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PEURBACH AND MARĀGHA ASTRONOMY? THE EPHEMERIDES OF JOHANNES ANGELUS AND THEIR IMPLICATIONS

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... when they come to model heav'n
And calculate the stars, how they will wield
The mighty frame, how build, unbuild, contrive
To save appearances, how gird the sphere
With centric and eccentric scribbled o'er,
Cycle and epicycle, orb in orb.
Milton, *Paradise Lost*, Book VIII

1. INTRODUCTION

In 1510 and 1512 Johannes Angelus (also called Engel), a Master of Arts, medical doctor and mathematician in Vienna, published annual ephemerides. Providing daily planetary positions on the verso and daily planetary aspects on the recto sides of the folios, Angelus's almanacs exactly mimic the printed format established for this genre by the 1474 ephemerides of Johannes Regiomontanus and continued by Johann Stöffler and Jacob Pflaum in their *Almanach nova* of 1499.[1] Yet Angelus entitled his works *Almanach novum atque correctum* and in their prefaces boasted that they offered planetary longitudes more accurate than the "common almanacs" because he had computed his from "new tables of planetary equations". Angelus claimed to have found these new tables partially completed in a manuscript "corrected and emended" by Georg Peurbach, and "with the help of the Almighty" to have finished them himself. Furthermore, Angelus promised "a more extensive work begun a long time ago" that would reveal the errors in existing tables and his proposals for correcting them.[2] But Angelus died in September 1512, and neither his "more extensive work" nor his "new tables of planetary equations" has been found.

Angelus's claim to have "corrected" planetary theory did not go unnoticed. Already in December 1511, a member of the Faculty of Arts in Cracow, Stanislaus Cracoviensis (also called Aurifaber), severely criticized Angelus for departing from the accepted Alfonsine tradition. Georg Tanstetter Collimitius, medical professor at the University of Vienna and a prolific writer of astrological prognostications and calendars, lauded Angelus in his 1514 *Viri mathematici* as an "excellent astronomer", skilled "in correcting books and tables", who had "tried to complete the tables of planetary motions of Master Georg Peurbach". In the

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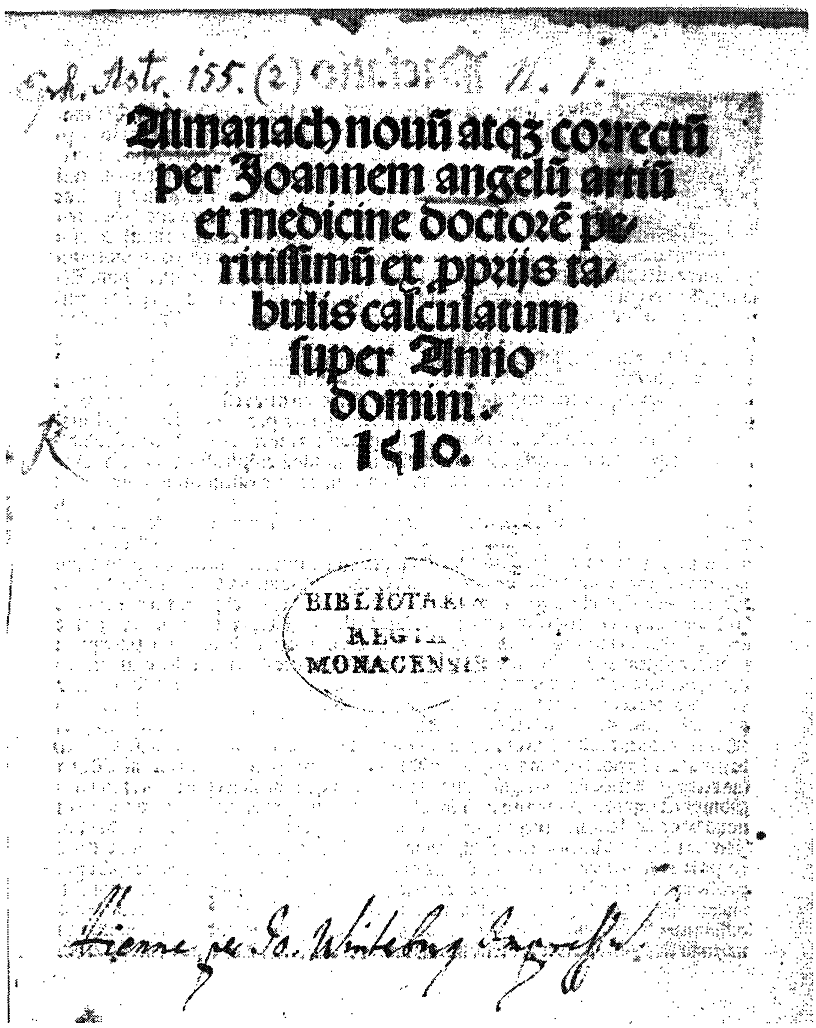

