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Multiple fragmentation of comet Machholz 2 (P/1994 P1)

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Abstract. Discovered in August of 1994, periodic comet Machholz 2 consisted of five condensations, A–E, of which D later became double. They were lined up along their common heliocentric orbit (with A being the leading and brightest component) and connected by a trail of material, suggesting that the comet's nuclear fragmentation was accompanied by a copious release of large dust particles. The earliest breakup is found to have occurred in late 1987, $\sim$600 days before the comet's 1989 perihelion, giving birth to fragment B and the grand precursor of A. The precursors of A and D and fragments A and C appear to have originated, respectively, $\sim$5 days prior to and right at perihelion. The last breakup episode during that same return to the Sun was the separation of E, probably from the precursor of D, $\sim$600 days after perihelion. The division of D into D_1 and D_2 is the only event analyzed in this paper that occurred one revolution later, in 1994. The circumstances and implications of this fragmentation sequence are examined in detail and predictions are presented for 1999/2000.

Key words: comets: general – comets: individual: P/Machholz-2

1. Description of the comet's appearance

Only 17 days after D. E. Machholz's discovery of his second periodic comet (with an orbital period of 5.2 years) on Aug. 13, 1994, a report was issued on M. Jäger's detection of another nearby comet, of the same apparent motion and $0^\circ.8$ to the northeast of the former (Lüthen 1994a). Prediscovery images of this second object were later found on Jäger's photographs exposed on Aug. 19 (Lüthen 1994b). On Sept. 2–3, independent detections of another companion were reported by Pravec (1994a) and by Johnson (1994). Finally, Pravec (1994a) discovered two more diffuse objects on Sept. 4, one of which was also found by Johnson (1994) and by others. I use for these condensations the nomenclature introduced by Green (1994), who referred to the southernmost and usually the brightest fragment as component A and to the four fainter objects as components B, C, D, and E in the order of their increasing distance from A northward. On Sept. 5.0 UT, for example, fragments B–E were, respectively, 5.1, 31.9, 32.5, and 37.5 arcmin from A, all aligned in a position angle of $\sim 23^\circ$. The five comets thus formed two clusters of objects separated by a large gap: a southern group of two (A and B) and a northern group of three (C, D, and E).

The space between the condensations, including the gap, was occupied by a trail of material. This trail was reported by Jäger (Lüthen 1994b) to have been 40 arcmin long on Sept. 5, extending northward from component A and connecting all five fragments. It was also detected by Nakamura (1994a), who remarked on a faint bridge of dust extending from A to E on Sept. 13.8 UT. The trail may have displayed a local brightening around component D, near which it was noticed by Pravec (1994b, c) on short exposures taken on Oct. 5.1 UT to extend for 2 arcmin in a position angle of 190° and for 5 arcmin in 10°. The line connecting the condensations was swinging in the sky like a very slow pendulum, first from the northwest-southeast to the northeast-southwest (until Sept. 12) and then back again, crossing the meridian on Aug. 21 and Oct. 13.

During this period of time, condensation D was observed to experience major changes in its appearance. It was described by Hale (1994a, b) as faint and vague on Sept. 9.5 UT, but 0.7 mag brighter and more condensed on Sept. 16.5 UT. Comparing sets of CCD images from Sept. 10.1 and 23.1, Pravec (1994c) found that the component brightened fully by $\sim$3 mag during the two weeks and that it developed a large coma and tail. This apparent flare-up was also confirmed by other observers. The fragment brightened by 2.5 mag between Sept. 11.5 and 24.5 and by another 0.2 mag one day later, according to Morris (1994); by 1.6 mag between Sept. 13.8 and Oct. 2.8, according to Nakamura (1994b); and by 2.4 mag between Sept. 6.1 and 28.1, according to Bouma (1995). The most significant development was reported by Pravec (1994b, c), who on the CCD images obtained on Oct. 5.1 noticed that D was double, its components D_1 (eastern) and D_2 being of similar brightness and 7 arcsec apart in a position angle of 292°, that is, essentially along the tail. Pravec added that D was elongated to about the same extent and in the same direction on Sept. 2.1 and 4.1 and, to a lesser degree, on Aug. 30.1 UT. He further remarked (Pravec 1995)

Send offprint requests to: Zdenek Sekanina that D was again elongated on Nov. 2.18 UT, with the separation between its components estimated at 9 arcsec in a position angle of 280°.

Component A was reported to display significant morphology between discovery and early September. Mikuž (1994) commented on the presence of a prominent but short-lived 3-arcmin-long and slightly curved jet as early as Aug. 16.0 UT. On Aug. 23.1, Pravec (1994c) found the comet to be ∼2–3 mag brighter than 6–7 days earlier; on Aug. 28.4, Bortle (1994) reported that the comet was much brighter than expected and suggested that an outburst was in progress; and on Sept. 1.0, Černis (1995) noticed a starlike nucleus, of mag 8. Up to three distinct jets were described independently by Pravec (1995) and by Viscome (1995) on images taken on Sept. 5.1 and 5.4, respectively. Activity at last began to subside on the subsequent days.

2. Diagnostic characteristics of the observations

Fig. 1 shows a sketch of the condensations. The relative sizes and distances are reproduced only approximately.

Dynamically, the alignment of components A, B, C, D (later D_1), and E and the correspondence between their connecting line and the trail of material are the most significant pieces of information that the observations offer at first glance. The direction of this line is found *invariably* to coincide with that of the comet's heliocentric orbit, as projected onto the plane of the sky. Component A was the leading fragment, with the other condensations trailing behind. The extensively tested model for split comets (Sekanina 1977, 1978, 1982), which is applied to P/Machholz 2 in the following sections, affirms that the rate at which a companion is seen to recede from the principal component after their breakup is determined primarily (though not entirely) by the slight difference between the contributions from directed outgassing to the orbital momenta of the two fragments. The net effect is modelled as a continuous radial nongravitational deceleration of the less massive component relative to the more massive one. The conservation of momentum law then requires that, after breakup, the decelerated companion fall ever farther behind the principal component in its radial motion and that it gradually turn away from the prolonged radius vector toward the orbit in its angular motion. Hence, a companion observed *shortly* after separation is expected to be located near the principal fragment along the prolonged radius vector, while a companion observed *very long* after separation should be situated far from the more massive component and behind it in the orbit. The configuration of the condensations of P/Machholz 2 clearly indicates that D_2 satisfies this rule-of-thumb condition for a companion that recently detached from D, whereas C, D, and E satisfy the condition for companions that broke away from a common parent with A or B very long before the observations. Component B also appears to satisfy this same condition with respect to fragment A, but the proximity of condensations A and B implies that, for some reason, they were subjected to almost identical nongravitational forces. The following modelling of the fragmentation hierarchy for P/Machholz 2 suggests that this first-approximation scenario has a number of attractive features, but that it oversimplifies the problem and is not correct in its entirety. Nevertheless, it deserves to be mentioned because it provides some useful insight into the process of this comet's progressive disintegration.

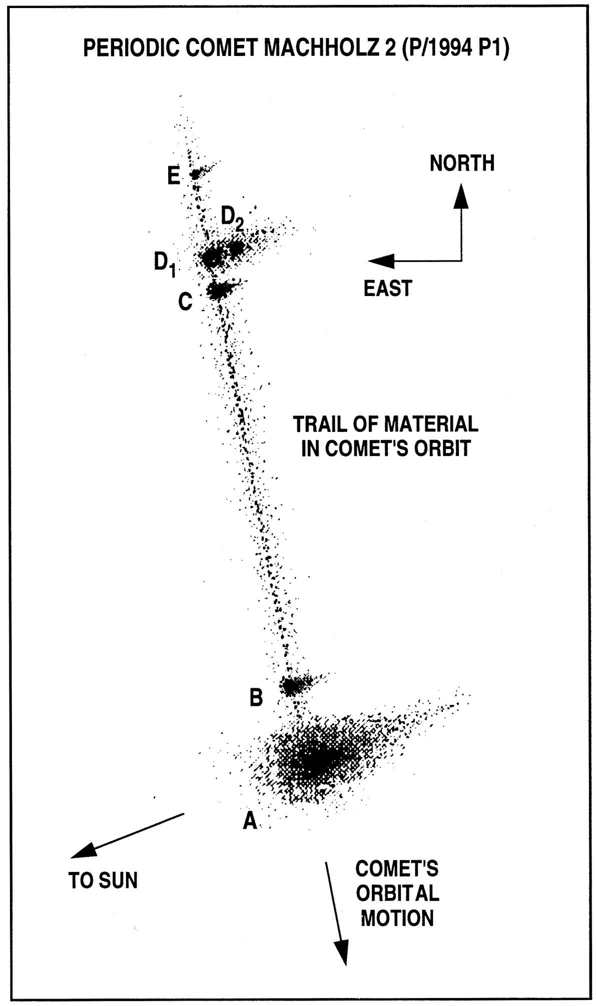

Fig. 1. Schematic representation of P/Machholz 2 as it appeared in early October 1994, when the distance between components A and D was ∼9 arcmin. All six condensations were apparently observed at about this time. Although component E was last measured for position on Sept. 10, it allegedly was examined for brightness as late as Oct. 9. The duplicity of component D was first noticed on Oct. 5 and D_2 was measured for position only until Oct. 11. The trail of material was reported on several dates between early September and early October.

