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Model of planetary motion in the works of Kerala astronomers

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Abstract. In this report, we discuss the significant advances in planetary theory made by the Kerala astronomers during 14th to 18th centuries. A geometrical picture of planetary motion, where the five planets Mercury, Venus, Mars, Jupiter and Saturn go around the Sun, was developed by the Kerala school, starting with the works of Parameswara (1380-1460). Parameswara explains the procedures for the calculation of planetary longitudes with a geometrical picture in which the five planets Mercury, Venus, Mars, Jupiter and Saturn move around the Sun, which in turn goes around the Earth. Nīlakaṇṭha Somayāji (1444 - 1550), the renowned Kerala astronomer describes such a "heliocentric" model in many of his works along with a detailed justification for the same. He arrives at the correct formulation of the equation of centre for Mercury and Venus, described in his great work *Tantrasangraha* (1500 AD).

As the planets move in orbits inclined to the ecliptic, the true longitude can be obtained only after projecting the position of the planet on the ecliptic. This procedure is discussed in Nilakanta's *Āryabhaṭīya-bhāṣya* and also in greater detail in *Yuktibhasha* of Jyeshtadeva (c. 1550 AD). It is to be noted that such an understanding of latitudinal effects on planetary longitudes came much later in the European tradition of astronomy.

Key words : Kerala astronomy, heliocentric model

1. Introduction

It is quite common to find statements in secondary works and works on history of Indian astronomy which essentially convey that mathematics and astronomy in India did not develop much after the famous Bhaskaracarya or Bhaskara II (c. 1150). This is due to lack of knowledge of the later developments in mathematics and astronomy, especially in Kerala. Many important works, some in Malayalam and some in Sanskrit have not yet been investigated. Of the few works which have been published, only a small fraction of them have been translated and edited with notes in English. Most of the works are available only as manuscripts.

There has been a continuous tradition of astronomical calculations and observations in India, from time immemorial till the recent past (till perhaps the beginning of this century). *Vedanga-*

Jyotisha is probably the earliest astronomical text available to us today which could have been compiled during 1350 - 1150 BC. After *Vedanga-Jyotisha*, one comes across an entirely different class of astronomical works called *Siddhantās, Tantras,* and *Karanas* around 500 AD. According to tradition there are 18 earlier *Siddhantās*, five of which are summarised in Varāhamihira's *Pañchasiddhāntikā* (505 AD). *Siddhantās* are one class of texts which give a detailed account of the mathematical solutions to astronomical problems, namely calculation of positions of the Sun, Moon and the Planets in the back ground of stars, sunrise and sunset times, duration of day for a particular latitude, measurement of time, the time required for a *rasi* to rise at the observer's location, the exact declination of the Sun on a particular day, the determination of the latitude of the observer from the shadow of a gnomon, eclipses, visibility of the planets, their retrograde motion and so on. For a general review of Indian astronomy see Shukla and Sen (1985) and Subbarayappa and Sarma (1985).

Āryabhaṭīya composed in 499 AD by Aryabhata (b.476 AD) is the earliest available astronomical text belonging to the class of *tantra* texts. In the earlier part of this work, Aryabhata has laid down the framework for the mathematical treatment of astronomical problems. Aryabhata considers a *Mahayuga* of 4320000 years. The number of revolutions made by the planets in the stellar background in this *Mahayuga* is given in all the Indian texts, directly or indirectly. These numbers would vary slightly from text to text, but the periods of revolutions obtained from these texts are all close to the values given in current texts of astronomy.

In the following section, we present the conventional scheme adopted for the calculation of longitudes, before we proceed to discuss the advances made by the Kerala astronomers regarding the calculational procedure as well as the picture of planetary motion.

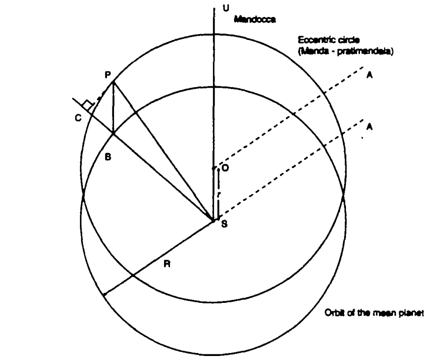
2. Calculation of longitudes

The conventional scheme followed in the Indian astronomical tradition, atleast from the time of Aryabhata (499 AD), for obtaining the geocentric longitudes is as follows. First, the mean longitude (called the *madhyamagraha*) is calculated for the desired day be computing the number of mean civil days elapsed since the epoch (this number is called *ahargaṇa*), and multiplying it by the mean daily motion of the planet. Then two corrections namely *manda-samskara* and *śīghra-samskara* are applied to the mean longitude to obtain the true longitude (called the *sphutagraha*), See D.A. Somayuji (1972)

2.1 Manda-samskara

The *manda-samskara* can be explained with reference to Fig. 1(a-c). In Indian astronomy, longitudes are always measured with respect to a fixed point in the Zodiac known as the *Nirayana Meshadi* denoted by A in the figures.

In the Fig. 1a, 's' is the centre of the mean planet's orbit, which is assumed to coincide with the earth, and 'B' is the *madhyamagraha* at a distance R from it. SU is the direction of the *mandocca*



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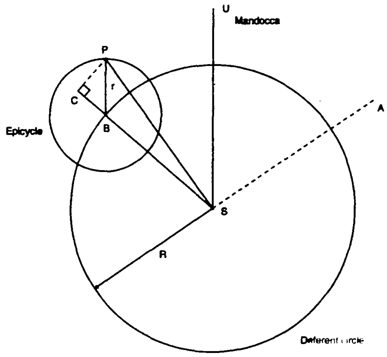

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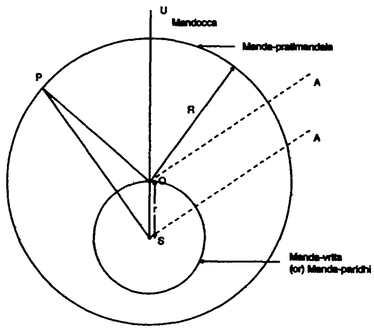


(c)

Figure 1. *Manda-samskara* for a planet. a) Eccentric model b) Epicyclic model c) Inverted epicyclic model.

which is the apogee in the modern terminology. By drawing a line BP = r parallel to SU and along the direction of *mandocca*, 'P', the *mandasphutagraha* or the manda corrected planet is obtained.

