

A NEW PULSATION SPECTRUM AND ASTEROSEISMOLOGY OF δ SCUTI

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ABSTRACT

We present the results of a five-year Strömgren y photometric campaign on δ Scuti. Our data set consists of 6515 discrete differential magnitudes, and spans the period of 1983 June to 1988 September. We found the primary pulsation mode at $59.731129 \pm 0.000002 \mu\text{Hz}$, in close agreement with the frequency determination of Fitch (1976, IAU Colloquium, 29, 167), but we find our best-fit observed frequencies for other pulsation modes differ by 0.5–2 cycles per year from Fitch's results. In the case of the second strongest pulsation mode, we found a frequency of $61.936104 \pm 0.000009 \mu\text{Hz}$ —one cycle per year off of the commonly quoted frequency. All of the other modes not classified as harmonics or beating modes were identified in our data, as well as a new pulsation frequency at $96.21443 \pm 0.00005 \mu\text{Hz}$ discovered in both Strömgren y and b observations. We measured the phase differences between our Strömgren y data and a short string of Strömgren b data taken during the 1987 multisite campaign, and find phase differences ranging from 0 to 0.33 radians, suggesting that there are modes of different spherical harmonic order present in δ Scuti. Finally, we evolved a set of $M=1.8$ – $2.4 M_{\odot}$ models with solar abundances ($X=0.7$, $Z=0.02$) and two ($M=2.2$ and $M=2.4 M_{\odot}$) models with solar abundances scaled to ($X=0.66$, $Z=0.06$), using recent opacity and reaction rate data, and applied linear, nonadiabatic pulsation analysis to models in the shell hydrogen burning phase. The $Z=0.02$ model which best fit the observed spectral type of F2 III, the *Hipparcos* absolute magnitude of $M_V=1.0$, and the radius estimate of Cugier and Monier of $R=4.1 R_{\odot}$, and which has a pure radial mode at $59.731 \mu\text{Hz}$ has a mass of $2.1 M_{\odot}$, with $T_{\text{eff}}=6894 \text{ K}$, $R=4.14 R_{\odot}$, and $M_V=1.0$. The best-fit $Z=0.06$ model has $M=2.4 M_{\odot}$, $T_{\text{eff}}=6827 \text{ K}$, $R=4.28 R_{\odot}$, and $M_V=1.0$. For the best-fit models, the highest amplitude observed mode is the radial fundamental mode, with several nonradial modes being simultaneously excited. This contradicts the results of Balona *et al.* that the strongest mode of δ Scuti is first overtone. In addition, the second radial overtone falls at $\nu=97.4 \mu\text{Hz}$ for both models, thus eliminating the two observed frequencies at 96.2 and 99.4 μHz as candidates for the second radial overtone, *unless* second-order rotation effects are considered. We are unable to perform a more comprehensive asteroseismological analysis because the theoretical nonradial modes overlap in frequency when rotational splitting is taken into account, thus making it impossible to uniquely determine the mode type without more observational identifications of the pulsation mode types. However, the mode spacing of the $Z=0.06$ model is less degenerate than the $Z=0.02$ model, so with more observational identifications, it may be possible to actually make definitive model fits. We discuss the implications of our results on δ Scuti star modeling in some detail. © 1997 American Astronomical Society. [S0004-6256(97)02710-6]

1. INTRODUCTION

The study of the multiperiodic behavior of δ Scuti stars provides an important test of our understanding of stellar

interiors and stellar evolution. δ Scuti stars have masses in the range of 1.5 to $2.5 M_{\odot}$ and so are important for helping us extend our understanding of stellar structure beyond the $1 M_{\odot}$ regime. Because they can pulsate in several discrete

radial and nonradial modes, we can study the current state of their interiors using the technique of asteroseismology. Asteroseismological analysis has been applied with great success to the Sun (Christensen-Dalsgaard *et al.* 1996) and white dwarf stars (Winget *et al.* 1991; Kawaler & Bradley 1994; Winget *et al.* 1994) and is now being applied to several multi-mode δ Scuti stars (Breger *et al.* 1995; Guzik & Bradley 1995; Bradley & Guzik 1996; Handler *et al.* 1996). In the current work, we hope to apply this technique to the prototype of the δ Scuti class of variables.

Fitch (1976) reported nine periodicities in his analysis of white light data by Fath (1940) and his own Strömgren *b* observations, five of which were linearly independent pulsation modes and four of which were Fourier harmonics or beat modes. At that time, this established δ Scuti as having one of the most complex pulsation spectra in this class of variable star. Since then, the majority of work done on the multiperiodic nature of δ Scuti itself has focused on line profile variations and phase differences between light and radial velocity curves, in an attempt to determine the spherical harmonic indices of the most prominent modes (cf. Smith 1982; Cugier & Monier 1993). Until now, very little work has been done to use all of the known pulsation modes to develop a precise stellar model of δ Scuti. We believe our data and the spectrum obtained from it can allow us to develop a reasonable model for the physical properties of δ Scuti.

Most published papers with pulsation spectra of δ Scuti claim that the strongest variability occurs at 59.731 μ Hz. Results vary as to the next-strongest pulsation mode, with the two most-commonly reported frequencies of 61.936 μ Hz and 61.972 μ Hz (Balona *et al.* 1981, and references therein) which differ by one cycle per year. Several other modes with much weaker amplitudes are thought to exist in addition to these three, including at least three other pulsation modes and four combination or Fourier harmonic modes. Here, we apply the technique of asteroseismology to the prototype of the δ Scuti class in order to gain a better understanding of moderate-mass stellar interiors and evolution. We seek to confirm and better constrain our understanding of the multiperiodic nature of δ Scuti by observationally identifying all of the pulsation modes of δ Scuti and comparing these with model frequencies. Ultimately, we hope to apply the techniques developed for this specific case to the study of other multiperiodic variables to obtain a better, more general model for stellar interiors and evolution.