The trail of material connecting the individual condensations apparently represents a continuous or quasi-continuous stream of dust particles, released long before the observations. This ejection process is likely to have been associated with the discrete breakup episodes, but it may have continued in the periods of time between the individual events as well. The force primarily responsible for the distribution of dust along the trail is proposed to have been solar radiation pressure. From the limited breadth of the feature, it can be inferred that the particles involved had been ejected from one or more nuclear fragments at low velocities and then subjected to very low radiation pressure accelerations, as will become apparent from further analysis (Sect. 7). Low values of radiation pressure are generally typical for massive particulate ejecta, whose size depends on their bulk density. Because of the unknown temporal profile of this process, however, it is not straightforward to derive a specific model for the location-dependent size and mass distributions of the material along the trail. Nevertheless, there is no doubt that this phenomenon is of the same nature as the dust trails discovered in the far infrared by the IRAS satellite along the orbits of several short-period comets (e.g., Sykes and Walker 1992). Considering that the reports of the trail of P/Machholz 2 are based on the observations with instruments as small as 20 cm in aperture and with fairly short exposures in the optical region (thus disadvantaged, in comparison with the far infrared regions, because of a low projected area-to-mass ratio of the ejecta), the detections represent a considerable accomplishment and point to extraordinarily large amounts of particulate material in the trail. While the comet's relatively small heliocentric and geocentric distances, $\sim$0.8 AU and 0.5–0.8 AU, respectively, were favorable to the detections, Earth's moderately large angular distances from the comet's orbital plane, 26° in early September and 12° in early October, were certainly of no assistance.

Because of the implied edge-on projection, the optical depth of the dust trail should have reached a maximum at the time of Earth's transit across the comet's orbital plane. Unfortunately, the transit occurred as late as on Nov. 28.6 UT, at which time P/Machholz 2 already was 1.21 AU from Earth and 1.34 AU from the Sun. As far as I am aware, the only images of the comet taken during the critical span of a few days around the time of transit appear to be those exposed at the Ondřejov Observatory on Dec. 1. According to Pravec (1998, personal communication), they were taken under rather unfavorable conditions and their inspection shows no evidence for any narrow trail, although a tail is present.

3. The objectives, model, approach and data

The objectives of this investigation are to determine, to the extent possible, the sequence of breakup events that led to the observed distribution of the fragments, to establish the conditions at each such event and the dynamical history of the observed components, and thus to describe the hierarchy of the parent comet's fragmentation and the subsequent evolution of the system.

Application of the standard model for split comets (Sekanina 1978, 1982) allows the user to employ a set of offsets in right ascension and declination between any two components to derive the model parameters and examine the degree of correspondence that the model provides. The model has five parameters: (i) the time of splitting $t_{\rm split}$; (ii) the three components of a velocity $V_{\rm total}$ with which the companion (the less massive or the *secondary* component) separates from the principal (or the *primary*) fragment (that is assumed to acquire no measurable impulse) at time $t_{\rm split}$; and (iii) the continuous differential nongravitational deceleration γ of the companion relative to the primary, directed radially away from the Sun and varying inversely as the square of heliocentric distance. The three components of the separation velocity are defined by the comet's heliocentric orbit plane and the orientation relative to the Sun at the time of splitting: the radial component $V_{\rm radial}$, positive in the direction away from the Sun; the transverse component $V_{\rm transv}$, positive in the direction of the comet's orbital motion; and the normal component $V_{\rm normal}$, completing the right-handed RTN coordinate system. The gravitational attraction between the fragments is neglected. The influence of the planets is also ignored because of the low sensitivity of the solutions to minor variations in the comet's orbit.

The model parameters are determined by applying a least-squares, iterative, differential-correction procedure. An important feature of this optimization technique is an option to solve for any combination of fewer than the five parameters, so that 31 different versions of the procedure are available. This option is indispensable both in the early phases of the iterative process, when the solution is far from being optimized, and in the cases of convergence difficulties. The convergence is always checked by comparing the residuals "observation minus model" from the normal equations with those from the orbit.

Experience with other split comets shows that companions always survive for only a limited time. As a rule, the appearance of a companion undergoes more rapid changes than that of the main component, and its terminal fading often sets in rather suddenly. Its nuclear condensation disappears first, the coma expands gradually and in some instances becomes progressively elongated, and eventually the entire object vanishes before the eyes of the observers. In most cases this whole process has been defined sufficiently well that it is meaningful to characterize a companion's longevity. Since companions are known to survive generally longer the farther they are from the Sun, an appropriate characteristic introduced to measure their longevity quantitatively is *endurance* E, defined as an interval of time from breakup to the companion's final observation, weighted by the inverse-square power law of heliocentric distance. Thus, the endurance essentially measures the time of the object's exposure to solar radiation (Sekanina 1977, 1982) and is expressed in *equivalent* days, that is, in days for a hypothetical object located at 1 AU from the Sun. The endurance was shown to correlate with the nongravitational deceleration (Sekanina 1982), even though the scatter is fairly large and the relationship's predictive capabilities are limited. Selection effects may be involved, as the final-sighting dates for some companions are determined primarily by unfavorable observational conditions (proximity to the Sun in the sky, excessive faintness because of a large geocentric distance, etc.) rather than by intrinsic dissipation. Hence, the derived values of the endurance represent only lower bounds to the actual longevity for at least some fragments.

In the case of P/Machholz 2, with more than two components involved, the essential part of the solution is to establish the identity of each pair of fragments that share a common parent. This is a difficult task that usually requires that a large number of the possible combinations of the primary and secondary fragments be examined and tested. As with any other data-processing technique, the result depends, to some extent, on the data sample used. In orbit-determination problems the makeup of the sample is dictated by the choice of the rejection cutoff for positional residuals left by the offsets. However, the cutoff-dependent scatter in the resulting parameters will be shown to be generally much smaller than the uncertainties in their values.

The data sample consists of 279 astrometric positions of condensations A–E, reported by 11 groups of observers and published mostly in Nos. 23884–25352 of the *Minor Planet Circular* in 1994-95. Of the 279 positions, 98 refer to condensation A (covering the period from Aug. 30 to Dec. 8), 43 to B (Aug. 30–Nov. 10), 22 to C (Aug. 30–Oct. 11), 91 to the optocenter of D (Aug. 30–Dec. 8), 12 to D_1 (Oct. 5–Nov. 2), six to D_2 (Oct. 5–11), and seven to E (Aug. 30–Sept. 10). Only positions communicated by the observers who measured at least two condensations on the same night have been collected. Three observers account for more than two thirds of the total: Pravec (1994d, 1998), with his group at the Ondřejov Observatory, contributed 138 positions; Nakamura (1994c, 1995), observing at Kuma Kogen, 34 positions; and Scotti (1994, 1995), at Kitt Peak, 20 positions, which extended the observed arc of component D by more than a month. The astrometry for D_1, D_2, and E has been reported only by Pravec.

Because of the enormous projected separation between the two groups of condensations (A–B vs. C–E), especially during the first weeks after their discovery, they could not all be imaged on a single exposure, except with wide-field cameras. Consequently, the offsets of a secondary fragment from the primary, which are required as input to the model, were not available for all combinations of the condensations directly from the published data. The necessary offsets in such instances had to be derived by converting the position for the primary from its listed time to the time of the position for the secondary on the same night. The developed procedure employed an ephemeris computer code that used the orbital elements for the presumed primary components, which were published by Marsden (1994). This approach was applied nearly universally to extract the offsets in the course of September and during much of October. Several positions of the primary at slightly different times were often available, in which case the individual corrected offsets for the secondary could be averaged. The involved time differences never exceeded 0.06 day. When offsets could be calculated from the positions for the primary and secondary fragments on the same exposure or on two exposures that were extremely close to one another in time, they were preferred to the offsets derived by averaging.

4. Search for the optimized orbital solutions

The following description of the orbital calculations faithfully reproduces the actual chronology of this investigation, with the merits of the various birth scenarios evaluated separately for each secondary fragment. This approach is deemed preferable to that based on the chronology of the fragmentation sequence, both for the benefit of the reader and for an illustration of the flow of this presentation. The results for the individual components are summarized in the subsections below, while the findings concerning the hierarchy of this comet's progressive fragmentation are presented in Sect. 5.

A major issue is the relationship between the two clusters of condensations, the $\{A, B\}$ group on the one hand and the $\{C, D, E\}$ group on the other hand. Since an orbital solution's quality depends on the length of the common observed arc of the primary and secondary fragments, and since the longest observation spans are available for condensations A and D, it appears that the first case to examine is a possible relationship between these two masses. Unfortunately, there is a complication caused by the elongation of condensation D, presumably associated with its splitting into D_1 and D_2. This problem needs to be clarified before an investigation into the history of component D is initiated.

4.1. Component D_2

Even though the duplicity of fragment D was reported only by Pravec (1994b, c), the implied elongation of this condensation must have influenced its astrometric positions on images taken by all observers in the critical period of time. In a response to my inquiry, Pravec (1998, personal communication) has pointed out that the astrometric positions of condensation D published by him in 1994 referred to the optocenter of D_1 and D_2, which was located somewhere between the two components. The optocenter's location depended on their brightness ratio (which varied rapidly and in an irregular fashion with time) and on the distribution of light in their common coma. Dynamically, therefore, the optocenter's positions were essentially meaningless. If not filtered out, these effects would introduce systematic errors into the positions of fragment D, and orbital solutions relative to any other component would significantly be degraded (especially in right ascension), if based on such a set of observations.

