# Efficient Method for Calculating the Time of Sunrise and Sunset

Robin G. Stuart
Randall Physics Laboratory, University of Michigan, Ann Arbor, MI 48190-1120
email : stuart@thoth.physics.lsa.umich.edu

**Abstract**

*An efficient method for calculating the time of sunrise and sunset is given without recourse to sidereal time. The same method may be used to calculate the times of the beginning and end of twilight.*

## 1 Introduction

For most celestial bodies, calculation of the times of rising and setting requires knowledge of the object's declination and its hour angle, which, in turn, is determined from its right ascension and the local sidereal time. In the case of the Sun, as noted by Sinnott (1994), things are quite different. The mean solar day forms the basis for civil timekeeping and hence the time of sunrise and sunset should be computable without recourse to sidereal time.

In this article compact expressions are derived for the times of rising and setting of the Sun. The method described here forms the basis of a short program that appeared previously (Stuart, 1995).

The Mean Sun is an imaginary body that travels along the celestial equator at a uniform rate. The real Sun and the Mean Sun pass through the vernal equinox at the same moment. The centre of the Mean Sun rises at 6:00 at an azimuth of 90° and sets at 18:00 at an azimuth of 270° all year round. This statement implicitly assumes that the observer is at the centre of his or her timezone and ignores the effect of atmospheric refraction. Also it ignores complex and unpredictable irregularities in the Earth's rate of rotation that have to be compensated for by the addition of leap seconds. Since this is never allowed to grow to much more than a second, it will not be considered further. The times of rising and setting of the real Sun, $T_R$ and $T_S$, can be obtained from those of the mean Sun by applying an number of corrections.

Expressed in hours

$$T_R = 6 + C_1 - C_2 + C_3 \qquad (1)$$

$$T_S = 18 + C_1 - C_2 - C_3 \qquad (2)$$

$C_1$ is the correction for the observer's longitude within their timezone,

$$C_1 = T_Z - \frac{\lambda}{15} \tag{3}$$

where $T_Z$ is the observer's timezone in hours and $\lambda$ is the observer's longitude in degrees. (Both are taken to be negative west of Greenwich.)

The real Sun travels along the ecliptic and may lead or lag the Mean Sun by an amount given by the equation of time which we denote as $C_2 = l - \alpha$, with $l$ being the mean longitude and $\alpha$ is the right ascension of the real Sun.

In equations (1) and (2), $C_3$ is a correction for the rising and setting times due to the declination of the Sun and the effect of atmospheric refraction. Sunrise or sunset is deemed to occur when the centre of the Sun is 50′ below the horizon. Mean refraction then elevates the upper limb to graze the horizon. When an object is at an altitude $h_0$, its hour angle, $H$, is

$$\cos H = \frac{\sin h_0 - \sin \Delta \sin \phi}{\cos \Delta \cos \phi} \tag{4}$$

where $\Delta$ is the object's declination and $\phi$ is the observer's latitude. Since the centre of the Mean Sun rises due East and sets due West, the correction $C_3$ is then the complement of $H$. Thus

$$\sin C_3 = \frac{\sin h_0 - \sin \Delta \sin \phi}{\cos \Delta \cos \phi} \tag{5}$$

When the right hand side of equation (5) is greater than 1 the Sun remains below the horizon and remains above when it is less than −1.

The R.A. and declination of the Sun required in the above formulas can be found from the relations

$$\tan \alpha = \cos \epsilon \tan (l + \upsilon - M) \tag{6}$$

$$\sin \Delta = \sin \epsilon \sin (l + \upsilon - M) \tag{7}$$

where $\epsilon$ is the obliquity of the ecliptic and $\nu$ and $M$ are the Sun's true and mean anomalies respectively. The quantity $l - M$ is the argument of perihelion denoted $\omega$. Note also that the true anomaly may be represented as a trigometric series in the mean anomaly

$$\upsilon - M = (2e - \frac{1}{4} e^3) \sin M + \frac{5}{4} e^2 \sin 2M + \frac{13}{12} e^2 \sin 3M + \ldots \tag{8}$$

in which $e$ is the eccentricity of the Earth's orbit. The azimuth of sunrise or sunset, measured from North is given by

$$\cos A = \frac{\sin \Delta - \sin \phi \sin h_0}{\cos \phi \cos h_0} \tag{9}$$

The quantities $l$, $M$, $e$ and $\epsilon$ can be obtained from Meeus (1991)

$$l = 280.466° + 36000.770°\ T \tag{10}$$
$$M = 357.529° + 35999.050°\ T \tag{11}$$
$$e = 0.016709 - 0.000042\ T$$
$$\epsilon = 23.439° - 0.013°\ T \tag{12}$$

where $T$ is the time in Julian centuries since J2000.0,

$$T = JD - \frac{2451545.0}{36525}\ . \tag{13}$$

Ideally the corrections $C_2$ and $C_3$ should be evaluated at the moment of sunrise and sunset. This requires iteration of the formulae Equation (1) - Equation (8) until satisfactory convergence is obtained. For most purposes, however, it is sufficient to evaluate them at local mean noon.