We have,

$\angle ASB = \Theta_m$ = *madhyamagraha* (known)

$\angle ASU = \Theta_A$ = longitude of *mandocca* (known)

$\angle ASP = \Theta_{ms}$ = *mandasphustagraha* (to be known)

$\angle BSP = \Theta_m - \Theta_{ms}$ = *mandaphala* (manda correction)

The difference between the longitudes of the *madhyamagraha* and *mandocca* namely,

$$M = \Theta_m - \Theta_A \;(= \angle USB = \angle PBC) \tag{1}$$

is called the *mandakendra* (anomaly) in Indian astronomy. Draw PC perpendicular to the extention of the line SB. From the triangle SPC, we can easily obtain the result

$$\sin \Delta\Theta = \sin(\Theta_m - \Theta_{ms}) \approx r/R \sin M \tag{2}$$

when r/R << 1 (which is true).

From the knowledge of r/R and Θ_A which are specified in the texts, one can obtain the *mandasphutagraha* using equation (2). We have,

$$\Theta_{ms} = \Theta_m - \sin^{-1}(r/R \sin M). \tag{3}$$

It can be easily seen that the locus of 'P' is again a circle of radius R, whose centre is O, which is at a distance 'r' from 'S' along the direction of *mandocca*. Hence, the planet moves in an 'eccentric'; circle (*manda-pratimandala*) and O is the point around which the angular velocity of the planet is constant. Thus, the *manda-samskara* takes the eccentricity of the planet's orbit into account.

Incidentally, it is interesting to note that, Nilakantha, while describing the procedure for finding "*Tatkalika-gati*" (the instantaneous motion) from *madhya-gati* (mean motion) of a planet in his *Tantrasangraha*, (see S. K. Pillai, 1958) gives the formula

$$\textit{Tatkalika-gati} = \textit{madhya-gati} - \frac{r/R \cos M \; dM/dt}{[1 - (r/R \sin M)^2]^{1/2}} \tag{4}$$

The additional factor in RHS of equation (4) exactly corresponds to the derivative of $\sin^{-1}$ (r/R sin M). In *Yuktideepika*, commentary of *Tantrasangraha* in the form of verses, Sankara Varier states -

प्रतिक्षणं प्रभिन्नैव स्फुटभुक्ति : द्युचारिणाम् ।

'The motion of the planets varies every second.'

The Indian astronomers have also explained *manda-samskara* with reference to Fig. 1b and 1c. In Fig. 1b, the circle centred around S and B are referred as deferent circle and epicycle respectively. SU is the direction of *mandocca*. When the planet is along the direction of *mandocca*, the mean and the true planets B & P, lie along the same direction as viewed from 'S'. From there, B moves anticlockwise on the deferent circle and P moves clockwise on the epicycle such that both of them trace the same angle. It can be easily seen that P traces an eccentric circle. Fig 1c is just the inversion of Fig.1b viz., the location of epicycle and the deferent circle are inversed. In this figure, the epicycle is referred as *mandaparidhi* and the deferent circle as *manda-pratimandala*. The rate of motion of the planet on *manda-pratimandala* is the mean angular motion of the planet.

In arriving at equation (2), used in *manda-samskara*, we have taken a simplified approach. Actually the Indian astronomers have used an epicycle with a variable radius. Moreover, in some major texts, the procedure for locating the planet is such that, equation (2) can be shown to be exact, without any approximation.

Different computational schemes for the *manda-samskara* are discussed in Indian astronomical literature. However, the *manda* correction in all these schemes coincides, to first order in eccentricity, with the equation of centre currently calculated in astronomy. The *manda*-corrected mean longitude is called *mandasphutagraha*. For the planets, the *mandasphutagraha* is the same as the true heliocentric longitude as we understand today.

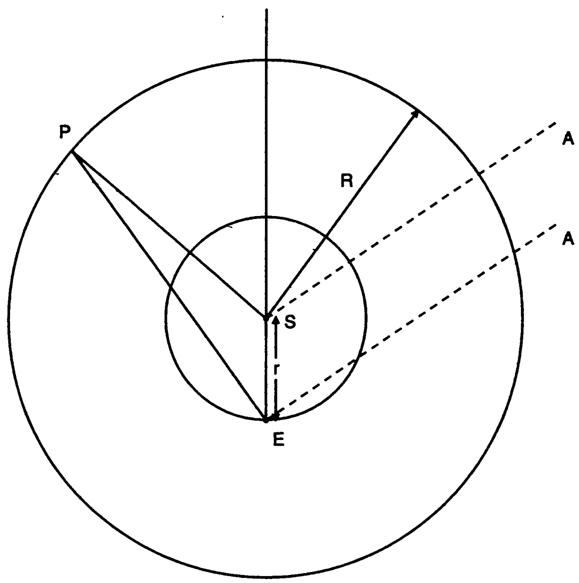
2.2 *Śīghra-samskara*

The *Śīghra-samskara* is carried out to the *mandasphutagraha* to obtain the *sphutagraha*. In modern heliocentric theory the *śīghra-samskara*, is applied to obtain the geocentric longitude from the true heliocentric longitude. The *śīghra-samskara* for the exterior and the interior planets are discussed below separately.

A : Exterior planets

In the case of Mars, Jupiter and Saturn, referred as exterior planets, the mean heliocentric sidereal period is identical with the mean geocentric sidereal period. Thus, the mean longitude calculated prior to the *manda-samskara* is the same as the mean heliocentric longitude of the planet. As the *manda-samskara* is applied to this longitude to obtain the *manda-sphutagraha,* the latter will be the true heliocentric longitude of the planet.

The *śīghra-samskara* for the exterior planets can be explained with reference to Fig.2b. E is the Earth and K is the *mandasphutagraha* at a distance R from it. S is the mean Sun referred to as the *śīghrocca* for an exterior planet. Draw KP = r' parallel to ES. Then P corresponds to the true planet.



