# THE UNIVERSE OF PTOLEMY REVISITED

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## ABSTRACT

In this work we will present an alternative method to teach the two first Kepler's laws, using basic concepts of geometry, but achieving conclusions in the important notions of reference systems and the equivalence between heliocentrism (Copernican-Keplerian system) and geocentrism (Ptolemaic system). We used this method for the students of junior (last year), high school and for graduation courses (Physics and Mathematics).

## 1. INTRODUCTION

The historical development of Astronomy, specially in the period between the transition from Ptolemaic to Copernican conception of the universe, found in the person of Johannes Kepler its maximum exponent. In the high school's teaching and, also, in the first years of graduation courses, the description of planetary motions is reduced using simple memorization of the Kepler's laws, without a true phenomenon comprehension. The mathematical relations involved in this kind of teaching are not so simple to encourage the teachers to treat this theme with a different form from that presented in common (and, frequently, very complicated way) astronomical textbooks.

It is a great disappointment for students use the memorization of laws without a real and effective comprehension of the studied phenomenon. Some authors (Brehme, 1976; Neves & Arguello, 1986) worked with Kepler's laws using alternative methods, based in science history or in geometrical arguments. The present work will treat this important and forgotten chapter of the Physics using geometry as tool to try to find out an alternative way to teach Mathematics (geometry), Astronomy and History of Science.

Our method consist in using two first of the Kepler's laws in a close form: replacing the ellipsis by non-centered circumferences, and using, also, the very old equant's point instead the area's velocity to calculate the planetary trajectories as seeing in the Earth's sky. Results were confronted with that presented in current bibliography and, at the end, it was made a change of the reference systems (heliocentrism to geocentrism). This method was tested with junior (last year), high schools and college students.

## II. NON-CENTERED CIRCUMFERENCE'S METHOD

### II.1. KEPLER'S FIRST LAW IN A SIMPLIFIED FORM

Kepler's first law (or orbit's law) states that motion of the planets are ellipsis with sun posed in the focus of these ellipsis.

We know that the planetary eccentricities are very small. Nevertheless, we can replace the elliptical shape of a planetary motion for a circular shape. But, this circumference will not be centered, e.g., with the sun occupying the center of this new circular orbit. The sun must be shifted from the center $O$ until the distance $s = R \cdot \varepsilon$, where $R$ is the circumference radius (figure 1).

The point occupied by the sun is called *solar focus F*, and the other point, simmetrically located around the center $O$, *angular focus F'*.

The equivalence between non-centered circumferences and ellipsis with very low eccentricities can be shown using the ellipse's equation in its polar form:

$$r = R / (1 - \varepsilon \cos \Theta) \qquad (1)$$

The equation (1), using a very low eccentricity ($\varepsilon^2 \ll 1$), becomes:

$$r = R + R \varepsilon \cos \Theta \qquad (2)$$

Observing figure 1 we can write the following mathematical relation:

$$x = s \cos \Theta \qquad (3)$$

And, finally,

$$r = R + s \cos \Theta \qquad (4)$$
$$r' = R - s \cos \Theta \qquad (5)$$

As we know that $\varepsilon = s / R$, we can finally write (4) in the following form:

$$r = R + R \varepsilon \cos \Theta$$

that correspond to equation (2) valid for an ellipse with low eccentricity and, for our purpose, also valid for a circumference of radius $R$ having two "focuses", shifted both of the center by the distance $s = R \cdot \varepsilon$.

## II.2. KEPLER'S SECOND LAW IN A SIMPLIFIED FORM

Kepler's second law (or area's law) states that areas swept in equal units of time by a planet orbiting around the sun are always the same.

For our purpose, we will enunciate this law in an alternative way, using the concept of the angular velocity respect to angular focus $F'$, or, as we can denominate it, *equant point*.

Using the equation (1) and putting in it $\varepsilon = s / R$, and making use of the relations (4) and (5), we can find:

$$r = R^2 / r'$$

We know that $R$ is constant (corresponding in planetary motion, the mean distance between sun and planet, so, we have that:

$$r \cdot r' = constant \qquad (6)$$

From the figure 2a e 2b we can find also:

$$L_1 \, r_1 = L_2 \, r_2 \approx 2 \cdot Area$$

(where L is the length of the arc traversed by the planet).

Using (6), we can finally arrive to:

$$L_1 / r_1 = L_2 / r_2' \cong \zeta \qquad (7)$$

where $\zeta$ is the angle, respect to equant point (focus $F'$), that remains always constant in equal time intervals.

