

Strange stars: how dense can their crust be?*

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Abstract. The strong electric field at the surface of a strange star is discussed, and a self-consistent model is proposed to calculate its capability of supporting a “normal” nuclear material crust. We find that the electric field is already not able to support the crust even when the bottom density of the crust is still considerably lower than the neutron drip point, which means that it is not the neutron drip effect that limits the maximum crust density of a strange star. The maximum crust density is probably only about $8.3 \times 10^{10} \text{ g cm}^{-3}$, so that a typical strange star ($1.4 M_{\odot}$) can not have a crust more massive than $\sim 3.4 \times 10^{-6} M_{\odot}$. Considerable limitations are also presented for strange dwarfs.

Key words: dense matter – elementary particles – stars: interiors; neutron; white dwarfs

1. Introduction

The true ground state of the hadrons may be “strange matter”, not ^{56}Fe (Witten 1984), which, if true, would have implications of fundamental importance for cosmology, astrophysical compact objects, and laboratory physics. From the theoretical viewpoint, strange matter is as plausible a ground state as the confined state of hadrons (Witten 1984; Farhi & Jaffe 1984). For a complete bibliography up to 1991, see Glendenning (1990) and Madsen & Haensel (1991).

If Witten’s hypothesis is true, then there is the very intriguing possibility of the existence of strange–quark matter stars (strange stars) (Alcock et al. 1986a). And if strange stars exist, then all the neutron stars might have been converted to strange stars since the whole Galaxy is likely to be contaminated by strangelets (Glendenning 1990; Madsen & Olesen 1991; Caldwell & Friedman 1991; Medina–Tanco & Horvath 1996). Glendenning et al. (1995a,b) went a step further to suggest that a kind of so-called strange dwarfs might survive, which, as a counterpart of white dwarfs, contain nuclear material up to $\sim 4 \times 10^4$

times denser than in ordinary white dwarfs of average mass, $M \sim 0.6 M_{\odot}$, and still about 400 times denser than in the densest white dwarfs. Owing to the influence of its strange core, such a dense strange dwarf is stable against acoustical vibrations.

Theoretical and observational researches of strange stars might provide scientific basis to test the hypothesis about the true ground state of the hadrons. The most urgent need is to know how to make a distinction between strange stars and normal neutron stars when we are observing. Unluckily it is very difficult to do so, since the maximum mass and radius limits of a strange star are similar to those of a neutron star. However, some clues are still available to help us decide whether we have observed a strange star. First, the stable neutron star mass generally decreases with radius (Baym et al. 1971), while the strange star mass increases in the same mass range (Glendenning et al. 1995a,b). Such a clue is not practical at present time, it seems impossible to measure the radius and mass of a pulsar precisely. In spite of such difficulties, an attempt has been made by Li et al. (1995) recently, who, based on the $M \sim R$ relation of strange stars, proposed that Her X–1 might be a strange star. Second, the minimum rotating period (P_k) of a neutron star can hardly be less than 1 ms, while that of a strange star falls in a smaller range, $0.55 \text{ ms} \leq P_k \leq 0.8 \text{ ms}$ (Glendenning & Weber 1992; Glendenning et al. 1995b; Weber et al. 1994). If we observed a fast pulsar with period less than 1 ms, it would be most probably a strange star.

Some other phenomena related to strange stars might also provide important criterions. For example, once a strange-matter seed presents in a neutron star, the whole star will be quickly converted to a strange star, with a few of 10^{51} ergs of energies being liberated in a detonation mode (Dai et al. 1993), which might be a possible origin of γ -ray bursts (Chen & Dai 1996). In fact, the original aim of the early papers of Wang & Lu (1984, 1985) was to provide an energy source for γ -ray bursts. Alcock et al. (1986b) also tried to explain the γ -ray burst on March 5, 1979 with the help of a strange star. The nuclear crust of a strange star can have a moment of inertia large enough to account for the magnitude of pulsar glitches, although it is still quite unclear whether strange stars can account for all phenomena associated with glitching such as the healing times and recurrence rates

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(Glendenning & Weber 1992). And the cooling mechanism of strange stars is quite different from that of neutron stars, so their surface temperature will be obviously lower, especially for the age of less than 10^5 years. Based on this reasoning, Dai et al. (1993b) proposed that PSR0656+14 might be a strange star.

