Mon. Not. R. Astron. Soc. **304**, 743–750 (1999)

Meteoroid stream identification: a new approach – I. Theory

G. B. Valsecchi,[1★] T. J. Jopek[2] and Cl. Froeschlé[3]

[1]*IAS – Planetologia, Area di Ricerca CNR, via Fosso del Cavaliere, I-00133 Roma, Italy*
[2]*Obserwatorium Astronomiczne UAM, Sloneczna 36, PL-60286 Poznań, Poland*
[3]*Observatoire de la Côte d'Azur, CNRS/UMR 6529, Bd de l'Observatoire, BP 4229, F-06304 Nice, France*

Accepted 1998 November 2. Received 1998 October 29; in original form 1998 July 16

ABSTRACT

We introduce a new approach to meteoroid stream identification, based on a distance function involving four geocentric quantities that are directly linked to observations; the new distance function is thus defined in a space that has as many dimensions as the number of independently measured physical quantities, at variance from the conventional orbital similarity criterion of Southworth & Hawkins. Two of the new variables turn out to be near-invariant with respect to the principal secular perturbation affecting meteoroid orbits, the one associated with the cycle of ω.

Key words: celestial mechanics, stellar dynamics – meteors, meteoroids.

1 INTRODUCTION: WHY A NEW CRITERION?

The procedures to identify meteoroid streams normally involve the use of a measure of the similarity of the orbits of meteoroids; for this purpose, perhaps the most common and widely used such tool is the orbital similarity criterion D_{SH} introduced by Southworth & Hawkins (1963), which has been in use for more than 30 yr. This criterion has not been without critics in the recent past; for instance, Štohl & Porubčan (1987) have questioned the relative weight of the various terms in it. In fact, past attempts to improve on D_{SH}, by Drummond (1981) and Jopek (1993), have not changed the basic approach, still using the conventional orbital elements but just weighing the contributions in different ways; Jopek (1993) has reviewed these attempts.

In essence, D_{SH} is a generalized measure of distance between two orbits in the five-dimensional space of the conventional orbital elements q (perihelion distance), e (eccentricity), i (inclination), ω (argument of perihelion) and Ω (longitude of ascending node):

$$D_{\mathrm{SH}}^2 = [e_2 - e_1]^2 + [q_2 - q_1]^2 + \left[2\sin\frac{I_{21}}{2}\right]^2 + \left[\left(\frac{e_2+e_1}{2}\right)\left(2\sin\frac{\pi_{21}}{2}\right)\right]^2, \qquad (1)$$

where

$$\left[2\sin\frac{I_{21}}{2}\right]^2 = \left[2\sin\frac{i_2-i_1}{2}\right]^2 + \sin i_1 \sin i_2 \left[2\sin\frac{\Omega_2-\Omega_1}{2}\right]^2 \qquad (2)$$

and

$$\pi_{21} = \omega_2 - \omega_1 + 2\arcsin\left[\cos\frac{i_2+i_1}{2}\sin\frac{\Omega_2-\Omega_1}{2}\sec\frac{I_{21}}{2}\right], \qquad (3)$$

and where the suffixes 1 and 2 refer to the two orbits being compared.

However, there is one dimension in D_{SH} that we should get rid of: as meteoroids are observed at heliocentric distance $r \simeq 1$ au, we have approximately that either

$$1 = \frac{a(1-e^2)}{1+e\cos(-\omega)} \qquad (4)$$

or

$$1 = \frac{a(1-e^2)}{1+e\cos(180^\circ-\omega)}, \qquad (5)$$

depending on the value of $\omega + f$. These relations are valid for orbits with non-zero inclination, which are essentially all those of practical interest.

The problem of measuring the similarity of meteor orbits therefore has only four dimensions, and one wonders whether there is a suitable set of four quantities, possibly easily computable starting from the usual orbital elements, that would allow meteoroid stream identification in a 'natural' way; it would be better still if these quantities were directly deducible from observed quantities, without passing through the derivation of the orbital elements.

Southworth & Hawkins themselves were somewhat concerned in this respect by their choice of the orbital elements with which to compare meteor orbits, as they wrote in their 1963 paper that 'several attempts, based upon a variety of principles, were made to classify the meteors into streams and a "sporadic" remainder. None of the classifications based on geocentric quantities – radiants, velocities, and dates of occurrence – was satisfactory. It became clear that a comprehensive quantitative criterion was required that would embrace all the elements of the orbit'.

Regarding the fact that D_{SH} is defined in a five-dimensional space, they remarked that 'the fact that all observed meteors pass close to the orbit of the Earth imposes one constraint on the

★E-mail: giovanni@saturn.iasrm.cnr.it

observed five elements, and thus reduces the number of independent elements to four. The Earth is not important in the evolution of meteor streams, however, so that this constraint is not relevant to the differences within streams. Accordingly, we disregard it in formulating D.'

It turns out that one *can* use a set of geocentric quantities to classify meteors in streams, and such a set with the desirable properties mentioned above is described in Section 2; it is based on the components of the geocentric velocity at encounter that are essential for Öpik's theory of close encounters.

In Section 3 the time behaviour of the new variables is examined, and it is shown that two of them are near-invariant with respect to the most important secular perturbation affecting meteoroid orbits; numerical checks of this property are also given. The discussion (Section 4) and the conclusions (Section 5) then follow.

2 CHANGING THE VIEWPOINT: FROM HELIOCENTRIC TO GEOCENTRIC QUANTITIES

2.1 A digression: the geometric setup of Öpik's theory of close encounters

Öpik's theory of close encounters is described in his book (Öpik 1976); here we only borrow from it the geometric setup, following Carusi, Valsecchi & Greenberg (1990) and Valsecchi (1992).

We assume that the Earth moves on an unperturbed circular orbit, of radius equal to 1, lying on the ecliptic; a massless meteoroid is on an orbit, with orbital parameters a (semimajor axis), e, i, ω and Ω, that crosses that of the Earth at (at least) one of the nodes. We put both the constant of gravity and the mass of the Sun equal to 1, and impose that the heliocentric velocity of the Earth is $v_\oplus = 1$, instead of $v_\oplus = \sqrt{1 + \mathrm{M}_\oplus}$, disregarding the effect of the terrestrial mass $\mathrm{M}_\oplus$ on its orbital velocity.