Nevertheless, considering the angles described by the planets always constant with respect to $F'$, and linking the points on the circumference until the solar focus $F$ (where is placed the sun), is obtained equal areas in equal time intervals (this fact can be proved using the Hero's formula to calculate the area of a triangle of unequal sides $a$, $b$ and $c$ $\rightarrow Area = [p \, (p - a) \, (p - b) \, (p - c)]^{1/2}$, where $p$ is the semiperimeter $= (a+b+c)/2)$.

It is very interesting observe that until now we worked with three different concepts of velocity: angular velocity, constant respect to focus $F'$; area's velocity, constant respect to focus $F$; and the linear velocity, inversely proportional to the distance planet-sun.

## II.3. USING THE NON-CENTERED CIRCUMFERENCES METHOD TO CALCULATE THE ORBITS OF MERCURY AND EARTH

To draw the Mercury orbit (valid also to any other planet) we need know the following orbital elements: longitude $\varpi$, ascendent node $\Omega$, longitude $\lambda$ in the epoch (here, epoch = 1980.0) and the period $T$ of a complete revolution of the planet around the sun.

The final drawing of the orbit is possible transfering orbital elements in spatial representation for a plane representation. Figure 3 shows the final result obtained [NEVES & ARGUELLO, 1986]. The most external circumference corresponds of fixed stars, with an infinite radius when it is compared of the radius of the planet orbits.

## II.4. DRAWING THE APPARENT TRAJECTORY OF THE MERCURY PLANET

To obtain the apparent behaviour (resulted of motion composition: Earth and planet) of the "Mercury motion around the Earth", we must work with a specific coordinate system. In general, the system most used is the equatorial or absolute coordinate system (right ascension $\alpha$ and declination $\delta$). However, for this specific method developed here, it will be used ecliptical coordinates: ecliptical longitude $\lambda$ and ecliptical latitude $\beta$.

Taking the longitude $\lambda$ close to right ascension $\alpha$, we can calculate the ecliptical latitude $\beta$ using the following formula:

$$\beta = i \, (DSP / DEP) \, sen \, (\lambda - \Omega) \qquad (8)$$

where $i$ is the inclination of planetary orbit respect to ecliptic, $DSP$ is the distance sun-planet, and $DEP$ is the distance Earth-planet.

Knowing $\lambda$ and, obviously, $\alpha$, corresponding specific epoch, 1980.0, and using precedent results, it is possible to determine the position of the Earth and the planet (in present section, Mercury) to sucessive posterior times (in this example, 1985). The geocentric apparent trajectory is obtained finding the ecliptical longitude, e.g., reducing to a point the radius of the Earth and the planet orbit, considering the radius of the fixed stars circumference as infinite (MENZEL, 1964).

Using a graph right ascension $\alpha$ versus declination $\delta$, and drawing the ecliptic "line" (that is our reference frame), we can obtain the geocentric apparent motion made by Mercury in the 1985 year. Figure 4 shows this trajectory. Some of the assumptions made here seems to be not so accurate, but the results obtained are the best validity justification of the method developed here, using non-centered circumferences [ANUARIO ASTRONOMICO, 1985]. Using this method we can change the origin of coordinate systems to any other planet. So, in analogous way, we should be suggested as an exercise, what would be the trajectory of the Earth by an hipotetic habitant of Saturn.

## III. CONCLUSION: FROM KEPLER TO PTOLEMY

The present work is restricted until this point to the presentation of an alternative method, using a basic and simple geometry to calculate and make previsions about the apparent trajectories of the planets in the sky. It seems to be a contradiction with the title: *from Kepler to Ptolemy*. However, this contradiction is only apparent, because using the equant's point we are working with a powerful tool chactheristic of the Ptolemaic system (geocentric). The use of *equant's law* as we presented in the previous section is an alternative and analogous way to reach to the Kepler's second law (area's law).

However, observing figure 3, and linking the points where Mercury and Earth were in the year 1985 (or, if we wish, to any other year), as illustrated in figure 5, and registrated the length of segments Earth-planet (DEP), and angles respect to the ecliptic, we can obtain a geocentric or geostatic system as imagined by Ptolemy in his Ancient Greece. Figure 6 illustrate final result after this change of reference system.

Observing the model of the universe as conceived by Ptolemy, in a very simplified way, the retrogradation motions were explained using a great deferent circumference, with the Earth placed at point *E* (figure 7). The retrogradation motions like that presented in figure 6 were obtained using an epicycle (litttle circumference on the deferent), with a constant angular velocity respect to equant point *D*.

