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An $\alpha\Omega$-dynamo in accretion disks with linear force-free coronae

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Abstract. We investigate the generation of a large-scale magnetic field by an $\alpha\Omega$-dynamo in turbulent accretion disks surrounded by linear force-free coronae. The standard α-disk model with parameters relevant to a typical disk around a compact star is assumed. The dynamo-generated magnetic field is calculated in both kinematic and nonlinear regimes. The critical dynamo number is $D_c \approx 10$ and the lowest normal mode has a quadrupole symmetry, just like dynamo models calculated for disks surrounded by a vacuum. The equilibrated magnetic field also has quadrupole symmetry and its distribution inside the disk is almost the same as in a disk with vacuum exterior. The novel feature is the existence of a toroidal field in the exterior of the disk which, permits the corona to transmit magnetic stresses between the inner and outer parts of the disk. However, all dynamical effects of the large-scale magnetic fields are small, due to the restricted magnitude and the specific configuration of the magnetic field generated by the $\alpha\Omega$-dynamo.

Key words: Accretion, accretion disks – MHD

1. Introduction

Magnetic fields have become a topic of great interest in research related to accretion disks in various astrophysical contexts. The main reason for this curiosity is the opinion that magnetic fields may be a major factor responsible for the structure and dynamical balance of those disks. Specifically, magnetic fields seem to be involved in disks' angular momentum redistribution, a process responsible for driving a disk whose nature is difficult, if not impossible, to understand without invoking magnetic fields. Because accretion disks are geometrically thin, they experience large magnetic losses (resistive and anomalous diffusion in cool and dense protoplanetary disks, mostly anomalous diffusion in hotter, high-energy accretion disks), and the magnetic field has to be contemporaneously generated and/or amplified at a high enough rate to balance the losses. There are two, conceptually distinct, conceivable mechanisms to maintain the magnetic field; interaction between the disk and an externally maintained magnetic field, and an internal generation of magnetic field. First, the field can be continuously captured from the surrounding environment and advected inward by the accretion flow, thus undergoing deformation leading to an increase of its strength. However, Lubow et al. (1994) and Reyes-Ruiz & Stepinski (1996) show that in the likely scenario of a disk driven primarily by anomalous viscosity characterized by a magnetic Prandtl number (ratio of turbulent magnetic diffusivity to turbulent viscosity) of the order of unity, the field lines that thread the disk are not efficiently advected by the accretion flow, and the magnetic field is not amplified. Thus, at least in the paradigm of viscous disks, the substantial magnetic field must be internally generated.

Until relatively recently, the theoretical underpinning of an internal generation process was provided by the $\alpha\Omega$-dynamo model pioneered by Parker (1955) and Steenbeck et al. (1966). In the $\alpha\Omega$ model the toroidal field is regenerated from the poloidal field through differential rotation, whereas the poloidal field is regenerated from the toroidal field by the preexisting turbulence (so-called α-effect). Admittedly, the $\alpha\Omega$ model is rather heuristic. It depends on postulated turbulence of unspecified origin, the magnitude of the α-effect can be only roughly estimated, and the theory is intrinsically linear with fluid forces being purely hydrodynamic, although, for computational purposes, so-called α-quenching formulas were invented to mimic the back reaction of a generated field on turbulence (see Sect. 4.). On the other hand, the $\alpha\Omega$ model provides a very transparent link between the small-scale, tangled magnetic field produced directly by turbulence, and the mean, large-scale field, the quantity which can be potentially measured, and which is of primary theoretical importance. Consequently, most existing models of the global structure of magnetic fields generated in accretion disks are based on the $\alpha\Omega$ formalism.

Balbus & Hawley (1991) have rediscovered the existence of a magnetorotational instability, which given a weak initial magnetic field can destabilize a Keplerian disk, simultaneously providing turbulence and small-scale, tangled magnetic fields. The magnetorotational instability acts like a dynamo that, however, generates its own turbulence. Thus, such a process is intrinsically nonlinear: turbulence and magnetic field are inseparable. It currently appears that the nature of dynamo activity in accretion disks is better understood in the framework of the magnetorotational instability than in terms of the $\alpha\Omega$-dynamo model. However, at present, computational complexity of MHD simulations

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rule out the construction of global disk models based directly on the magnetorotational instability concept. Only studies simulating *local* shearing boxes (Brandenburg et al. 1995, Hawley et al. 1996) are available. One of the most interesting features of these simulations is that the character of the generated magnetic field is somewhat similar to that predicted by the $\alpha\Omega$-dynamo (Brandenburg et al. 1995) despite some fundamental differences between the two paradigms. One of the major difference seems to be the magnitude of the large-scale equilibrated magnetic field, which can be of the order of the equipartition with the gas pressure in the magnetorotational instability simulations, but is of the order of the equipartition with the kinetic energy of the turbulence in the $\alpha\Omega$-dynamo calculations.

In this paper we *assume* that the generation of the magnetic field in an accretion disk proceeds via the $\alpha\Omega$-dynamo process. This is dictated by the global character of our problem. Our calculations are self-consistent within the $\alpha\Omega$-dynamo framework, but we hypothesize that they may also be applicable to the actual disks, where magnetic field is probably generated by the magnetorotational instability.

Throughout this paper we focus on the large-scale field. However, in both, the $\alpha\Omega$-dynamo and the magnetorotational instability paradigms, a small-scale field is also present. In the $\alpha\Omega$-dynamo context it has been demonstrated (Pudritz 1981) that the contribution of the large-scale magnetic field to the Maxwell stress is not significant. Likewise, in the context of the magnetorotational instability, results of Brandenburg et al. (1995) suggest that the large-scale field is not a dominant contributor to the Maxwell stress. Thus the large-scale field is not a major contributor to a disk's "viscous" evolution. On the other hand, the large-scale magnetic field pervading the closed corona provides a means of angular momentum exchange between disjoined and possibly distant segments of the disk. This may lead to dynamical consequences that are qualitatively different from those due to the viscous torque. Heyvaerts & Priest (1989) investigated such a scenario and concluded that strong enough large-scale magnetic fields lead to significant non-Keplerian behavior. However, they left open the question of evaluating the actual strength of a dynamo generated magnetic field in the disk. One of the goals of our work is to see whether the magnetic field obtained from a more self-consistent calculation is capable of inducing any dynamical behavior above and beyond that caused by the viscous torque.