With these conventions the geocentric velocity of the meteoroid when crossing the Earth's orbit is

$$U = \sqrt{3 - T}, \tag{6}$$

where T is the well-known Tisserand parameter:

$$T = \frac{1}{a} + 2\sqrt{a(1-e^2)}\cos i. \tag{7}$$

In a reference frame centred on the Earth, with the z-axis perpendicular to the plane of the ecliptic, the y-axis in the direction of the Earth's velocity and the x-axis pointing away from the Sun, which is therefore located at $x = -1$, $y = 0$, $z = 0$, the unperturbed geocentric encounter velocity $\boldsymbol{U}$ of the particle has components

$$U_x = U \sin\theta \sin\phi, \tag{8}$$

$$U_y = U \cos\theta, \tag{9}$$

$$U_z = U \sin\theta \cos\phi, \tag{10}$$

where θ is the angle between $\boldsymbol{U}$ and the y-axis (i.e. the direction of motion of the Earth), and ϕ is the angle between the y–z plane and that containing $\boldsymbol{U}$ and the y-axis (for encounters at the ascending node $-90^\circ < \phi < 90^\circ$, while at the descending node $90^\circ < \phi < 270^\circ$).

The components of $\boldsymbol{U}$ can be expressed in terms of a, e and i:

$$U_x = \pm\sqrt{2 - \frac{1}{a} - a(1-e^2)}, \tag{11}$$

$$U_y = \sqrt{a(1-e^2)}\cos i - 1, \tag{12}$$

$$U_z = \pm\sqrt{a(1-e^2)}\sin i; \tag{13}$$

and a, e and i in terms of the components of $\boldsymbol{U}$:

$$a = \frac{1}{1 - U^2 - 2U_y}, \tag{14}$$

$$e = \sqrt{U^4 + 4U_y^2 + U_x^2(1 - U^2 - 2U_y) + 4U^2U_y}, \tag{15}$$

$$i = \arctan\frac{U_z}{1 + U_y}; \tag{16}$$

note that in equation (14) the minus sign must be used for pre-perihelion encounters, and in equation (16) for encounters at the descending node.

2.2 The new variables

Let us now come back to the meteoroid hitting our idealized Earth. The longitude of the latter, $\lambda_\oplus$, at the date and time of the meteor observation, obviously gives us the longitude λ of the meteoroid, as well as the position of the nodal line of its orbit.

We would then like to have three more quantities to characterize the orbit completely and we do not want to use the other orbital parameters a (or q), e, i and ω because, as seen in the Introduction, they are four quantities, three of which are related by a constraint, either (4) or (5).

On the other hand, the geometry of the encounter suggests to us what to do: use the modulus of the unperturbed geocentric velocity U and the two angles that give its direction, which in Öpik's theory are θ and ϕ, given by

$$\theta = \arccos\frac{U_y}{U}, \tag{17}$$

$$\phi = \arctan\frac{U_x}{U_z}. \tag{18}$$

Actually, θ and ϕ are the angles that define the direction *opposite* to that from which the meteoroid is seen to arrive (after removal of the effect of the Earth's gravity), i.e. opposite to the geocentric radiant, with the further important difference that the latter is defined with respect to fixed stars, whereas θ and ϕ identify the antiradiant at the place where the meteor is observed, since they are defined in the instantaneous geocentric reference frame in which the x–y plane coincides with the ecliptic and the x-axis coincides with the heliocentric position vector of the Earth.

Let us discuss the potential use of θ and ϕ, in turn, in a new stream identification criterion.

First, we note that θ depends only on U and a, and not on e and i. Moreover, relative to the geocentric sphere of radius U, θ plays the role of a latitude, since a uniform distribution of points over the surface of this sphere would give a uniform distribution in $\cos\theta$; this fact, coupled with the circumstance that in terms of U and a we have

$$\cos\theta = \frac{1 - U^2 - 1/a}{2U}, \tag{19}$$

i.e. that $\cos\theta$ is directly proportional to $-1/a$, the orbital energy of the meteoroid, suggests that we use $\cos\theta$, and not θ, as a variable for the new distance function.

Coming to ϕ, let us consider the ranges of values that it takes, as a consequence of the sign of U_x and U_z, for the four basic geometries of encounter of a meteoroid with the Earth, summarized in Table 1.

Table 1. The sign of U_x and U_z, and the corresponding ranges of ϕ, for the four basic geometries of encounter of a meteoroid with the Earth.

	Ascending node			Descending node		
Post-perihelion	$U_x > 0$	$U_z > 0$	$0° < \phi < 90°$	$U_x > 0$	$U_z < 0$	$90° < \phi < 180°$
Pre-perihelion	$U_x < 0$	$U_z > 0$	$270° < \phi < 360°$	$U_x < 0$	$U_z < 0$	$180° < \phi < 270°$

Actually, in order to have that set (U, $\cos\theta$, ϕ, λ) maps one-to-one with (a, e, i, ω, Ω), given the constraint on the nodal distance, one could reduce ϕ and λ to, say, the first and fourth quadrants, and in this way one could recognize as belonging to the same meteoroid stream meteors belonging to showers like the Orionids and the η Aquarids, observed at two nodes of nearly the same orbit, when approaching perihelion and when receding from it.

However, such a choice would have an undesirable consequence for a nearly ecliptical stream like, for instance, the Taurids, which are characterized by $\phi \approx 270°$: ϕ of the southern branch would be put close to $-90°$ and ϕ of the northern branch close to $90°$, thus very far away from each other, whereas in reality their velocity vectors would differ only by a small angle.