This last figure states correspondence between heliocentrism and geocentrism, using two different reference frames. Kuhn (1974) writes that Ptolemy and Copernicus were chosen equally justifiable processes to describe the position of the planets and the Earth.

Using non-centered circumferences method and observing figure 6 we can, intuitively, make predictions about the number of the retrograde motions for each planet in the Earth's sky. In the case of external visible (with naked eye) planets (Mars, Jupiter and Saturn), number of retrogradations corresponds to the number of rotations realized by the Earth during a complete orbital period of that planets around the sun. For example, the orbital period of Jupiter is 12 years. So, the number of the retrogradations is around twelve, for each twelve years orbital periods of the Earth around the sun.

In the case of internal planets (Venus and Mercury), it is valid the same scheme. For example: to Mercury, which rotates around the sun in 88 days, it is predictable, at least, three retrogradations, because this number of days is inside duration of a complete revolution of the Earth around the sun.

Using this method in schools we can introduce geometrical notion (perspective and spatial views in an applied form), History of Science and physical principles of reference frames. The most important thing that we can show using this method is that both reference systems, Ptolemaic and Copernican-Keplerian systems, are equally valid and correct to describe the phenomena of the observable world.
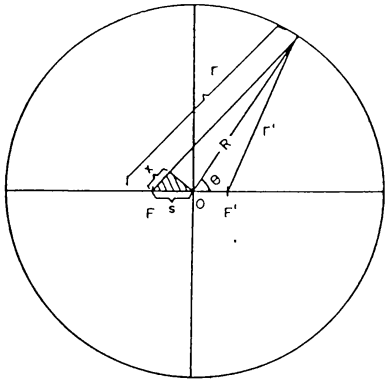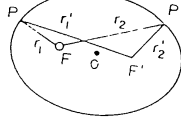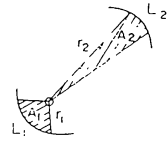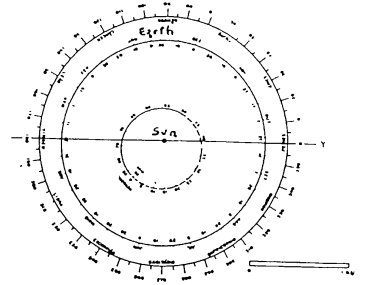
Figure 1

Figure 2

(a)

(b)
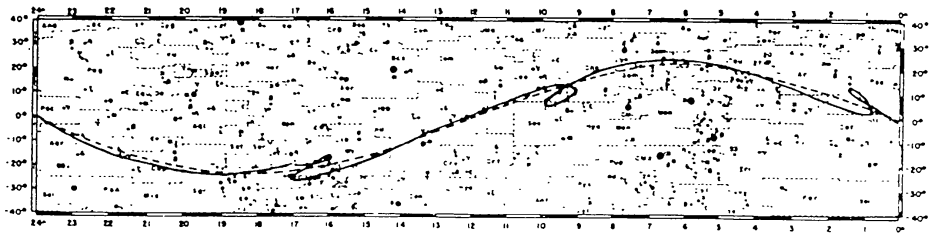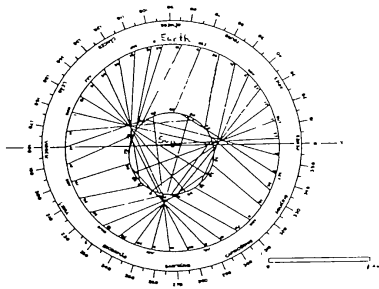
Figure 3

Figure 4

Figure 5
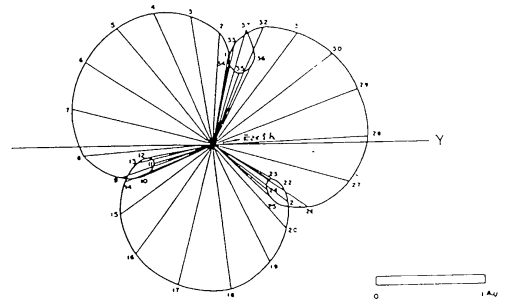
Figure 6

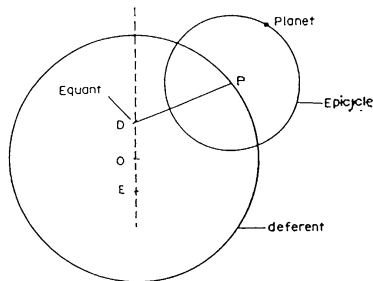Figure 7

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