When the nuclear crust of a strange star being concerned, a maximum bottom density leading to neutron drip was usually assumed in previous theoretical works (Alcock et al. 1986a; Glendenning & Weber 1992; Glendenning et al. 1995a,b; Weber et al. 1994). In this work, we will point out that in fact the electric field is already unable to support the crust even when the bottom density (ρ_c) is still far below neutron drip point (ρ_{drip}). We find the maximum density is probably only about $8.3 \times 10^{10} \text{ g cm}^{-3} \approx \rho_{drip}/5$. This means, the maximum crust mass of a typical strange star ($1.4 M_\odot$) is $\sim 3.4 \times 10^{-6} M_\odot$, one order of magnitude lower than that usually thought based on the neutron drip condition. Our results may be important to the identification of strange stars. Considerable limitations will also be provided for strange dwarfs in the following.

The electric field model of Alcock et al. (1986a) is summarized in Sect.2. The weakness of their model is stressed. Our refined model is derived in Sect.3. We present our numerical results in Sect.4. And the discussions will be given in Sect.5.

2. Normal material crust of a strange star

2.1. Strong electric field at the surface

Because the strange quark mass is larger than that of the up and down quarks, the dense strange matter at equilibrium will contain an approximately equal mixture of all three flavors of quarks, with a slight deficit of strange quarks (Kettner et al. 1995). Since the Coulomb interaction is so much stronger than the gravitational, a star must be charge neutral to very high precision, $\sim 10^{-37}$ net charge per baryon (Glendenning & Weber 1992). The net positive quark charge therefore must be balanced by electrons. At the surface of a strange star, the quarks bound by the confinement of strong interaction have very sharp surface with thickness of the order of 1 fm. On the contrary, the electrons bound by the Coulomb force can extend several hundred fermis beyond the quark surface. So, in a thin layer of several hundred fermis thick above the strange matter surface, a strong electric field is established, it is estimated to be about $\sim 10^{17} \text{ V cm}^{-1}$ and outwardly directed (Alcock et al. 1986a).

The large outward-directed electric field is capable of supporting some normal material, which gives birth to a thin crust. Strange star may acquire its crust during its creation in a supernova, or by accretion from the interstellar medium (Glendenning & Weber 1992; Glendenning et al. 1995b).

2.2. The model of Alcock et al.

A concise model was developed by Alcock et al. (1986a) to describe the gap between the strange core and the crust of a strange star. They proposed that the electric field should be described as follows: (in the unit system of $c = \hbar = 1$)

$$\frac{d^2 V}{dz^2} = \begin{cases} \frac{4\alpha}{3\pi}(V^3 - V_q^3), & z \leq 0, & (1a) \\ \frac{4\alpha}{3\pi}V^3, & 0 < z \leq z_G, & (1b) \\ \frac{4\alpha}{3\pi}(V^3 - V_c^3), & z_G < z, & (1c) \end{cases}$$

where z is a space coordinate measuring the height above the quark surface, α the fine-structure constant, $V_q^3/3\pi^2$ the quark charge density inside the quark matter, V the potential, and V_c the electron Fermi momentum near the base of the crust, z_G the gap width between the strange matter and the crust.