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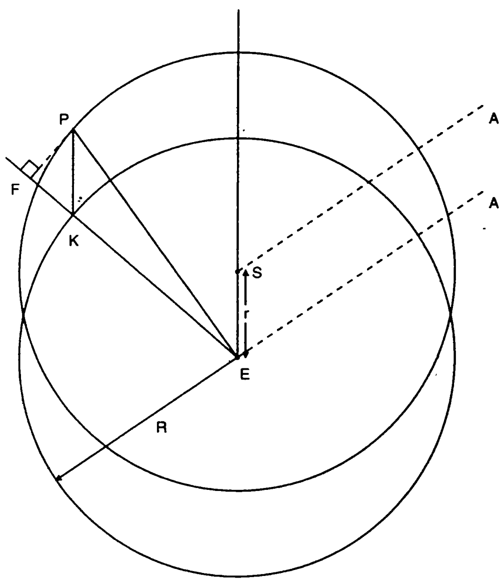


(b)

Figure 2. *Śīghra-samskara* for an exterior planet. a) Heliocentric model b) Eccentric model.

we have,

$\angle AEK = \Theta_{ms}$ = *Mandasphuta* (known after *mandasamskara*)

$\angle AES = \Theta_s$ = Longitude of *śīghrocca* (mean Sun)

$\angle AEP = \Theta$ = Geocentric longitude of the planet

$\angle KEP = \Theta - \Theta_{ms}$ = *Śīghra* correction

The difference between the longitudes of the *śīghrocca* and the *mandasphuta*, namely,

$$\sigma = \Theta_s - \Theta_{ms}$$

is called the *śīghrakendra* (anomaly of conjunction) in Indian astronomy. Draw PF perpendicular to the extension of the line EG. From the triangle EPF we can easily obtain the result

$$\sin(\Theta - \Theta_{ms}) = \frac{r'\sin\sigma}{[(R + r'\cos\sigma)^2 + (r'\sin\sigma)^2]^{1/2}} \qquad (6)$$

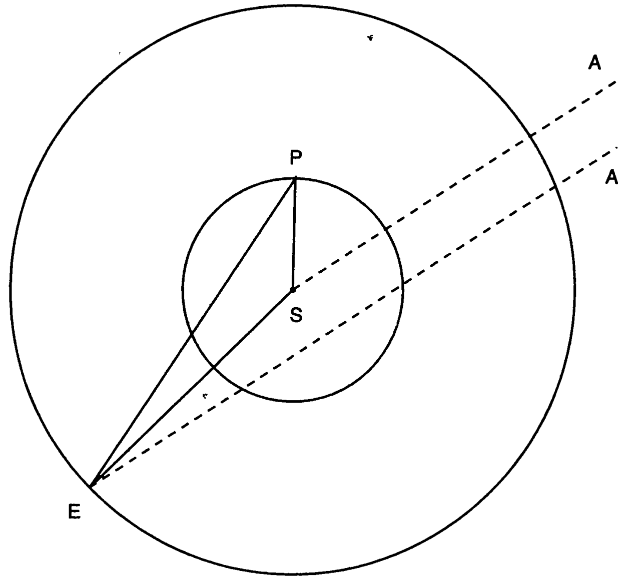
which is the *śīghra* correction formula given by Indian astronomers to calculate the geocentric longitude of an exterior planet.

The above formula can also be obtained from triangle ESP of Fig.2a. This is easily seen, by observing that, this triangle is identical with the triangle EKP of Fig. 2b. Thus from figures 2a and 2b, it is clear that the *śīghra-samskara*, is to obtain the *sphutagraha*, the geocentric longitude from the *mandasphutagraha* the true heliocentric longitude. But equation (6) is still an approximation because *madhyama-arka* the mean Sun is used in the calculation instead of *sphuta-arka* the true Sun.

B: Interior planets

In the case of exterior planets, since the mean geocentric sidereal period is equal to the mean heliocentric sidereal period, the *mandasphutagraha* calculated corresponds to the true heliocentric longitude. However, for the interior planets Mercury and Venus, the mean geocentric sidereal period corresponds to that of the mean Sun and the Indian astronomers at least from the time of Aryabhata (499 AD) till Nilakantha (1500 AD), took the mean Sun as the *madhyamagraha* or the mean planet and applied *manda-samskara* to it. Since the ancient Indian astronomers adopted this procedure, it is not to be understood that they were not aware of the mean heliocentric periods of Mercury and Venus. Their heliocentric periods were known, but were referred as the period of revolution of an associated *śīghrocca*.

The *śīghra-samskara* for the interior planets can be explained with reference to Fig.3b. Here E is the Earth and S is the *mandasphutagraha*. Draw SP = r'' parallel to EK where K is the *śīghrocca*. Then P corresponds to the true planet.



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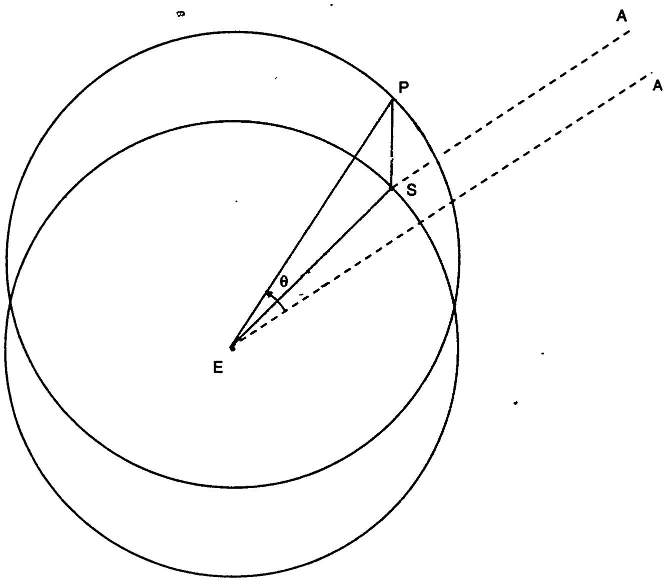

(b)

Figure 3b. *Śīghra-samskara* for an interior planet. a) Heliocentric model b) Eccentric model.

We have,

$\angle$AES = Θ_{ms} = *Mandasphuta-graha*

$\angle$AEK = Θ_s = Longitude of *Śīghrocca*

$\angle$AEP = Θ = True geocentric longitude of the planet

$\angle$SEP = $\Theta - \Theta_{ms}$ = *Śīghra* correction.

Again, the *śīghrakendra* is defined as the difference between the *śīghrocca* and the *mandasphutagraha*. Thus,

$$\sigma = \Theta_s - \Theta_{ms}. \tag{7}$$

Let PF be perpendicular to the extension of the line ES. From the triangle EPF we get the same formula :

$$\sin(\Theta - \Theta_{ms}) = \frac{r'\sin\sigma}{[(R + r''\cos\sigma)^2 + (r''\sin\sigma)^2]^{1/2}} \tag{8}$$

which is the *śīghra* correction given in the earlier Indian texts to calculate the geocentric longitude of an interior planet.

The above formula can also be obtained from triangle ESP of Fig.3a. For interior planets, the identity of Figs 3a, b are more obvious than the exterior planets.