2. DATA REDUCTION AND ANALYSIS

2.1 Data Acquisition and Reduction

Data for this project were obtained at three different sites: Tortugas Mountain Observatory (NM, USA), Cerro Tololo Interamerican Observatory (CTIO), and the South African Astrophysical Observatory. All data from 1983 through 1985 were obtained at the New Mexico site; data from 1987 June were obtained from all three sites in a multisite campaign; data from 1988 September were obtained at CTIO. Observations followed an alternating sequence of δ Scuti and comparison star through a Strömgren *y* filter, with observations

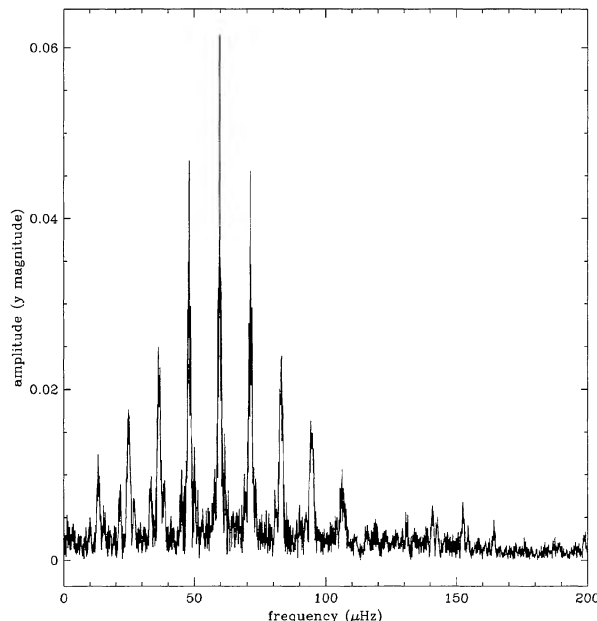


FIG. 1. The raw spectrum of the Strömgren *y* data set between 0 and 200 μ Hz. The largest amplitude peak is located at 59.73 μ Hz. Weaker variability components (see Table 1) are only visible after pre-whitening with the high-amplitude components.

temporally separated by a few minutes. For the 1987 data, observations were made in the same way, but were alternated between Strömgren *y* and *b* filters. We then converted raw photometer counts from a comparison star to magnitudes, and computed a solution for extinction per airmass. Differential (δ Scuti, comparison) magnitudes were then computed for the δ Scuti measurements. We used HR 7094 (HD 174464), an F2 star not known to be variable, as our comparison star. The constancy of the comparison star was location dependent, with the lowest rms scatter (~ 5 mmag) at CTIO, and with slightly higher rms scatter (from 5 to 10 mmag) at SAAO and the Tortugas site. After reduction of the photometry, we then converted observation times from Universal Time to heliocentric Julian date. After deletion of ≈ 20 obviously discrepant data points, the Strömgren *y* data set consists of 6515 discrete photometric measurements, spanning 1928.6 days. The Strömgren *b* data set consists of 679 data points spanning 15.4 days.

2.2 Time-series Analysis

Our primary method for obtaining the pulsation spectrum of δ Scuti is a combination of discrete Fourier transform (DFT) and nonlinear least-squares sinusoid fitting (NLSQ). Initially, we perform a DFT at low resolution to pick out regions in the spectrum with most of the power and to determine the level of sidelobe contamination of the spectrum (see Fig. 1). The figure shows significant pulsation amplitude at ~ 60 μ Hz, with other peaks blended with the large sidelobes of the 60 μ Hz pulsation; there is significant aliasing at 1 cycle per day (11.57 μ Hz) and 1 cycle per year (0.03 μ Hz), so the weaker pulsation frequencies are not obviously apparent without prewhitening.

For each suspected frequency in the pulsation spectrum, we perform a DFT around the suspected peak at the highest resolution,

$$\delta\nu = \frac{3\sigma_N}{\sqrt{N}(t_N - t_0)a_i}, \quad (1)$$

where σ_N is the standard deviation per point *after* prewhitening with all eight frequencies, N is the number of data points, t_0 and t_N are the observation times of the first and last data points, respectively, and a_i is the amplitude of each periodicity. (See Horne & Baliunas 1986, and references therein.) Although this equation is derived specifically for evenly sampled data with no data gaps, the only effect of data gaps will be to increase the power in the sidelobes, *not* to broaden the peak of the central frequency. However, as we discuss later, the low-amplitude modes *do* suffer from some sidelobe confusion, making their frequency estimates subject to error higher than the purely statistical error.

Once we identify a peak in the Fourier spectrum, we input that frequency into a least-squares sinusoid fitting routine. The routine fits a sine wave to the data—allowing the frequency, amplitude, and phase to vary—by minimizing the least-squares residuals of the subtraction of the fitted wave from the light curve, where the residuals are computed by

$$x(t) = x_0(t) - \sum_{i=1}^n a_i \cos(\omega_i t - \phi_i). \quad (2)$$

We use the DFT as a filter for the data, because our least-squares routine is not global; it only seeks a local minimization in the residuals. As confirmation of the validity of the approach, we note that the amplitudes and phases obtained by NLSQ were very close to those obtained via the DFT.

Once we obtain a frequency, amplitude, and phase solution for each suspected pulsation frequency, we prewhiten the data with that solution (and all previous solutions) and reapply our DFT and NLSQ analysis to obtain the next pulsation mode. We note that for each iteration, we apply the least-squares routine to *all* frequencies obtained, rather than using the least-squares routine for one frequency at a time on prewhitened data. If we do not try to fit all frequencies simultaneously, progressively weaker pulsations would be much more dependent upon prewhitening parameters, yielding leading to less accurate fits or spurious signals. Prewhitening is then applied to raw data with all frequencies at once.