The recent availability of separate astrometric positions for D_1 and D_2 on three dates between Oct. 5 and 11, 1994 and for D_1 also on Oct. 18 and Nov. 2 (Pravec 1998) has considerably facilitated a solution to this problem. The motion of D_2 relative to D_1 could then be modelled, and the fragment that has the common parent with D could be searched for with greater confidence (Sect. 4.2), because the poorly defined positions for D have been replaced with the clearly defined positions for D_1.

The total number of offset pairs (in right ascension and declination) of D_2 relative to D_1 from Pravec's measured images is six. This low number is a result of unmeasurably small separation distances between the two components on the images exposed before Oct. 5 and the excessively faint and diffuse appearance of D_2 on the images taken after Oct. 11. In fact, D_2 was always more diffuse than D_1, but on Oct. 5 it was about as bright as D_1 (Pravec 1994c). Under these circumstances (scarce data, difficult measurements, short arc), it would be unrealistic to apply the full five-parameter model. Instead, I opted (Sect. 3) to solve first for just the two basic parameters: the time of splitting and the deceleration.

The parameters of this solution are listed in Table 1 as Solution I. Solving for three parameters, with the normal component of the separation velocity added, proved meaningless, as

Table 1. Solutions for component D_2 separating from D.

Parameter	Solution			
	I	II	III	IV
Time $t_{\rm split}$ (days from perihelion) (1994 UT)	-6.6 ± 5.6 Sept. 12.6	-8.5 ± 4.4 Sept. 10.7	(-7.0) (Sept. 12.2)	(-7.0) (Sept. 12.2)
Deceleration γ (10^{-5} solar attraction)	17.4 ± 7.4	15.9 ± 4.8	16.9 ± 0.8	17.7 ± 0.6
Mean residual (arcsec)	±0.80	±0.57	±0.77	±0.54
Number of offsets used	6	5	6	5

the value of $V_{\rm normal}$ came out to be essentially indeterminate, -0.11 ± 0.10 m/s, and the fit was not improved. Further experimentation confirmed that any attempt to solve for more than two parameters would indeed be futile.

One of the positions of D_2 on Oct. 7 left a residual of 1.7 arcsec in right ascension, while all the others could be fitted to within $\sim$1 arcsec. Considering the difficulties with bisecting D_2 (Pravec 1998, personal communication), this residual is not anomalously large. Yet, an alternative solution was searched for by eliminating this position from the set. The result is listed as Solution II in Table 1, which shows that the differences between the two solutions are much smaller than the errors involved.

Either solution suggests that D broke up most probably in, or shortly before, mid-September 1994, that is, a few weeks after its discovery. Since this component was observed to brighten dramatically in the second half of September, it is distinctly possible that the flare-up was a signature of the disruption event. To explore this possibility, the light curve of D between the beginning of September and the end of October was investigated, using 31 visual-brightness estimates and 10 CCD magnitudes. To minimize the degree of scatter among the magnitude scales of the visual observers, the quantity plotted in Fig. 2 is the magnitude difference between components D and A. Since the brightness of A was not subjected to any major, rapid variations during the two-month period, the plotted magnitude differences provide a good approximation to the temporal profile of the flare-up of D. There is a high degree of correlation between the visual magnitudes and the CCD magnitudes with a V filter. The CCD magnitudes with no filter require a correction of -0.3 mag, indicating perhaps a color effect. The best fit suggests the outburst to have commenced most probably on Sept. 12, or 7 days before perihelion. It could not have started before Sept. 11.4 UT, and it appears to have already been in progress on Sept. 13.8 UT. Thus, it indeed is highly likely that the outburst and the breakup were triggered by the same event, whose onset (and, by implication, the time of splitting) is determined with an error of only about ±1 day. The orbital solutions, in which the time of splitting was forced to have taken place on Sept. 12.2 UT (that

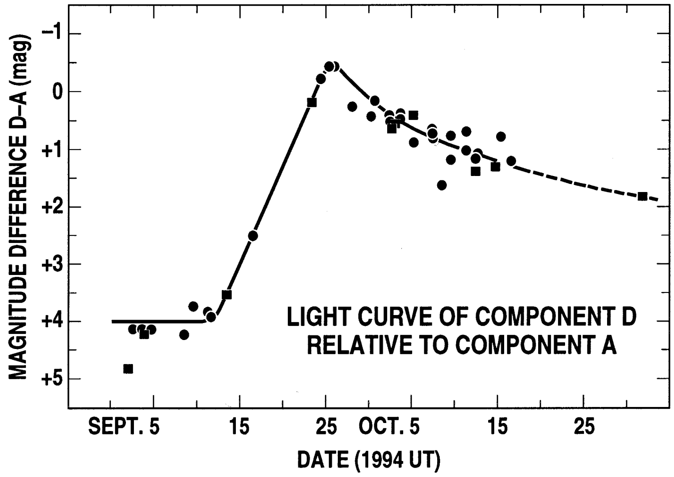

Fig. 2. Temporal brightness variations between components A and D in September–October 1994. A positive difference indicates that D was fainter and vice versa. The 31 magnitude estimates by eight experienced visual observers are depicted by circles, the 10 CCD measurements by three observers are shown as squares. The CCD observations with a V filter are used with no correction to the visual scale, those with no filter require a correction of -0.3 mag. An outburst of component D is found to have commenced most probably on Sept. 12.

is, exactly 7 days before perihelion of D) and which are based on, respectively, the six and the five offset pairs, are listed in Table 1 as Solutions III and IV. The time interval covered by all four solutions is Oct. 5–11.

Since these are one-parameter solutions, the formal error in the deceleration is reduced substantially and the mean residual slightly as compared with, respectively, Solutions I and II. Yet, there is a common envelope to the four solutions listed in Table 1, which, together with the information on the outburst, allows one to make two important conclusions: (i) the images of D on Aug. 30, Sept. 2, and Sept. 4, reported by Pravec (1994b, c), refer to times that were too early for the observed elongation to be related to D_2; and (ii) at the time of Pravec's (1995) observation on Nov. 2, D_2 should have been about 22 arcsec away from D_1 in a position angle of 292°, so that the companion that Pravec detected marginally at $\sim$9 arcsec from D_1 in 280° cannot be D_2. Since the elongations at these times are not in doubt, the only plausible conclusion is that between late August and early November 1994 Pravec witnessed manifestations of three different breakup events of component D. It is estimated that the first episode occurred approximately in mid-August, and the third some time in the second half of October. It is possible that the dramatic brightening of D by 2.5 mag in 9 days, apparent from a comparison of its images on Aug. 19 and 28 (Lüthen 1994a, b), was due to an outburst accompanying the first presumed breakup. There are no brightness data available on D between Oct. 16 and 31, so no flare-up potentially associated with the third inferred event can be documented.

The endurance of companion D_2 is estimated at about 65 equivalent days. This implies the expected observability of D_2 until about Oct. 24, at which time its separation distance from D_1 should have been $\sim$15 arcsec. The estimated longevity of D_2 is consistent with Pravec's (1998, personal communication) finding that on his images of Oct. 18 it was most probably still present but no longer measurable because of its projection onto the background of densely distributed field stars. The endurances of the other two inferred minor fragments are expected to be much shorter still, probably just several days.

4.2. Component D (later D_1)

The configuration of components D_1 and D_2 was distinctive both in orientation and in that their projected separation distance was increasing with time. By contrast, the overall extent of the comet's fragmented appearance was getting smaller. One reason for the shrinking was the increasing distance from Earth ever since early August, before discovery. However, this fact does not account for the whole effect. For example, the projected distance between components A and D decreased from 39.7 arcmin on Aug. 31.0 UT to 32.5 arcmin on Sept. 5.0, that is, by a factor of 1.22. On the other hand, the geocentric distance increased by only a factor of 1.11. The remaining effect was due to the gradual increase in the foreshortening, that is, in the degree of alignment between the Earth-comet configuration and the separation vector of the fragments. In space the distances between any two fragments were increasing at all times.

I first considered component D to have a common parent with fragment A. No satisfactory orbital solution was obtained from the offsets of the optocenter of D, obviously because of the condensation's elongation. All the positions reported for the dates of Sept. 27 through Nov. 9 left prominent, systematic negative residuals of several arcsec in right ascension and less prominent, but still systematic, positive residuals in declination. The implied effect, toward the west-northwest from D_1, strongly suggested that it was due to D_2 in late September and during most of October and a product of the third event (Sect. 4.1) in late October and early November. Similar but somewhat smaller systematic residuals were also noticed for the optocenter's offsets in the span of Sept. 1–6, apparently related to the first event.

Next, all the optocenter's offsets, relative to A, between Sept. 27 and Nov. 9 were rejected. Instead, a solution was searched for by linking the offsets based on the newly measured positions of D_1 (Pravec 1998) with the optocenter's offsets from the times, when the central condensation of component D displayed either no elongation at all or only a very slight one. Obviously, *all* the positions after Nov. 9 must have referred to D_1. For an assumed rejection cutoff of $\pm$2.5 arcsec, most of the offsets from early September could be retained and the 50 employed offset pairs yielded a fairly satisfactory solution, which is identified in Table 2 as Solution I.