For the nuclear material at the neutron drip point (that is, $\rho_c = \rho_{drip} \sim 4.3 \times 10^{11} \text{ g cm}^{-3}$, $A = 118$, $Z = 36$), Alcock et al. evaluated V_q as 20 Mev, V_c as 10 Mev, and calculated the electric field in their model. They began their calculation at some place $z < 0$ fm, integrating Eq.(1a) with initial values determined by the potential at $z = 0$ fm ($V(z = 0 \text{ fm}) = 0.75V_q + 0.25V_c^4/V_q^3$). After passing through the $z = 0$ fm point, one should integrate Eq.(1b). Their numerical calculation proceeded until $V = V_c$ was obtained at some outer place. They denoted this place as the base of the crust, and the corresponding z as z_G . Because of perfectly neglecting the gravitation, they regarded the crust as completely neutral. Thus V would be constant within the crust, and the Eq.(2c) be in fact unnecessary in their calculating procedure. From numerical results, they drew the following conclusions: (i) the electric field at the surface of a strange star can be as strong as $\sim 5 \times 10^{17} \text{ V cm}^{-1}$; (ii) for nuclear matter of $\rho_c = \rho_{drip}$, $z_G \approx 328$ fm; (iii) the rate for the ions at the base of the crust to penetrate the gap through tunnel effect is very small, so that the crust can safely exist; (iv) the maximum density at the base of the crust is ρ_{drip} , beyond which free neutrons will be present, they will fall into the strange core and be converted into the strange matter. The crust of a strange star is only the counterpart of the outer crust of a normal neutron star. The strange star can not have inner crust.

But such a model is in fact inconsistent. Alcock et al. noticed only the continuity of potential at the base, they derived the gap width z_G without considering the mechanical balance of the system. Although the gravitational forces are tiny compared to the electrical forces across the gap so that we can ignore the gravitational force for any single particle, we can not ignore the gravitational force on the crust as a whole, it should be balanced by an outward-directed force, which stems from the electric field not very strong as the material is nearly neutral. We propose that z_G should be derived by considering the balance between gravitational and electrical force on the crust. Alcock et al. regarded the crust as an equipotential object, they can not consider the mechanical balance.

For the sake of establishing a more consistent model, we will modify the model of Alcock et al., by taking into account the mechanical balance to give the gap width z_G . We find that the electric field is already unable to support the crust when ρ_c is still notably less than ρ_{drip} . So, it is the electric field, not the neutron drip effect, that constrains the maximum mass of the crust.

3. Our revised model

3.1. Supporting effect of electric field to nuclear matter

Based on the work of Alcock et al., we can give a self-consistent model to determine the gap width by taking the balance between the electrical and the mechanical forces into account. Instead of Eq.(1), we should now have:

$$\frac{d^2V}{dz^2} = \begin{cases} \frac{4\alpha}{3\pi}(V^3 - V_q^3), & z \leq 0, & (2a) \\ \frac{4\alpha}{3\pi}V^3, & 0 < z \leq z_G, & (2b) \\ \frac{4\alpha}{3\pi}(V^3 - V_c^3), & z_G < z \leq z_p, & (2c) \\ 0, & z_p < z, & (2d) \end{cases}$$

where z_p is an effective height above which the crust can indeed be regarded as equipotential ($V = V_c$). For $z < z_p$, the potential $V > V_c$, and the electric field $E \neq 0$. Being strongly polarized, nuclear matter between z_p and z_G would thus be pressed outward by the electric field. $z_p - z_G$ is the thickness of the polarized layer which is usually of several hundred fermis thick only.

Now we consider the condition of mechanical balance. The gravitation on the whole crust should be balanced by the force of the electric field on the non-zero charge (very small, net charge of nuclei and electrons) of this thin layer. We could also consider the balance in terms of pressure locally, as the pressure is in fact determined by the gravitation on the whole crust. Consider the thin layer of non-zero charge between z_G and z_p . There is the vacuum below this layer, the corresponding pressure will be zero. On the upper side, the pressure should almost be equal to the pressure (P_c) at the base of the crust, as the thickness of this layer is several hundred fermis only. Thus, the force downward on a unit area of this layer is just the pressure P_c which should be balanced by the “electric pressure” P_E with P_E being defined as the force by the electric field on the charge within a unit area of the thin layer. Here, the gravitation of the strange core on this thin layer has been neglected. Please note, the gravitation on the whole crust can not be neglected, it should balance the electric force, but the gravitation on this thin layer can certainly be neglected. So, the balance condition can simply be written as $P_c = P_E$.