Large variations in atmospheric refraction near the horizon make an accuracy of better than about a minute of dubious value (Schaefer, 1990). The foregoing formulae give the position of the Sun to about 1″, but considerable simplification may be obtained by assuming that sunrise and sunset occur at the same time on any given day of the year. If $N$ is the day of the year then, in radian measure, the mean longitude at local mean noon is given by

$$l = 4.8857 + 0.017202\ (N + 0.5 - \lambda/360). \tag{14}$$

with $\lambda$'s being expressed in degrees. The first term on the right hand side is the mean longitude, in radians, of the Sun at $12^h$ on 1993 December 31, a time roughly midway between two leap-year days. The constant 0.017202 is the Sun's mean daily motion, also in radians. $N$ for a given day $d$ in month $m$ can be computed from the formula (Nautical Almanac, 1978)

$$N = int\left(\frac{275\,m}{9}\right) - 2\ int\left(\frac{m + 9}{12}\right) + d - 30 \tag{15}$$

that assumes that the year is a common year. The errors generated by this approximation will thus be greatest in leap years.

To an excellent approximation, it follows from Equations (10) and (11) that $M = l + 77.06°$, or in radians, $M = l + 1.3450$. Hence Equation (8) becomes

$$\upsilon - M = 0.3342\ \sin\ (l + 1.3450) \tag{16}$$

keeping only terms linear in $e$. The R.A. and declination of the Sun can be obtained from Equations (6) and (7),

$$\tan\ \alpha = 0.9175\ \tan\ (l + \upsilon - M) \tag{17}$$
$$\sin\ \Delta = 0.3978\ \sin\ (l + \upsilon - M).$$

and hence the equation of time, $C_2$, may be written in a form that is convenient for machine evaluation as

$$C_2 = \tan^{-1} \tan (l + \upsilon - M) - \tan^{-1} (0.9175 \tan (l + \upsilon - M)) - (\upsilon - M). \qquad (18)$$

All angles here are expressed in radians and $C_2$ may be converted to hours by multiplying by $12\pi$. The use of the sequential $\tan^{-1} \tan$ ensures that the first and second terms lie in the same quadrant and can be safely subtracted.

Finally $C_2$ may be obtained from Equation (5) using the substitution $\sin h_0 = -0.0145$. The result in radian measure can again be converted to hours by multiplication by $12\pi$.

The formula (Equation (9)) for the azimuth of sunrise or sunset becomes

$$\cos A = \frac{\sin \Delta = 0.0145 \sin \phi}{\cos \phi} \qquad (19)$$

where to sufficient accuracy we have set $\cos h_0 = -1$.

The procedure for calculating the time of sunrise and sunset may now be summarized as follows: 1) calculate $C_1$, the correction for the observer's longitude and timezone, as given in Equation (3); 2) calculate the day of the year, $N$, using Equation (15); 3) from $N$ calculate the Sun's mean longitude, $l$, using Equation (14) and from it, the difference between the true and mean anomalies, $\nu - M$, by means of Equation (16); 4) calculate the equation of time, $C_2$, from $l$ and $\nu - M$ using Equation (18); 5) determine the Sun's declination from $l$ and $\nu - M$ using Equation (17) and from it the correction for the declination and refraction, $C_2$, by means of Equation (5); and 6) when $C_3 > 1$ the Sun never rises and when $C_3 < -1$ it never sets. If $|C_3| \leq 1$ then the local time of sunrise and sunset are given by Equations (1) and (2) by substituting for $C_1$, $C_2$ and $C_3$. A short BASIC program implementing these steps can be found in Stuart (1995).

The procedure as described above may be used to calculate the beginning and end of civil, nautical and astronomical twilight by substituting $-6°$, $-12°$ and $-18°$ respectively for $h_o$ or equivalently $-0.1045$, $-0.2079$ and $-0.3090$ for $\sin h_o$.

## 2 References


Meeus, J., 1991, *Astronomical Algorithms*, Willman-Bell, Richmond.

Nautical Almanac Office, 1978, *Almanac for Computers for the Year 1978*, U.S. Naval Observatory, Washington).

Schaefer, B. E. and Liller, W., 1990, *Publ. Astron. Society of the Pacific*, **102**, 796.

Sinnott, R.W., 1994, *Sky & Telescope*, **88**, 84.

Stuart in Sinnott, R.W., 1995, *Sky & Telescope*, **89**, 84.