2.3 *Significance of śīghra-samskara*

The *śīghra-samskara* as explained earlier, is to arrive at a transformation from the heliocentric to the geocentric frame. This procedure would have been successful in achieving the objective only if, the value specified for r"/R for each planet were very nearly equal to the ratio of the Planet-Sun and Earth-Sun distances for the interior planet and vice versa for the exterior planet. In Table.1 we give Aryabhata's values for both the exterior and interior planets along with the currently accepted values for the mean Earth-Sun and Planet-Sun distances. Different astronomers have choosen slightly different values for r"/R, which they found to be yielding better results. In fact, the Indian astronomers do mention in their texts that the parameters used for the calculations have to be modified appropriately, if the observation and the calculation are not in concordance, see K. V. Sarma (1977a).

Though the Indian astronomers have used a picture in which the Sun moves around the Earth, as far as the calculation of the geocentric longitudes is concerned it makes little difference whether one assumes the Earth to be moving around the Sun or vice versa. This could be easily conceived with the help of Figs 2 and 3.

Table 1. Comparison of r"/R (variable) in *Āryabhaṭīya* with modern values (ratio of the mean values of Earth-Sun and Planet-Sun distances for exterior planets and the inverse ratio for interior planets).

Planet	*Āryabhaṭīya*	Modern value
Mercury	0.361 to 0.387	0.387
Venus	0.712 to 0.737	0.723
Mars	0.637 to 0.662	0.656
Jupiter	0.187 to 0.200	0.192
Saturn	0.114 to 0.162	0.105

Since the *manda* correction or equation of centre for an interior planet was applied to the longitude of the mean Sun instead of the mean heliocentric longitude of the planet, the accuracy of the computed longitudes of the interior planets as per the older Indian planetary models would not have been as good as that achieved for the exterior planets. Even with this limitation, the earlier texts would have given fairly accurate results, since the magnitude of the equation of centre correction itself is fairly small.

3. Computation of the planetary latitudes

Planetary latitudes (called *vikshepa* in Indian astronomy), play an important role in the prediction of planetary conjunctions, occultation of stars by planets etc. In Fig. 4, P denotes the planet moving in an orbit inclined at angle 'i' to the ecliptic, intersecting the ecliptic at the point N, the node (called *pata* in Indian astronomy). If β is the latitude of the planet, Θ_h its heliocentric longitude, and Θ_n the heliocentric longitude of the node, then for small 'i' we have

$$\sin\beta = \sin i \sin(\Theta_h - \Theta_n) \approx i \sin(\Theta_h - \Theta_n). \tag{9}$$

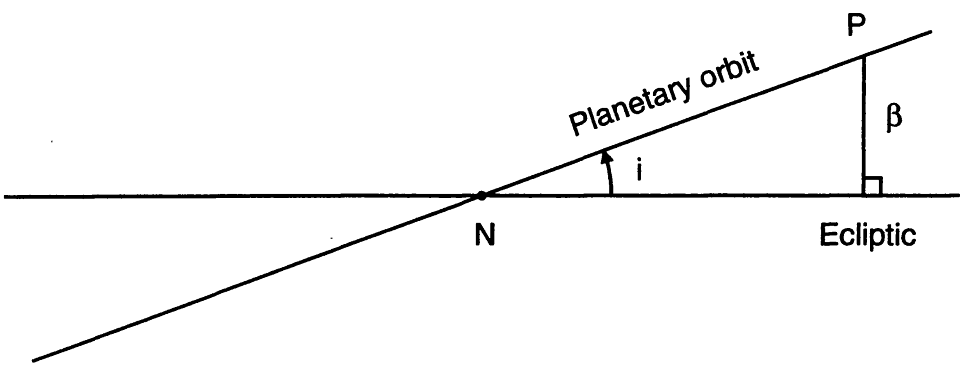

Figure 4. Latitude of a planet.

This is essentially the rule for calculating the latitude, as given in Indian texts, at least from the time of Aryabhata. For the exterior planets, it was stipulated that

$$\Theta_h = \Theta_{ms} \tag{10}$$

the *mandasphutagraha*, which as we saw earlier, coincides with the heliocentric longitude of the exterior planet. The same rule applied for interior planets would not have worked, because according to the earlier Indian planetary model, the *mandasphutagraha* for the interior planet has nothing to do with its true heliocentric longitude.

However, all the older Indian texts on astronomy stipulated that, for interior planets, the latitude is to be calculated from equation (9) with

$$\Theta_h = \Theta_s + \textit{manda}\text{ correction,} \tag{11}$$

the *manda*-corrected longitude of the *śīghrocca*. Since the longitude of the *śīghrocca* for an interior planet, as we explained above, is equal to the mean heliocentric longitude of the planet, equation (11) leads to the correct identification, that even for an interior planet, Θ_h in equation (9) has to be the true heliocentric longitude.

Thus, we see that, the earlier Indian astronomical texts did provide a fairly accurate theory for the planetary latitudes. But they had to live with two entirely different rules for calculating latitudes, one for the exterior planets (equation [10]), and an entirely different one for the interior planets.

Though the peculiarity was noted by the earlier astronomers (see Shukla, 1976) they were adopting this procedure because it led to *drigganitaikya*, or predictions which are in conformity with observations, see Chaturvedi (1981).

4. Nilakantha's revision of the planetary model

Nilakantha Somayaji observing the peculiarity in having two entirely different rules for the calculation of planetary latitudes, in his *Aryabhateeya-bhashya* (see S. K. Pillai, 1957 p.8) states :

शीघ्रवशाच्च विक्षेप : उक्त : । कथमेतद्युज्यते ?
ननु स्वबिंबस्य विक्षेप : स्वभ्रमणवशादेव भवितुमर्हति ।
न पुनः अन्य भ्रमणवशात् । सत्यम् । न पुनः अन्य भ्रमणवशात् अन्यस्य विक्षेप : उपपद्यते ।
तस्मात् बुध : अष्ताशीत्यैव दिनै : स्वभ्रमणवृत्तं पूरयति ।

'The variation in latitude has been stated due to *śīghrocca*. How is it possible?'

'The variation in the latitude of a planet should be attributed to its own revolution. True. The latitude of an object cannot be due to the revolution of some other (object). Therefore, Mercury completes (its revolution in) its own orbit, in 88 days'.

In the above, Nilakantha is explaining that the latitude of an interior planet like Mercury can arise only from the deflection of the planet (from the ecliptic) and not from that of a *śīghrocca* which is different from the planet. Since the period in which the latitude of Mercury completes one cycle of northwards and southwards movement from the ecliptic in 88 days, the mean revolution period of Mercury in its own orbit should also be 88 days (this was construed to be the period of revolution of *śīghrocca* earlier). Thus he concludes that, what was thought of as being the *śīghrocca* of an interior planet should be identified with the mean planet itself and the *manda* correction is to be applied to this mean planet, and not to the mean Sun.