Also making a change of variable to, say, the angle $\chi = \phi - 90° = \arctan U_z/U_x$ would not be a good idea, as it would solve the problem for the southern and northern branches of nearly ecliptical streams, but would make the same problem reappear for a stream with $q \approx 1$ au, like the Quadrantids; in this case $U_x \approx 0$ and $\phi \approx 180°$, so that $\chi \approx 90°$: for $\phi \gtrsim 180°$, we would have $\chi \gtrsim 90°$ and, by the reduction to the first and fourth quadrant, $\chi \gtrsim -90°$, i.e. the new angle would again jump by nearly 180° for meteors, the geocentric velocity vectors of which are in reality rather close to each other.

We therefore keep ϕ and λ as variables for the classification, and stipulate that they must not be reduced to just two quadrants, but can take any value between 0° and 360°, as shown in Table 1; the various problems that we have just described can be solved by considering the differences in ϕ and λ, using the procedure described in the next subsection.

The three quantities U, θ and ϕ can be obtained from the components U_x, U_y, U_z, which in turn, can be calculated from the directly measurable quantities that characterize an observed meteor: the geocentric velocity V_G and the equatorial coordinates of the meteor radiant α_G and δ_G. We have

$$\begin{pmatrix} U_x \\ U_y \\ U_z \end{pmatrix} = \hat{\mathbf{r}}(\lambda)\cdot\hat{\mathbf{p}}(\varepsilon)\cdot\frac{V_G}{29.7}\begin{pmatrix} -\cos\delta_G\cos\alpha_G \\ -\cos\delta_G\sin\alpha_G \\ -\sin\delta_G \end{pmatrix}, \tag{20}$$

where $\hat{\mathbf{p}}(\varepsilon), \hat{\mathbf{r}}(\lambda)$ are rotational matrices around the x- and z-axes, respectively, and the angle ε denotes the inclination of the ecliptic plane to the plane of the celestial equator.

Incidentally, the above expression shows that, if all the particles in a stream share the same values of a, e, i [and thus ω, because of (4) or (5)], and just differ in Ω, there is a unique curve in the sky along which the radiant may lie. The radiants of observed streams move during the period of observation, and a comparison between the actual movement of a radiant and the one obtained from (23) and the hypothesis of constancy of a, e and i may give some information about the distribution of orbital elements within a stream.

2.3 The new criterion

Having identified four quantities with the desired properties, we can now proceed to define a new orbital similarity criterion for meteoroid orbits. We formulate the new criterion as follows:

$$D_N^2 = [U_2 - U_1]^2 + w_1[\cos\theta_2 - \cos\theta_1]^2 + \Delta\xi^2, \tag{21}$$

where

$$\Delta\xi^2 = \min\left[w_2\Delta\phi_I^2 + w_3\Delta\lambda_I^2, w_2\Delta\phi_{II}^2 + w_3\Delta\lambda_{II}^2\right], \tag{22}$$

$$\Delta\phi_I = \left[2\sin\frac{\phi_2 - \phi_1}{2}\right], \tag{23}$$

$$\Delta\phi_{II} = \left[2\sin\frac{180° + \phi_2 - \phi_1}{2}\right], \tag{24}$$

$$\Delta\lambda_I = \left[2\sin\frac{\lambda_2 - \lambda_1}{2}\right], \tag{25}$$

$$\Delta\lambda_{II} = \left[2\sin\frac{180° + \lambda_2 - \lambda_1}{2}\right], \tag{26}$$

and w_1, w_2, w_3 are suitably defined weighting factors; note that $\Delta\xi$ is small if $\phi_1 - \phi_2$ and $\lambda_1 - \lambda_2$ are either both small or both close to 180°. In this second case, the two meteors would belong to the two showers corresponding to the two node crossings of essentially the same orbit, characterized by $q(1 + e) \approx 1$ and $\omega \approx 90°$ or $\omega \approx 270°$, as is the case for the Orionids and the η Aquarids.

In the application of the new criterion to a set of precisely measured photographic meteor orbits given in an accompanying paper (Jopek, Valsecchi & Froeschlé 1999, Paper II in this issue) we have put the three weighting factors equal to 1 but, of course, other choices are possible.

3 A REDUCED CRITERION BASED ON SECULARLY NEARLY INVARIANT QUANTITIES

3.1 Useful, directly observed secular near-invariants

Many factors influence the dynamical evolution of meteoroid stream particles, and some of them result from forces other than gravitation, especially for meteoroids of very small size. However, over not too long time-scales, and in the absence of planetary close encounters, we can assume that only planetary secular perturbations affect meteoroid orbits.

The most important secular perturbation to take into account in this case is the one related to the cycle of ω, first described by Kozai (1962). Assuming that all the planets are on circular and coplanar orbits, that there are no close encounters, and that the meteoroid body is not near mean motion or secular resonances, it leaves invariant not only the so-called Kozai integral K, upon which we will comment later, but also the z-component of the orbital angular momentum,

$$L_z = \sqrt{a(1 - e^2)}\cos i, \tag{27}$$

and, because of the assumed absence of close encounters, one can

Table 2. Orbital parameters of the meteoroid streams associated with 96P/Machholz 1, according to Babadzhanov & Obrubov (1993); rows denoted with T contain the data computed by those authors, while rows denoted with O give the observed data.

Stream name		q(au)	e	$i(°)$	$\Omega(°)$	$\omega(°)$	U	$\cos\theta$
Quadrantids	T	0.92–1.02	0.65–0.72	67–72	278–290	159–180	1.33	−0.41
	O	0.97–0.98	0.65–0.70	69–73	280–283	167–170	1.35	−0.43
Ursids	T	0.92–1.02	0.66–0.72	68–73	271–282	180–198	1.35	−0.42
	O	0.94–0.95	0.64–0.85	54–67	260–283	187–206	1.21	−0.30
Carinids	T	0.92–1.02	0.66–0.71	72–80	86–109	344–360	1.43	−0.48
	O	0.98	0.61	79	108	354	1.45	−0.52
κ Velids	T	0.92–1.02	0.68–0.71	74–80	99–109	0–16	1.45	−0.49
	O	0.97	0.51	77	102	18	1.39	−0.52
Northern	T	0.04–0.11	0.96–0.99	19–40	104–136	326–340	1.41	−0.47
δ Aquarids	O	0.06–0.12	0.95–0.98	14–21	128–143	323–334	1.34	−0.44
Southern	T	0.03–0.12	0.96–0.99	20–40	296–322	141–160	1.41	−0.47
δ Aquarids	O	0.07–0.14	0.96–0.99	23–32	304–322	139–152	1.39	−0.42
Daytime	T	0.03–0.12	0.96–0.99	20–40	72–87	20–37	1.41	−0.47
Arietids	O	0.04–0.10	0.94–0.98	18–46	77–89	19–30	1.34	−0.51
α Cetids	T	0.04–0.12	0.96–0.99	19–36	249–263	203–216	1.40	−0.45
	O	0.06–0.18	0.89–0.99	20–31	255–269	194–214	1.27	−0.44

