PERFORMANCE EVALUATION OF SPACECRAFT PROPULSION SYSTEMS IN RELATION TO MISSION IMPULSE REQUIREMENTS

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ABSTRACT

The paper deals with the performance evaluation of propulsion systems with regard to mission impulse and velocity-increment requirements. For the determination of related propulsion system performance capabilities, a reference number called the 'System-specific Impulse' is introduced, which defines the delivered impulse per kilogram of system mass:

$$I_{ssp} = \frac{I_{tot}}{m_{PS}} \left[\frac{Ns}{kg}\right]$$

This reference number takes into account all parameters influencing the impulse-dependent part of the propulsion system mass, such as the mass of the propellant storage and electric power supply systems, and therefore it characterises the performance of propulsion systems better than the 'Thruster-specific Impulse', I_{sp}, alone. Consequently, the 'System-specific Impulse' provides an ideal basis for an objective and comparative evaluation of propulsion systems of different designs, and with different propellants.

Mathematical formulas, derived for 'System-specific Impulses', are presented for systems containing compressed gases, vaporising liquids, monopropellant hydrazine, bipropellant MMH/N_2H_4, and for electric propulsion systems.

With the help of the 'System-specific Impulse', the velocity-increment and total impulse capabilities of today's commonly available spacecraft propulsion system designs are evaluated and compared with previously built and flown systems.

Nomenclature

C	tank filling ratio (V_p/V_T)
F	force [N]
I	impulse [Ns]
K	tank performance factor ($P_{op}\, V_T/m_T$) [m^2/s^2]
m	mass [kg]
M	molecular weight [$kg/kmol$]
N	electrical energy [W]
P	pressure [N/m^2]
R	gas constant 8.314 [$kJ/°K/kmol$]
S	safety factor [P_b/P_{op}]
T	temperature [$°K$]
v	velocity [m/s]
V	volume [m^3]
z	gas compressibility factor
Δv	velocity-increment [m/s]
γ	specific power [W/kg]
η	overall energy conversion efficiency (N_e/N)
ρ	specific mass of propellant [kg/m^3]
τ	thrust time [s]

Subscripts

b	burst (pressure)
e	exhaust (effective)
El	electric (system)
op	operating
opt	optimal
P	propellant
pr	pressurising gas
PS	propulsion system
PSS	propellant storage system
S/C	spacecraft
T	tank
tot	total
sp	specific
ssp	system-specific

1. INTRODUCTION

For the selection of propulsion systems for spacecraft missions of given impulse and velocity-increment requirements, there is a need to determine system performance quantitatively in the form of their impulse capability. Thus, a system reference number is needed, describing delivered impulse per kilogram of system mass which permits an objective and comparative evaluation of systems of different designs and with different propellants.

This paper deals with the evaluation of the impulse capability and resulting mass of common chemical and electric spacecraft propulsion systems. It is assumed that, especially for missions which require high total propulsion impulse (e.g. geostationary and interplanetary missions), the mass of the corresponding auxiliary propulsion system may represent an important fraction of the overall mass of the spacecraft. Attempts to minimise the mass of propulsion systems have therefore to concentrate on parameters, which

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characterise the system's propulsive performance capabilities. Hence, a system reference number has to be defined, describing those design parameters which influence system mass in relation to delivered impulse. With the help of this reference number, the total impulse and velocity-increment capabilities of today's common spacecraft propulsion system designs are evaluated and compared with previously flown systems.

2. DEFINITION OF "SYSTEM-SPECIFIC IMPULSE"

A reference number is introduced, which defines the total impulse, I_{tot}, delivered by the system, divided by the system mass m_{PS}:

$$I_{ssp} = \frac{I_{tot}}{m_{PS}} \left[\frac{Ns}{kg}\right] \tag{1}$$

Because of the resulting dimension, delivered impulse per kilogram of system mass, this number is called 'System-specific Impulse'.