The study of dynamo-generated magnetic fields can be conceptually divided into two parts: a purely kinematic part, which involves finding criteria (critical dynamo numbers) for the field generation, and a semi-dynamic part, which involves computation of the overall configuration of the equilibrated magnetic field. In both phases the results can, in principle, depend very strongly on the assumed properties of the medium surrounding the disk. All existing studies assume a conductivity, σ, of the exterior medium. Most settle for $\sigma = 0$, which leads to a so-called vacuum boundary condition. Stepinski & Levy (1990) also consider the case of $\sigma = \infty$, which leads to the magnetic field that is contained entirely within the disk. Rüdiger et al. (1995) consider also the intermediate cases, $0 < \sigma < \infty$. Conductivity-based boundary conditions simplify calculations, but they are not very realistic and may lead to spurious results. In particular, vacuum boundary conditions lead to a *potential* magnetic field in the disk exterior, and thus explicitly exclude magnetic configurations capable of the non-local exchange of angular momentum.

The gas in an accretion disk is concentrated around the equatorial plane and its density drops sharply in the vertical direction. Therefore, there exist imaginary surfaces that divide the part of space where gravity and gas pressure dominate over the Lorentz force (the interior of the disk) and the part where the magnetic field is dominant (the exterior of the disk). This suggests that dynamics-based, rather than conductivity-based, boundary conditions are more appropriate. If the system is in a steady state, and there is no wind, then, from a dynamical point of view, the magnetic field in the exterior of the disk has a force-free configuration. The assumption of force-free boundary conditions models well many actual situations and is the simplest approximation compatible with the possibility of non-local magnetic transport of angular momentum.

The goal of this paper is to calculate the configuration of the global magnetic field generated by the $\alpha\Omega$-dynamo in a disk surrounded by a force-free medium (corona). In Sect. 2. we discuss how to accommodate the force-free exterior into the $\alpha\Omega$-dynamo, and present the analytic solution for the magnetic field outside the disk. In Sect. 3. we present results of kinematic calculations, the growth rates for normal modes of various symmetries are calculated, and the eigenmodes with fastest growth rates are determined. In Sect. 4. we calculate the overall configuration of the equilibrated magnetic field in a disk with the force-free exterior. In Sect. 5. we assess the ability of the obtained magnetic field to transport angular momentum between distant parts of the disk, and in Sect. 6. we summarize our results and present our conclusions.

2. A model for an accretion disk corona

We consider a portion of the space that, in the cylindrical coordinates (R, ϕ, Z), is a hollow cylinder of radii R_i and R_o extending infinitely in the Z direction. The disk occupies a thin portion of this cylinder located symmetrically around the midplane $Z = 0$ and bounded by disk's surfaces defined by $Z = \pm H$. We define the region $|Z| > H$, where the gas density falls off rapidly, as the corona. This tenuous, highly ionized region, permeated by magnetic fields generated in the disk, is quite similar to the solar corona, hence the name. The entire disk-corona system is assumed to be axisymmetric. Driven by the motion of their footpoints at disk's surfaces, field lines in the corona are continuously stressed and simultaneously relaxed by resistive processes (see Heyvaerts & Priest 1989 for details). We assume that the coronal large-scale magnetic field relaxes in a short time to the minimum energy state presumed to have a linear force-free configuration (Woltjer 1958) given by the solution to the equation

$$\nabla \times \mathbf{B} = \mu\, \mathbf{B} \qquad (1)$$

where μ is a constant that reflects the magnetic helicity of the coronal field (Heyvaerts & Priest 1989 and references therein). As it is very difficult to establish the value of μ from the first principles, we consider μ to be a free parameter and seek to determine the effects, if any, of different values of μ on the magnetic field structure inside and outside the disk. Note, however, that for a sufficiently large value of $|\mu|$, a linear force-free field may not necessarily minimize the energy of the coronal field (Aly, 1993), nonaxisymmetric fields with smaller energies can be constructed, but a linear force-free configuration still minimizes the energy of axisymmetric fields.

Because of the assumed axisymmetry, the magnetic field can be expressed as $\mathbf{B} = B\mathbf{e}_\phi + \nabla \times A\mathbf{e}_\phi$, where $A\mathbf{e}_\phi$ is a vector potential of the poloidal field. Using this representation Eq. (1) reduces to

$$\left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R^2} + \frac{\partial^2}{\partial Z^2} + \mu^2\right] A \;=\; 0 \qquad (2)$$
$$B \;=\; \mu A$$

Eq. (2) can be solved analytically subject to the following boundary conditions:

$$A(R_i, Z) = B(R_i, Z) = 0, \quad A(R_o, Z) = B(R_o, Z) = 0 \quad (3)$$

$$A(R, Z), B(R, Z) \to 0 \text{ as } Z \to \infty \qquad (4)$$

where R_i and R_o are the inner and the outer disk radii, respectively. In the upper half-space ($Z > H$) the solution has the following general form:

$$A(r, z) \;=\; \Sigma_{n=1}^{\infty} C_n \mathrm{e}^{-\sqrt{k_n^2 - m^2}z} \times \left[\mathrm{J}_1(k_n r) - \frac{\mathrm{J}_1(k_n r_i)}{\mathrm{Y}_1(k_n r_i)}\mathrm{Y}_1(k_n r)\right] \qquad (5)$$

where J_1 and Y_1 are the 1st order Bessel functions of the first and the second kind, $r = R/R_0$ and $z = Z/R_0$ are coordinates in units of a normalization measure R_0 (not necessarily equal to the outer radius R_o), r_i and r_o are the disk inner and outer radii in such units, and $m = \mu R_0$. The dimensionless wavenumbers k_n, which arise from the radial boundary conditions, are a solution of

$$\mathrm{J}_1(k_n r_o)\mathrm{Y}_1(k_n r_i) - \mathrm{J}_1(k_n r_i)\mathrm{Y}_1(k_n r_o) = 0 \qquad (6)$$

and the unknown coefficients C_n are determined by matching this solution to the magnetic field inside the disk at $Z = H$, assuming the continuity of all magnetic field components across the disk's surface. In the lower half-space ($Z < H$) the solution is analogous, but possibly with different coefficients C_n.