When the rejection cutoff was tightened to $\pm$2.0 arcsec, seven additional offset pairs of the optocenter had to be eliminated from the sample, most of them in the span of Sept. 2–4. The remaining 43 offset pairs, including all those involving D_1, served to derive Solution II, which is also displayed in Table 2. This set represents an improvement over Solution I and is clearly preferable. Nevertheless, the parameters of the two sets are seen mostly to overlap, and the differences between them do not appear to be significant.

Table 2. Solutions for component D as companion to A or B.

Parameter	Component D as companion to		
	component A		component B
	Solution I	Solution II	Solution III
Time $t_{\rm split}$ (days from perihelion in 1989)	-6.4 ± 0.8	-5.3 ± 0.7	-8.9 ± 1.9
Velocity of separation (m/s):			
$V_{\rm total}$	1.40 ± 0.21	1.34 ± 0.14	1.29 ± 0.60
$V_{\rm radial}$	$+1.14 \pm 0.05$	$+1.25 \pm 0.05$	-0.56 ± 0.10
$V_{\rm transv}$	-0.77 ± 0.37	-0.45 ± 0.38	-1.14 ± 0.68
$V_{\rm normal}$	-0.23 ± 0.09	-0.16 ± 0.09	$+0.23 \pm 0.17$
Deceleration γ (10^{-5} solar attraction)	5.7 ± 1.6	4.3 ± 1.6	6.1 ± 2.9
Mean residual (arcsec)	$\pm$1.22	$\pm$1.03	$\pm$1.52
Number of offsets used	50	43	35
Dates 1994 covered	8/30 – 12/8	8/30 – 12/8	8/30 – 11/10

Assuming component D to have, instead, a common parent with B led to solutions that were distinctly inferior. The match to the data was especially poor at both ends of the orbital arc of the data sample. The best achievable result is listed in Table 2 as Solution III.

It is thus fair to conclude that fragment D had a common parent with A and that the breakup occurred only several days before the *previous* perihelion passage, in mid-1989. The excellent fit provided by the positions of D_1 indicates that fragmentation of D subsequent to this episode had no measurable effect on the motion of fragment D_1 and that the mass of D_2 (and the other two probable fragments causing the elongation of D in 1994) was considerably smaller than the mass of D_1.

To illustrate the quality of match by the three solutions in Table 2 and the degree of refinement that was introduced by the measurements of D_1, Table 3 lists the residuals $o-c$, or "observed minus computed", left by the positions of D_1 and the optocenter of D, as measured by Pravec on his exposures between Oct. 5 and Nov. 2. For comparison, the offsets of D_2 from D_1 in, respectively, right ascension and declination predicted from Solution IV in Table 1 are -5.4 and +2.5 arcsec on Oct. 5, -6.2 and +2.9 arcsec on Oct. 7, -8.0 and +3.6 arcsec on Oct. 11, -11.5 and +5.0 arcsec on Oct. 18, and -20.3 and +8.3 arcsec on Nov. 2.

Besides the systematic trends in the residuals left by the positions of the optocenter of D, one also notices the fairly

Table 3. Residuals left by Solutions I–III in the positions of D_1 and the optocenter of D, as measured by Pravec.

Date of observation 1994 (UT)	Residual $o-c$ (arcsec)											
	Solution I (companion to A)				Solution II (companion to A)				Solution III (companion to B)			
	component D_1		optocenter of D		component D_1		optocenter of D		component D_1		optocenter of D	
	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.
Oct. 5.142	—	—	−4.2	+3.4	—	—	−4.0	+3.3	—	—	−3.2	+1.4
5.145	−0.6	+2.0	—	—	−0.4	+1.9	—	—	+0.2	+0.1	—	—
5.149	−0.6	+1.7	—	—	−0.4	+1.6	—	—	+0.2	−0.2	—	—
5.154	−0.6	+1.8	—	—	−0.4	+1.7	—	—	+0.2	−0.1	—	—
7.139	−0.2	+0.8	−4.9	+1.6	0.0	+0.7	−4.7	+1.5	+4.2	−1.7	−0.1	−1.8
7.147	−1.8	+0.6	−5.6	+1.9	−1.6	+0.5	−5.4	+1.9	+1.9	−2.1	−0.7	−1.3
11.145	0.0	+0.1	−3.1	+0.4	+0.3	0.0	−2.9	+0.3	−0.8	+2.1	−4.0	+2.3
11.150	+0.7	−0.5	−2.5	−0.3	+1.0	−0.6	−2.2	−0.5	+0.8	+1.2	−2.4	+1.3
11.166	+0.8	−0.6	−2.2	+0.2	+1.1	−0.8	−1.9	0.0	+1.0	+0.4	−1.9	+1.3
18.156	+0.1	0.0	−5.1	+0.7	+0.4	−0.2	−4.8	+1.5	+3.1	−3.0	−2.1	−1.4
18.159	—	—	−6.4	+1.3	—	—	−6.1	+1.1	—	—	−3.4	−1.8
18.183	—	—	−10.5	+1.4	—	—	−10.3	+1.2	—	—	−7.2	−1.5
Nov. 2.155	0.0	−1.4	−3.9	+1.0	+0.4	−1.8	−3.5	+0.6	−2.4	+0.6	−6.3	+3.0
2.165	—	—	−3.8	−0.7	—	—	−3.4	−1.0	—	—	−6.2	+1.9
2.168	−0.4	+0.2	—	—	+0.1	−0.1	—	—	−2.7	+2.8	—	—
2.171	−1.8	−1.3	—	—	−1.4	−1.7	—	—	−4.2	+1.3	—	—
2.189	—	—	−3.8	+1.0	—	—	−3.4	+0.7	—	—	−4.7	+2.0

high degree of scatter, from position to position, over the span of less than one hour on the night of Oct. 18. This kind of phenomenon may be due to major short-term variations in the brightness ratio between D_1 and D_2. Perhaps the most dramatic illustration of this suspected effect is provided by the positions of component D reported by Meyer et al. (1994) for Oct. 16. On the average, the two positions yielded residuals of −12.1 arcsec in right ascension and +6.2 arcsec in declination. The separation of D_2 from D_1 in the two coordinates predicted for this time is −10.5 and +4.6 arcsec. This coincidence suggests with a high probability that Meyer et al. measured D_2, which at the time of their observation must have been brighter than D_1 to the extent that it satisfactorily approximated the optocenter of D.

The endurance of component D (and later D_1) is estimated from the observations at ∼410 equivalent days, which is somewhat less than the value found for the maximum longevity of the persistent fragments in the past (Sect. 6).

4.3. Component C

The scenarios under consideration included those with component C sharing a common parent with A, B, or D. The premise of C separating from the precursor of A yielded solutions that were the most consistent with the data. At a rejection cutoff of ±3 arcsec, these solutions matched 21 of the total of 22 offset pairs; at a cutoff of ±2 arcsec, 19. The solutions based on the assumption of a common parent for B and C yielded only a slightly inferior fit, but implied an improbably high separation velocity, in excess of 4 m/s. At a rejection cutoff of ±3 arcsec, these solutions could accommodate 13 of the total of 15 offset pairs available; at ±2 arcsec, only 10. The solutions based on the premise that C and D had a common parent offered the least satisfactory results, yielding an acceleration, rather than a deceleration, for C relative to D. Of the total of 22 offset pairs, the rejection cutoffs of ±3 and ±2 arcsec reduced the number of data that could be satisfied by this hypothesis down to, respectively, 17 and 13.

To illustrate the parametric scatter among the best achieved solutions, three of them are compared in Table 4. They all indicate that component C separated from the precursor of A right at the time of the 1989 perihelion, that is, only several days after the event that involved component D. On the other hand, the solutions based on the less likely fragmentation hypotheses, with B or D in the capacity of A, yielded the time of splitting in a range of 20–30 days after the 1989 perihelion.

The endurance of component C is estimated at approximately 350 equivalent days, taking Oct. 11, 1994 (Pravec 1998) as the date of its final sighting. This estimate is near the upper limit to the expected longevity for fragments similar to C, and it is rather unlikely that this component will ever be detected again.

4.4. *Component E*

All four other components, A–D, were considered as potential participants in a breakup episode that gave birth to companion E. Unfortunately, because of the faintness, diffuseness, and lack of condensation of E (Pravec 1994a), and also because of the small number of positions measured (a total of seven) and the short interval of time covered (11 days), only two-parameter solutions could successfully be derived. When more than two parameters were solved for, either their errors were found to be unacceptably high or such solutions altogether failed to converge.

The optimized two-parameter solutions for the four scenarios are presented in Table 5. If fragment E was related to one of the two components in the southern group, then E had begun its existence several months *before* the 1989 perihelion passage. On the other hand, if E was related to one of the other components in the northern group, then it was the product of the last fragmentation event of the 1989 return and its birth had taken place more than $1\frac{1}{2}$ years after the 1989 perihelion. While it cannot be determined with certainty which of the four scenarios is the correct one, the common parentage of D and E is suggested as the likeliest. This hypothesis satisfies the seven positions most closely and is also preferable because it provides a better match to the approximate positions of component E reported, but not measured, by Pravec (1994c) on Sept. 23 and Oct. 5.

4.5. *Components A and B*

The described succession of breakup episodes points to a scenario in which components A and B almost certainly shared a common parent.