In the case of missions which require a high total propulsive impulse, it can be further assumed, that the bulk of the mass of chemical propulsion systems is due to the combined mass of propellant and its corresponding tankage, which is the propellant-storage system mass, m_{PSS}, while for electric propulsion, the combined mass of the power supply and power processing systems, m_{El}, has to be considered too. Whereas the mass of this equipment is proportional to the propulsion impulse, the propulsion hardware mass, such as piping and engines, is independent of impulse. Consequently, the system performance evaluation will be concentrated on the impulse related mass of equipment, assuming that the impulse independent hardware mass is comparatively small and is therefore proportionately contained in the impulse dependent mass of the system.

I_{ssp} has to be defined for the two kind of systems applied commonly for spacecraft propulsion: chemical propulsion systems, comprising cold gas and hot gas systems:

$$I_{ssp} = \frac{I_{tot}}{m_{PSS}} \tag{2}$$

and electric propulsion systems:

$$I_{ssp} = \frac{I_{tot}}{m_{PSS} + m_{El}} \tag{3}$$

The above noted mathematical formulas have in common the same numerator, $I_{tot} = m_p v_e$, representing the total impulse delivered by the propellant contained in the propellant tank. The denominator, representing the impulse related system mass, varies with the kind and design of the propulsion system. By taking into account the parameters describing the nominator and denominator, mathematical formulas can be derived for the I_{ssp} of the various above noted propulsion systems. A more detailed presentation of derived mathematical formulas for I_{ssp} is given in the Annex to this paper. These formulas, denoted in the Annex by (A), will be used for the performance evaluation of today's most commonly used spacecraft propulsion system designs.

3. "SYSTEM-SPECIFIC IMPULSE" OF COMMON SPACECRAFT PROPULSION SYSTEM DESIGNS

The I_{ssp} will be evaluated for systems operating with cold- and hot gas while the I_{ssp} of electric propulsion systems will be looked at in general. Furthermore, evaluated I_{ssp} data will be compared with those of previously flown propulsion systems.

3.1 Cold Gas

Systems operating with cold gas, comprise compressed (inert) gas, and vaporising liquids (high vapour pressure hydrocarbons).

Especially compressed cold gas systems used for auxiliary propulsion of satellites (attitude and orbit control), although of moderate impulse capability, are still of interest in view of their simplicity, high reliability, repeatability of impulse bit and low system costs.

The 'System-specific Impulse', as derived in (A5) for compressed cold gas systems, is:

$$I_{ssp} = \frac{I_{tot}}{m_p + m_T} = \frac{v_e}{1 + \frac{zRT}{KM}} \tag{4}$$

From Equation (4) it is obvious, that the thruster exhaust velocity v_e as well as the type of propellant and the tank performance factor K influences the I_{ssp}. To achieve high values of I_{ssp}, high values for v_e, M and K are desired, while z should be small.

Values of I_{ssp} calculated for systems operating with different potential compressed gases, as well as I_{ssp} values of previously flown spacecraft propulsion systems are noted in Table 1. In addition, related mission average values of 'Thruster-specific Impulse', I_{sp}, are noted in Table 1 with the understanding of:

$$v_e \left[\frac{m}{s}\right] \equiv I_{sp} \left[\frac{Ns}{kg}\right] \tag{5}$$

PROPELLANT		THRUSTER SPEC.-IMPULSE I_{sp} (mission average) (Ns/kg)	SYSTEM SPEC.-IMPULSE I_{ssp} (Ns/kg) Tank Material Ti 6Al 4V ($K=6.9\cdot10^4$ m^2/s^2)	Kevlar ($K=1.12\cdot10^5$ m^2/s^2)	REMARKS/REFERENCES
Hydrogen	H_2	2668	122	194	calculated
Methane	CH_4	980	338	454	calculated
Ammonia	NH_3	950	700 for C = 0.8		calculated
Propane	C_3H_8	618	486 for C = 0.8		calculated
Nitrogen	N_2	706	291	378	calculated
		706	281	N.A.	COS-B satellite [1]
Argon	A	490	261	319	calculated
		490	218	N.A.	TD-A1 satellite [2]
Freon 14	CF_4	441	348	377	calculated