Once the polarized layer be defined and the balance condition given, a self-consistent solution is then available. Our detailed calculating procedure is as follows: for a given density ρ_c (then V_c) and the corresponding pressure P_c (pressure at the crust base), we give an initial value for z_G arbitrarily and begin our integrating of Eq.(2) outward. When $z \leq 0$ fm, what we do is just like that by Alcock et al.. When $0 \text{ fm} < z \leq z_G$, Eq.(2b) is integrated. When $z_G < z$, Eq.(2c) will be integrated until V equals V_c at some place. We stop there and denote the corresponding z as z_p . The layer between z_G and z_p is just the polarized layer, which is not neutral but weakly positively charged. We calculate the positive charge density of nuclear matter in this layer from its nuclide composition (ratio of protons to neutrons or Z/A), assuming the nuclear matter being

thoroughly ionized. We compute the electrostatic force on the polarized layer and obtain the supporting pressure P_E . Usually $P_E \neq P_c$, so we have to adjust z_G , and repeat the aforementioned procedure. We should not stop our procedure until P_E equals P_c , then we get the self-consistent solution.

Obviously, if the total mass of the crust (M_{crust}) is not too large, the needed supporting pressure can be easily provided, and the gap width keeps to be some z_G steadily. If M_{crust} increases, z_G and z_p will decrease, so that the polarized layer will be immersed in a stronger electric field to get a larger supporting pressure. But if M_{crust} is so large that the needed supporting pressure could not be met even if z_G decreases to zero, then the crust would contact with the strange core and nuclear matter would be converted to strange matter.

3.2. Our results

We have recalculated the electric field according to our revised model. In our calculation, the following parameters are assumed (except specially stated): the mass of the strange core is $M_{core} = 1.4 M_\odot$, the radius is $R_{core} = 1.097 \times 10^6$ cm; the bag constant B is $B^{1/4} = 145$ Mev ($B = 57$ Mev/fm³); V_q and V_c corresponding to ρ_{drip} are evaluated to be 20 Mev, 10 Mev respectively, just like what Alcock et al. have done in their paper. V_c values corresponding to other densities are calculated according to the relation: $V_c \propto \rho_c^{1/3}$.

We employ Runge-Kutta-Fehlberg algorithm to get the numerical solution of Eq.(2). The same method is employed when we have to solve the Tolman-Oppenheimer-Volkoff equation (TOV) (also see Shapiro & Teukolsky 1983) inside the strange core and the crust. At the base of the crust, ρ_c , P_c and nuclide composition (Z, A) are evaluated correspondingly according to the results of Baym et al.(1971). Their equation of state (EOS) for low density nuclear matter ($\rho < \rho_{drip}$) is also introduced in our procedure. We assume that the nuclear matter at the base of the crust is thoroughly ionized. The equation of state for strange matter according to the bag model is $P = (\rho - 4B)/3$.