3. THE PULSATION SPECTRUM OF δ SCUTI

Using the procedure outlined in Sec. 2.2, we identified what we believe to be six distinct modes, one Fourier harmonic, and one beating mode (see Fig. 2). Table 1 contains our eight frequencies, with amplitudes, phases, and the variance of least-squares residuals after prewhitening with that mode and those done before it. The dominant frequency is at $f_0 = 59.731129 \pm 0.000002 \mu\text{Hz}$, with a semi-amplitude of 62.3 ± 0.3 mmag, which is consistent with the results of Fitch both for the 1936–40 white light and 1972–73 Strömgren *b* data sets; the Strömgren *b* and white light semi-amplitudes

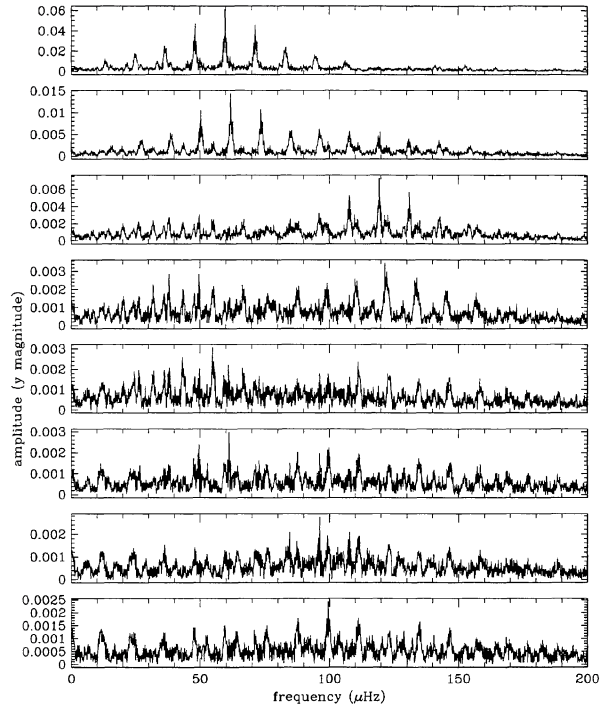


FIG. 2. Prewhitened DFTs for all eight frequencies listed in Table 1, in the same order as the table from top to bottom.

are ~ 80 – 120 mmag, and the amplitude of the pulsation is expected to be higher in *b* and in broad-band white light than in *y*.

We also find the $2f_0$ Fourier harmonic at $119.46187 \pm 0.00002 \mu\text{Hz}$, consistent with our determination of f_0 . Our other frequencies disagree with those of Fitch by between 1 cycle per 200 days ($0.058 \mu\text{Hz}$) and 1 cycle per 2 years ($0.016 \mu\text{Hz}$). However, we note that the value of σ_N decreases for each mode that we use to prewhiten the data, indicating that we are reducing the level of variability in the data set with each iteration of prewhitening, and that we are detecting real periodicities, not random noise peaks. Furthermore, because our data set spans a longer period of time and suffers less from annual sidelobe confusion, we believe our frequency measurements for these modes are an improvement over those of Fitch. This will be discussed in detail in the following sections. Finally, we note that the precision of our frequency identifications is crucial for studies of period changes but is not important for the asteroseismology done here.

3.1 The $f_1=61.936 \mu\text{Hz}$ Mode

For the second-strongest mode, $f_1 = 61.936 \mu\text{Hz}$, we find our best-fit frequency differs by one annual sidelobe from most previous measurements. We believe our identification of this frequency is correct for three reasons. First, the fully sampled DFT made after prewhitening with the $59.731 \mu\text{Hz}$ frequency (Fig. 3) clearly shows that the peak at $61.936 \mu\text{Hz}$ is significantly higher than neighboring peaks. Previous results indicated that the annual sidelobes about $61.936 \mu\text{Hz}$ were at the same significance level as the main peak, so we

TABLE 1. The pulsation spectrum of δ Scuti. Column 4 contains the least-squares residuals for each prewhitening step. Column 5 contains the signal-to-noise for each peak, computed using the method of Breger *et al.* (1995).

Frequency (μHz)	Amplitude (y magnitudes)	ϕ (radians) ^a	σ_{fit} (y magnitudes)	S/N
59.731129 $\pm$ 2	0.0623 $\pm$ 3	4.9934 $\pm$ 0.0063	0.0141	112.2
61.936104 $\pm$ 9	0.0142 $\pm$ 3	2.9203 $\pm$ 0.0282	0.0098	25.1
119.46188 $\pm$ 2	0.0073 $\pm$ 3	2.6306 $\pm$ 0.0573	0.0084	12.3
121.66699 $\pm$ 4	0.0034 $\pm$ 3	0.5100 $\pm$ 0.1146	0.0080	6.2
54.82807 $\pm$ 4	0.0030 $\pm$ 3	4.2241 $\pm$ 0.1507	0.0077	4.6
61.16792 $\pm$ 4	0.0031 $\pm$ 3	0.5131 $\pm$ 0.1099	0.0074	6.4
96.21443 $\pm$ 5	0.0028 $\pm$ 3	5.2450 $\pm$ 0.1457	0.0071	4.9
99.44722 $\pm$ 5	0.0024 $\pm$ 3	1.6977 $\pm$ 0.1627	0.0069	4.3

^aPhases relative to JD 2445943.5.

believe that in those cases, the investigators chose a peak one annual sidelobe off of the true frequency. Secondly, we find a beating mode at 121.667 μHz , which is precisely $f_0 + f_1$, suggesting that our value of f_1 is the real value. Finally, as mentioned in the introduction, Paddock & Struve (1954) also measured $f_1 = 61.936 \mu\text{Hz}$ using radial velocity measurements, so our result does have precedence in published studies. Therefore, we are confident that our measurement is the correct frequency.

3.2 Lower Amplitude Pulsation Modes

The four other pulsation modes we found have less accurate frequency solutions, and may be in error by 0.016 to 0.058 μHz . We calculate the error in amplitude per mode (see Table 1) by determining the standard deviation of the Fourier amplitudes, after prewhitening with all eight frequencies. We calculate the standard deviation only for frequencies between 62.5 and 69.4 μHz in the middle of our band of frequencies, away from strong sidelobe interference. If we assume this noise level is correct, then our weakest pulsation has a signal-to-noise of 4.3, using the method of Breger *et al.* (1995) to determine the signal-to-noise. We caution that these are only the *statistical* errors; because of sidelobe confusion, the cumulative errors could be higher. The errors in frequency on this level are not significant for the purposes of asteroseismology, however, because the sidelobe errors are less than 0.1 μHz and the first-order rotational splitting values for the theoretical nonradial modes are as high as 2.7 μHz for $l=2$ modes.