Nilakantha has presented his improved planetary model for the interior planets in his treatise *Tantrasangraha*, which according to Nilakantha's pupil Sankara Varier, was composed in 1500 AD (see S. K. Pillai, 1958 p.2). We shall describe here, the main features of Nilakantha's model in so far as they differ from the earlier Indian planetary model for the interior planets.

In the first chapter of *Tantrasangraha*, while presenting the mean sidereal periods of planets, Nilakantha gives the usual values of 87.966 days and 224.702 days (which are traditionally ascribed to the *śīghroccas* of Mercury and Venus), but asserts that these are *svaparyayas*' i.e., the mean revolution periods of the planets themselves (see S. K. Pillai, 1958, p.8). As these are the mean heliocentric periods of these planets, the *madhyamagraha* as calculated in Nilakantha's model will be equal to the mean heliocentric longitude of the planet, for the case of interior planets also.

In the second chapter of *Tantrasangraha*, Nilakantha discusses the *manda* correction or the equation of centre and states (see S. K. Pillai, 1958, pp. 44-46) that this should be applied to the *madhyamagraha* as described above to obtain the *mandasphutagraha.* Thus, in Nilakantha's model, the *mandasphutagraha* will be equal to the true heliocentric longitude for both the interior and exterior planets.

Subsequently, the *sphutagraha* or the geocentric longitude is to be obtained by applying the *śīghra* correction. While Nilakantha's formulation of the *śīghra* correction is the same as in the earlier planetary theory for the exterior planets, his formulation of the *śīghra* correction for the interior planets is different and is explained below.

According to Nilakantha the mean Sun should be taken as the *śīghrocca* for interior planets also, just as in the case of exterior planets. In Fig.5, P is the *manda*-corrected planet, E is the Earth and S the *śīghrocca* or the mean Sun.

We have,

$\angle AES$	=	Θ_s	=	*Śīghrocca* (mean Sun)
$\angle ASP$	=	Θ_{ms}	=	*Mandasphuta*
$\angle AEP$	=	Θ	=	True geocentric longitude of the planet
$\angle SEP$	=	$\Theta - \Theta_s$	=	*śīghra* correction

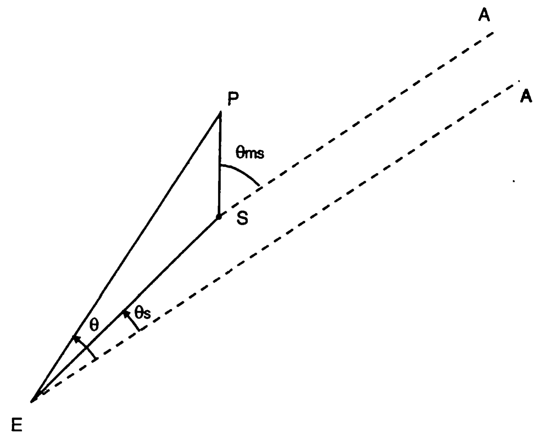

Figure 5. True longitude of an interior planet according to Nilakantha

The *śīghrakendra* σ is defined in the usual way by :

$$\sigma = \Theta - \Theta_{ms} \tag{12}$$

as the difference between the *śīghrocca* and the *mandasphutagraha*. Then from triangle ESP, we get the relation :

$$\sin(\Theta - \Theta_S) = \frac{-r''\sin\sigma}{[(R + r''\cos\sigma)^2 + (r''\sin\sigma)^2]^{1/2}} \tag{13}$$

which is the *śīghra* correction given by Nilakantha for calculating the geocentric longitude Θ of the planet. Comparing equations (12) and (13) with equations (7) and (8), and Fig.5 with Fig.3, we notice that they are the same, except for the interchange of the *śīghrocca* and the *mandasphutagraha*. The *manda* correction or the equation of centre is now associated with P whereas it was associated with S earlier.

Nilakantha, by 1500 A.D., had thus arrived at a consistent formulation of the equation of centre and a reasonable planetary model which is applicable also to the interior planets, perhaps for the first time in the history of astronomy. By this modification, he has also solved the longstanding problem in Indian astronomy, of there being two different rules for the calculation of latitudes.

Just as was the case with the earlier Indian planetary model, the ancient Greek planetary model of Ptolemy and the planetary models developed in the Islamic tradition during the 8th-15th centuries postulated that the equation of center for an interior planet should be applied to the mean Sun rather than to the mean heliocentric longitude of the planet, as we understand today (Dreyer, 1953, Swerdlow and Neugebauer, 1984). In fact, Ptolemy seems to have compounded the confusion by clubbing together Venus along with the exterior planets and singling out Mercury as following a slightly deviant geometrical model of motion, see Hutchins (1952).

Even the celebrated Copernican revolution brought about no improvement in the planetary theory for the interior planets. As is widely known now, (Dreyer, 1953) the Copernican model was only a reformulation of the Ptolemaic model (with some modifications borrowed from the Maragha School of Astronomy of Nasir ud-Din at-Tusi (1201-74 AD), Ibn ash-shater (1304-75) and others) for a heliocentric frame of reference, without altering his computational scheme in any substantial way for the interior planets. The same holds true for the geocentric reformulation of the Copernican system due to Tycho Brahe. Indeed, it appears that the correct rule for applying the equation of centre for an interior planet, to the mean heliocentric planet (as opposed to the mean Sun) was first enunciated in European astronomical tradition only by Kepler in the early 17th century.

5. Geometrical model of planetary motion

The Indian astronomers were mainly interested in the successful computations of the geocentric longitudes, rising and setting times of the planets, *lagnas*, eclipses, etc., which had direct relevance to the day to day practices of the common people. They were not seriously preoccupied with proposing models of the Universe. However they did discuss the geometrical model implied by their computations sometimes, perhaps as an aid to elucidate them.

5.1 Geometrical model corresponding to the conventional scheme

The renowned Kerala astronomer Parameswara, who is often referred as *Paramaguru* by Nilakantha, has discussed the geometrical model of motion, as implied by the conventional procedure used for the calculation of longitudes, in his super commentary *Siddhantadeepika* (Kuppanna Sastry, 1957). He observes :

स्वाश्रयवृत्तात् बाहये यस्माद्भूमि : ज्ञशुक्रयो : ।
तस्मात् भूम्याश्रितवृत्तगतं स्वोच्चं मध्यं स्वमध्यं स्वोच्चम् ॥

'As the earth is (always) outside (and not encircled by) the orbit in which they (Mercury and Venus) move, hence their (so called) *śīghrocca* is the mean planet which moves on a circle which is dependent on the earth (*manda* eccentric circle) and the mean (heliocentric) planet is (what is hither to known as) *śīghrocca.*

Later, Parameswara notes :-

शीघ्रोच्चं सर्वेषां भवति रवि : ।

'Sun is the *śīghrocca* for all the planets.'