Tab.1: Comparison of Cold Gas Propulsion Systems Performances

Finally, propellant tank *K*-factors of constructed tank designs are noted in Table 1. In comparison with propellant tanks, which are made commonly from titanium alloy (Ti 6Al 4V), tanks, which are made from fibre-reinforced materials, achieve high values of tank *K*-factors with resulting higher values of I_{ssp}. The presented values of I_{ssp} clearly show, that high values of I_{sp} (high v_e) do not necessarily result in systems with high impulse capabilities, as seen especially for the case of H_2. However, if we look back to Equation (4), we see, that both z and M, the most important characteristics of a given gas, have a decisive influence on I_{ssp}. Therefore, in practise, compressed cold-gas systems utilising N_2 are being commonly used for missions with low impulse requirements.

The vaporising liquid system is characterised by a liquid propellant, e.g. propane, pressurised by its own equilibrium vapour pressure and the expulsion of its vapour through a nozzle. While no great improvement over inert gas thruster exhaust velocity can be obtained, considerable savings in propellant storage mass result from the propellant's high density and low pressure. With regard to the I_{ssp}, as derived in (A8), it is obvious that, as in the case of compressed gas, the type of propellant as well as the tank *K*-factor influences the I_{ssp}.

3.2 Hot Gas

For increasing absolute levels of thrust and impulse requirements for space propulsion, cold-gas systems are inadequate and more energetic propellants such as monopropellant hydrazine (N_2H_4), storable bipropellant (MMH/N_2O_4) and solid propellants are required.

In contrast to cold gas, liquid propellants need to be pressurised in their storage tank to feed the thrusters with propellant. Consequently, for the evaluation of the I_{ssp}, the mass of pressurising gas and, if necessary, an extra tank for the pressurising gas have to be considered too. The I_{ssp} of systems operating with liquid propellants has been derived in the Annex for both, the blow-down pressurisation and the pressure constant modes.

Because of its inherent simplicity, the blow-down mode is the most widely used mode of tank pressurisation for monopropellant hydrazine and therefore will be discussed first for hot gas systems.

The 'System-specific Impulse', as derived in (A11) for systems operating in the blow-down mode, is:

$$I_{ssp} = \frac{I_{tot}}{m_T + m_p + m_{pr}} =$$

$$= \frac{v_e}{1 + \frac{P_{op}}{C\rho K}\left(1 + \frac{K(1-C)M}{zRT}\right)} \qquad (6)$$

Equation (6) shows, that both the type of propellant (represented by v_e, ρ - a high v_e and a high ρ are desirable) and the propellant storage conditions (propellant-storage pressure, tank-filling ratio, type of pressurising gas, tank performance factor) influence the 'System-spec. Impulse'.

In the case of the constant pressure mode, the tank-filling ratio is usually close to 1 (0.95), and the mass of the tank for the pressurising gas has to be added to the tankage mass. The constant pressure mode is adopted for systems operating with bipropellants. With regard to the I_{ssp} , as derived in (A15), it is obvious that, as in the case of the blow-down mode, both the type of propellant and the propellant storage conditions have an effect on the I_{ssp}.

Values of I_{ssp} calculated for systems operating with monopropellant hydrazine and with bipropellants are noted together with assumed typical mission average values of thruster I_{sp} in Table 2.