FIG. 1. Angelus, *Almanach novum atque correctum* (Vienna, 1510), title page.

Narratio prima (1540), Georg Joachim Rheticus noted that Copernicus had promised to "draw up astronomical tables with new canons and that if his work had any value he would not keep it from the world, as was done by Johannes Angelus among others".[3] Nothing in these brief comments implies that Aurifaber, Tanstetter or Rheticus knew anything about Angelus's efforts beyond what they had read in the prefaces to the 1510 and 1512 almanacs. All subsequent secondary literature has simply repeated Angelus's own claims or those of Tanstetter concerning the "corrections" in the almanacs. To the best of our knowledge, no one has analysed Angelus's almanacs or attempted to determine whether he did indeed modify or "correct" the received Alfonsine planetary tables.

This paper will offer such an examination of Angelus's ephemerides. In the second section of this essay, we will seek to reconstruct the missing planetary tables from which Angelus computed his ephemerides. By comparing his planetary positions with those of Stöffler's and Pflaum's Alfonsine-based almanacs and with positions generated from the first published edition of the so-called *Alfonsine tables* (1483), we will show that Angelus's results deviate significantly from the Alfonsine longitudes, just as he claimed in his prefaces. Furthermore, we will suggest that these non-Alfonsine longitudes may be reproduced by adding several mechanisms of harmonic motion to the standard Ptolemaic geometry, and will indicate how such mechanisms can be implemented within the computational procedures of the *Alfonsine tables*. In the third section of this essay, we will discuss various geometrical (or physical) alterations to the Ptolemaic planetary models which might generate the non-Ptolemaic harmonic motions revealed in Angelus's reconstructed tables. In the absence of any textual evidence, we will speculate that Peurbach (rather than Angelus) probably constructed the modified models in their entirety, and that he may well have employed mathematical techniques from the Marāgha School (the Ṭūsī couple or Ibn al-Shāṭir's double epicycle). This may indicate that the use of harmonic operators, as elaborated by Marāgha astronomers, spread through Vienna to such early sixteenth-century Latin astronomers as Angelus, Johann Werner and Copernicus.

2. RECONSTRUCTING ANGELUS'S LOST PLANETARY TABLES

At first glance, Angelus appears to be simply one of the many medical astrologers active around 1500.[4] In 1468, he began his university studies in Vienna, earning a baccalaureate in 1471. In 1472, he matriculated at the newly established university in Ingolstadt, where two years later he received his Master of Arts degree and began lecturing on topics in logic and grammar. As Christoph Schöner recently has discovered, in 1475 Angelus and several other Ingolstadt masters founded a short-lived *communitas* in which, independently from the university, they hoped to offer lectures on humanistic topics to subscribers. The society soon fell apart for financial reasons, and by 1479 Angelus had matriculated in Ingolstadt's medical faculty. Continued financial problems, however, forced him to leave the university in 1484, when he published the first of his single-sheet astrological calendars. His best-known work, the *Astrolabium planum in tabulis ascendens* (Augsburg, 1488, ²1494, ³1502), was compiled largely from the astrological writings of Julius Firmicus Maternus and Pietro d'Abano. For the Augsburg printer, Erhard Ratdolt, Angelus emended Regiomontanus's *Opus tabularum directionum profectionumque* (1490) and several important astrological treatises — Albumasar's *De magnis conjunctionibus* (1489), Pierre d'Ailly's *Concordantia astronomiae cum theologia* (1490), and Guido Bonatti's *Decem tractatus astronomiae* (1491). In 1492, Angelus was appointed to the recently created chair for mathematics at the university in Ingolstadt, where he presumably taught astrology to medical students. By 1498, he was practising medicine in Krems. Angelus moved to Vienna around 1500, where he apparently