In the above, Parameswara is essentially stating that what has been called as the *śīghrocca* of an interior planet in conventional planetary model should be identified as the planet itself and the mean Sun should be taken as the *śīghrocca* for all the planets, while computing the *śīghra* correction.

Thus many of the basic ideas which were used by Nilakantha in formulating his new model were already present in the work of Parameswara. The geometrical model described by Parameswara, to elucidate the procedure adopted in the conventional scheme of calculation of longitudes, is shown in Figs 6a and b.

In Fig.6a, the circles with S and E as center are the same as those shown in Fig.2b. In drawing these circles Parameswara prescribes that the distance of separation between the points S and E should be equal to the radius of *śīghra* epicycle and that the point S should be choosen along the direction of *śīghrocca* as viewed from E. Similarly he states that the distance of separation between S and O should be equal to the radius of *manda* epicycle and that as viewed from S, the point O should be along the direction of *mandocca*. Thus in Parameswara's model the pair of circles with S and O as centers, are the deferent and eccentric circles of the planet, used in *manda-samskara*. The circle with E as center together with the one with S as centre are used to explain the *śīghra-samskara* , which is essentially the transformation from the heliocentric frame to the geocentric frame.

In Fig. 6b, the circles with E and O as centre are similar to those shown in Fig.1a. Since astronomers prior to Nilakantha used to apply *manda* correction to the Sun, for interior planets, S represents the manda corrected Sun. For interior planets, Parameswara states that, with S as centre and *śīghra* epicycle as radius, a circle has to be drawn. This circle will be the orbit of the planet (*svasrayavrtta*). Thus we find Parameswara himself describing a model, in which the planets Mercury and Venus move around the Sun.

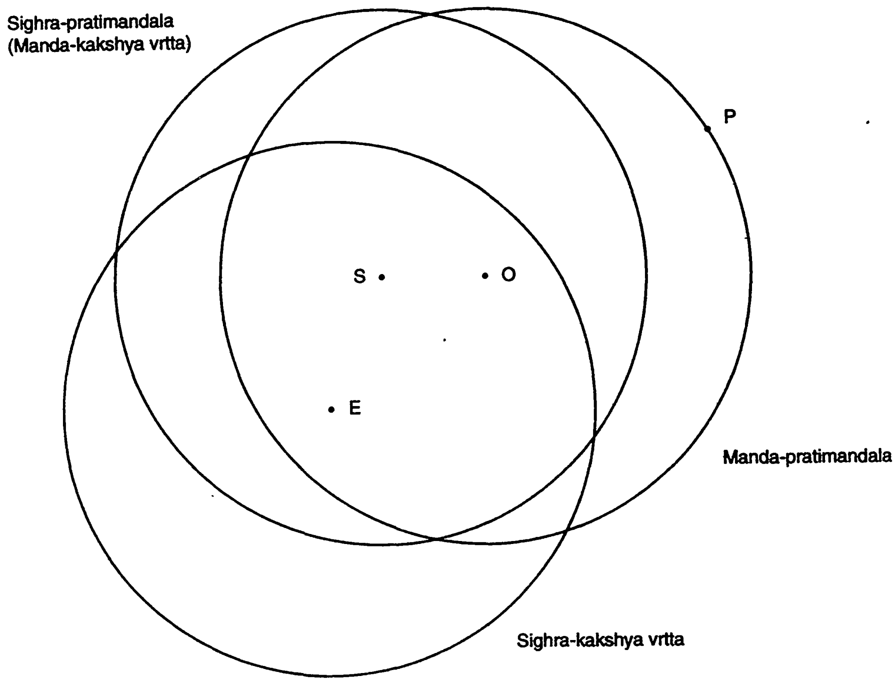

Fig. 6(a)

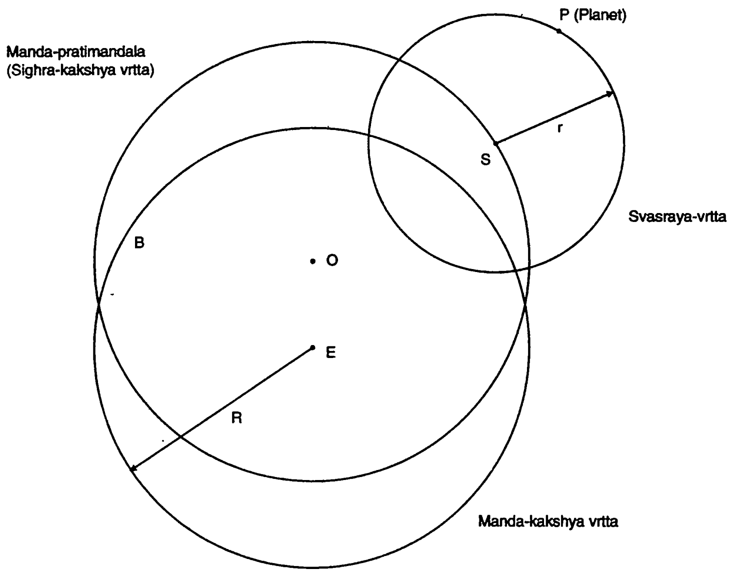

Figure 6. Geometrical model of planetary motion according to *Bhatadeepika* of Parameswara. a) Exterior planets b) Interior planets.

5.2 *Geometrical model of Nilakantha*

Nilakantha describes the geometrical picture associated with his model of planetary motion in his works *Golasara, Siddhantadarpana* (with his own commentary), and in much greater detail in his *Āryabhaṭīya-bhāṣhya*. There is also a tract of his, on planetary latitudes, *Grahasphutanayane Vikshepavaasana* (K.V.Sarma, 1979) which deals with this topic.

In his *Āryabhaṭīya-bhāṣhya*, while discussing the planetary latitudes, Nilakantha points out that the exterior planets are deflected north or south of the ecliptic and that this deflection is to be calculated using the *mandasphuta-graha* in the same way as for the Moon. But this value of the latitude differs from the view point of an observer at the centre of the *Bhagola* (centre of the Earth). He asks why this difference arises and answers thus (S.K. Pillai, 1957, p.5)

भगोलमध्यनाभिकस्य कात्स्न्र्येन अपमण्डलमार्गगस्य शीघ्रवृत्तस्य परिधौ य :
शघ्रोच्चसम : प्रदेश : तद्धि मन्दकर्मणि कक्ष्यामण्डलकेन्द्रमिति कालक्रियापादे एवं उक्तम् ।

'The place where the *śīghrocca* lies on the circumference of the *sighravrtta* whose centre is the centre of the celestial sphere (Earth) and which lies totally in the plane of the ecliptic was stated as the centre of the *kakshya-mandala* (manda deferent circle) in Kalakriyapada in *Mandakarma* (while describing *manda* process)'.