PROPELLANT	THRUSTER SPEC.-IMPULSE I_{sp} (mission average) (Ns/kg)	TOTAL IMPULSE I_{tot} (Ns)	PROPULS. SYSTEM MASS m_{PS} (kg)	SYSTEM SPEC.-IMPULSE I_{ssp} (Ns/kg)	REMARKS/REFERENCES [3] if not otherwise noted
Monopropellant Hydrazine; N_2H_4	2150	-	-	1860	Calculated for tank with diaphragm, C=0.75, $K=2\cdot10^4$ m^2/s^2
	2170	$2.61\ 10^5$	142	1838	ECS
	2134	$6.4\ 10^5$	375	1707	ERS-1
	2237	$1.34\ 10^6$	740	1811	EURECA
	2110	$6.41\ 10^4$	38	1687	GEOS
	2163	$1.49\ 10^5$	80	1862	GIOTTO
	2178	$6.97\ 10^4$	40	1743	HIPPARCOS
	2168	$2.36\ 10^5$	130	1815	MARECS
	2168	$3.05\ 10^6$	167	1820	TELECOM-1
Bi-Propellant; MMH/MON	2950	-	-	2730	calculated for surface tension tank, C=0.95, $K=2.6\cdot10^4$ m^2/s^2
	2963	$2.22\ 10^6$	849	2615	DFS
	2962	$2.89\ 10^6$	1101	2625	EUROSTAR
	2900	$3.10\ 10^6$	1170	2650	EUTELSAT-2
	2930	$5.05\ 10^6$	1839	2746	OLYMPUS
	2960	$3.05\ 10^6$	1147	2659	TV-SAT/TDF1/TELE-X
	2962	$3.34\ 10^6$	1253	2666	TELECOM-2
Solid Propellant	2880	$1.41\ 10^6$	528	2670	MAGE 2 Apogee Kick Motor [4]

Tab. 2: Comparison of Hot Gas Propulsion Systems Performances

For comparison, values of I_{ssp} of actual spacecraft propulsion systems are also presented in Table 2, showing an overall good agreement with calculated nominal values of I_{ssp}.

3.3 Electric Propulsion

This technology, although still under development, has proven to achieve thruster exhaust velocities v_e an order of magnitude higher than the best performing chemical propulsion systems. Therefore, electric propulsion is essential for further reduction of system (propellant) mass, enabling higher payload mass and coping best with future high energy mission requirements.

Electric propulsion relies on externally provided electric power to create or augment the kinetic energy of the exhaust jet. Therefore, for the evaluation of the 'System-spec. Impulse', the mass of the electric power (supply- and processing) system has to be considered in addition to the propellant storage system, already dealt with for chemical propulsion systems. The 'System-specific Impulse', as derived in (A21) for electric propulsion systems operating with gaseous propellants (e.g. Xenon), is:

$$I_{ssp} = \frac{I_{tot}}{m_P + m_T + m_{El}} = \frac{v_e}{1 + \frac{zRT}{KM} + \frac{v_e^2}{2\eta\gamma\tau}} \qquad (7)$$

From Equation (7) it is obvious, that, as for chemical propulsion, the type of propellant and the tank performance factor influence the I_{ssp}. In addition, the design of the electric power supply system (represented by the specific power γ[*W/kg*] and the overall energy conversion efficiency η (N_e/N)), together, with the thruster operation time τ (s) will affect the I_{ssp}.

The I_{ssp} will be a maximum for the optimal thruster exhaust velocity of:

$$v_{e_{op}} = \sqrt{2\eta\gamma\tau\left(1 + \frac{zRT}{KM}\right)} \qquad (8)$$

which has been derived by observing the first and second derivatives of (7) with respect to v_e.

A precise quantitative determination of the I_{ssp} of electric propulsion systems is more difficult than with chemical propulsion systems. In the case of electrical propulsion, it has to be noted, that the electrical power can be shared (partly and/or timely) with the payload of a spacecraft. Therefore, the value of I_{ssp} is dependent on the kind and operative conditions of a spacecraft mission.

Table 3 lists values of I_{ssp} for electrothermal systems (hydrazine resistojets and arcjets), which have been calculated without taking into account the power supply systems, assuming the systems are operating in the power-sharing mode. In addition, it was assumed that the electrothermal thrusters are operated with monopropellant hydrazine in the pressure constant (limited blow-down) mode. Therefore, (A15) was used for the calculation of the I_{ssp}.