I first postulated that A was the primary fragment and B the secondary. All solutions with the transverse component of the separation velocity assumed to be zero implied for B a very slight deceleration, but left an entirely unacceptable distribution of residuals, with strong systematic trends reaching a maximum of $\sim$10 arcsec in right ascension. Once $V_{\rm transv}$ was solved for, the match improved dramatically, but the deceleration changed into an acceleration. Simultaneously, the calculated time of splitting moved back in time from $\sim$100 days before the 1989 perihelion passage in the runs without $V_{\rm transv}$ to $\sim$600 days before perihelion in the improved solutions.

Thus, the calculations somewhat unexpectedly suggest that the principal component of this breakup event is to be identified with condensation B. This identity is also implied by the solutions in which A was from the beginning *assumed* to be the secondary fragment, with three representative sets of parameters listed in Table 6. Solution I results for a rejection cutoff of $\pm$3 arcsec, while the cutoff is $\pm$2 arcsec for Solutions II and III. The normal component of the separation velocity is found to be for all practical purposes zero, and this value is forced in Solution III, judged to be the best of the three.

The solutions in Table 6 consistently indicate that the event involving components A and B was the earliest breakup episode for any of the fragments observed in 1994. The endurance of component A, if reckoned from the time of birth of its precursor shortly before the 1989 perihelion, is estimated at a minimum of $\sim$580 equivalent days.

Table 4. Solutions for component C as companion to A.

Parameter	Solution I	Solution II	Solution III
Time $t_{\rm split}$ (days from perihelion in 1989)	+0.2 ± 1.6	−0.3 ± 1.4	−0.3 ± 1.3
Velocity of separation (m/s):			
$V_{\rm total}$	2.43 ± 1.04	2.72 ± 0.98	2.83 ± 0.50
$V_{\rm radial}$	+1.63 ± 0.22	+1.66 ± 0.19	+1.66 ± 0.19
$V_{\rm transv}$	−1.80 ± 1.39	−2.15 ± 1.23	−2.29 ± 0.60
$V_{\rm normal}$	+0.17 ± 0.60	+0.06 ± 0.51	(0.00)
Deceleration γ (10^{-5} solar attraction)	9.4 ± 5.8	11.0 ± 5.1	11.5 ± 2.5
Mean residual (arcsec)	±1.19	±0.99	±0.98
Number of offsets used	21	19	19

Table 5. Comparison of various solutions for component E.

Parameter	Component E as companion to A	B	C	D
Time $t_{\rm split}$ (days from perihelion in 1989)	−117 ± 8	−166 ± 8	+649 ± 33	+592 ± 29
Deceleration γ (10^{-5} solar attraction)	6.2 ± 0.3	7.2 ± 0.3	5.5 ± 0.4	4.3 ± 0.3
Mean residual (arcsec)	±3.1	±3.0	±3.9	±2.8
Number of offsets used	7	7	7	7

5. Fragmentation sequence and the hierarchy of progressive splitting

It is now appropriate to summarize the sequence of nuclear fragmentation for comet P/Machholz 2 as described by the optimized orbital solutions derived in Sect. 4.

The string of condensations observed after the comet was discovered in 1994 indicates that the fragmentation process began some 600 days prior to the comet's previous passage through perihelion. The earliest breakup is found to have occurred in late 1987, when the parent comet was at a heliocentric distance of $\sim$4.75 AU and $\sim$0.3 AU south of the ecliptic. For

Table 6. Solutions for component A as companion to B.

Parameter	Solution		
	I	II	III
Time $t_{\rm split}$ (days from perihelion in 1989)	−644 ± 43	−614 ± 32	−600 ± 24
Velocity of separation (m/s):			
$V_{\rm total}$	1.86 ± 0.11	1.99 ± 0.10	2.05 ± 0.09
$V_{\rm radial}$	+1.85 ± 0.11	+1.98 ± 0.10	+2.03 ± 0.09
$V_{\rm transv}$	−0.15 ± 0.10	−0.22 ± 0.08	−0.27 ± 0.07
$V_{\rm normal}$	−0.04 ± 0.02	−0.02 ± 0.02	(0.00)
Deceleration γ (10^{-5} solar attraction)	2.8 ± 2.6	4.8 ± 2.1	5.9 ± 1.7
Mean residual (arcsec)	±1.47	±1.05	±1.03
Number of offsets used	42	30	29

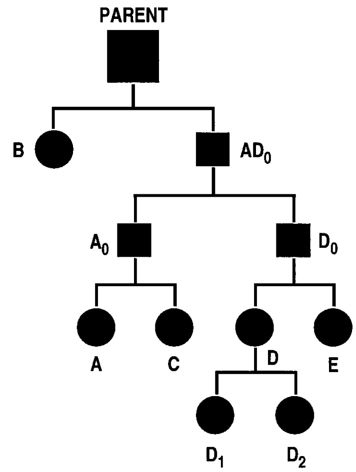

Fig. 3. Fragmentation hierarchy proposed for P/Machholz 2. The PARENT is the original comet, whose existence was terminated in late 1987, when it split into components B and AD_0. Component AD_0 existed for about 600 days, before it broke up into components A_0 and D_0 near the comet's 1989 perihelion. None of the four objects depicted by the squares has ever been observed. Component A_0 was a short-lived fragment that divided, after only some 5 days, into components A and C. On the other hand, component D_0 survived for about 600 days before it split into components D and E. Component D broke up into components D_1 and D_2 near the 1994 perihelion. Fragments A, B, C, D (later D_1 and D_2), and E, which are depicted by the circles, were all observed in 1994.

comparison, the comet's aphelion distance is 5.3 AU. This early splitting involved component B and the grand precursor of fragments A and D. The next breakup event took place ∼5 days before the 1989 perihelion, when the grand precursor divided into two pieces, the precursors of A and D. Several days later, right at perihelion, a new disruption episode gave birth to components A and C. Components D and E were born approximately 600 days later, in early 1991, at 4.75 AU from the Sun and 0.8 AU south of the ecliptic. There is no evidence for any other fragmentation event until shortly before the 1994 perihelion passage. The elongated appearance of condensation D suggests a rapid sequence of three potential events involving this component, the first some 5 weeks before perihelion, the second a week before perihelion, and the third perhaps some 5 weeks after perihelion. Only the second of these three episodes produced an observed fragment, when D_2 was detected on three nights alongside the much more persisting condensation D_1. There is no doubt whatsoever that fragments similar to D_2, having separated from any of the components during the 1989 return, could not survive until 1994. Thus, we remain unaware of any such breakup events, because the comet was not observed during its 1989 return on account of an extremely poor geometry.

Referring to the precursor of A and C as component A_0, to the precursor of D and E as component D_0, and to the precursor of A_0 and D_0 as component AD_0, the proposed fragmentation hierarchy for P/Machholz 2 is presented schematically as a family tree in Fig. 3.

The startling feature of this sequence of breakup events is an extreme asymmetry, in that only one of the two initial components of the parent nucleus has gone on to split progressively into ever more fragments. The proposed interpretation of available evidence suggests that it is component AD_0, the presumably less massive of the two initial fragments, that has continued to break up. The other piece, identified as component B in 1994, appears to have undergone not a single disruption episode. Considering the suggested correlation between nuclear splitting and activity (Sect. 4.1), the intriguing question is whether the intrinsic faintness of component B (in comparison with A and D, for example) could be an inevitable consequence of its resistance to splitting. The implied relationship between splitting and activity, apparent from numerous examples in the past, has traditionally been explained by the sudden exposure of a formerly protected surface and by the resulting increase in the sublimation of newly excavated volatile substances that have become subjected to the effects of impinging solar radiation. If there is no splitting, no ices are exposed, hence no significant activity.

Since August 1982, when P/Machholz 2 approached Jupiter to approximately 1 AU, no closer encounter between the two bodies has taken place (Marsden 1998, personal communication). Consequently, the entire sequence of the comet's fragmentation events is definitely nontidal in nature, including the earliest episode near Jupiter's orbit. It is known that one attribute of nontidally split comets, which has repeatedly been confirmed by observations, is the leading position of the principal, most massive nucleus (Sekanina 1997). Thus, contrary to the conclusion based on the dynamical analysis, this *rule of thumb* suggests that it is fragment A that is descended from the initial principal component of P/Machholz 2.