Note that condition (4) requires $|\mu| < \mu_{crit} = k_1/R_0$, where k_1 is the smallest root of Eq. (6). For $\mu = 0$ the magnetic field has a familiar, potential field configuration with $B = 0$ and the poloidal field is characterized by closed, untwisted field lines. The characteristic scale of $\nabla \times \mathbf{B}$ distribution, defined as $l_c = |\mathbf{B}|/|\nabla \times \mathbf{B}| \sim |\mu|^{-1}$, is infinity for the current-free field. Increasing the value of $|\mu|$ corresponds to shearing field lines in the $R - \phi$ space toward the ϕ coordinate and decreasing the value of l_c. Thus, the field lines of the force-free field acquire "swirls" as compared with the current-free field. At $|\mu| = \mu_{crit}$ the field lines are infinitively sheared in the $R - \phi$ space, yielding $B_R = 0$. The magnetic field has a constant strength with open field lines spiraling to infinity, and $l_c = R_0/k_1$, which is of the order of R_o.

The value of μ can be, in principle, positive or negative. Moreover, as the upper corona is separated from the lower corona by the presence of the disk, the sign and/or value of μ can be different in these two regions. In particular, it is easily seen that for the coronal field to have dipole symmetry (A even and B odd with respect to the midplane) or quadrupole symmetry (A odd and B even), μ must be odd with respect to the midplane. In addition, the coefficients C_n must be even with respect to the midplane for the dipole field and odd for the quadrupole field. Because we consider only pure dipole or pure quadrupole magnetic field configurations, we can restrict our calculations to the upper half-space and use the symmetry properties to obtain the field in the lower half-space.

3. Normal modes

Within the kinematic approximation, the induction equation for the large-scale magnetic field is linear and the solutions are in the form of normal modes. Establishing the generation threshold and the symmetry of the lowest mode is of primary interest because the actual magnetic field will retain these properties (see the next section). It has been shown (Rüdiger et al. 1995) that the lowest mode of the dynamo-generated field in a disk surrounded by the "halo" of finite conductivity has a quadrupole symmetry. Only in the case of the disk surrounded by the perfect conductor does the lowest mode have dipole symmetry (Stepinski & Levy 1990, Meinel et al. 1990). The halo, as defined by Rüdiger et al. (1995), is a tenuous medium that is, however, capable of generating a magnetic field via the $\alpha\Omega$ process. Thus, by definition, the motion of the plasma in the halo governs the behavior of the magnetic field there. On the other hand, the corona, as defined in the previous section, is the medium where magnetic fields govern the behavior of the plasma. We seek to determine whether changing a disk's surroundings from halo to corona results in a change of generation threshold and/or symmetry of the lowest mode.

The induction equation for the large-scale magnetic field in the axisymmetric thin disk separates into the following poloidal and toroidal components (for details see for example Stepinski & Levy 1991):

$$\frac{\partial A}{\partial t} = \alpha B + \eta \left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R^2} + \frac{\partial^2}{\partial Z^2}\right] A \qquad (7)$$

$$\frac{\partial B}{\partial t} = \frac{3\Omega_k}{2}\frac{\partial A}{\partial Z} + \eta \left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R^2} + \frac{\partial^2}{\partial Z^2}\right] B \qquad (8)$$

where Ω_k is the Keplerian angular velocity. A function α, known in dynamo theory as the so-called α-effect, is responsible for producing a poloidal field out of a toroidal field by means of turbulence. In a thin Keplerian disk α is given by (Stepinski & Levy 1991)

$$\alpha = \alpha_{ss}\Omega_k Z \tag{9}$$

where α_{ss} is a dimensionless turbulent viscosity parameter of Shakura & Sunyaev (1978).

The total magnetic diffusivity η is dominated by turbulent diffusivity, which in a thin Keplerian disk is given by (Stepinski & Levy 1991)

$$\eta = \alpha_{ss} H^2 \Omega_k \tag{10}$$

Eqs. (7) and (8) supplemented by boundary conditions at the inner and outer radii (see Eq. (3)), at the midplane $Z = 0$ (specified by the condition that the field has either dipole or quadrupole symmetry), and at the disk's surface $Z = H$ (set by the condition that all components of the field continuously join the respective components of the force-free field in the corona) describe an eigenvalue problem. The eigenvalues are the growth rates of the normal modes. It has been shown that the dependence of these eigenvalues on the properties of the disk can be encapsulated to their dependence on only two parameters, the so-called dynamo number,

$$\mathcal{D} = \frac{\alpha_0 \Omega_{k,0} H_0^3}{\eta_0^2} \tag{11}$$

and

$$\lambda = \frac{H_0}{R_0} \tag{12}$$

where the subscript 0 denotes characteristic values of corresponding quantities.