PROPELLANT	THRUSTER SPEC.-IMPULSE I_{sp} (mission average) (Ns/kg)	SYSTEM SPEC.-IMPULSE I_{ssp} (Ns/kg)	REMARKS/REFERENCES
Power Augmented Catalytic Thruster (PACT)	3 000	2 780	Calculated; Tank with surface tension device, P_{op} = 17.5 bar
ARC-JET	5 000	4 540	Calculated; Monopropellant Hydrazine; Tank with surface tension device, P_{op} = 17.5 bar
Stationary Plasma Thruster SPT-100	15 700	6 375	GALS (Russian Telecommunication Satellite); Propellant: Xenon , m_{ps} = 128 kg, m_p = 52 kg, I_{tot} = 816 kNs, [5]

Table 3: Comparison of Electric Propulsion Systems Performances

The 'Stationary Plasma Thruster' system (SPT-100) for the Russian 'GALS' telecommunication satellite, is an example for an electrostatic system. The I_{ssp} of this system has been calculated according to the design parameters given in [5], which are noted in Table 3.

Although, when compared with chemical propulsion, limited data of built electric propulsion systems are available, Table 3 reveals that both the arcjet and the SPT operated systems are clearly superior to chemical propulsion with regard to their high values of I_{sp} and I_{ssp}.

However, further evaluation of the I_{ssp} needs to be performed as soon as more spacecrafts with electric propulsion have been realised. The 'System-spec. Impulse' will be indispensable for an objective comparison of system performances.

4. PROPULSION SYSTEMS PERFORMANCES IN RELATION TO SPACECRAFT MISSION IMPULSE REQUIREMENTS

To assess the suitability of spacecraft propulsion systems with spacecraft mission impulse requirements, the velocity increment, Δv, capability of the various systems will be estimated, based on the investigations performed in Section 3.

In general, the total impulse delivered by a certain amount of propellant is:

$$I_{tot} = v_e m_P = v_e m_{S/C}\left[1 - e^{-\frac{\Delta v}{v_e}}\right] \qquad (9)$$

With (1) and (5) follows the overall propulsion system mass fraction:

$$\frac{m_{PS}}{m_{S/C}} = \frac{I_{sp}}{I_{ssp}}\left[1 - e^{-\frac{\Delta v}{v_e}}\right] \qquad (10)$$

while for small values of $\Delta v/v_e$ (e.g. $\leq$ 0.2) the propulsion system mass fraction is approximately ($\leq$10% different from(10)):

$$\frac{m_{PS}}{m_{S/C}} \cong \frac{\Delta v}{I_{ssp}} \qquad (11)$$

It is now easy to derive from Equation (10) the dependence of the propulsion system mass fraction, $m_{PS}/m_{S/C}$, on mission velocity increment Δv for any given value of I_{sp} (v_e) and I_{ssp}. Curves of $m_{PS}/m_{S/C}$ plotted as a function of Δv for different propulsion system designs with values of I_{sp} and I_{ssp}, which are summarised in Tables 1 to 3, are shown in Figure 1.

An important consideration for the selection of a suitable propulsion system for given mission impulse requirements will be the trade-off between its velocity increment capability and propulsion system mass. In addition, an important requirement of the spacecraft designer will be that the mass of the propulsion system shall not exceed a certain percentage of the overall mass of the spacecraft. Therefore, Figure 1 gives a first and also important indication for the selection of propulsion systems.

If we assume $m_{PS}/m_{S/C} \leq 0.3$, we can read directly from Figure 1:

- for low $\Delta v \leq 150$ m/s, compressed cold gas and vaporising liquid propulsion systems seem to be the best choice, because they meet the requirement and have the lowest cost;

- for $150 < \Delta v \leq 650$ m/s, monopropellant hydrazine fed propulsion systems are the best choice, because of their inherent simplicity (reliability) and potential low cost, while still meeting the requirement;

- For high $\Delta v > 650$ m/s, bipropellant systems, monopropellant hydrazine fed resistojet systems (power-augmented thrusters, arcjets), and electrostatic (electromagnetic) systems will satisfy the Δv-requirements best.

