_{SH}.

(viii) D_N seems to work rather differently from D_{SH} when applied to near-ecliptical streams possessing both a northern and a southern branch: D_N detects a southern α Capricornid, missed by D_{SH}, and the northern branch of the χ Orionids, also missed by D_{SH}; moreover, of the streams identified only by D_N two, the α Virginids and the ε Piscids, are near-ecliptical and possess both branches; also the two streams identified by D_{SH} and not by D_N possess both branches, but their dispersion in the U–$\cos\theta$ plane, as noted before, casts some doubts on their reality; the difference of behaviour between the two distance functions shows up for $i \lesssim 10°$, and is very likely caused by the different spaces in which the variables are defined.

(ix) The reduced distance function D_R can be successfully applied to search for meteoroid streams originating multiple showers, because in our computations the Quadrantids and the southern and northern δ Aquarids have been identified as potential members of the same meteoroid stream.

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REFERENCES

Babadzhanov P. V., Obrubov Yu. V., 1991, Astron. Vestn., 25, 387
Babadzhanov P. V., Obrubov Yu. V., 1992a, in Harris A., Bowell E., eds, Asteroids, Comets, Meteors 1991. Lunar and Planetary Inst., Houston, p. 27
Babadzhanov P. V., Obrubov Yu. V., 1992b, Astron. Vestn., 26, 70
Babadzhanov P. B., Obrubov Yu. V., 1993, in Stohl J., Williams I. P., eds, Meteoroids and their parent bodies. Slov. Acad. Sci., Bratislava, p. 49
Hawkins G. S., Southworth R. B., 1958, Smithson. Contrib. Astrophys., 2, 349
Hawkins G. S., Southworth R. B., 1961, Smithson. Contrib. Astrophys., 4, 85
Jacchia L. G., Whipple F. L., 1961, Smithson. Contrib. Astrophys., 4, 97
Jopek T. J., Froeschlé Cl., 1997, A&A, 320, 631
Jopek T. J., Valsecchi G. B., Froeschlé Cl., 1996, in Manara A., Migliorini F., Paolicchi P., eds, Planet. Sci.: First Italian Meeting. Alenia, Torino, p. 100
Lindblad B. A., 1971a, Smithson. Contrib. Astrophys., 12, 1
Lindblad B. A., 1971b, Smithson. Contrib. Astrophys., 12, 14
Lindblad B. A., 1987, in Ceplecha Z., Pecina P., eds, Proc. 10th Eur. Regional Astron. Meeting, Interplanetary Matter, Vol. 2. Czech. Acad. Sci., Prague, p. 201
Lindblad B. A., 1991, in Levasseur-Regourd A. C., Hasegawa H., eds, Origin and Evolution of Interplanetary Dust. Kluwer, Dordrecht, p. 311
Southworth R. B., Hawkins G. S., 1963, Smithson. Contrib. Astrophys., 7, 261
Valsecchi G. B., Jopek T. J., Froeschlé Cl., 1996, in Proc. Planet. Sci.: First Italian Meeting, Bormio, Italy, p. 111
Valsecchi G. B., Jopek T. J., Froeschlé Cl., 1999, MNRAS, 304, 743 (Paper I, this issue)
Whipple F. L., 1954, AJ, 59, 201

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