In addition to the 61.1679, 54.8281, and 99.4472 μHz modes noted by Fitch, we found another mode at 96.2144 μHz in both our 1983–1988 Strömgren *y* and 1987 Strömgren *b* data sets. Because it is so close to the suspected second radial overtone at 99.4472 μHz it raises the possibility that it may in fact be the actual second radial overtone, or it may be driven by the second radial overtone. Our analysis of phase shifts could not conclusively determine that this is a nonradial mode. We present this analysis in the following section.

In Fig. 4, we present the fits of our calculated light curve to the observed light curve of δ Scuti. The computed light curve shows very good agreement with much of the data, even through several thousand cycles. The majority of points where the observed light curve deviates strongly from the

calculated light curve (e.g., $t = 2.4, 125.18, 456.25$) are due to photometric errors, although some points in the light curve with better photometry (e.g., $t = 1474.35$) may be due to errors in the fit. However, we again state that this set of best-fit frequencies yields the lowest rms residuals (6.9 mmag) of all those tested. Thus we are confident of the accuracy of our results.

Finally, we found evidence of a weaker ($S/N \sim 3$) periodicity at 59.81 μHz , but chose not to include it in our list of pulsation modes. It was not previously reported by other observers and it lies close in frequency to the large radial mode at 59.731 μHz ; we suspect that it is either a noise peak, or may be related to our prewhitening technique. No other Fourier peaks appear above this level in our analysis of the Strömgren *y* data set.

3.3 Phase Differences between *y* and *b*

To measure the phase shifts between the 1983–1988 Strömgren *y* and 1987 Strömgren *b* light curves, we use the

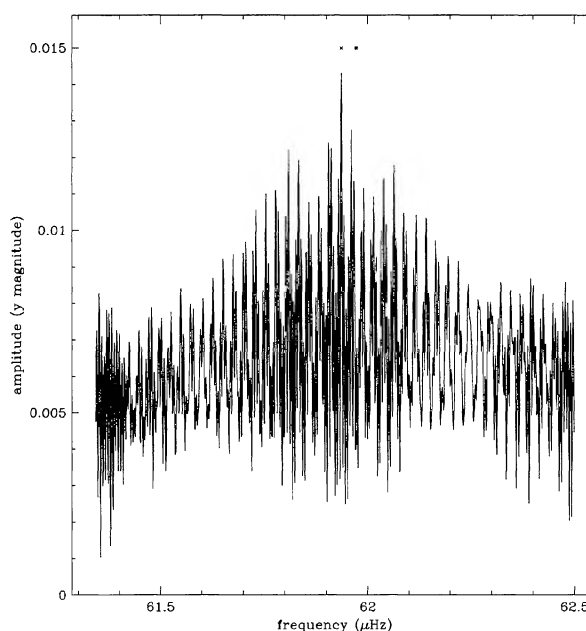


FIG. 3. High-resolution spectrum of the 61.936 μHz mode. Our best-fit frequency is marked with an "x," while the 61.972 μHz mode reported by Fitch is marked by an "*."

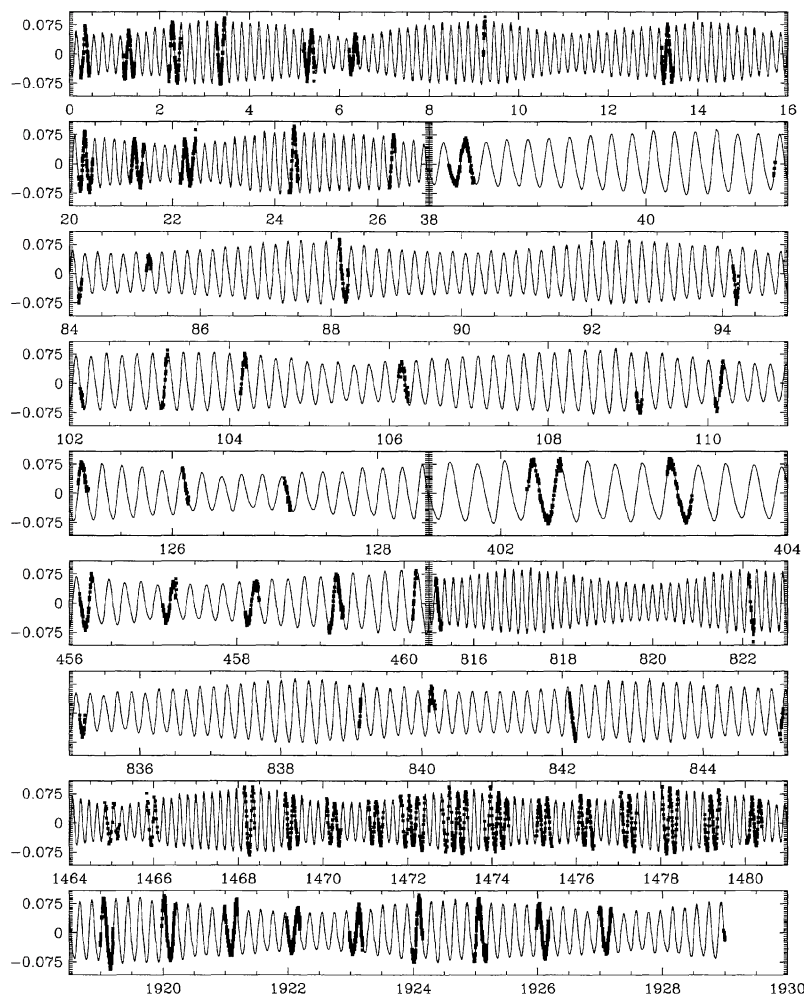


FIG. 4. Observed vs calculated light curves for all of our δ Scuti data. Observed data points are open circles; calculated light curve (solid line) generated using all eight of the best-fit pulsation modes. Time is in days from JD 2445493.5. Note that the time scale changes from window to window, so that all of the data could be displayed on one page.

spectrum obtained from the y data and apply the frequencies directly to the b data. Since the two data sets are on the same time frame ($t_0 = \text{JD } 2445493.5$), the phases obtained from Fourier analysis give the relative phase shifts in each filter passband.