also count on the invariance of the orbital energy,

$$E = -\frac{1}{2a}. \tag{28}$$

Then, however,

$$T = \frac{1}{a} + 2\sqrt{a(1-e^2)}\cos i = 2(L_z - E) \tag{29}$$

is constant, and therefore so is U, because of (6). Finally, if a and U are conserved, so is $\cos\theta$, because of (19). Note that here we are dealing with U and θ as *quantities computed from the orbital elements*, using Öpik's expressions; these expressions are very valuable in this context because of the interpretation of U and θ in terms of observed quantities in the case of meteors.

The secular invariance of U and $\cos\theta$ suggests to us that we use these quantities to identify as possibly being originated from the same parent body meteor showers with orbits having different q, e, i, ω and Ω, as a result of secular perturbations on a single meteoroid stream (Babadzhanov & Obrubov 1991, 1992a,b, 1993).

A possible definition for such a criterion is

$$D_{\mathrm{R}}^2 = [U_2 - U_1]^2 + w_1[\cos\theta_2 - \cos\theta_1]^2 \tag{30}$$

where, as before, w_1 is a suitably defined weighting factor.

To illustrate the goodness of U and $\cos\theta$ as indicators of a possible common parent body for streams on widely separated orbits, let us examine the case of the meteoroid stream possibly associated with comet 96P/Machholz 1 (as proposed by Babadzhanov & Obrubov 1993; note that Jenniskens et al. 1997 and Williams & Collander-Brown 1998 strongly doubt that 96P/Machholz 1 is the parent body of the Quadrantids).

Babadzhanov & Obrubov (1993) computed the secular evolution of 96P/Machholz 1 over an extended time span, obtaining its orbital elements at the crossings of the Earth's orbit. They found that there are eight such crossings, which take place in two groups of four orbits that are similar from the point of view of a, e and i; meteor showers are associated with each crossing, and Table 2 contains data from their paper, supplemented with the corresponding values of U and $\cos\theta$. The Quadrantids and the δ Aquarids are the most conspicuous representatives of each group of showers, and the possibility of their common origin had been suggested long before the discovery of 96P/Machholz 1 by Hamid & Whipple (1963).

As it is possible to see, although the two groups of orbits cluster about $q \approx 0.97$ au, $e \approx 0.69$, $i \approx 72°$ and $q \approx 0.07$ au, $e \approx 0.97$, $i \approx 30°$, respectively, thus spanning practically all the available range of q, and a considerable fraction of the ranges available in e and i, their values of U and $\cos\theta$ are much more tightly clustered.

Fig. 1 helps to illustrate this point better. It represents the plane U–$\cos\theta$, and in it are reported the streams discussed by Babadzhanov & Obrubov (1993) as well as two lines: the upper line separates solar system meteoroids from hyperbolic ones, and thus is the practical upper bound to the U–$\cos\theta$ range available to meteoroid stream orbits, while the lower line corresponds to $a = 1$ au.

In the figure are also marked the positions of comet 96P/Machholz 1, the presumed parent body of all these streams according to Babadzhanov & Obrubov (1993), asteroid (5496) 1973 NA, suggested as possible parent of the Quadrantids by Williams & Collander-Brown (1998), comet C/1490 Y1=1491 I, with the eccentricity adjusted to 0.77, also suggested as possible parent of the Quadrantids by Williams & Wu (1993), and comet 8P/Tuttle, which has been for some time in the past considered the parent body of the Ursids, as also remarked by Kresák in the discussion following the paper of Babadzhanov & Obrubov.

The figure in fact shows vividly how most of the shower orbits, either calculated or observed, are close to each other in this type of diagram; 96P/Machholz 1, (5496) 1973 NA and the adjusted orbit of C/1490 Y1, all being very close to the cluster, are obvious parent candidates. The figure also shows how peripheric the observed Ursids are with respect to the other observed showers, and how much closer they are to 8P/Tuttle, thus pointing to the necessity of further investigating the problem of their origin.

Having seen how clustered in U–$\cos\theta$ are orbits that have undergone a substantial secular evolution, it is of some interest to look at how the orbits of asteroids and comets are arranged, in the same plane, as well as those of individual photographic meteors.

Concerning the latter, Fig. 2 contains the 865 most precise photographic orbits, including 139 small-camera meteors (Whipple 1954), 413 Super Schmidt meteors (Jacchia & Whipple 1961), and

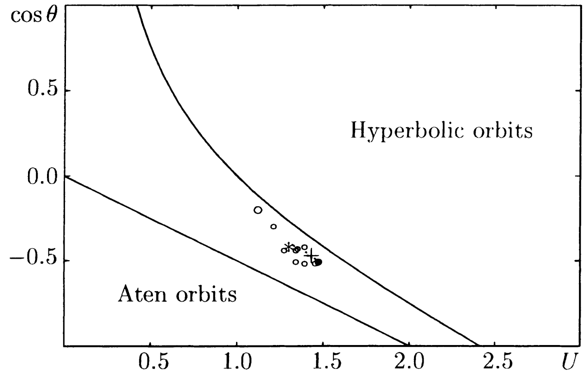

Figure 1. The calculated (small dots) and the observed positions (small open circles) of the streams discussed by Babadzhanov & Obrubov (1993) in the plane U-cos θ; the large dot corresponds to 96P/Machholz 1, the large open circle to comet 8P/Tuttle, the asterisk to asteroid (5496) 1973 NA, and the cross to comet C/1490 Y1 (see text).