Fig. 4 compares the observed separations among the various fragments with the accepted dynamical solutions. The paramet-

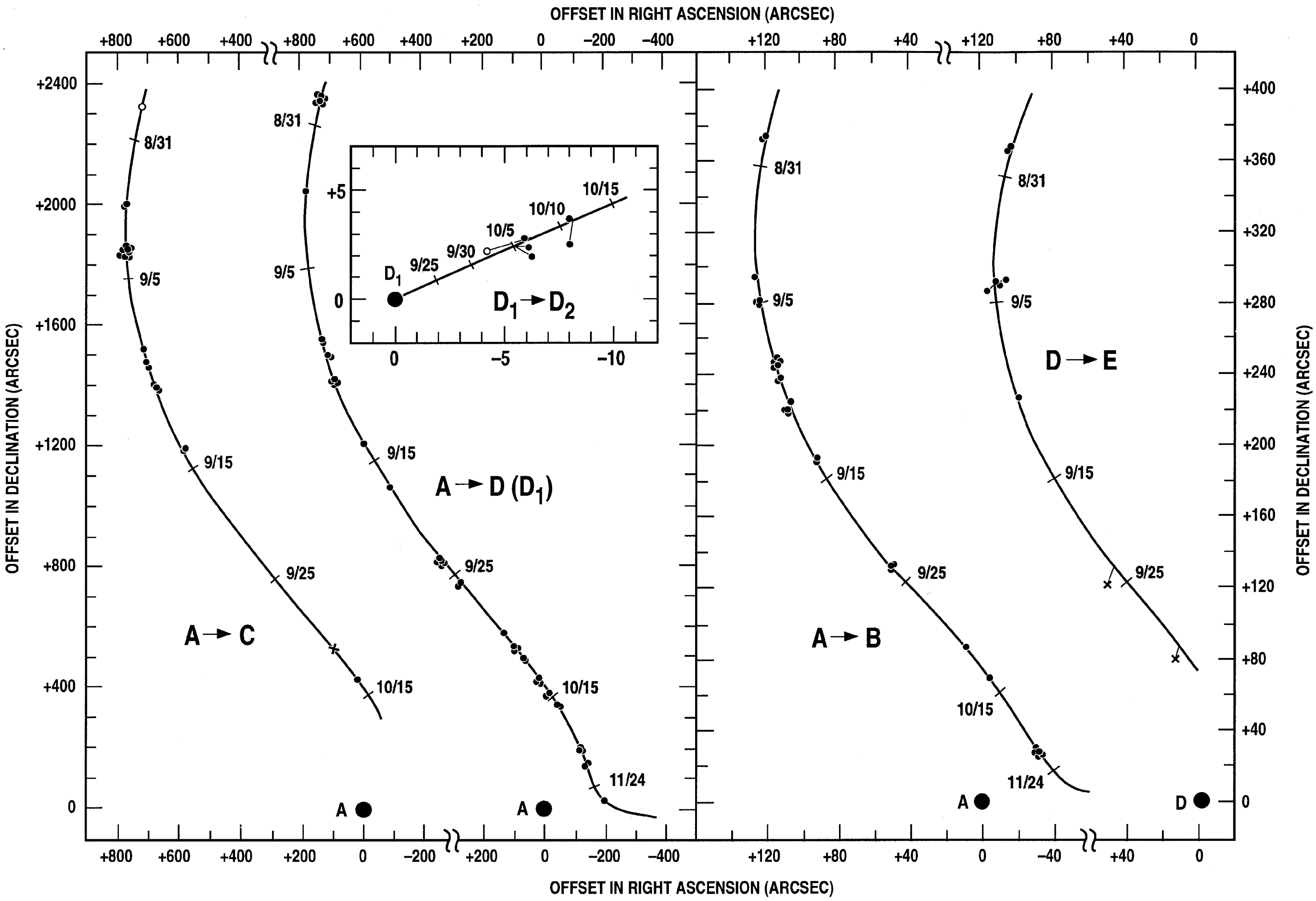

Fig. 4. Comparison of the optimized solutions for the breakup events of P/Machholz 2 with the astrometric positions of the fragments. From left to right, the offsets in right ascension and declination refer to: component C relative to component A; D (later D_1) relative to A; B relative to A; and E relative to D. In the inset:: D_2 relative to D_1. The large circles are the fixed positions of the reference component from which the offsets of the other fragment are reckoned. The dots are the offsets employed in the solutions. The open circles are some of the offsets excluded from the solutions because they have left unacceptably large residuals. The crosses are approximate offsets, reported but not astrometrically determined.

ric sets used are Solution III from Table 6 for the event involving fragments A and B, Solution III from Table 4 for the birth of component C, Solution II from Table 2 for component D (later D_1), the rightmost solution from Table 5 for component E, and Solution IV from Table 1 for component D_2. The offsets of B relative to A are simply the offsets of A relative to B plotted with the opposite signs. With the exception of the two approximate positions of E on Sept. 23 and Oct.5, the match is entirely satisfactory for all the fragmentation events.

The left panel of Fig. 5 is an overview of the orbital evolution of fragments B through E relative to A, in projection onto the plane of the sky, until the beginning of 1995. The trajectories are dominated by large loops, which are confined mostly to August 1994, around the time of the comet's close approach to Earth shortly before perihelion. The right panel is a closeup view of the region of small separations.

Investigating the evolution of the products of secondary fragmentation of comet Shoemaker-Levy 9, Sekanina et al. (1998) found that the separation-velocity vectors of the secondary fragments were distributed very nonuniformly. The velocity vectors were arranged essentially along a great circle, in a configuration that was interpreted to be a product of the approximately conserved angular momentum of the original comet at the time of initial disruption. Although the separation-velocity vectors are available only for three breakup events of comet Machholz 2, it still is of interest to test whether and to what degree they satisfy the condition to which the fragments of comet Shoemaker-Levy 9 conformed so closely. The examination of the vectorial distribution of the separation velocities of comet Machholz 2 is also warranted by their striking nonrandomness in the RTN coordinate system (Tables 2, 4, and 6). The radial component of the separation velocity (exceeding 1 m/s) is always positive, its transverse component is always negative, and its normal component is always near zero. This last piece of evidence implies that the separation-velocity vectors are approximately confined to the comet's orbital plane.

The equatorial coordinates $\{\alpha_{\rm vel}, \delta_{\rm vel}\}$ of a companion's separation-velocity vector, whose tabulated magnitude and RTN components are, respectively, $V_{\rm total}$, $V_{\rm radial}$, $V_{\rm transv}$, and $V_{\rm normal}$, can be calculated from:

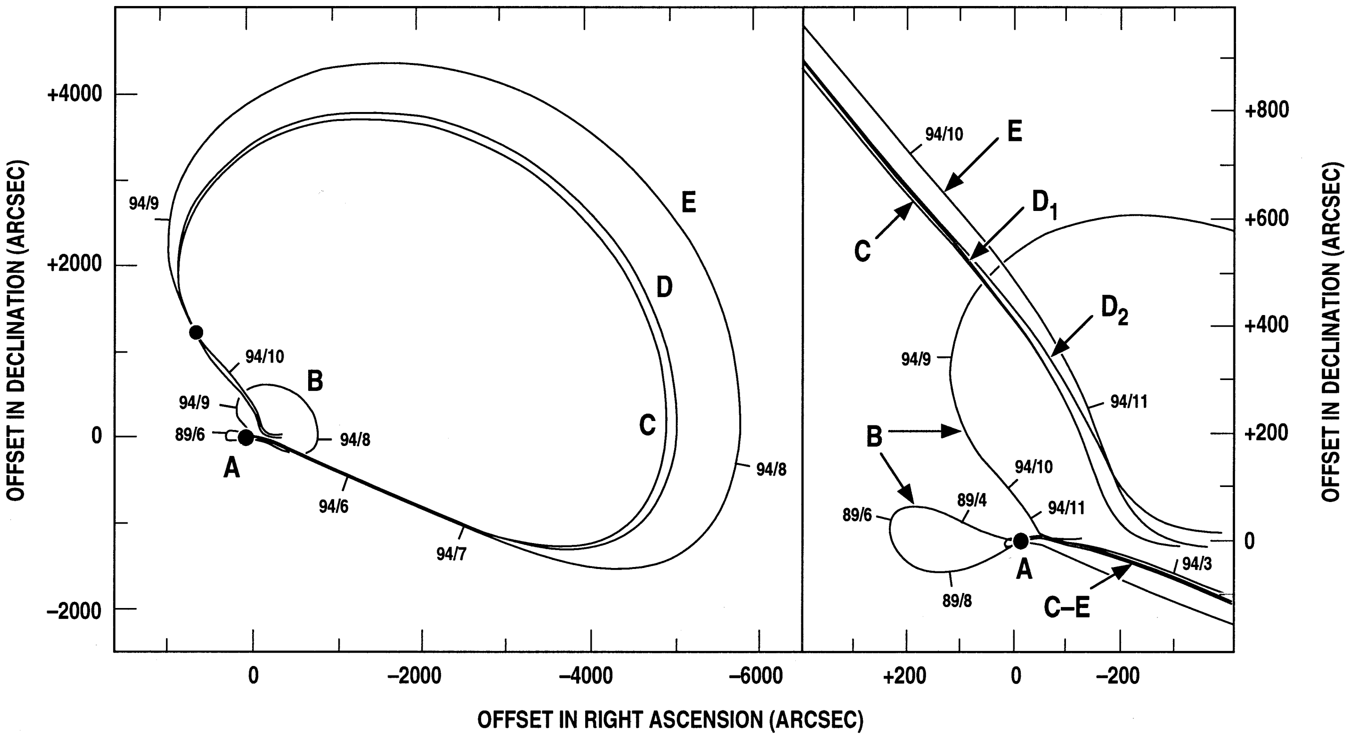

Fig. 5. Projected motions of fragments B–E relative to A (large solid circle) of P/Machholz 2. A global view of the evolution until early 1995 is on the left, a closeup on the right. A small solid circle on the left is the birth point of D_2. The tick marks refer to the beginning of the indicated month. For example, 94/8 stands for August 1, 1994.

$$\begin{pmatrix} \cos\alpha_{\rm vel}\cos\delta_{\rm vel} \\ \sin\alpha_{\rm vel}\cos\delta_{\rm vel} \\ \sin\delta_{\rm vel} \end{pmatrix} = \frac{1}{V_{\rm total}} \begin{pmatrix} P_x & Q_x & R_x \\ P_y & Q_y & R_y \\ P_z & Q_z & R_z \end{pmatrix} \times \begin{pmatrix} \cos v_{\rm split} & -\sin v_{\rm split} & 0 \\ \sin v_{\rm split} & \cos v_{\rm split} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_{\rm radial} \\ V_{\rm transv} \\ V_{\rm normal} \end{pmatrix}, \tag{1}$$

where $v_{\rm split}$ is the true anomaly at the time of splitting and $P_x, \ldots, R_z$ are the relevant equatorial components of the unit vectors $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ directed, respectively, to the perihelion point, to the point in the orbit plane at true anomaly of +90°, and to the northern orbital pole.