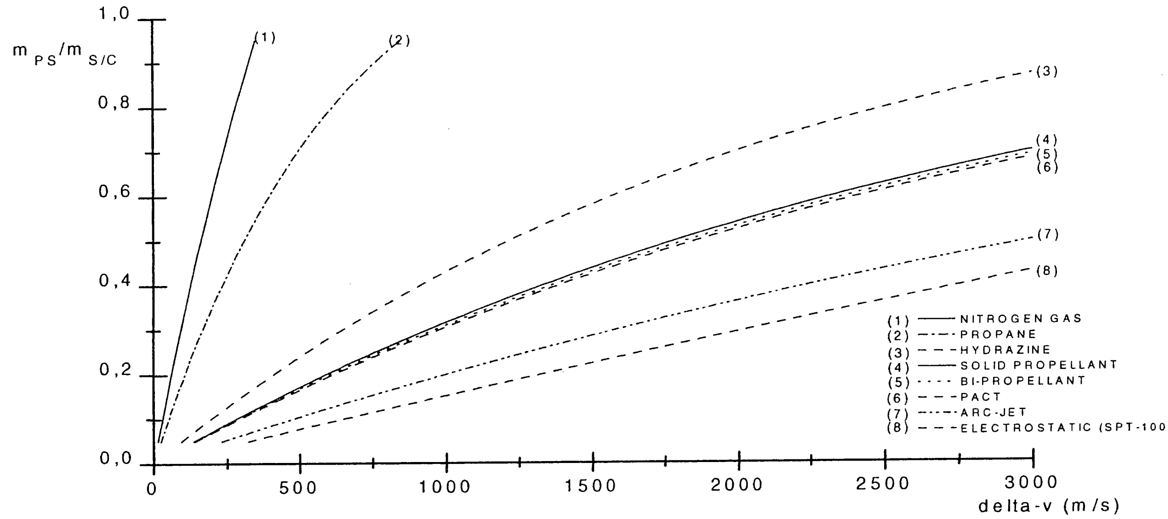

Fig. 1: Delta-v Performance of Spacecraft Propulsion Systems

Finally, for any given value of total impulse I_{tot}, the mass of the propulsion system m_{PS} can, of course, be calculated directly from values of I_{ssp}.

5. CONCLUSIONS

The introduction of the 'System-specific Impulse' allows a more accurate determination of the propulsive performance of spacecraft propulsion systems than the commonly used 'Thruster-specific Impulse', which only includes the propellant mass.

With the assumption that for high impulse mission requirements the impulse related mass of propellant and its corresponding tankage determine mainly the mass of chemical propulsion systems, the 'System-specific Impulse' allows the analysis of those parameters which influence the propulsive performance of these systems. The analysis shows, that the type of propellant, the tank performance factor K, and in addition for systems operated with liquid propellants, the propellant storage conditions (tank filling ratio, propellant feed pressure, type of pressurising gas, etc.) determine the propulsive performance of propulsion systems.

The determination of the 'System-specific Impulse' of electric propulsion systems requires also the consideration of the contained mass of the power supply and power processing systems. The 'System-specific Impulse' allows an objective and comparative evaluation of electric propulsion systems, considering their integral functioning (power sharing) within spacecraft systems. With the development of more spacecrafts equipped with electric propulsion, further evaluation of their 'System-specific Impulse' needs to be performed.

Values of the 'System-specific Impulse', which have been mainly evaluated for chemical propulsion systems, are in an overall good agreement with those of actual systems.

Therefore, tables and the graph which resulted from these investigations should facilitate a preliminary selection of propulsion systems (chemical, electric) for spacecraft missions of given impulse and velocity-increment requirements.

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ANNEX:

DERIVATIONS OF "SYSTEM-SPECIFIC IMPULSE"

GENERAL

Equations (2) to (3) noted in section 2 for I_{ssp} of the various propulsion systems, have in common the same numerator, representing the total impulse delivered by the propellant contained in the propellant tank, which is:

$$I_{tot} = m_P v_e \quad (A1)$$

while the denominator, representing the impulse related system mass (m_{TS}, m_{El}), varies with the kind and design of propulsion systems.