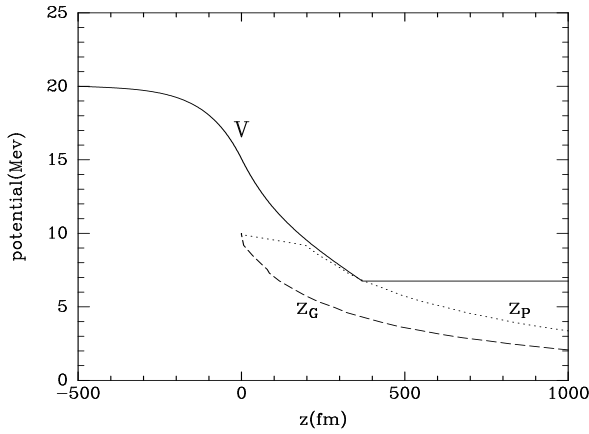
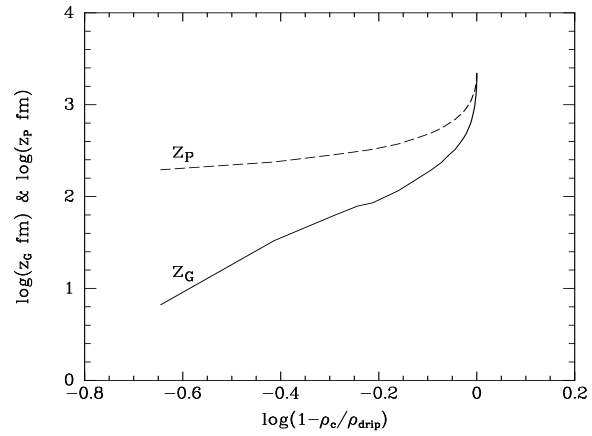
In Fig. 1 we show our solution to Eq.(2) with $\rho_c = 1.32 \times 10^{11}$ g cm⁻³. z_G and z_p thus derived are 116 fm and 374 fm respectively. Dashed and dotted lines show how z_G and z_p are related to V_c . Relations of z_G and z_p to ρ_c are plotted in Fig. 2. Notice that abscissa of 0 corresponds to $\rho_c \rightarrow 0$, while abscissa of $-\infty$ corresponds to $\rho_c \rightarrow \rho_{drip}$ and -0.6 corresponds to $\rho_c \sim (3/4)\rho_{drip}$. We see that z_G is already very small when ρ_c is still far from ρ_{drip} . In fact, when ρ_c equals ρ_{drip} and V_c equals 10 Mev, a meaningful z_G (positive) can not be found. The maximum density at the base of the crust is restricted by the electric field rather than the neutron drip phenomenon.

Some of our numerical results are presented in Table 1, in which ρ_c , P_c , V_c , Z and A are parameters at the base of the crust.

We show in Fig. 3 how the mass of the crust (M_{crust}) varies with z_G , which is also a parameter of great interest. It is clear that z_G is relatively large when M_{crust} is small. For example, z_G is 1258 fm when M_{crust} equals $2.0 \times 10^{-8} M_\odot$, and z_G is 62 fm when $M_{crust} = 1.0 \times 10^{-5} M_\odot$. z_G is only of order of 6.6 fm when M_{crust} reaches $1.8 \times 10^{-5} M_\odot$, in this case, the

Table 1. Some of our numerical results

V_c (MeV)	ρ_c (g cm ⁻³)	P_c (dynes cm ⁻²)	Z	A	z_G (fm)	z_p (fm)	Crust Mass ($M_\odot$)
2.7	8.312×10^9	5.662×10^{27}	32	82	755.2	1230.3	1.71×10^{-7}
3.1	1.318×10^{10}	1.048×10^{28}	32	82	610.3	1070.3	3.18×10^{-7}
3.6	2.09×10^{10}	1.938×10^{28}	32	82	483.1	912.7	5.89×10^{-7}
4.2	3.313×10^{10}	3.404×10^{28}	30	80	377.5	764.8	1.04×10^{-6}
5.0	5.254×10^{10}	5.949×10^{28}	28	78	284.3	626.3	1.81×10^{-6}
5.8	8.332×10^{10}	1.1×10^{29}	28	78	193.2	490.8	3.36×10^{-6}
6.8	1.322×10^{11}	2.033×10^{29}	28	78	116.4	374.8	6.24×10^{-6}
7.3	1.664×10^{11}	2.597×10^{29}	26	76	85.4	325.5	7.99×10^{-6}
7.5	1.844×10^{11}	2.892×10^{29}	42	124	78.8	309.5	8.88×10^{-6}
7.9	2.096×10^{11}	3.29×10^{29}	40	122	62.6	285.5	1.01×10^{-5}
8.5	2.64×10^{11}	4.473×10^{29}	40	122	33.0	237.5	1.38×10^{-5}
9.2	3.325×10^{11}	5.816×10^{29}	38	120	6.6	195.8	1.8×10^{-5}

**Fig. 1.** Electrostatic potential (V) vs. height (z) near the surface of a strange star (full line). Density at the base (ρ_c) is chosen as 1.32×10^{11} g cm⁻³. The dashed line shows V_c vs. gap width (z_G) and the dotted line shows V_c vs. z_p .**Fig. 2.** $\log(z_G)$ (the full line) and $\log(z_p)$ (the dashed line) vs. $\log(1 - \rho_c / \rho_{drip})$. Notice that abscissa of 0 corresponds to $\rho_c \rightarrow 0$ while abscissa of $-\infty$ corresponds to $\rho_c \rightarrow \rho_{drip}$.

crust is very close to the strange core. If M_{crust} got even larger, no positive z_G could be found.