We have calculated eigenvalues (and corresponding normal modes) as functions of $\mathcal{D}$. Fig. 1 shows the growth rate of the lowest mode as a function of $\mathcal{D}$ for a disk of constant thickness ($\lambda = 0.01$, which is a typical value for disks in cataclysmic variables) surrounded by a corona characterized by $|\mu| = 0.95\mu_{crit}$. The assumption of constant thickness is done solely for the purpose of comparing our results to those obtained by Stepinski & Levy (1991), who calculated the growth rates in a disk of constant thickness surrounded by a vacuum using approximate boundary conditions suggested by Zeldovich et al. (1983). However, we have found that the condition of constant disk thickness can be relaxed without appreciable differences in the values of the growth rate or symmetry properties of the excited modes.

To calculate eigenvalues for a given symmetry condition we discretize Eqs. (7) and (8) and the boundary conditions with second-order finite differences on a uniform mesh covering the interior of the disk and having 30 points in the radial direction and 20 points in the vertical direction. This produced a matrix for which eigenvalues were calculated using EISPACK subroutines based on the QR algorithm. In the process we also determine coefficients C_n in Eq. (5). The summation in Eq. (5) has to be truncated after 30 terms, the number equal to the number of grid points in the radial direction.

Examination of Fig. 1 reveals that, in general, the consideration of a force-free exterior has no significant effect on the growth rates of the lowest modes. In fact, the difference between our results and those of Stepinski & Levy (1991) is predominantly due to the fact that Stepinski & Levy assumed $B_R = 0$ at the disk surfaces, and not because they use vacuum rather than a force-free exterior. The maximum growth rates for disks surrounded by a vacuum and those surrounded by a force-free corona, using the full solution outside the disk, are virtually the same. The critical dynamo number (a threshold below which only decaying normal modes exist) is about 10, and the lowest dynamo mode has a quadrupole symmetry. The quadrupole mode is dominant for all dynamo numbers smaller than ~ 650. For dynamo numbers greater than ~ 650, oscillatory dipole modes are the fastest growing. Note that $\mathcal{D}$ as given by Eq. (11) is $\sim \alpha_{ss}^{-1}$, thus only a magnetic field with quadrupole symmetry can be generated unless α_{ss} is smaller than about 10^{-3}. However, the actual magnetic field does not resemble any normal mode, as its structure is shaped by the nonlinear interaction with the disk. Nevertheless, it has been demonstrated (Rüdiger et al. 1995) that the equilibrated magnetic field preserves a symmetry of the lowest mode, in our case the quadrupole symmetry, regardless of how small the value of α_{ss} is.

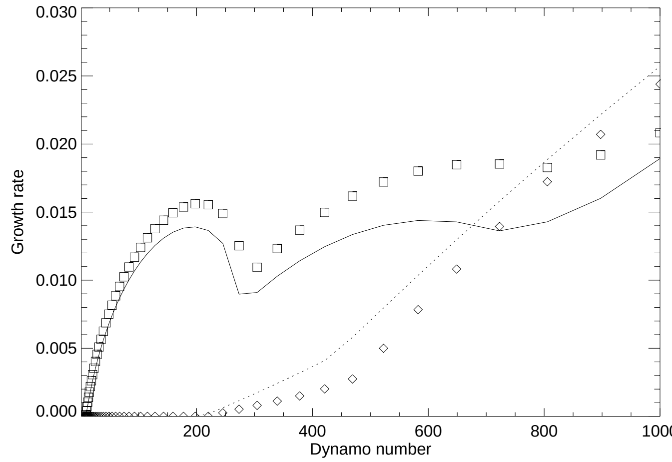

Fig. 1. Maximum dynamo growth rate as a function of dimensionless dynamo number for a disk with constant scale-height and $\lambda = 0.01$. The continuous and dashed lines show the growth rate, respectively, of quadrupole and dipole modes in a disk with $|\mu| = 0.95\mu_{crit}$. For comparison, triangles and squares show the corresponding maximum growth rates using Zeldovich's thin disk approximation for the boundary conditions at H.

4. Nonlinear dynamos

Within a paradigm of a linear dynamo, a seed magnetic field will grow without limit providing that $\mathcal{D} > \mathcal{D}_{\rm crit}$. In reality, the Lorentz force due to a generated magnetic field modifies the amplification sources, halting the further growth of the magnetic field. We performed nonlinear calculations considering the back-reaction of the magnetic field on the α-effect, but ignoring the back-reaction of the magnetic field on the turbulent diffusion (see Field 1995 for a review of such a possibility) and the large-scale velocity field. Note that this last assumption must be checked *a posteriori*, and it would be inconsistent if the obtained large-scale field leads to a non-Keplerian behavior.

Nonlinear calculations require a model of the back-reaction of the large-scale field on the α-effect. We assume that this nonlinearity has the form of an α-quenching, which can be introduced by replacing Eq. (9) by

$$\alpha = \frac{\alpha_{ss}\Omega_k Z}{1+\frac{|\mathbf{B}|^2}{B_{eq}^2}} \tag{13}$$

where B_{eq} is the magnitude of the magnetic field in equipartition with the turbulent motion, $B_{eq}^2 = 4\pi\alpha_{ss}\rho C_s^2$. Here ρ is the gas density, and C_s is the speed of sound. The turbulent velocity is assumed to be equal to $\alpha_{ss}^{1/2} C_s$. Our choice of a specific α-quenching function is arbitrary, but the fact that the quenching scales with $|\mathbf{B}|/B_{eq}$ is a result derived from the mean-field theory (Rüdiger & Kichatinov 1993). It can be shown (Reyes-Ruiz & Stepinski, 1997), that an α-quenching formula (13) leads to the equilibrated large-scale magnetic field of the magnitude

$$B \approx B_{eq}\left(\frac{0.12}{\alpha_{ss}} - 1\right)^{1/2} \approx B_{pr}\ 0.25\left(1-\frac{\alpha_{ss}}{0.24}\right)^{1/2}, \tag{14}$$

where B_{pr} denotes the magnitude of magnetic field in equipartition with the gas pressure. Thus, in the framework of the $\alpha\Omega$-dynamo theory, the saturated large-scale magnetic field is *always* subthermal. However, the small-scale component of the saturated magnetic field is larger (Krause & Roberts, 1976) and may be in the equipartition with the gas pressure. This is in contrast to the saturated magnetic field produced by the magnetorotational instability, where both the small-scale and the large-scale components can be, at their peak values, in equilibrium with the gas pressure (Brandenburg et al. 1995).