COLD GAS

The simplest expression for the denominator can be noted for cold gas systems, such as compressed gas and vaporising liquids. Starting with compressed cold gas systems, usually the cold gas used is stored at high pressures in a tank. Therefore, for calculating the gas mass content in the tank, the gas law applies as follows :

$$P_{op} V_T = m_P z \frac{R}{M} T \quad (A2)$$

For calculating the tank mass , the so-called "Tank-Performance Factor" usually defined as:

$$K = \frac{P_{op} V_T}{m_T} \quad (A3)$$

is to be used. With Equations (A2) and (A3) the combined mass of the tank and propellant is:

$$m_{PSS} = m_P + m_T = m_P \left[1 + \frac{zRT}{KM} \right] \quad (A4)$$

Therefore, with Equations (2), (A1) and (A4), the "System-Specific Impulse" for **COMPRESSED GASES** becomes:

$$I_{ssp} = \frac{I_{tot}}{m_P + m_T} = \frac{v_e}{1 + \frac{zRT}{KM}} \quad (A5)$$

Further, for vaporising liquids, the mass of liquids in a tank is:

$$m_P = V_P \rho \quad (A6)$$

In order to allow a certain ullage, the volume of propellant is a certain fraction (0.5 to 0.9) of the available tank volume. Therefore, with $V_P = CV_T$ and Equations (A3) and (A6) for the combined mass of tank and propellant we get:

$$m_{PSS} = m_T + m_P = m_P \left[1 + \frac{P_{op}}{C \rho K} \right] \quad (A7)$$

With Equations (2), (A1) and (A7), the "System-Specific Impulse" for **VAPORISING LIQUIDS** becomes:

$$I_{ssp} = \frac{I_{tot}}{m_P + m_T} = \frac{v_e}{1 + \frac{P_{op}}{C \rho K}} \quad (A8)$$

HOT GAS

For systems operating with liquid propellants, the denominator of Equation (2) has to be further determined. In contrast to compressed gas and vaporising liquids, liquid propellants need to be pressurised to feed the thrusters with propellant. Consequently, the mass of the pressurising gas m_{pr} and, if necessary, an extra tank m_{Tpr} for the pressurising gas have to be considered too.

No extra tank for the pressurising gas is needed for the blow-down mode, which is the most widely used means of tank pressurisation for monopropellant hydrazine. At the beginning of a mission the volume of the propellant is a certain fraction C (mostly 0.75 for a blow-down ratio of 4:1) of the internal tank volume. Consequently, the volume of pressurising gas in the propellant tank will be $V_{pr} = (1-C)V_T$. The mass of the gas can be derived easily from the gas law and will be with Equations (A2) and (A3):

$$m_{pr} = \frac{K m_T (1-C) M}{zRT} \quad (A9)$$

With Equations (A7) and (A9) the combined mass of tank, propellant and pressurising gas is given by:

$$m_{PSS} = m_T + m_P + m_{pr} = m_P \left[1 + \frac{P_{op}}{C \rho K} \left(1 + \frac{K(1-C)M}{zRT} \right) \right] \quad (A10)$$

With (2), (A1) and (A10) we obtain the final expression for the I_{ssp} of systems operating with stored liquid and with contained pressurising gas in the propellant tank, representing the "**BLOW-DOWN MODE**":

$$I_{ssp} = \frac{I_{tot}}{m_T + m_P + m_{pr}} = \frac{v_e}{1 + \frac{P_{op}}{C \rho K} \left(1 + \frac{K(1-C)M}{zRT} \right)} \quad (A11)$$