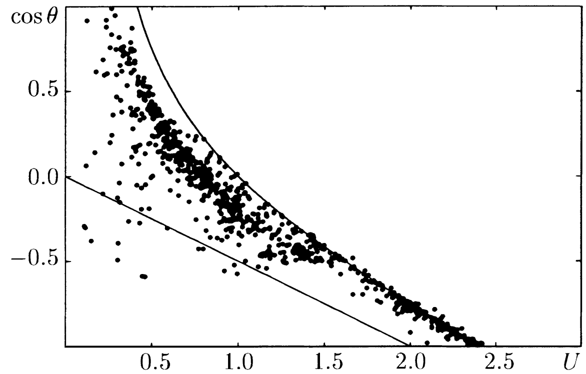

Figure 2. The positions of 865 precisely measured photographic meteor orbits in the plane U-cos θ.

313 additional Super Schmidt meteors (Hawkins & Southworth 1958, 1961); many of them are close to the parabolic limit, while only a minority are situated below the $a = 1$ au condition.

Fig. 3 shows all the comets taken from Marsden & Williams (1996) as dots, and all the Apollo–Amor–Aten asteroids taken from the listings of the Minor Planet Center World Wide Web server as open circles. There is relatively little overlap between the two populations; it is only along a strip parallel to the parabolic limit and not very far from it. It is interesting to compare the two figures, as one immediately gets a rather clear impression of the proportion of meteors belonging to the sample of 865 precise photographic orbits that are probably of asteroidal origin; in doing so, however, one must take into account that slow meteors are disfavoured, because of the larger size that they must have in order to reach a given magnitude. In any case, from Fig. 3 we learn that, for the range of U and $\cos\theta$ occupied only by open circles, even an approximate determination of just these two quantities allows us to infer a probably asteroidal origin for a given sporadic meteor.

3.2 Can we use additional near-invariants?

The secular problem in which a meteoroid orbit is affected by perturbations caused by one or more planets moving in circular orbits, all in the same plane, admits one further constant of motion besides E and L_z, namely the Kozai integral K (Kozai 1962; Thomas & Morbidelli 1996). One may then think of completing D_R, adding

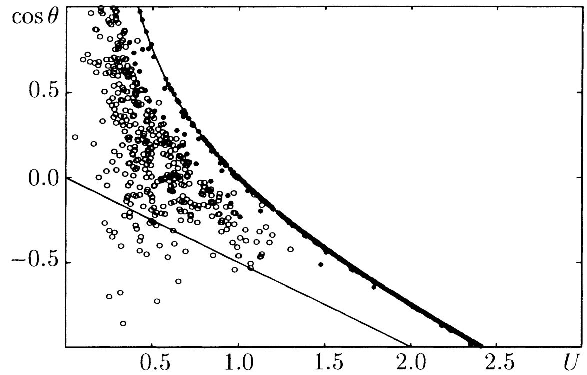

Figure 3. The positions of Apollo, Aten and Amor asteroids (open circles) and comets (dots) in the plane U-cos θ.

to it the separation in K between the two orbits being compared. This is not as useful as it might seem at first glance, for several reasons.

(i) There is no closed expression to calculate K; it is actually computed, as described in Thomas & Morbidelli (1996), using the orbital elements, so that one has to give up the direct link with observed quantities enjoyed by the variables used in D_N and D_R.

(ii) K is not defined for the hyperbolic orbits that one sometimes finds for meteoroids associated with long-period comets.

(iii) While the conventional orbital elements as well as U and $\cos\theta$ exhibit small variations for dispersions typical of meteoroid streams, the behaviour of K in this respect is rather bizarre, and examples of this are given below.

(iv) Even if D_R were not very small for a pair of orbits, a check made with an accurate model would still be necessary, given the previous point, so why bother with the more complex criterion if it would not save us the necessity of checking?

The third point is particularly important. To illustrate it, we have compiled Table 3, where we have collected the typical orbital parameters of some well-known streams, together with values of U, $\cos\theta$ and K; in addition, for each stream the same quantities have been recomputed after adding, at perihelion, $\pm 1\,000$ m s^{-1} in the direction of motion ('forwards' and 'backwards' in the table), of the radius vector ('outwards' and 'inwards'), and of the normal to the orbital plane ('upwards' and 'downwards').

Actually, meteors are ejected from comets not only at perihelion and, anyway, at far lower velocities, as demonstrated by the observations of meteor storms and by the cometary dust trails discovered by *IRAS* (Kresák 1992, 1993); however, planetary perturbations and measuring errors cause an observed dispersion of meteoroid stream orbits comparable to that they would have if the actual ejection velocities were in the km range. In Table 3 we just want to show how the observed orbital dispersion within streams reflects on individual quantities.

As one can easily see, K suffers a serious shortcoming: at variance with the behaviour of U and $\cos\theta$, the variations of K can be very large in relative terms, and this makes the use of K as a classification parameter questionable; in fact, while a closeness in K, U and $\cos\theta$ of the orbits of two meteoroids would mean that they might have a common parent, the opposite is not always true. In fact, according to Table 3, if two orbits were close to each other in U and $\cos\theta$, but rather widely separated in K, they could still be compatible with ejection from the same parent body.