In fact, a minimum value of $z_G \sim 200$ fm was established as the lower bound on z_G necessary to guarantee the crust's security against strong interactions with the strange core (Alcock et al. 1986a; Weber et al. 1994; Kettner et al. 1995). If we adopt 200 fm as a criterion for z_G , then the maximum value of M_{crust} is $3.4 \times 10^{-6} M_\odot$, which is about a magnitude less than that for $\rho_c = \rho_{drip}$, which is about $2.5 \times 10^{-5} M_\odot$. Other corresponding parameters at the base are: $\rho_c = 8.332 \times 10^{10}$ g cm⁻³ $\sim \rho_{drip}/5$, $P_c = 1.1 \times 10^{29}$ dynes cm⁻², $Z=28$, $A=78$, $z_G = 193$ fm. We emphatically point out that $\rho_c \ll \rho_{drip}$ here.

In our model as well as that of Alcock et al., V_q and V_c corresponding to ρ_{drip} are two parameters which can not be determined accurately and might vary in a wide range. We have also considered the effect of their uncertainties. ρ_c versus z_G is plotted in Fig. 4 with V_q equals 20 MeV while V_c equals 5, 8, 10, 12, 15 MeV respectively. The same relation for $V_c = 10$ MeV while $V_q = 15, 18, 20, 25, 30$ MeV is displayed in Fig. 5.

These two figures show clearly that z_G gets less than 200 fm when ρ_c is still less than ρ_{drip} although V_q and V_c vary in wide ranges. So we emphasize that it is a convincing conclusion that nuclear matter at the base of the crust can not be so dense as to reach neutron drip point. It is the electric field, not the neutron drip effect, that limits the crust mass.

4. Conclusions

In previous works, when the crust of a strange star was mentioned, it was generally assumed that the density at the base was limited by neutron drip point. However it is only a specious assumption. We have shown that it is in fact another factor that takes effect. Our results might provide important clues for the observation and identification of strange stars.

As the counterparts of white dwarfs, Glendenning et al. (1995a,b) suggested that there might exist a kind of strange dwarfs with strange matter core enveloped by nuclear matter. A particular character of strange dwarfs is that they might contain nuclear material up to $\sim 4 \times 10^4$ times denser than the ordinary

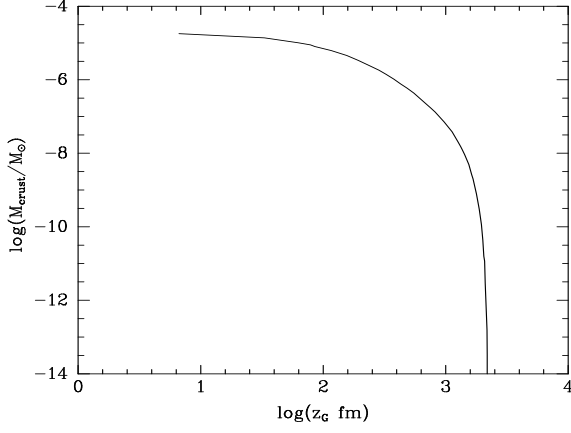


Fig. 3. $\log(M_{crust})$ vs. $\log(z_G)$. The mass of the strange core (M_{core}) is $1.4 M_\odot$, its radius (R_{core}) is 1.097×10^6 cm, which is evaluated by solving TOV equation numerically