To obtain the phase information, we proceed in a slightly different manner with the Strömgren b data than with the y data. The frequencies obtained from the y data are more precise than the frequencies obtainable from the b data, because the b data only spans 15.4 days. Therefore, we determine the b band phase at the y band frequencies using the DFT alone, rather than by performing least-squares fits. The error in phase is then only a function of the local noise power,

$$\Delta\phi_{y-b} = \frac{1}{\sqrt{2 \cdot P}}, \quad (3)$$

where P is the pulsation power normalized by the local noise power. This is simply the half width at half maximum for the Gaussian power peak in the complex plane. (For a derivation, see Middleditch 1976.) However, because the mode

amplitudes are so small, our phase errors are large enough such that we can only check the consistency of our results with theory, and then only with the highest-amplitude mode. The results are shown in Table 2.

The phase shift associated with the $59.731 \mu\text{Hz}$ pulsation has an upper limit of only 0.003 ± 0.010 radians, which is consistent with the radiative transfer models of Cugier & Monier (1993, their Fig. 2). According to their models, an $l=0$ mode should have only a very slight positive phase shift $\Delta\phi_{y-b}$ between the y (550 nm) and b (470 nm) passbands. Within the errors, the observed phase shift is essentially zero. Our result is therefore consistent with previous determinations of the purely radial nature of the $59.731 \mu\text{Hz}$ mode.

Our other phase measurements produce more ambiguous results, primarily because of the short duration of the Strömgren b data set, which results in a higher local noise power and a larger value of $\Delta\phi_b$. The second-strongest mode at $61.936 \mu\text{Hz}$ has a 3σ positive phase shift of 0.139 radians, and the $54.828 \mu\text{Hz}$ mode has a 1.25σ positive phase shift of 0.328 radians. These are both inconsistent with Cugier and Monier's results. However, because our errors

TABLE 2. Phase comparison of Strömgen γ and b data of δ Scuti, with b data from 1987 June 11 to 27.

Frequency (μHz)	P_γ	ϕ_γ (radians) ^a	P_b	ϕ_b (radians) ^a	$\Delta\phi_{\gamma-b}$ (radians)
59.731129 $\pm$ 2	1.26 $\cdot 10^4$	4.9934	8.63 $\cdot 10^3$	4.9904	+0.003 $\pm$ 0.010
61.936104 $\pm$ 9	6.29 $\cdot 10^2$	2.9203	3.70 $\cdot 10^2$	2.7815	+0.139 $\pm$ 0.046
119.46188 $\pm$ 2	1.52 $\cdot 10^2$	2.6306	1.11 $\cdot 10^2$	2.5310	+0.100 $\pm$ 0.088
121.66699 $\pm$ 4	3.81 $\cdot 10^1$	0.5100	2.39 $\cdot 10^1$	0.3182	+0.192 $\pm$ 0.184
54.82807 $\pm$ 4	2.20 $\cdot 10^1$	4.2241	1.12 $\cdot 10^1$	3.8957	+0.328 $\pm$ 0.259
61.16792 $\pm$ 4	4.14 $\cdot 10^1$	0.5131	3.32 $\cdot 10^1$	0.5789	-0.066 $\pm$ 0.165
96.21443 $\pm$ 5	2.36 $\cdot 10^1$	5.2450	3.00 $\cdot 10^1$	5.4582	-0.208 $\pm$ 0.195
99.44722 $\pm$ 5	1.89 $\cdot 10^1$	1.6977	1.86 $\cdot 10^1$	1.5038	+0.195 $\pm$ 0.231

^aPhases relative to JD 2445943.5.

are so large, and because the predicted difference in phase between the γ and b filters is so small for most models, we do not feel this conflict is significant. We note that the mode at 96.214 μHz has a slight *negative* phase shift, which would be consistent with a nonradial mode, but it is only a 1.2σ result, so we need confirmation of this before we can claim this as a nonradial mode.

4. ASTEROSEISMOLOGY OF δ SCUTI

We now present our seismological analysis of δ Scuti. Here, we use the pulsation spectrum from section three to select candidates from a set of evolutionary models we constructed using evolution and pulsation codes previously developed for the Sun and for other δ Scuti stars. δ Scuti is an interesting test of the asteroseismological implications of metallicity, because the photosphere is metal rich with $[\text{Fe}/\text{H}] = 0.49 \pm 0.04$ (Russell 1995). We are investigating the effect of higher metallicity on the seismology of δ Scuti stars in general and we will report more comprehensive results in a future paper. Furthermore, δ Scuti is highly evolved, so it is an excellent test of our ability to model the deep interiors of evolved stars. In Sec. 4.1, we describe our models and modeling technique in detail. In Sec. 4.2, we present our results, and discuss their implications both for δ Scuti itself and for evolved δ Scuti-type stars in general.

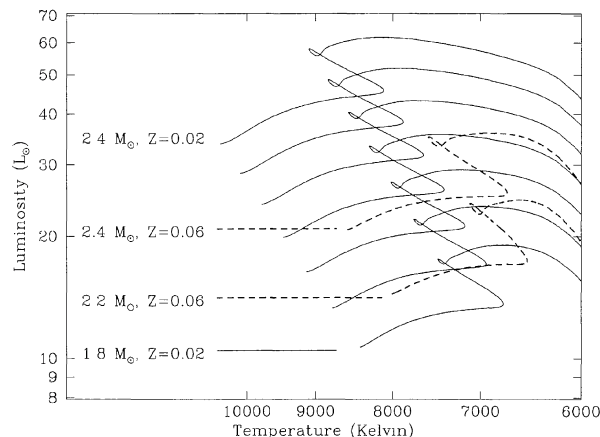


FIG. 5. Evolution tracks for nine of our ten stellar models—the 2.15 $M_\odot$ track is omitted for clarity.

4.1 Models and Initial Constraints

To perform asteroseismology on δ Scuti, we first constructed eight evolutionary models for stars ranging from 1.8 to 2.4 $M_\odot$ with initial compositions of $(X, Y, Z) = (0.7, 0.28, 0.02)$, and two models ($M = 2.2$ and $2.4 M_\odot$) with initial compositions of $(X, Y, Z) = (0.66, 0.28, 0.06)$. Our evolution code uses the most recent OPAL opacity tables (Iglesias & Rogers 1996), the Alexander & Ferguson (1994) low-temperature opacity tables, and the Grevesse & Noels (1993) solar mixture data. We also use the Eggleton *et al.* (1973) equation of state with corrections for Coulomb effects, and the Caughlan & Fowler (1988) nuclear reaction rates. All ten models were evolved through the main sequence to the base of the subgiant branch (see Fig. 5).