B_{eq} is a function of disk quantities and a complete model of an unmagnetized steady state disk is needed to perform nonlinear dynamo calculations. We use the "standard" disk model (Frank et al. 1992), which has parameters corresponding to typical conditions in cataclysmic variables. Dynamo calculations require radial profiles of disk half-thickeness H and B_{eq}, which for this model are

$$H = 1.7\times 10^8\ \alpha_{ss}^{-1/10}\dot{M}_{16}^{3/20}M_1^{-3/8}R_{10}^{9/8}\mathrm{cm} \tag{15}$$

$$B_{eq} = 634\ \alpha_{ss}^{1/20}\dot{M}_{16}^{17/40}M_1^{7/16}R_{10}^{-21/16}\mathrm{gauss} \tag{16}$$

where the radial distance R_{10} is measured in units of 10^{10} cm, the accretion rate is measured in units of 10^{16} g sec^{-1}, and stellar mass is measured in units of 1 $M_\odot$.

We consider a model characterized by $\alpha_{ss} = 0.1$, $\dot{M}_{16} = 1$, $M_1 = 1$, $R_i = 5\times 10^9$ cm, and $R_o = 10^{10}$ cm. The ratio $R_i/R_o = 0.5$ is smaller than expected in the actual disks to save the time required by our numerical algorithm. The evolution of the magnetic field inside the disk, governed by Eqs. (7) and (8), is computed using an explicit finite difference scheme, second-order accurate in space and first-order accurate in time, limited by the Courant stability condition. The evolution of the field outside the disk, governed by Eq. (2) is determined by the changing boundary conditions at $|Z| = H$ as the field inside the disk evolves. At each timestep the exterior field is computed using a relaxation procedure. The entire computational procedure is analogous to the one described by Reyes-Ruiz & Stepinski (1997).

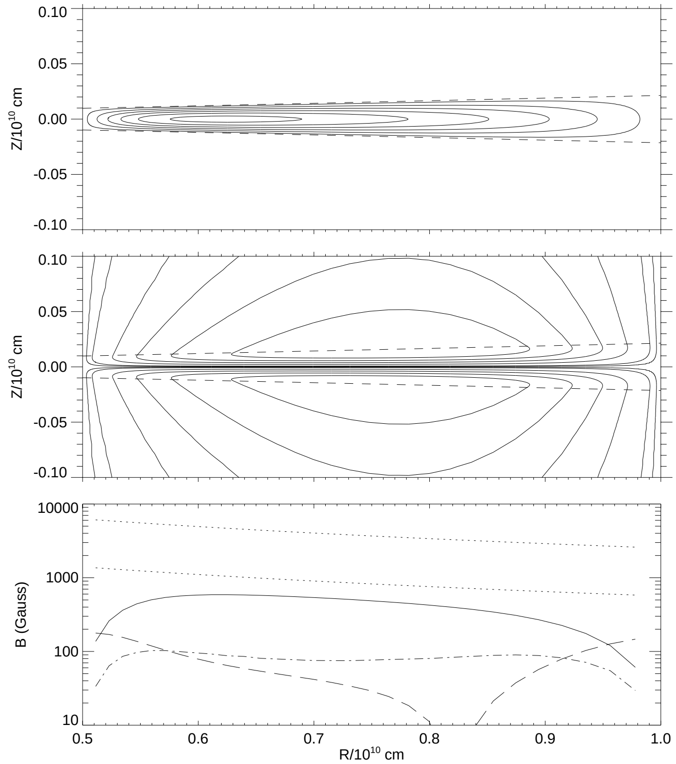

Fig. 2. Dynamo-generated magnetic field in an accretion disk with a corona characterized by $\mu = 0$. The *top panel* shows contours of the toroidal magnetic field intensity, solid lines denote positive (into the $R-Z$ plane) toroidal field, and dotted lines indicate negative toroidal field. Poloidal field lines are shown in the *middle panel*. Dashed lines indicate the disk surfaces in both cases. The *bottom panel* shows absolute values of magnitudes of individual components of magnetic field, the solid line corresponds to $B_\phi(Z=0)$, the dashed line to 30 $B_Z(Z=H)$, and the dash-dotted to 30 $B_R(Z=H)$. For comparison, the lower dotted line shows the strength of B_{eq}, and the upper dotted line shows the strength of magnetic field in equipartition with the gas pressure.

Figs. 2, 3, and 4 show the configuration of the equilibrated dynamo-generated large-scale magnetic field calculated for various values of μ. As expected, the field has quadrupole symmetry in all cases. The sense of the magnetic field is arbitrary; we assume that A is positive inside the disk. Because the field lines must be closed, the sign of A does not change in the corona and the poloidal field "rotates"clockwise in the $R-Z$ plane. Thus, B_R must change its sign from the negative (toward the Z-axis) near the disk midplane to the positive in the corona. Inside the disk the dynamo equations dictate that B_ϕ and B_R have opposite signs, thus the toroidal field inside the disk "rotates" counterclockwise (in a positive direction) around the Z-axis. In the corona the deformation of the positive B_R by the motion of foot points embedded in the counterclockwise-rotating disk produces negative (rotating clockwise around Z-axis) B_ϕ. This