In the case of the constant pressure mode, which is the common mode of tank pressurisation for storable bipropellants , C is usually close to 1 (e.g. 0.95) and the mass of the tank containing the pressurising gas has to be added to the tankage mass. To include the constant pressure mode in our calculations, (A10) has to be modified to include the mass of the extra gas storage tank. For the mass of the pressurising gas plus the extra gas storage tank we get with Equation (A4) - as already derived for compressed gases:

$$m_{pr} + m_{Tpr} = m_{pr}\left[1 + \frac{zRT}{K_{pr}M}\right] \qquad (A12)$$

The pressurising gas will have to fill the propellant tank plus the gas storage tank at the end of the spacecraft mission. Therefore, with the gas storage tank estimated to have a volume of about 10% of that of the propellant tank, the mass of the pressurising gas can be calculated with the help of the gas law (see Equation (A2)):

$$m_{pr} = \frac{P_{op}1.1V_T M}{zRT} \qquad (A13)$$

With help of (A12) and (A13), Equation (A10) can be now expanded to:

$$m_{PSS} = m_I + m_p + m_{pr} + m_{Tpr} = \qquad (A14)$$

$$= m_p\left[1 + \frac{P_{op}}{C\rho K_p}\left(1 + \frac{K_p(1-C)M}{zRT}\right) + \frac{1.1P_{op}M}{\rho zRT}\left(1 + \frac{zRT}{K_{pr}M}\right)\right]$$

With (2), (A1) and (A14) we obtain the final expression for the "System-Specific Impulse" of systems operating with liquid propellants in the "**CONSTANT PRESSURE MODE**":

$$I_{ssp} = \frac{I_{tot}}{m_I + m_p + m_{pr} + m_{Tpr}} = \qquad (A15)$$

$$= \frac{v_e}{1 + \frac{P_{op}}{C\rho K_p}\left(1 + \frac{K_p(1-C)M}{zRT}\right) + \frac{1.1P_{op}M}{\rho RT}\left(1 + \frac{zRT}{K_{pr}M}\right)}$$

ELECTRIC PROPULSION

To describe the performance of electric propulsion systems, the denominator of Equation (3) has to be further determined. Both, the mass of the propellant storage system and the mass of the electric power supply system have to be considered.

For systems operation with gaseous propellants, e.g. Xenon, the combined mass of tank and propellant is calculated according to Equation (A4) as derived for cold gas systems above. The mass of the electric power (supply and conditioning) system is calculated with the system specific power $\gamma\,[W/kg]$:

$$m_{El} = \frac{N}{\gamma} \qquad (A16)$$

where $$N = F\frac{v_e}{2\eta} \qquad (A17)$$

is the input electrical energy and $\eta = N_e/N$ is the overall energy conversion efficiency.

With Equation (A4), (A16) and (A17) the combined mass of the propellant storage system and the electric power system is calculated for systems operating with gaseous propellants:

$$m_{PSS} + m_{El} = m_P + m_T + m_{El} =$$

$$= m_P\left[1 + \frac{zRT}{KM} + \frac{Fv_e}{m_P 2\eta\gamma}\right] \qquad (A18)$$

The "System-Specific Impulse" becomes with Equation (3) and (A18):

$$I_{ssp} = \frac{I_{tot}}{m_P + m_T + m_{El}} = \frac{v_e}{1 + \frac{zRT}{KM} + \frac{Fv_e}{m_P 2\eta\gamma}} \qquad (A19)$$

And for:

$$m_P = \frac{Ft}{v_e} = \frac{F\tau}{v_e} \qquad (A20)$$

with $t = \tau$, which is the thruster operating time, the "System-Specific Impulse" for **ELECTRIC PROPULSION SYSTEMS, operating with gaseous propellants,** becomes finally:

$$I_{ssp} = \frac{v_e}{1 + \frac{zRT}{KM} + \frac{v_e^2}{2\eta\gamma\tau}} \qquad (A21)$$

The I_{ssp} will be a maximum for the optimal thruster exhaust velocity of:

$$v_{e,o} = \sqrt{2\eta\gamma\tau\left(1 + \frac{zRT}{KM}\right)} \qquad (A22)$$

which has been derived by observing the first and second derivatives of Equation (A21) with respect to v_e .

In a similar way, the "System-Specific Impulse" and the optimal thruster exhaust velocity can be determined for electric propulsion systems operating with liquid propellants, by taking into account the relevant expressions for mass of the propellant storage systems as derived for hot gas systems above.