To test each evolutionary model, we first select a single timestep from each of the ten evolutionary models, and use the composition, mass, and luminosity information to generate a pulsation model for input into a linear nonadiabatic pulsation code developed by Pesnell (1990). We strongly bias the zoning of our models towards the hydrogen-burning shell, with nearly 70% of the zones covering the inner 10% of the mass of the star (Fig. 6). This is necessary because (see the following section) the majority of nonradial modes

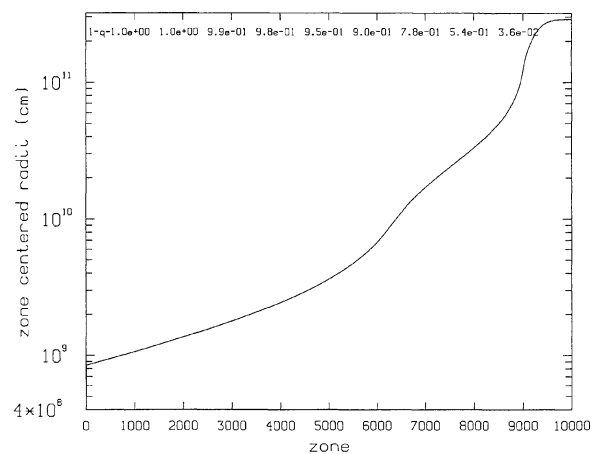


FIG. 6. The mass-zone distribution for our pulsation tests. The majority of zones (more than 70%) are located in the inner 10% of the mass star. The values at the top of the graph are $(1-q)$ —mass fraction. The curve is not a smooth function because we adjust the mass zoning of different parts of the star to achieve the best resolution of the eigenfunctions possible with reasonable computation times.

TABLE 3. Stellar parameters for the eight pulsation models tested for nonradial pulsations, having radial fundamental modes close in frequency to the highest-amplitude observed mode. The metal abundance and radial order (fundamental or first overtone) of the 59.731 μHz mode are noted for each model.

Mass ($M_{\odot}$)	Radius ($R_{\odot}$)	Luminosity ($L_{\odot}$)	T_{eff} (K)	Z	Mode type F/1H	Frequency (μHz)
2.00	4.01	28.0	6652	0.02	F	59.802
2.10	4.14	34.4	6894	0.02	F	59.718
2.15	4.18	38.4	7039	0.02	F	59.834
2.20	4.19	42.4	7201	0.02	F	60.333
2.30	4.29	51.1	7465	0.02	F	59.864
2.30	5.07	47.4	6728	0.02	1H	59.503
2.40	5.16	57.2	6997	0.02	1H	59.607
2.20	3.93	24.2	6482	0.06	F	59.731
2.40	4.30	35.5	6822	0.06	F	59.728

in the frequency range of interest (50–100 μHz) have as many as 70 g -type nodes in the hydrogen-burning shell and burned-out core; to properly resolve the eigenfunctions, the pulsation code requires several mass zones per node. To test the frequency accuracy of our modeling results, we compare frequencies obtained using the eigenvalue and eigenvector of the pulsation matrix, and the difference yields the error. We find that errors for theoretical nonradial pulsation frequencies of interest are of order $\sigma_T \sim 10^{-2-3} \mu\text{Hz}$. This is slightly higher than the errors for low overtone radial modes, but higher accuracy requires higher resolution of mass zones and an increase in computation time; we typically use between 2000 and 10000 mass zones to achieve the best possible resolution of the eigenmodes with reasonable computation time. Errors in theoretical radial pulsations are at least a factor of ten less, because these modes sample the entire star, rather than being concentrated in the hydrogen-burning shell and core.

To begin our pulsation modeling we initially test each evolution model at the evolution timestep with an effective temperature closest to $T_{\text{eff}}=7000$ K for radial pulsation modes at or near the observed $l=0$ mode at 59.731 μHz . If the resulting theoretical radial modes do not fall near the observed mode, we choose a new timestep using the period-mean density relation:

$$\Pi \left(\frac{M_*}{\frac{4}{3}\pi R_*^3} \right)^{1/2} = Q, \quad (4)$$

where Π is the pulsation period, M_* and R_* are the stellar mass and radius, and Q is a constant over several hundred Kelvin in effective temperature (Cox & Giuli 1968). For the 1.8 and 1.9 $M_{\odot}$, $Z=0.02$ models, we were unable to obtain models with radial modes pulsating at 59.731 μHz that agreed with the current spectral designation of δ Scuti; δ Scuti is classified as an F2 III star (Hoffleit & Jaschek 1982), which would have $T_{\text{eff}}=6870 \pm 200$ K (Lang 1990). This immediately places a lower limit of 1.9 $M_{\odot}$ on the mass of δ Scuti if we assume a solar metallicity. Furthermore, these two models are underluminous (assuming a *Hipparcos* absolute magnitude $M_V=1.0$ for a luminosity of $\sim 33 L_{\odot}$). We summarize the stellar parameters of mass, radius, luminosity, T_{eff} , and fundamental or first overtone pulsation frequency for six models having a fundamental or first overtone at 59.731 μHz in Table 3.

If the published spectral classification and luminosity are correct, then the mass is constrained to lie near 2.1 $M_{\odot}$ with $Z=0.02$ abundance and the above restriction on radial modes. In this case, the 59.731 μHz mode is the fundamental rather than first radial overtone. Models with masses higher than 2.1 $M_{\odot}$ produce acceptable frequency fits with first overtone modes to the observed mode at 59.731 μHz , but they would produce models with luminosities well in excess of 60 $L_{\odot}$, which are inconsistent with the known absolute magnitude. Higher mass models pulsating in the fundamental mode are too hot by several hundred Kelvin for the observed spectral type. For the $Z=0.06$ models, the best-fit model has a mass of 2.4 $M_{\odot}$, using similar arguments as for the $Z=0.02$ models.