Here he is stating that the centre of the deferent circle of the orbit of an exterior planet is the *śīghrocca* which is the Sun. That is why the computed value of latitude differs from what is observed from the Earth.

For the case of interior planets, the following is a graphic description of their motion given by Nilakantha in his *Āryabhaṭīya-bhāṣya* (S.K. Pillai, 1957, p.9)

तयो : भ्रमणवृत्तेन न भू : कबलीक्रियते। ततो बहिरेव सदा भू : ।
भगोलैकपार्क्ष्वे एव तद्वृत्तस्य परिसमाप्तत्वात् तद्भगणेन न द्वादशराशिषु चार : स्यात्।
तयोरपि वस्तुतः आदित्यमध्यम एव शीघ्रोच्चम्। शीघ्रोच्चभगणत्वेन पठिता एव स्वभगणा : ।
तथापि आदित्य भ्रमणवशादेव द्वादशराशिवु चार : स्यात्।

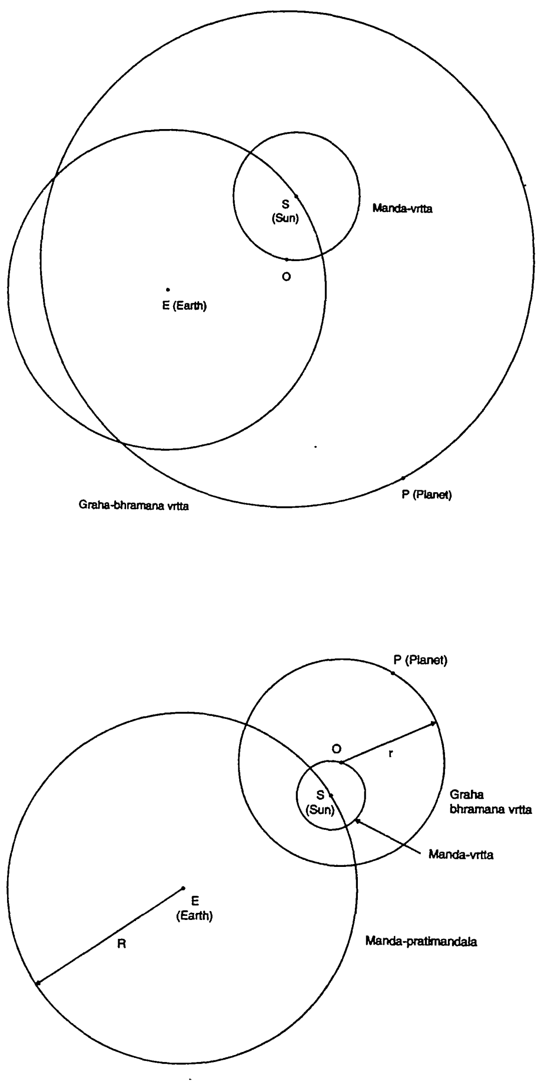

Figure 7. Geometrical model of planetary motion according to *Siddhantadarpana* of Nilakantha. a) Exterior planets b) Interior planets.

'The earth is not circumscribed by their (i.e. the interior planets, Mercury and Venus) orbits. The Earth is always outside their orbit. Since their orbit is always confined to one side of the geocentric celestial sphere, in completing one revolution they do not go around the twelve signs (*raśīs*). For them also really the mean Sun is the *śīghrocca.* It is only their own revolutions which are stated to be the revolutions of the *śīghrocca* (in ancient texts such as the *Āryabhaṭīya*). It is only due to the revolution of the Sun (around the Earth) that they (i.e., the interior planets, Mercury and Venus) complete their movement around the twelve *raśīs* (and complete their revolution of the Earth).

The geometrical picture described by Nilakantha in his *siddhanta-darpana* is shown in Figs. 7a,b. There are several other graphic descriptions of this geometrical picture in other works of Nilakantha (see K. V. Sarma, 1976 and K. Ramasubramanian et al. 1994). In Nilakantha's planetary model, Mercury, Venus, Mars, Jupiter and Saturn, are assumed to move in eccentric orbits around the *śīghrocca*, which is the mean Sun going around the Earth. The planetary orbits are tilted with respect to the orbit of the Sun or the ecliptic and hence cause the motion in latitude.

Nilakantha's modification of the conventional planetary model of Indian astronomy seems to have been adopted by most of the later astronomers of the Kerala school. This is not only true of Nilakantha's pupils and contemporaries such as Sankara Varier (1500 - 1560), Chitrabhanu (1530), Jyeshtadeva (1500), who is the author of the celebrated *Yuktibhasha*, but also of later astronomers such as Achyuta Pisarati (1550-1621), Putumana Somayaji (1660-1740) and others. They not only adopt Nilakantha's planetary model, but also seem to discuss further improvements. In the next section, we discuss the effect of latitude on the calculation of longitudes as explained in *Yuktibhasha.*

6. The effect of latitudinal deflection on the longitudes

Yuktibhasha (K. V. Sarma, to be published) by Jyeshtadeva is a milestone in the history of Indian mathematics and astronomy for several reasons. Its original version is in Malayalam, and it is one of the important scientific works in that language. There is also a sanskrit version of the text which we have made use of here. *Yuktibhasha* is also one of the special works solely devoted to proofs and demonstrations. In the second part of *Yuktibhasha* which is devoted to astronomy, Jyeshtadeva explains the planetary model developed by Nilakantha in detail using geometrical constructions.

There is a separate section on the *Yuktibhasha* on the effect of the inclination of a planet's orbit on its longitude, to which there is a brief allusion by Nilakantha in his *Āryabhaṭīya-bhāṣya.* Jyeshtadeva having described the procedure to calculate the latitude at any given instant, proceeds to describe how to find the true longitude of the planet when it has latitudinal deflection. He states :

अनन्तरं मन्दकर्णवृत्तव्यासार्धस्य वर्गे विक्षेपवर्गं त्यक्त्वा मूलित्वा विक्षेपकोटिं उत्पादयेत् ।

Now calculate the *vikshepakoti* (cosine *vikshepa*) by subtracting the square of the *vikshepa* from the square of the *mandakarnavyasardha* and calculating the root (of the difference).