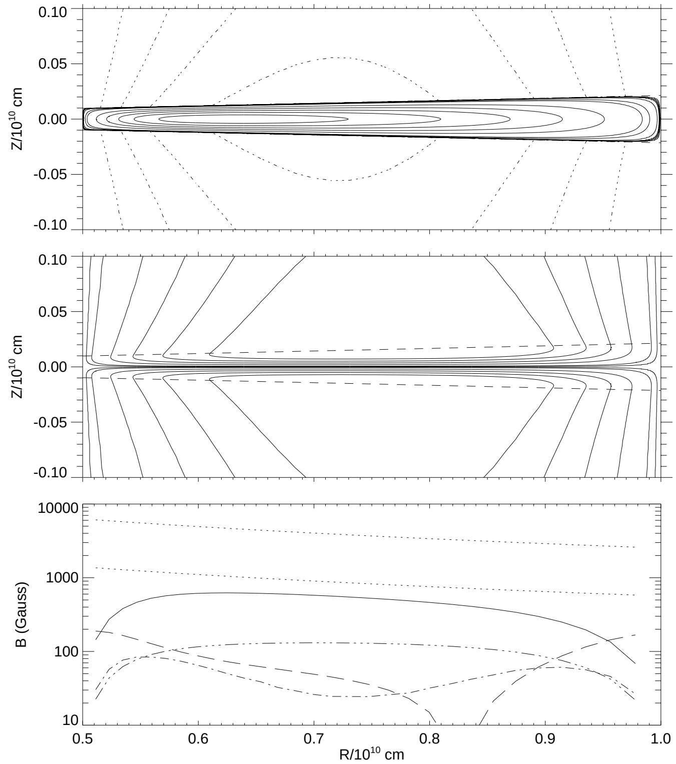

Fig. 3. Same as Fig. 2 but for an accretion disk with corona characterized by $|\mu| = 0.85\mu_{crit}$. We have also added a dash-triple-dot line to denote the value of 30 $B_\phi(Z = H)$.

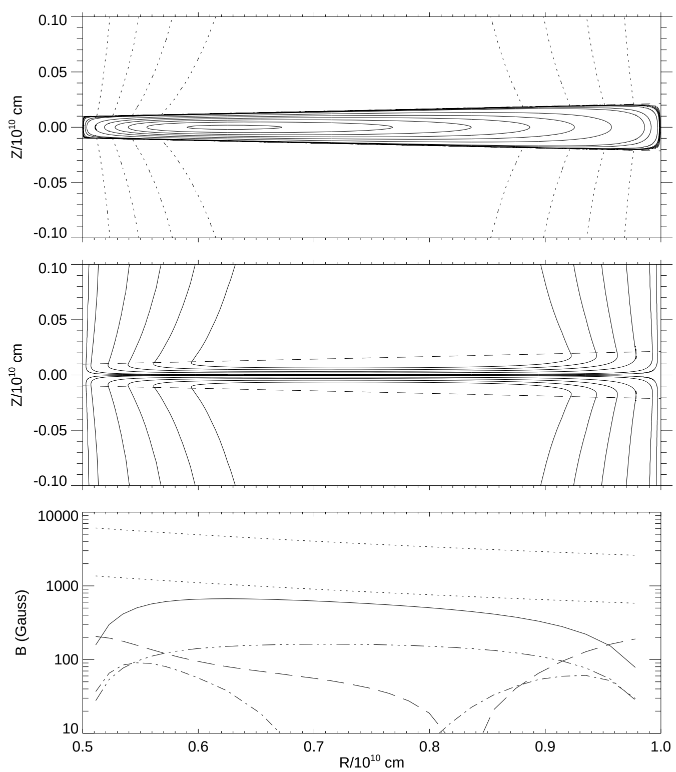

Fig. 4. Same as Fig. 2 but for an accretion disk corona characterized by $|\mu| = 0.95\mu_{crit}$. We have also added a dash-triple-dot line to denote the value of 30 $B_\phi(Z = H)$.

means that the force-free parameter μ must be negative in the upper half-space, and the toroidal field has to change sign near the surface of the disk.

Fig. 2 shows the configuration of the magnetic field corresponding to $\mu = 0$, a condition equivalent to the vacuum boundary condition, whereas Figs. 3 and 4 show configuration of the magnetic field for $|\mu| = 0.85\mu_{crit}$ and $|\mu| = 0.95\mu_{crit}$, respectively. Force-free boundary conditions do not introduce significant changes to the structure and the magnitude of the field inside the disk. However, outside the disk the force-free fields differ significantly from the vacuum field. The biggest difference is the existence of the coronal toroidal field; however, this field is weak, and vanishes near the disk's surface. The existence of the force-free corona also results in "swelling" of the poloidal field configuration, but the field lines remain closed as required by the assumptions of our model.

Magnetic field configurations shown here are representative and their features are robust. Similar configurations are obtained for the standard disk model characterized by different values of α_{ss}, M_{16}, and M_1, as well as for different disk models and different α-quenching models. In particular, the configuration of the equilibrated magnetic field for disks characterized by $\alpha_{ss} < 10^{-3}$, for which the lowest-order normal mode is an oscillatory dipole, is still a quadrupole steady state like those shown on Figs. 2–4. This is because the back-reaction of the magnetic field on the α-effect, as modeled by the α-quenching, effectively decreases the magnitude of the dynamo number to its critical value, thus restoring the character of the magnetic field to the steady state with quadrupole symmetry.

Note, however, that in order to obtain a magnetic field configuration with the magnitude of $B_\phi(Z = H)$ comparable to the magnitude of the poloidal field, values of $|\mu|$ close to its critical value $\mu_{crit} = 6.393/R_0$ are needed. It is possible that at such high values of $|\mu|$ the relaxed coronal magnetic field is nonaxisymmetric and does not have a linear, force-free configuration (Aly, 1993).

5. Dynamical effects of the generated magnetic field

The Maxwell stresses due to the dynamo-generated large-scale magnetic field can affect the structure and the dynamical evolution of the accretion disk by 1) the contribution of the magnetic pressure to the total pressure, 2) the contribution of the $\phi - R$ component of the Maxwell stress to the radial redistribution of angular momentum, or 3) the non-local transport of angular momentum via magnetic field pervading the corona.