4.2 Asteroseismology and Modeling Results

We tested both the $Z=0.02$ and $Z=0.06$ best-fit models for nonradial pulsation modes to try and match the observed pulsation frequencies. We have found that both models have unstable modes in the frequency range of 55 to 215 μHz , which is the region where the known pulsations occur (54 to 99 μHz). Table 4 gives the theoretical pulsation modes for $l=0, 1$, and 2 for the $Z=0.02$ and $Z=0.06$ models, respectively. Several things are apparent from both sets of frequencies. First, the pure radial ($l=0$) mode frequencies are roughly the same for both metallicities. In particular, the second radial overtone falls at $\sim 97.3 \mu\text{Hz}$ for both models. Therefore it does not seem likely that either of the observed modes at 96.2 or 99.4 μHz can be the second radial overtone, as was claimed by Fitch (1976), and that these modes are probably both nonradial modes.

Another fact which is immediately apparent is the extremely close frequency spacing of the nonradial modes, particularly the $l=2$ modes; at 60 μHz , the average mode spacing is roughly 0.7 μHz . This is well within our observational and theoretical errors, but it is significantly less than the maximum first-order rotational splitting possible for each mode. The $v \sin i$ of δ Scuti was determined recently by Solano & Fernley (1997) to be 30.1 km/s. Using the theoretical radii for the two models, this rotation yields Ω_{rot} of 1.7 and 1.6 μHz for the $Z=0.02$ and $Z=0.06$ models, respectively. This yields a maximum first-order frequency splitting of $\pm 0.9 \mu\text{Hz}$ for $l=1$ and up to $\pm 2.7 \mu\text{Hz}$ for $l=2$. (The calculated C_{lk} values for $l=1$ and $l=2$ are very

TABLE 4. Theoretical pulsation modes for $l=0, 1$, and 2 for (a) the $2.1 M_{\odot}$, $Z=0.02$ model, (b) the $2.4 M_{\odot}$, $Z=0.06$ model. Growth rates are $\Delta E/E$ per cycle.

l	# of p -nodes	# of g -nodes	ν (μHz)	growth rate/ 10^{-5}	l	# of p -nodes	# of g -nodes	ν (μHz)	growth rate/ 10^{-5}
(a)					(b)				
0	0	0	59.75	4.941	2	2	46	84.97	2.642
0	1	0	77.76	31.361	2	2	45	86.71	5.394
0	2	0	97.44	126.250	2	2	44	88.35	10.975
1	0	43	53.40	-0.054	2	2	43	89.91	10.813
1	0	41	54.65	-0.047	2	2	42	91.79	7.328
1	0	41	55.98	-0.048	2	2	41	93.89	8.302
1	0	39	57.45	-0.011	2	2	40	95.94	15.720
1	0	39	58.89	0.147	2	2	39	97.74	18.450
1	0	37	60.29	1.849	2	2	38	99.85	10.894
1	0	37	61.07	2.426	2	2	37	102.28	7.559
1	1	37	62.46	0.517	(b)				
1	1	35	64.22	0.286	0	0	0	59.73	2.141
1	1	35	65.92	0.341	0	1	0	77.37	10.413
1	1	33	67.72	0.367	0	2	0	97.22	32.754
1	1	33	69.53	0.351	1	1	23	49.03	-0.039
1	1	31	71.67	0.343	1	0	23	51.39	-0.037
1	1	31	73.88	0.501	1	0	21	54.06	-0.030
1	1	29	76.32	1.107	1	0	21	56.80	0.035
1	1	29	78.76	5.639	1	0	19	58.77	0.454
1	1	27	80.38	22.817	1	0	19	60.86	0.737
1	2	27	82.31	4.662	1	1	19	62.75	1.239
1	2	27	85.30	1.430	1	1	17	65.31	0.184
1	2	25	88.81	1.602	1	1	17	69.35	0.062
1	2	25	92.08	3.366	1	1	15	74.20	0.197
1	2	23	95.69	13.312	1	1	15	78.41	6.121
1	2	23	98.53	35.294	1	1	13	80.55	3.570
1	2	22	101.51	69.652	1	2	13	85.79	2.253
2	0	74	52.91	-0.187	1	2	11	89.94	2.367
2	0	73	53.61	-0.216	1	2	11	96.59	2.564
2	1	72	54.39	-0.223	1	2	10	101.49	34.596
2	0	71	55.19	-0.207	2	1	42	48.50	-0.136
2	0	70	56.01	-0.170	2	0	41	49.86	-0.129
2	0	69	56.81	-0.062	2	0	39	51.30	-0.105
2	0	68	57.51	0.219	2	0	38	52.41	-0.006
2	0	67	58.15	0.230	2	0	37	53.29	-0.054
2	0	66	58.93	0.057	2	0	36	54.80	-0.079
2	0	65	59.82	0.028	2	0	35	56.53	-0.050
2	0	64	60.75	0.090	2	0	34	58.03	0.275
2	0	63	61.65	0.350	2	0	33	58.92	0.313
2	0	62	62.47	0.671	2	0	32	60.60	0.038
2	0	61	63.30	0.475	2	0	31	62.58	0.092
2	1	61	64.27	0.238	2	1	31	64.36	0.649
2	1	60	65.32	0.191	2	1	30	65.59	0.503
2	1	59	66.43	0.231	2	1	29	67.81	0.119
2	1	58	67.54	0.415	2	1	28	70.37	0.286
2	1	57	68.63	0.770	2	1	27	72.48	2.116
2	1	56	69.70	1.013	2	1	26	73.97	1.175
2	1	55	70.84	1.101	2	1	25	76.81	0.381
2	1	54	72.03	1.386	2	1	24	79.96	0.882
2	1	53	73.20	1.450	2	1	23	82.30	3.362
2	1	52	74.48	1.165	2	1	22	84.75	1.313
2	1	51	75.90	1.294	2	2	22	88.62	1.273
2	1	50	77.32	2.543	2	2	21	92.07	10.148
2	1	49	78.55	3.765	2	2	20	94.13	5.281
2	1	48	79.90	2.319	2	2	19	98.82	1.658
2	1	47	81.51	1.572	2	2	18	103.67	6.967
2	1	46	83.22	1.681					

close to the $1/[l(l+1)]$ limit (Unno *et al.* 1979) for g -nodedominated modes.) Thus there is a severe overlap in theoretical mode frequencies for all of the observed mode frequencies, which prevents us from obtaining a unique fit of the l value and the number g and p nodes for the observed

modes. Table 5 shows the theoretical modes for the two models which could fit the observed modes if first-order rotational splitting is taken into account. The degeneracy in frequencies appears to be less severe with higher metallicity, which is due to the fact that the $Z=0.06$ model is slightly

TABLE 5. Possible theoretical matches to observed pulsation modes for (a) $M = 2.1 M_{\odot}$, $Z=0.02$ and (b) $M = 2.4 M_{\odot}$, $Z = 0.06$. The theoretical mode closest to the observed frequency is highlighted in bold.