In Fig.8, N refers to the ascending node. P is the planet on the *mandakarna-vrtta*, which is inclined to the ecliptic. From the longitude of the node and that of the planet known after *śīghra-samskara* , NP is calculated. With NP and 'i' (the inclination of the planetary orbit) β, the latitude (*vikshepa* of the planet is determined. From *vikshepa*, the *vikshepa-koti* is obtained. We have,

$$\textit{vikshepa-koti} = OM = (OP^2 - PM^2)^{1/2} \approx (OP^2 - \beta^2)^{1/2} \quad (14)$$

Having obtained the *vikshepa-koti*, Jyeshtadeva states :

पूर्वोक्तविक्षेपकोटिं व्यासार्धमिति च, अस्य मन्दकर्णमिति च कल्पयित्वा पूर्ववत् शीघ्रस्फुटं कुर्यात् ।

'Taking this *vikshepakoti* and assuming it to be the *mandakarna*, the true longtidue (*śīghrasphuta*) has to be calculated as before'.

Here Jyeshtadeva essentially states that the true longitude has to be measured along the planet of ecliptic, by assuming OM to be the radius of *mandakarna-vrtta*. From the above, it is obvious that the Indian astronomers have clearly understood that the longitudes obtained after *śīghra-samskara*, have to be corrected when the planets have latitudinal deflection. It is to be noted that the understanding of latitudinal effects in planetary longtitudes came only by the end of 16th (?) century in the European tradition of astronomy.

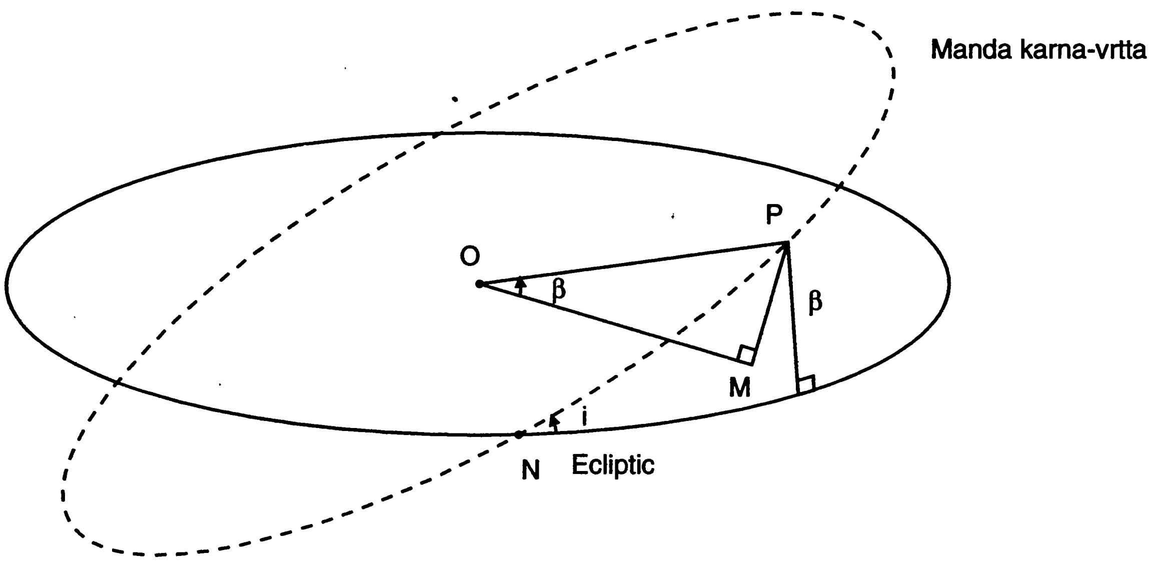

Figure 8. Longitude of the planet when there is latitudinal deflection.

Achyuta Pisarati in his *Sphutanirnaya-tantra* and *Rasigolasphutaneeti* (see K. V. Sarma, 1974-1977) also discusses in detail, the correction to planetary longitudes due to latitudinal effects by the method of reduction to the ecliptic.

There are other mathematician-astronomers of Kerala such as Sankara Varier (1500-1560), Chitrabhanu (1530), and Putumana Somayaji (1660-1740) who seem to have adopted Nilakantha modification of the conventional planetary model of Indian astronomy and improved upon it.

7. Summary and conclusion

1. a) Nilakantha was perhaps the first scientist in the history of astronomy to have arrived at a fairly correct understanding of the application of equation of centre for the inner planets.

b) As far as the Indian tradition of astronomy is concerned, he also gave a unified formulation of (i) *Śīghra-samskara* ; (ii) *Vikshepa* - Planetary latitudes.

2. Nilakantha's overall picture of planetary motion would be as follows :
The five planets go in eccentric orbits around the mean Sun and the Sun goes around the Earth.

3. In the following, we compare (briefly) the planetary models in Indian astronomy and elsewhere.

a) Ptolemy treated Venus along with outer planets singling out Mercury. Thus, a clear separation of inner and outer planets found in Indian planetary theory was absent in Greek tradition and others who followed them.

b) Ptolemy's model suffered from the same inaccuracy as the ancient Indian planetary models as regards the improper application of equation of centre for the inner planets.

c) While the ancient Indians had a fairly accurate formulation for the latitudes of the inner planets, Ptolemy's model was completely off the mark. It could not even give the latitudinal periods accurately.

d) The modification made by later Greek and Islamic astronomers, did not correct the inaccuracies of Ptolemy's theory as regards the computation of longitudes and latitudes of inner planets (including Copernicus and Tycho Brahe - though they had a different overall picture).

4. Thus, the modification made by Nilakantha around 1500 AD was the first fairly accurate formulation of the motion of inner planets. His work predates by over 100 years, the work of Kepler who seems to be the first astronomer in the Greko- European tradition to have departed from the Ptolemaic scheme for the interior planets, while formulating his theory of Planetary motion.

In conclusion, it may be noted that there is a vast literature on astronomy (including mathematics) both in Sanskrit and Malayalam, produced by the Kerala school, during the period 14th-19th century. Only a small fraction of it has been published and so far only a few studies of these texts have appeared. What seems to emerge clearly from the source-works already published is that, by the later part of the 15th century, if not earlier, Kerala astronomers had arrived at many of the discoveries in mathematical analysis and astronomy which are generally hailed as the important achievements of the scientific renaissance in Europe during the 16th and 17th centuries. Only more detailed investigations can lead to a correct appreciation and assessment of the work of the Kerala astronomers during the 14-16th centuries and their consequent developments.

Acknowledgements

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