If angular momentum has been conserved during the progressive fragmentation of the original comet, then the separation-velocity vectors of the nuclear components should satisfy a condition

$$A_{\rm rot}\cos\alpha_{\rm vel} + B_{\rm rot}\sin\alpha_{\rm vel} + \tan\delta_{\rm vel} = 0, \tag{2}$$

where $A_{\rm rot}$ and $B_{\rm rot}$ are rotation constants of the parent nucleus (see Sekanina et al. 1998 for details).

Application of condition (2) to the separation-velocity vectors derived from the three relevant fragmentation solutions for P/Machholz 2 indicates a good match, to within about 5°, even though the vectors are distributed along an arc ∼140° long. If the companions separated, like the secondary fragments of Shoemaker-Levy 9, from the dark side of their parents, the original comet's rotation was retrograde, the rotation pole was located at $\mathrm{R.A.} \simeq 60^\circ$, $\mathrm{Dec.} \simeq -70^\circ$, and the obliquity of the nucleus was near 170°. If the separation points were on the sunlit side, the rotation was prograde and the obliquity was close to 10°.

6. Comparisons with comet 3D/Biela and other split comets

Noteworthy cases of both similarity and discrepancy in the dynamical and physical properties of fragments are found when P/Machholz 2 is compared with other split comets, and especially with Biela's comet.

Besides the fortuitous coincidence between Machholz 2 and Biela in most, but not all, of their orbital elements, the two objects had other attributes in common.

In the first place, Machholz 2 and Biela are the only split comets with fragments that are known to have survived over time spans substantially exceeding one revolution about the Sun. (The splitting of 79P/du Toit-Hartley had probably taken place only one revolution before it was observed as a double comet in 1982; in any case, the assumption of two revolutions elapsed does not improve the match to the observations available.)

However, whereas the earliest breakup event of comet Machholz 2 occurred near aphelion, approximately $1\frac{1}{3}$ orbital periods before the comet's discovery, Biela's fragments were observed at its next return to the Sun, in 1846, as well as one revolution later, in 1852. And while the earliest breakup of Machholz 2 is found to have occurred about 1 year *past* aphelion, Biela split some $2\frac{1}{2}$ years *before* aphelion. The significance of this difference is obvious, as it implies disruptions at times of opposite thermal regimes at the nucleus surface: Machholz 2's was warming up, while Biela's cooling down.

The two objects also differ in that Machholz 2 is known to have broken up into a multitude of fragments, not just two components. Considering, however, the limited sensitivity of visual detection techniques of the mid-19th century, Biela's additional fainter companions may have been missed, so that the number of observed fragments is not necessarily an important aspect of this comparison.

The behavior common not only to P/Machholz 2 and 3D/Biela, but also to a number of other split comets, involves major short-term intrinsic-brightness and appearance variations among the fragments as well as the development, in the antisolar direction, of independent, nearly parallel tails, once the components are far enough apart that they no longer share the coma. I have already referred to sudden flare-ups of P/Machholz 2's component D, while erratic light curves for fragments of several split comets are presented in Figs. 8 and 9 of Sekanina (1982). A spectacular example of parallel tails was displayed by the recently defunct comet Shoemaker-Levy 9 (e.g., Weaver et al. 1995), but descriptions of a number of less impressive instances can be found throughout the literature.

For rapid changes in the brightness and appearance of Biela's fragments the reader is referred to Maury (1846), Peters (1846), Reslhuber (1847), and Struve (1848) during the 1846 apparition and to Secchi (1853) and Struve (1857) during the 1852 apparition. In the critical period of time in 1846, the tails pointed in position angles of $\sim 70^\circ$–80°, nearly perpendicular to the direction of the companion, which was in $\sim 320^\circ$–330° from the main component. In 1852, on the other hand, the direction of the projected orbit (along which the fragments were aligned) and the direction of the prolonged radius vector (along which the tails were extended) subtended an angle of only about 15°, and they were more difficult to distinguish.

The problem of material connecting Biela's nuclear condensations is a controversial one. At first glance, a wealth of supporting evidence was provided by Maury (1846). On Jan. 18, 1846, he remarked that a second tail of the companion was "reaching toward" the main fragment. On Jan. 23 this tail was again "reaching over to" the primary component or "just to the south of it." On Feb. 12, Maury "caught glimpses" of a tail extending from the companion to the principal nucleus "just above a straight line between the two, and in a sort of arch." Commenting on his Feb. 18 observation, Maury mentioned that the companion appeared "to have thrown a light arch of cometary matter from its head over to" the main component. On Feb. 22 he reported an "arch way of cometary matter between the two nuclei" and on Feb. 26 the principal nucleus was "darting" a tail at the companion. Thus, on as many as six occasions between mid-January and the end of February 1846 did Maury refer to cometary material that in one way or another connected the two components. This information was corroborated only to some extent by Peters (1846), who remarked on a small, very faint tail extending on Jan. 19 toward the northwest, which was the direction of the companion. Reslhuber's (1847) comment that only an extremely delicate nebulous envelope (*Nebelhülle*) was connecting both components together on Feb. 21 and the following dates appears to be too vague to offer any meaningful support to Maury's more detailed characterization of the observed phenomena. Even worse, Peters (1846) reported that on Feb. 20 he could not detect any nebulosity that would bridge the gap between the two condensations. And Schmidt (1846), who observed the comet only from Feb. 4 on, was adamant in his report that on Feb. 21 no material was connecting the two masses and that the space between them was completely dark. He remarked, though, that on Feb. 26 the main condensation had a slight fan-shaped extension on the side facing the companion, thereby filling out some of the dark space between the two components. On the whole, evidence for a dust trail bridging the space between the fragments of Biela's comet in 1846 should be considered as inconclusive.

In 1852, Biela's comet was fainter than in 1846 and, according to Struve (1857), both fragments were detected simultaneously on only four of the 16 days of observation. Major brightness variations and tails were again reported, but no arch of material between the two components. Interestingly, the comet's final observation, by Struve on Sept. 28, 1852, referred to the companion. Its endurance is estimated at $\sim$500 equivalent days, a near-record longevity among the split comets in the past.

During Biela's next return to the Sun, in 1859, observing conditions were extremely unfavorable, and neither fragment has ever been seen again. Another nontidally split short-period comet that vanished was the poorly documented case of comet Giacobini (D/1896 R2). On the other hand, three nontidally split short-period comets – 69P/Taylor, 79P/du Toit-Hartley, and 108P/Ciffréo – were, after duplicity, observed to return to the Sun with only a *single* condensation, which in each instance turned out to be the principal component. This inherent diversity – with Biela and Giacobini on the one side and the three comets on the other – should alone provide motivation for considering investigations of P/Machholz 2 in its forthcoming return to the Sun (Sect. 7).

Comparison of P/Machholz 2 with other split comets also addresses one aspect of the controversial issue of the initial principal fragment's identity. Among the 26 nontidally split comets on the updated list (Sekanina 1997), six (including P/Machholz 2) are known to have broken up into more than two pieces. Of these, two – C/1899 E1 (Swift) and C/1915 C1 (Mellish) – are "new" comets from the Oort Cloud; one (C/1975 V1 West) is an old comet, whose original orbit had a period of about 16,000 years (Marsden and Williams 1997); and the three remaining ones – 51P/Harrington, 73P/Schwassmann-Wachmann 3, and P/Machholz 2 – belong to short-period comets of the Jupiter family. For Swift, Mellish, and West, all companions were found to have broken away from the principal nucleus (Sekanina 1982). Preliminary studies suggest that this likewise was the case with P/Schwassmann-Wachmann 3 (Sekanina et al. 1996) and almost certainly also with P/Harrington (Sekanina, unpublished). Thus, if condensation B should indeed be the primary component of P/Machholz 2, this object's fragmentation hierarchy would very probably be without a precedent.

7. Conclusions and predictions

The major results of this investigation are: (i) the determination of nontidal nature of P/Machholz 2's nuclear splitting; (ii) the optimization of a model for the sequence of the comet's breakup events; and (iii) the description of the proposed hierarchy of its splitting that is consistent with the sequence. This hierarchy is lopsided, with only one of the two initial components of the parent nucleus continuing to split further. All but one of the comet's fragments observed in 1994 separated from their precursors during the previous return to the Sun, in 1989, when the comet was unobservable. Since P/Machholz 2 is currently a one-apparition comet, its orbital period is still to be refined following the next return to perihelion in late 1999. And since the separation-time determinations are relatively insensitive to the orbital period, their values for the events during the 1989 return are, in Tables 2 and 4–6, expressed relative to perihelion, rather than being identified by the date. The most uncertain aspect of the fragmentation sequence is the identity of the initial principal fragment; from the findings in Sects. 4.5 and 5, it should be either component B or the precursor of A.