Table 3. Orbital parameters of some meteoroid streams, and variations resulting from ejection at perihelion at 1 km s^{-1}

Stream & ejection dir.	q (au)	e	i (°)	U	$\cos\theta$	K ($\times 10^4$)
Leonids	0.98	0.92	162	2.35	−0.98	−0.35
backwards	0.98	0.83	162	2.32	−0.98	−0.71
forwards	0.98	1.01	162	2.38	−0.98	hyp.
inwards	0.98	0.92	162	2.35	−0.98	−0.34
outwards	0.98	0.92	162	2.35	−0.98	−0.34
downwards	0.98	0.92	163	2.36	−0.98	−0.35
upwards	0.98	0.92	161	2.35	−0.98	−0.34
Orionids	0.57	0.97	165	2.23	−0.91	−0.23
backwards	0.57	0.90	165	2.20	−0.91	−0.68
forwards	0.57	1.04	165	2.27	−0.90	hyp.
inwards	0.57	0.97	165	2.24	−0.91	−0.22
outwards	0.57	0.97	165	2.24	−0.91	−0.22
downwards	0.57	0.97	165	2.23	−0.91	−0.22
upwards	0.57	0.97	165	2.24	−0.91	−0.22
Perseids	0.95	0.95	113	2.00	−0.76	−0.22
backwards	0.95	0.86	113	1.97	−0.77	−0.56
forwards	0.95	1.04	113	2.03	−0.76	hyp.
inwards	0.95	0.95	113	2.00	−0.76	−0.22
outwards	0.95	0.95	113	2.00	−0.76	−0.22
downwards	0.95	0.95	114	2.02	−0.77	−0.22
upwards	0.95	0.95	112	1.99	−0.76	−0.22
Lyrids	0.92	0.99	80	1.59	−0.48	−0.05
backwards	0.92	0.90	80	1.56	−0.49	−0.42
forwards	0.92	1.08	80	1.62	−0.47	hyp.
inwards	0.92	0.99	80	1.59	−0.48	−0.04
outwards	0.92	0.99	80	1.59	−0.48	−0.04
downwards	0.92	0.99	81	1.60	−0.49	−0.04
upwards	0.92	0.99	79	1.57	−0.47	−0.04
S. δ Aquarids	0.08	0.97	27	1.38	−0.47	−1.04
backwards	0.08	0.94	27	1.26	−0.51	−0.96
forwards	0.08	1.00	27	1.50	−0.43	−0.17
inwards	0.08	0.97	27	1.39	−0.47	−1.04
outwards	0.08	0.97	27	1.39	−0.47	−1.03
downwards	0.08	0.97	27	1.39	−0.47	−1.04
upwards	0.08	0.97	27	1.38	−0.47	−1.04
Quadrantids	0.98	0.68	72	1.37	−0.44	−1.02
backwards	0.98	0.59	72	1.35	−0.46	−0.96
forwards	0.98	0.77	72	1.40	−0.42	−0.82
inwards	0.98	0.68	72	1.37	−0.44	−1.03
outwards	0.98	0.68	72	1.37	−0.44	−1.00
downwards	0.98	0.68	73	1.39	−0.46	−1.02
upwards	0.98	0.68	71	1.35	−0.42	−1.02
Geminids	0.14	0.90	24	1.16	−0.46	−0.96
backwards	0.14	0.87	24	1.05	−0.51	−0.95
forwards	0.14	0.93	24	1.26	−0.42	−1.01
inwards	0.14	0.90	24	1.16	−0.46	−0.96
outwards	0.14	0.90	24	1.16	−0.46	−0.96
downwards	0.14	0.90	24	1.16	−0.46	−0.96
upwards	0.14	0.90	24	1.16	−0.46	−0.96
α Capricorn.	0.58	0.78	6	0.77	0.01	−1.09
backwards	0.58	0.71	6	0.72	−0.01	−1.00
forwards	0.58	0.85	6	0.82	0.04	−0.99
inwards	0.58	0.78	6	0.78	0.01	−1.09
outwards	0.58	0.78	6	0.78	0.01	−1.09
downwards	0.58	0.78	6	0.78	0.01	−1.09
upwards	0.58	0.78	6	0.78	0.01	−1.09

3.3 How good are the secular near-invariants? A numerical check

To test the invariance of U and $\cos\theta$ over time-scales of interest for the meteoroid stream identification problem, we have integrated over 25 000 yr a fictitious object on an orbit similar to that of 96P/Machholz 1, and two other objects obtained by a velocity change of ± 5 m s^{-1} at perihelion in the forward and backward directions, in a simplified solar system consisting only of Jupiter and the Earth, both on circular coplanar orbits. The velocity difference of ± 5 m s^{-1} was chosen following Kresák (1993), who finds this value to be about the maximum compatible with the observations of meteor storms and *IRAS* dust trails. We have chosen a simplified solar system model, in which the only secular effect is that tied to the ω-cycle, in order to show the difference of behaviour between the conventional elements and our near-invariants in one of the simplest, and therefore one of the most generic, gravitational systems in which these differences are present. Of course, in a more realistic solar system there would be more sources of perturbation, and therefore the behaviour of our near-invariants would somewhat degrade.

The results of the evolution details we have computed are shown in Fig. 4, which gives the time behaviours of, from top to bottom, the heliocentric distances of the descending and the ascending node, Ω, ω, $\sin i$, e, U, $\cos\theta$, K (multiplied by 10 000) and q. The dots refer to the central body, the small open circles to the object ejected backwards, and the large open circles to the object ejected forwards.

While q, e and $\sin i$ exhibit the large variations correlated with the prograde rotation of ω that are to be expected for such an orbit, U, $\cos\theta$ and K are much more stable, demonstrating their quasi-invariance in this case; in fact, planetary encounters with the Earth and Jupiter are possible, but not very effective on the time-scale examined, because they take place at rather high velocity.

We can also consider the dispersion of the two ejected bodies with respect to the central one by examining the behaviour of $D_{\rm SH}$ and $D_{\rm R}$ (Fig. 5). The dispersion in the latter is rather small throughout the time span covered by the integration, never exceeding about 1/10 of the range of possible values for elliptical orbits, while the dispersion in $D_{\rm SH}$ can go up to 1/2 of its total range.

4 DISCUSSION

An important question concerning the new variables is the size of the error that we incur by using quantities that are defined for a circular orbit of the Earth, of radius equal to 1 au, lying on the mean ecliptic; moreover, the expressions that relate the components of the geocentric unperturbed velocity of the meteoroid to the a, e and i of its orbit are strictly valid if one disregards the effect of the terrestrial mass $M_\oplus$ on the orbital velocity of the Earth itself. This last error, the relative size of which is about 10^{-6}, is immaterial compared with the errors involved in the measurement of even the best photographic meteor orbits, which are of relative size of about 10^{-3}. Also, the errors resulting from the fact that the osculating semimajor axis and inclination of the Earth are not exactly 1 and 0 respectively are of little importance, both being below well below 10^{-3}. The only significant errors are caused by the ellipticity of the Earth's orbit.