It is immediately clear from our calculations that the first two contributions are independent from the assumed boundary conditions, inasmuch as the magnetic field inside the disk is insensitive to the particulars of the medium outside the disk. Stepinski et al. (1993) showed that the ratio of the magnetic pressure to gas pressure is

$$\frac{P_{mag}}{P_{gas}} \approx \frac{1}{2}\alpha_{ss}\left(\frac{B}{B_{eq}}\right)^2 \tag{17}$$

and the ratio of the $\phi - R$ component of the Maxwell stress to the $\phi - R$ component of the viscous stress is

$$\frac{\mathcal{F}_B}{\mathcal{F}_\nu} \approx {\alpha_{ss}}^{1/2} \left(\frac{B}{B_{eq}}\right)^2 \quad (18)$$

Figs. 2–4 show that for all cases considered here $B < B_{eq}$ and the contributions of the large-scale magnetic field to the total pressure and radial transport of angular momentum are small. Again, this is a robust result, applicable to a broad range of disk models and values of parameters (see Rüdiger et al. 1995). On the other hand, the small-scale field generated by the $\alpha\Omega$-dynamo may contribute significantly to both, the total effective pressure and the radial transport of angular momentum.

The new feature of the dynamo-generated magnetic field in a disk with the force-free corona is the possibility that the $\phi - Z$ component of the Maxwell stress transports angular momentum vertically from the disk to the corona (or from the corona to the disk), thus providing dynamical link between distant portions of the disk. In principle, this can lead to significant dynamical effects, providing that the value of $B_\phi^H B_Z^H$ is sufficiently high. From Figs. 3 and 4 it may appear that the toroidal field reverses its sign *on* the surface of the disk. Such a configuration would yield $B_\phi^H B_Z^H = 0$ and would fail to provide magnetic link between distant parts of the disk. However, closer examination of our results reveals that the surface defined by $B_\phi = 0$ is located beneath disk surface, so the value of $B_\phi^H B_Z^H$ is small but not equal to zero. We can assess the importance of this effect by comparing the vertical and radial fluxes of angular momentum due to the presence of a magnetic field.

$$\epsilon \sim 2\frac{R}{H} \frac{B_\phi^H B_Z^H}{|\langle B_\phi B_R \rangle|} \quad (19)$$

where $\langle\,\rangle$ denotes an average over the vertical extent of the disk. Fig. 5 shows $\epsilon(R)$. It is clear that the vertical flux of angular momentum is a small fraction of the radial flux of angular momentum due to the Maxwell stress, which, in turn, is a small fraction of the radial angular momentum flux due to the viscous stress.

The configuration of the magnetic field is such that $\langle B_\phi B_R \rangle$ is always negative, and consequently the radial angular momentum flux due to the magnetic stress is outward, just like the angular momentum flux due to viscous stress. In the steady-state disk with a fixed $\dot{M}$ the $\phi - R$ component of the magnetic stress causes a slight increase of V_R and a corresponding decrease in the magnitude of the surface density Σ. In the inner portion of the disk (where $B_Z^H > 0$), the vertical flux of angular momentum is negative, and this part of the disk is losing angular momentum to the corona. In the outer portion of the disk (where $B_Z^H < 0$), the disk is gaining angular momentum from the corona. Overall, there is no net change in the total angular momentum, just a non-local redistribution. In a steady-state disk with a fixed $\dot{M}$ the $\phi - Z$ component of the magnetic stress causes a slight increase of V_R and a decrease of Σ in the inner portion of the disk, but also a slight decrease of V_R and a increase of Σ in the outer portion of the disk. However, given the small magnitude of $B_\phi^H B_Z^H / |\langle B_\phi B_R \rangle|$, the changes, ΔV_R and $\Delta\Sigma$, are small.

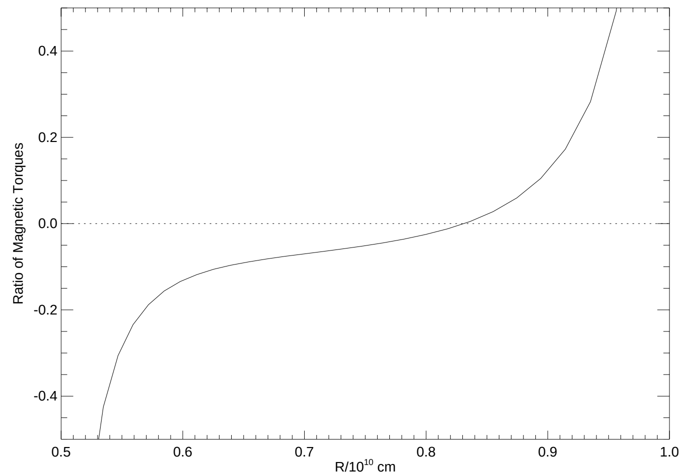

Fig. 5. The ratio of vertical to radial fluxes of angular momentum due to the stress of the large-scale magnetic field. The linear force-free corona with $|\mu| = 0.95\mu_{crit}$ is assumed.

Because of the existence of the pressure gradient, the gas velocity is slightly non-Keplerian even in a disk without any magnetic field. The presence of the magnetic field introduces magnetic pressure and the term proportional to $B_R^H B_Z^H$. We have already estimated (see Eq. (17) above) that $P_{mag} < P_{gas}$. Furthermore, P_{mag} and P_{gas} have the same radial dependence (due to the α-quenching model), so the contribution of magnetic pressure to the character of the large-scale velocity is negligible. To assess the contribution of the magnetic stress $B_R^H B_Z^H$ we compare the magnitude of $(B_R^H B_Z^H)/4\pi$ with the magnitude of $(H/R)P_{gas}$ (Heyvaerts & Priest 1989). From Figs. (3) and (4) we conclude that $(B_R^H B_Z^H)/4\pi$ is about 3 to 4 orders of magnitude smaller (recall that the values of B_R^H and B_Z^H on both figures have to be divided by the factor of 30) than $B_{eq}^2/8\pi$, which itself is an order of magnitude smaller than the gas pressure. On the other hand, $H/R \approx 10^{-2}$ for our disk, thus the magnetic term is two to three orders of magnitude smaller than the pressure term, and its contribution to the large-scale velocity is negligible. Overall, a dynamo-generated magnetic field cannot lead to non-Keplerian behavior.