Obs. freq.	Frequency	l -value	g -value	p -value	Comments
(a)					
59.73	59.75	0	0	0	
61.94	59.82	2	65	0	
61.94	60.75	2	64	0	
61.94	61.07	1	37	0	
61.94	61.65	2	63	0	(closest)
61.94	62.46	1	37	1	
61.94	62.47	2	62	0	
61.94	63.30	2	61	0	
61.94	64.27	2	61	1	
61.17	58.93	2	66	0	
61.17	59.82	2	65	0	
61.17	60.29	1	37	0	
61.17	61.07	1	37	0	(closest)
61.17	61.65	2	63	0	
61.17	62.47	2	62	0	
61.17	63.30	2	61	0	
54.83	52.91	2	74	0	
54.83	53.61	2	73	0	
54.83	54.39	2	72	1	
54.83	54.65	1	41	0	(closest)
54.83	55.19	2	71	0	
54.83	56.01	2	70	0	
54.83	56.81	2	69	0	
54.83	57.51	2	68	0	
96.21	95.69	1	25	2	
96.21	95.94	2	40	2	(closest)
96.21	97.74	2	39	2	
99.45	98.53	1	23	2	
99.45	97.74	2	39	2	
99.45	99.85	2	38	2	(closest)
99.45	102.28	2	37	2	
(b)					
59.73	59.73	0	0	0	
61.94	58.92	2	33	0	
61.94	60.60	2	32	0	
61.94	60.86	1	19	0	
61.94	62.58	2	31	0	(closest)
61.94	62.75	1	19	1	
61.94	64.36	2	31	1	
61.17	58.03	2	34	0	
61.17	58.92	2	33	0	
61.17	60.60	2	32	0	
61.17	60.86	1	19	0	(closest)
61.17	62.58	2	31	0	
54.83	52.41	2	38	0	
54.83	53.29	2	37	0	
54.83	54.06	1	21	0	
54.83	54.80	2	36	0	(closest)
54.83	56.53	2	35	0	
54.83	56.80	1	21	0	
96.21	94.13	2	20	2	
96.21	96.59	1	11	2	(closest)
96.21	98.82	2	19	2	
99.45	98.82	2	19	2	(closest)

less evolved in the core and has fewer g -type nodes in the deep interior. A more detailed discussion of the quantitative differences between models with different metallicities will be left for a later paper, but for this particular case, we can say that with the observational data available, we are unable to make a unique fit to δ Scuti with either model. However, we can now say with some certainty that our asteroseismological techniques are no longer limited by the accuracy of our models but by the physical nature of this highly evolved star.

One way in which the difficulty in mode fitting could be overcome is if unambiguous l values can be assigned to these modes observationally. In particular, if several of these modes can be identified as $l=1$ modes, then models would be easier to fit, since the mode spacing is lower for $l=1$ than for $l=2$. There is some evidence that the $61.936 \mu\text{Hz}$ mode is $l=2$, based both on the observations of Smith (1982) and Cugier & Monier (1993), and using geometric cancellation arguments between this mode and the $59.731 \mu\text{Hz}$ radial mode (since an $l=0, l=2$ combination is more likely than $l=0, l=1$ or an $l=1$ multiplet). There is now more research into l -value identification for these stars, and spectroscopic methods are becoming more and more reliable in determining the l values, so it may be possible to fit these models when such identifications become available.

Another possible remedy for this problem would be the observational detection of higher frequency modes (greater than $\sim 150 \mu\text{Hz}$). Our models predict that unstable modes exist up to $\sim 215 \mu\text{Hz}$, and the frequency spacing of these modes is as high as $10 \mu\text{Hz}$. A detection of these modes, along with a detection of their l value, would significantly improve our ability to distinguish between different models. We are currently analyzing shorter subsets of our data with better-behaved window functions to try to detect modes in this range, but there were no significant detections in our five-year data set. We encourage both reanalysis of previous multi-site data as well as new photometric campaigns to try and detect these modes.

5. CONCLUSIONS

We used a previously unpublished set of Strömgren y and b observations of δ Scuti to more accurately determine the observed pulsation spectrum of this star. Analysis of this data set yields a previously unknown mode at $96.214 \mu\text{Hz}$, bringing the number of known discrete, non-beating modes to six.

We also attempted an asteroseismological analysis of δ Scuti using the latest 1996 OPAL opacity tables and newer modeling codes, and determined a range of possible masses assuming both a solar abundance and an enhanced abundance with $[\text{Fe}/\text{H}]=0.49$. Our best-fit models are in good agreement with the observed spectral type (F2 III), luminosity ($33 L_{\odot}$), and radius ($4.1 R_{\odot}$) of δ Scuti, when we assume the highest amplitude mode at $59.731 \mu\text{Hz}$ is the radial fundamental mode to provide an additional model constraint. A detailed asteroseismological analysis of both models has not yielded strong constraints on either the metallicity or the internal structure of this star, because of the close frequency

spacing of the theoretical nonradial modes. However, we were able to confirm that all of the modes in this frequency region are unstable, and we were able to determine that δ Scuti does not have a visible second radial overtone mode, contrary to previous theoretical work. Further constraints such as observational l -value identifications or identifications of higher frequency pulsation modes will significantly enhance our ability to model this star with greater accuracy than is presented here.

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