Since $e_\oplus = 0.0167$, one obtains the heliocentric distance

$$0.9833 \text{ au} \lesssim r_\oplus \lesssim 1.0167 \text{ au}, \tag{31}$$

the heliocentric velocity

$$29.2\,\text{km s}^{-1} \lesssim v_\oplus \lesssim 30.2\,\text{km s}^{-1}, \tag{32}$$

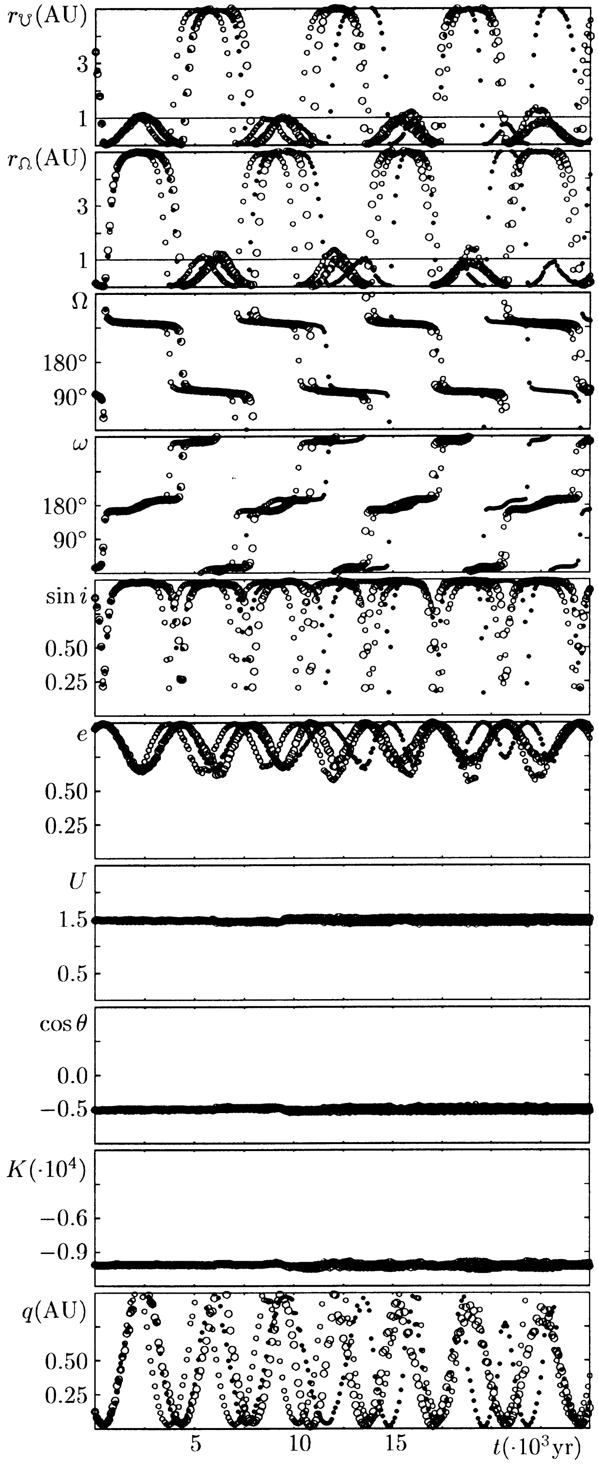

Figure 4. Time evolution of a 96P/Machholz 1 like orbit (dots) and of two similar other orbits, one obtained with a backwards ejection at perihelion (small open circles), and one obtained with a forwards ejection (large open circles), over 25 000 yr (see text); from top to bottom: heliocentric distances of the descending and the ascending node, Ω, ω, $\sin i$, e, U, $\cos\theta$, K (multiplied by 10 000) and q. The ranges of the ordinates are (sometimes approximately) proportional to the ranges spanned by each quantity.

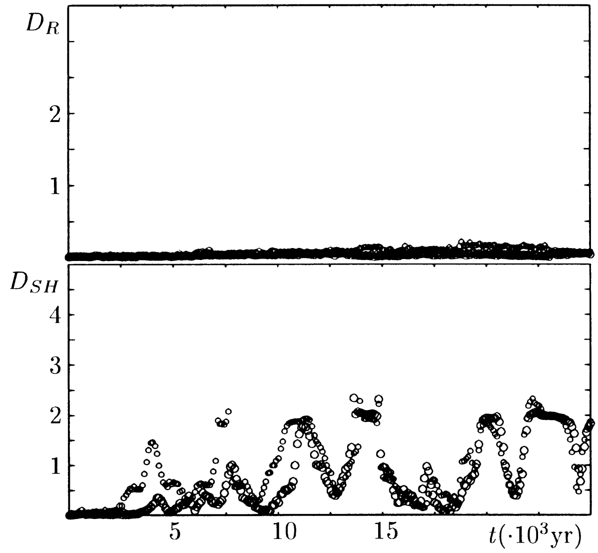

Figure 5. Time evolution of D_{SH} (bottom panel) and of D_{R} (top panel) of a body ejected backwards (small open circles) and one ejected forwards (large open circles), calculated with respect to the central body of Fig. 4. The ranges of the ordinates are proportional to the ranges spanned by each quantity.

and the angle between the directions of radius vector and velocity

$$89° \lesssim \arcsin \frac{a_\oplus \sqrt{(1-e_\oplus^2)}}{\sqrt{r_\oplus(2a_\oplus - r_\oplus)}} \lesssim 91°. \qquad (33)$$

In practice, we compute U from the tabulated V_{G} of the meteor, and θ and ϕ from the tabulated time of observation, α_{G} and δ_{G}, using the actual values of $r_\oplus$ and $\lambda_\oplus$.