6. Conclusions

We investigated the character of the large-scale, dynamo-generated magnetic field in a standard accretion disk. Unlike previous calculations that addressed this problem (for example Rüdiger et al. 1995), we assume that the disk is surrounded not by a vacuum, but rather by a linear force-free corona. The major objective of this study was to find out whether the force-free corona assumption changes the conditions of the magnetic field generation and the character of the generated field as compared to the models with the vacuum surroundings. We have found that conditions for a turbulent dynamo to generate a magnetic field in a disk with a force-free corona are practically identical to those obtained for a disk with a vacuum exterior. The normal modes are stationary and have quadrupole symmetry (providing that $\alpha_{ss} > 10^{-3}$). The field inside the disk is dominated by the toroidal component. However, the existence of the force-free corona allows the presence of the toroidal field in the disk exterior. The direction of the toroidal field in the corona is opposite to its direction in most of the disk. The reversal takes place in the disk, very near its surface.

We also performed nonlinear calculations, aimed at obtaining the structure of the equilibrated magnetic field. Again, inside the disk, the equilibrated magnetic field resembles the field calculated in a disk surrounded by the vacuum. In the exterior the character of the field is different, and depends on the value of the force-free parameter μ. All three components of the magnetic field are present, and the field lines display a "swirly" character, unlike the field lines in the vacuum, which have no twist. As $|\mu| \rightarrow \mu_{crit}$ the coronal field becomes more wound up, but it is also more vertical and extends farther up beyond the surface of the disk. The magnetic field at the surface of the disk has a strength of about one hundredth that of the field at the midplane. Unlike the midplane field, all components of the magnetic field on the surface are of the same order of magnitude.

We have found that the existence of the corona does not lead to non-Keplerian behavior or any other significant dynamical effect. This follows from the magnitude and the structure of the magnetic field generated by the $\alpha\Omega$-dynamo. The magnitude of the magnetic field is limited to $B \sim B_{eq}$ by its negative feedback on the α-effect. Therefore, magnetic pressure and magnetic radial transport of angular momentum are small in comparison with gas pressure and the viscous transport of angular momentum, respectively. The magnetic field is highly concentrated around the midplane, and its magnitude at the surface is much smaller than its magnitude at the midplane. The toroidal field reverses its sign near the surface, but it would have a small amplitude at the surface, even without such a reversal. Consequently the magnitude of $B\phi^H B_Z^H$ is small and vertical transport of angular momentum is not significant.

Our findings complement the results of HP89 showing that, within the framework of $\alpha\Omega$ dynamo, accretion disks with coronae operate in the low magnetization limit. Heyvaerts & Priest did not actually solve a dynamo problem, as is done here, instead, they *assumed* the radial distribution of B_Z and derived their results in terms of a parameter λ reflecting the magnitude of the large scale magnetic field in comparison to the gravitational energy in the disk. We find that, within our dynamo model, this parameter is very small and therefore deviations from keplerian rotation are insignificant.

It appears that, at least in the framework of the $\alpha\Omega$-dynamo concept, a self-generated, large-scale magnetic field in an viscous accretion disk does not contribute significantly to the dynamics of the disk. However, the small-scale component may be a significant factor in the radial transport of angular momentum. We stress that these conclusions are reached studying a disk model based on the $\alpha\Omega$-dynamo paradigm.

Guided by the qualitative similarity between the character of the large-scale magnetic field resulting from the magnetorotational instability simulations (Brandenburg et al. 1995), and the field resulting from $\alpha\Omega$-dynamo calculations, as far as the relative magnitude between components, one may hypothesize that an approximation of the large-scale magnetic field produced in the former scenario can be obtained by calculating the $\alpha\Omega$-dynamo. However, to do this one should replace the term $|\mathbf{B}|/B_{eq}$ by $|\mathbf{B}|/B_{pr}$ in Eq. (13) to account for the fact that the large-scale magnetic field may be roughly in equipartition with the gas pressure as results in some of the simulations of the magnetorotational instability. In the context of thin accretion disks, such a change in an α-quenching function leads to the saturated field with unchanged structure but appropriately increased magnitude (Rüdiger et al., 1995). This has been verified within our calculations.

If saturated, large-scale magnetic field has a magnitude $B \sim B_{pr}$, it contributes significantly to both, the total effective pressure and the radial transport of angular momentum (see Eqs. (17–18)). Such contribution would be comparable to that of the small-scale field also resulting from the instability. As for the vertical transport of angular momentum through the corona, increasing the magnitude of B would result in increasing the absolute magnitude of such transport, but its importance relative to the radial transport, as given by Eq. (19), would remain the same. It is also easy to estimate that magnetic field with the magnitude of the order of B_{pr} is still too weak to significantly alter disk's Keplerian rotation. However, an important point to consider before jumping to any conclusions drawn from this hypothesis, is that a large-scale magnetic field in equipartition with the pressure would most likely be unstable to buoyancy instabilities, a factor not considered in the formulation of the $\alpha\Omega$ dynamo framework. An evaluation of the potential importance of this effect in modifying the vertical distribution of the different field components is out of the scope of the present paper. Finally, there are still many additional aspects to be considered. For example, field lines could become open by the superposition of an external magnetic field or by the wind driven from the disk. It is feasible that a magnetocentrifugally driven wind (see Blandford 1993 for the review) may significantly change the dynamics of the disk. Such a change would be indirectly attributed to the generated large-scale magnetic field.

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