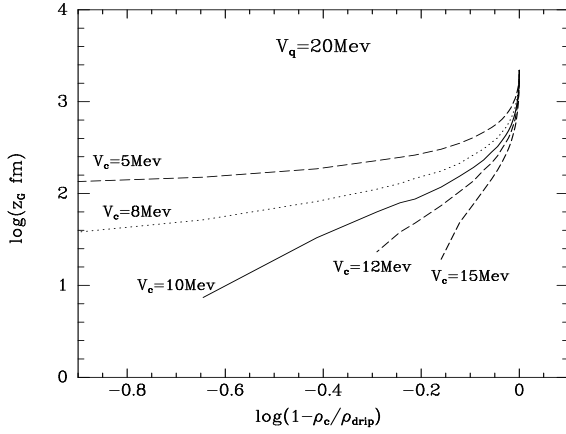


Fig. 4. $\log(z_G)$ vs. $\log(1 - \rho_c/\rho_{drip})$ for $V_q = 20$ Mev. This figure shows the influence of the uncertainty of parameter V_c on z_G . The five lines from top to bottom correspond to $V_c = 5, 8, 10, 12, 15$ Mev respectively

white dwarfs of average mass, $M \sim 0.6 M_\odot$, and still about 400 times denser than the densest white dwarfs. Such dwarfs are denoted as "dense strange dwarfs". Owing to their strange cores, dense strange dwarfs are stable to radial oscillations. They found the entire stable sequence that runs from the maximum-mass strange star to the maximum-mass strange dwarf.

The sequence extending from strange stars to strange dwarfs is characterized by two parameters: the central density of the core (ρ_{core}) and the density at the base of the crust (ρ_c). Glendenning et al. assumed ρ_{drip} to be the maximum value of ρ_c . Our results give a more stringent limitation for ρ_c : $\rho_c \leq \rho_{drip}/5$. We have investigated what such a limitation brings about for the sequence. The $M \sim R$ relation for the whole sequence is shown in Fig. 6 and the $M_{crust} \sim M_{core}$ relation in Fig. 7. In these two figures full lines correspond to $\rho_c = \rho_{drip}$ and dashed lines correspond to $\rho_c = \rho_{drip}/5$. We find that when $\rho_c = \rho_{drip}/5$, the maximum density in the crust will be only $\sim 8 \times 10^3$ times denser than at the center of an average-mass

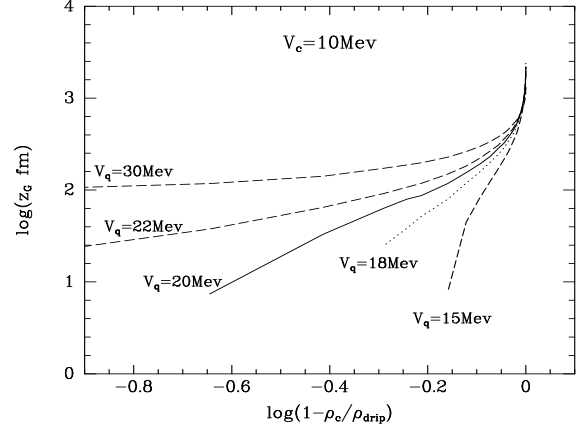


Fig. 5. $\log(z_G)$ vs. $\log(1 - \rho_c/\rho_{drip})$ for $V_c = 10$ Mev. This figure shows the influence of the uncertainty of parameter V_q on z_G . The five lines from top to bottom correspond to $V_q = 30, 25, 20, 18, 15$ Mev respectively

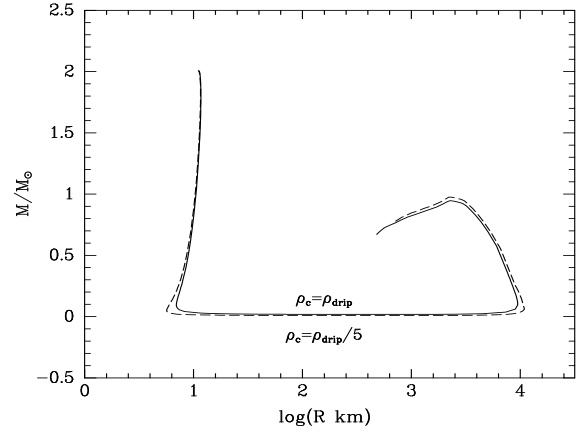


Fig. 6. Mass (M) vs. radius (R) for strange star–strange dwarf sequence. The full line is plotted with ρ_c equals ρ_{drip} and the dashed line with $\rho_c = 8.332 \times 10^{10} \text{ g cm}^{-3}$

white dwarf or ~ 60 times than in the densest white dwarfs. The minimum mass for the sequence is $0.009 M_\odot$, much less than $0.017 M_\odot$ for $\rho_c = \rho_{drip}$. The minimum radius of strange stars also shrinks so that strange stars can rotate even faster.