Although the relations between the orbital elements and U, θ and ϕ given in Section 2 are strictly valid only if the latter quantities were computed with reference to our idealized circular orbit of the Earth, we can see from the previously listed expressions that the errors incurred in our procedure are of the order of 1 km s^{-1} for the velocity and 1° for the angles; these errors are large if compared with those of the best photographic meteor orbits, but always less, and sometimes much less, than the typical dispersions within well-established meteoroid streams.

Turning to a more general question, it can be remarked that the problem of identifying meteoroid streams has several similarities with that of asteroid families. In both cases, a single parent is supposed to have originated many smaller bodies, and it is the relatively low ejection velocity, compared with the orbital velocity, that makes the orbital parameters still rather similar, allowing for recognition of the common origin.

On the other hand, the dynamics is different: asteroid orbits are far less chaotic than meteoroid orbits, because asteroid families are located in regions of the asteroid belt far separated from the orbits of the planets, while meteoroid orbits cross, by definition, the orbits of the Earth, and very often also the orbits of various other planets, so that the time-scales for the dissolution into the background of asteroid families can be of the order of the age of the solar system, while the equivalent ones for meteoroid streams cannot be much larger than 10^4–10^5 yr. Thus, suitably defined 'proper elements' (near-invariants of the assumed non-chaotic motion of asteroid family members) computed with various techniques (Knežević et al. 1995) allow us to identify families that are probably billions of years old, while the osculating elements of meteoroid streams remain concentrated over thousands of years. In this respect, the secular near-invariants introduced in this paper allow a certain prolongment of the time span over which meteoroid orbit clusters can possibly be recognized.

5 CONCLUSIONS

In this paper we have introduced a new distance function D_N, to be used to test the orbital similarity of meteoroid orbits, based on geocentric observed quantities. At variance from the usual distance function D_{SH}, the new one is defined in a space with a number of dimensions equal to the number of independently measured physical quantities.

In addition, two of the quantities that are used in D_N have been shown to be nearly invariant under the secular perturbation associated with the cycle of ω, allowing the introduction of a distance function D_R useful to pinpoint cases of widely separated osculating orbits that may, in fact, originate from a single parent body.

The new distance functions D_N and D_R are applied to a set of 865 precisely measured photographic meteoroid orbits in an accompanying paper in this issue (Jopek et al. 1999).

ACKNOWLEDGMENTS

GBV started working on this paper at the Observatoire de la Côte d'Azur, Nice, while holding the G. Colombo Fellowship of ESA; further short stays of GBV in Nice were funded by the EC. TJJ's work on this paper was partly supported by the KBN Project 2 PO3C00608, and his permanences in Nice were funded by the Observatoire de la Côte d'Azur. We thank P. Babadzhanov, J. Baggaley, Z. Ceplecha, A. Milani and A. Noullez for discussions/encouragement, and the referee, S. J. Collander-Brown, for helpful suggestions. We are particularly grateful to A. Morbidelli for giving us his subroutines for the Kozai problem, as well as for many useful discussions.

REFERENCES

Babadzhanov P. V., Obrubov Yu. V., 1991, Astron. Vestn., 25, 387
Babadzhanov P. V., Obrubov Yu. V., 1992a, in Harris A., Bowell E., eds, Asteroids, Comets, Meteors 1991. Lunar and Planetary Inst., Houston, p. 27
Babadzhanov P. V. Obrubov Yu. V., 1992b, Astron. Vestn., 26, 70
Babadzhanov P. B., Obrubov, Yu. V. 1993, in Stohl J., Williams I. P., eds, Meteoroids and their parent bodies. Slov. Acad. Sci., Bratislava, p. 49
Carusi A., Valsecchi G. B., Greenberg R. 1990, Celest. Mech. Dynamical Astron., 49, 111
Drummond J., 1981, Icarus, 45, 545
Hamid S. E., Whipple F. L., 1963, AJ, 68, 537
Hawkins G. S., Southworth R. B., 1958, Smithson. Contrib. Astrophys., 2, 349
Hawkins G. S., Southworth R. B., 1961, Smithson. Contrib. Astrophys., 4, 85
Jacchia L. G., Whipple F. L., 1961, Smithson. Contrib. Astrophys., 4, 97
Jenniskens P., Betlem H., de Lignie M., Langbroek M., van Vliet M., 1997, A&A, 327, 1242
Jopek T., 1993, Icarus, 106, 603
Jopek T. J., Valsecchi G. B., Froeschlé Cl., 1999, MNRAS, 304, 751 (Paper II, this issue)
Knežević Z., Froeschlé Ch., Lemaitre A., Milani A., Morbidelli A., 1995, A&A, 293, 605
Kozai Y., 1962, AJ 67, 591
Kresák, Ľ., 1992, Contrib. Astron. Obs. Skalnaté Pleso, 22, 123
Kresák, Ľ., 1993, A&A, 279, 646
Marsden B. G., Williams G. V., 1996, Catalogue of Cometary Orbits. Central Bureau for Astronomical Telegrams, Cambridge, MA
Öpik E. J., 1976, Interplanetary Encounters. Elsevier, New York
Southworth R. B., Hawkins G. S., 1963, Smithson. Contrib. Astrophys., 7, 261
Štohl J., Porubčan V., 1987, in Ceplecha Z., Pecina P., eds, Proc. 10th Eur. Regional Astron. Meeting, Interplanetary Matter, Vol. 2. Czech. Acad. Sci., Prague, p. 163
Thomas F., Morbidelli A., 1996, Celest. Mech. Dynamical Astron., 64, 209
Valsecchi G. B., 1992, in Fernández J. A., Rickman H., eds, Periodic Comets. Univ. de la República, Montevideo, Uruguay p. 81
Whipple F. L., 1954, AJ, 59, 201
Williams I. P., Collander-Brown S. J., 1998, MNRAS, 294, 127
Williams I. P., Wu Z., 1993, MNRAS, 264, 659

This paper has been typeset from a TEX/LATEX file prepared by the author.