Three points can be construed as evidence for the precursor of A to be equated with this principal fragment: (i) A was the leading component (cf. Sekanina 1997 for an interpretation); (ii) during most of the 1994 apparition it also was the brightest of the fragments and persisted the longest, having been identified by Marsden (1998, personal communication) with the only condensation observed at the end of March 1995 (Green et al. 1995), some $6\frac{1}{2}$ months after perihelion; and (iii) all the observed fragments except for B separated from the precursor of A. Unfortunately, the brightness (because of its variability) and the survival *within a single apparition* are not very diagnostic of the principal fragment, as companions (such as Biela's) are in these respects often on a par with the main mass or even exceed it. Of the other two points, the leading position appears to be a more significant one.

There are two arguments that favor the identity of the principal component with condensation B in the initial breakup of P/Machholz 2. One is the deceleration of fragment A relative to B, while the other is based on the radial and transverse components of the separation velocity, whose signs are consistent with those of companions D and C in the subsequent breakup events. This latter point is deemed important because it is strongly reminiscent of the rotation effect in the separation-velocity distribution for the products of secondary fragmentation of comet Shoemaker-Levy 9 (Sekanina et al. 1998). If fragment B were the companion to A, the signs of the separation-velocity components would be reversed and the significance of this coincidence would be lost.

If condensation B should be the initial breakup's principal component, the leading position of A could only be understood as an effect of the impulse acquired by this fragment in that event. Since the solutions in Table 6 indicate that A has been decelerating relative to B, the separation-velocity effects exceeded the deceleration effects as late as >1 revolution after the breakup. The separation-velocity vector of A in Table 6 has the correct direction to support this hypothesis. The tendency for cometary splitting to entail a subsequent increase in outgassing, which is commonly explained in terms of exposure of a "fresh" icy surface to solar radiation (Sect. 5), could account for the revitalization, due to recurring breakup episodes, of the activity of fragment A and its precursor, as well as for their nongravitational decelerations. By contrast, the resistance of component B to splitting can explain both its intrinsic faintness and imply the absence of nongravitational effects in its orbital motion.

The fairly high separation velocities, consistently derived from the orbital solutions for the events involving components A, B, C, and D, suggest that the nucleus of the original comet may have been relatively large. Assuming that fragmentation was facilitated, if not triggered, by rapid rotation, one can estimate the effective diameter of the parent nucleus at $\sim$10–15 km. The separation-velocity vectorial distribution along a great circle corroborates a rotation-driven scenario and suggests that at the time of initial disruption the parent comet's spin axis was nearly normal to the plane of the comet's heliocentric orbit. If one can draw analogy to the (nontidal) process of secondary fragmentation of Shoemaker-Levy 9, then the rotation sense of the parent nucleus of comet Machholz 2 was retrograde.

The dust trail, connecting the string of fragments and reported independently by three observers, provides compelling evidence for a copious production of large-size particulate debris during, between, and/or following the breakup episodes. Since the dust trail shared the same volume of space with the individual nuclear condensations, the upper limits to the sizes and masses of dust particles involved can be estimated by assuming that they were released *during the earliest* discrete breakup event, and by interpreting the derived magnitude of this nongravitational effect as due to solar radiation pressure. Typically, the radiation-pressure accelerations are on the order of 10^{-5} the solar gravity, thereby implying the presence in the trail of particulates of very respectable dimensions, in the submeter range and with masses significantly exceeding a kilogram. To estimate the lower limits to particle sizes and masses, one needs to study the trail's observed spatial characteristics as a function of the assumed particle ejection time and solar radiation-pressure acceleration. For example, the calculations made for the trail that does not deviate from the orbital path of the major fragments by more than $\sim$1 arcmin suggest that in Sept.–Oct. 1994 the contributing dust grains could not have been released – not even with a zero normal ejection velocity – more recently than several hundred days after the 1989 perihelion. The characteristic radiation-pressure accelerations of the particles confined to the orbital arc between fragments A and E could not have significantly exceeded $\sim$0.0002 the solar gravity. If their bulk density was as low as $\sim$0.2 g/cm^3, as generally expected, the particles would typically be at least a few centimeters in diameter and at least a few grams in mass. The bulk of smaller particles, ejected more recently than several hundred days after the 1989 perihelion, would be located farther to the west from the line that connected the fragments. A more comprehensive analysis would require a photometric examination of the light distribution both across and along the trail. Since one of the dis-

Table 7. Predicted separation distances and position angles for components A, B, and D of P/Machholz 2 during the comet's forthcoming apparition of 1999/2000.

Date ($0^{\rm h}$ UT)	B relative to A		D relative to A		Elongation from Sun
	Sep. (arcmin)	Pos. angle	Sep. (arcmin)	Pos. angle	
1999 July 1	4.8	254°	16.9	254°	162°
21	4.2	248	14.4	247	137
Aug. 10	3.3	238	11.4	237	114
30	2.7	228	9.5	226	94
Sept. 19	2.4	220	8.6	218	78
Oct. 9	2.3	217	8.5	216	66
29	2.3	217	8.7	219	57
Nov. 18	1.7	223	7.3	227	52
23	1.4	228	6.4	234	51
28	0.9	246	5.2	248	50
Dec. 3	0.8	297	4.6	278	50
8	1.6	333	6.4	311	49
13	2.9	341	10.8	326	50
18	4.9	338	17.6	328	50
23	7.7	332	27.5	325	52
28	11.7	323	42	318	54
2000 Jan. 2	17.3	313	62	310	59
7	24.6	304	87	302	64
12	32.2	296	115	295	72
17	37.8	290	137	289	81
22	39.6	285	147	285	90
27	37.9	282	144	282	97
Feb. 1	34.3	281	133	280	103
6	30.2	280	118	280	107
11	26.3	281	103	280	110
16	22.8	281	90	280	111
21	19.9	281	79	281	112
26	17.4	282	69	281	112
Mar. 2	15.3	283	61	282	111
7	13.6	283	54	283	110
12	12.1	284	48	283	109
17	10.8	285	43	284	107
22	9.7	285	38.3	285	105
27	8.8	286	34.6	285	103
Apr. 6	7.3	287	28.6	286	98
16	6.2	288	24.0	287	94
26	5.3	289	20.4	288	88

crete breakups apparently occurred as late as $\sim$600 days past the 1989 perihelion, the first-approximation model suggests that the observed dust trail consisted of material ejected during and/or in between the splitting episodes, but not in the wake of them. The trail consisted of centimeter- to submeter-sized particulate debris, which – if on a collision course with Earth – would give rise to a brief fireball shower.

The last issue addressed here is, appropriately, the future evolution of this unusual comet. Driven by the need for information on the 1994 fragments in case they persist for another revolution about the Sun, my predictions for the comet's forthcoming return of 1999/2000, a very favorable one, are presented in Table 7. The orbital elements calculated by Marsden (1996) indicate that the comet will be at perihelion in early December 1999. Fragments B and D will follow A to perihelion by about 0.21 and 0.82 day, respectively (Marsden 1998, personal communication). The comet's elongation from the Sun will exceed $\sim 50^\circ$ for a total of 17 months without interruption, from early February 1999 (3.4 AU from the Sun inbound) until early July 2000 (2.7 AU outbound). Earth's closest approach, to 0.32 AU, will occur in mid-January 2000, when the comet is slightly less than 1 AU from the Sun. The geometry is therefore ideal to conduct extensive searches for the fragments seen in 1994 and, if the comet's disintegration has been continuing, for more recent fragments as well.

Comparison with other split comets is too inconclusive to predict which fragments, if any, will survive and which will not. However, it seems unlikely that C, D_2, and E will be recovered in 1999, so only condensations A, B, and D_1 (called D) are considered in Table 7. Because of the unfavorable circumstances of comet Biela's 1859 return (Sect. 6), we will never know what the chance of its recovery would have been under propitious conditions. Since one revolution about the Sun in the orbit of comet Biela adds $\sim$300 equivalent days to the endurance, an 1859 detection of Biela's fragments would have increased their observed longevity to $\sim$800 equivalent days. Survival of the three brighter fragments of P/Machholz 2 through the 1999 perihelion passage requires their longevity to reach $\sim$780 equivalent days for components A and B and $\sim$650 equivalent days for D. Another argument that can serve to elevate our hope that searches for some of the fragments of P/Machholz 2 in 1999/2000 may not be entirely in vain is the enormous increase in the detector sensitivity of observing techniques since the mid-19th century.

With an optimistic frame of mind, further bolstered by the apparent tendency for underestimating the true longevity of comet fragments (Sect. 3) and by the erratic behavior (including unexpected flare-ups) of past nontidally split comets (Sect. 6), I show in Table 7 that condensation D may project up to $\sim 2^{\circ}\!.5$ and B up to $\sim 0^{\circ}\!.7$ away from A. Although uncertain, these predictions could serve four useful purposes. First, in the case that all three, or at least two, of the considered fragments will have survived, the listed separations should be of some assistance in the efforts to identify them. Second, if further breakup events have occurred following the 1994 apparition, such new fragments would be more closely spanned than indicated in Table 7, in which case the presented data could serve as a discriminator between the known and the more recent splitting episodes. Third, if only one fragment is detected in 1999/2000, the listed separations, combined with a standard ephemeris, should help determine or constrain its identity and thereby to assist in recognizing the principal component. And fourth, if recovery of even one of the fragments should by itself become a problem, the predictions provide at least some information on the probable length of the orbital arc along which the search should be intensified. In any case, concerted efforts aimed at recovering P/Machholz 2 and its fragments in 1999/2000 are bound to have beneficial effects in our quest for a better understanding of comets in general and the constitution of their nuclei in particular.

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