The crust of normal nuclear matter has strong impact on the thermal evolution of a strange star. This effect should be reconsidered since we have shown that M_{crust} is of a magnitude less than that of previously assumed.

5. Discussion

Alcock et al. discussed the possibility for a strange star to maintain a thin crust. They pointed out that the crust was mainly influenced by two factors. One is tunnel effect through which ions might penetrate the gap. They have shown, the penetrating rate is relatively small. The other factor is the neutron drip effect. The density at the base of the crust can not be denser than ρ_{drip} since free neutrons would come out of nuclei and fall onto

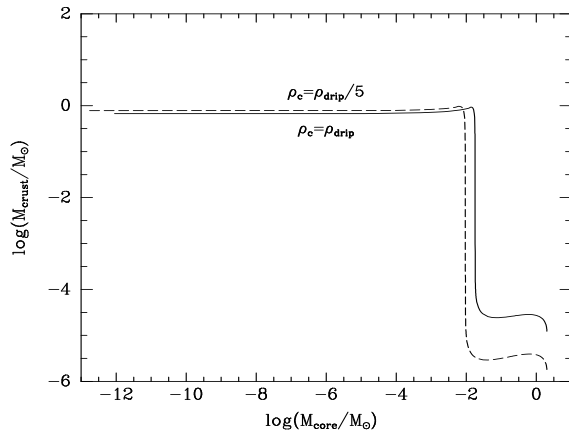


Fig. 7. Crust mass ($\log(M_{\text{crust}})$) vs. the mass of strange core ($\log(M_{\text{core}})$) for strange star–strange dwarf sequence. The full line is plotted with $\rho_c = \rho_{\text{drip}}$ and the dashed line with $\rho_c = \rho_{\text{drip}}/5 = 8.332 \times 10^9 \text{ g cm}^{-3}$

the strange core. We have revised their calculating model for the electric field and found that the second factor will also not be encountered. The electric field is already not able to support the crust before free neutrons begin to emerge enormously. If $z_G = 200 \text{ fm}$ is a lower bound for the crust to be secure, a possible criterion proposed by Alcock et al. (1986a), then the maximum density at the base of the crust is about $\rho_c = 8.332 \times 10^{10} \text{ g cm}^{-3} \sim \rho_{\text{drip}}/5$, giving a maximum mass of $\sim 3.4 \times 10^{-6} M_\odot$ for the crust, which is about one order of magnitude smaller than formerly assumed. However, we have plotted the curve in Fig. 2, giving the density at the base of the crust at various gap height. Our work is the first time to restrict ρ_c below ρ_{drip} through numerical calculation.

As the strange star accretes, the mass of its crust gets larger and larger. When the density at the base of the crust reaches the limit, how the crust will break and fall onto the strange core is a very interesting dynamical question. If it will suddenly collapse, then a burst will happen. This may be the origin of some astrophysical burst phenomenon and will be studied later.

More stringent limitations for the characters of strange stars–strange dwarfs sequence are provided in this work. The maximum density of nuclear matter in strange dwarfs will be only about $\sim 8 \times 10^3$ times denser than in average mass white dwarfs, or about 60 times denser than in the densest white dwarfs. The minimum mass for the sequence will be $0.009 M_\odot$, less than $0.017 M_\odot$, which corresponds to $\rho_c = \rho_{\text{drip}}$.

How to distinguish strange stars and normal neutron stars observationally is a difficult but important problem. Our work might provide an important clues for the puzzling problem, since most of the observable phenomena of strange stars should be related to their crusts.

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