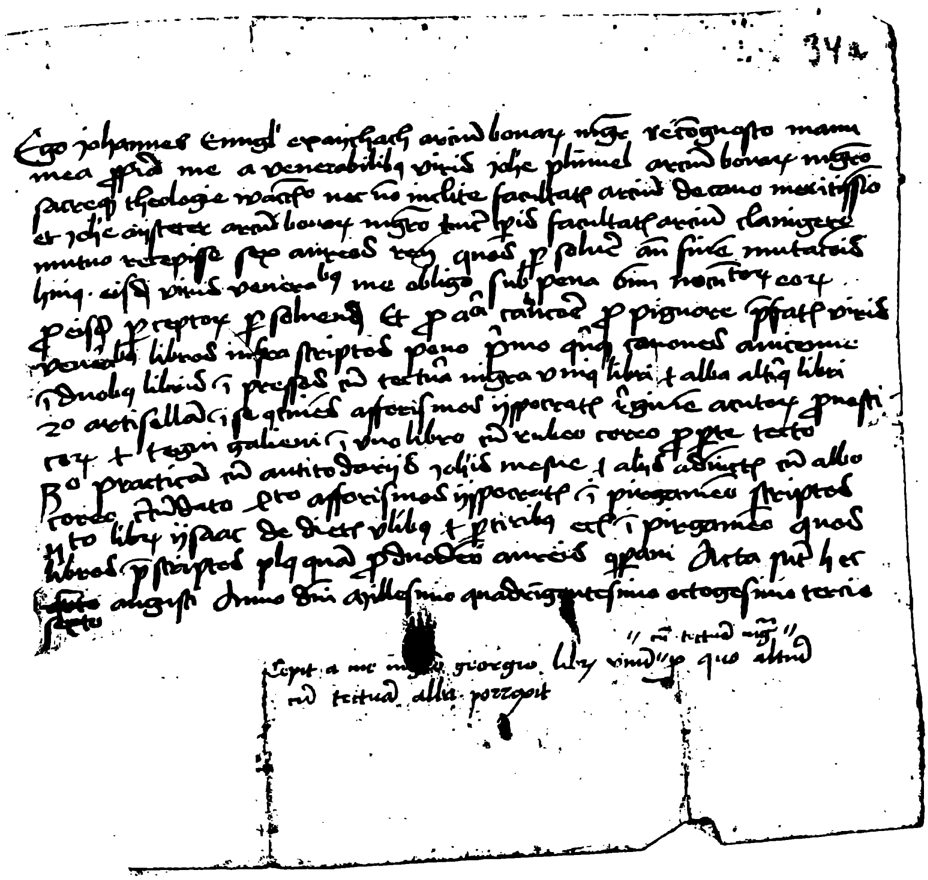

FIG. 2. Angelus, signed autograph, 1483, offering five medical books as collateral for a loan from the Ingolstadt Faculty of Arts. Munich, Ludwig-Maximilian-University Archive, O-V-I, f. 34a^r (reproduced with permission).

continued his medical practice. He did not formally teach at the university, but did become acquainted with the circle of humanist mathematicians around Conrad Celtis that included Andreas Stiborius and Georg Tanstetter.[5] While in Vienna, Angelus published the almanacs for 1510 and 1512; his iatromathematical treatise on the plague was printed in 1518 in Augsburg, six years after Angelus's death.

Prior to 1510, Angelus's printed works contain no criticisms of received astronomical traditions and reveal only a modicum of mathematical astronomy.[6] Between 1484 and 1497 he published at least thirteen single-sheet annual calendars or slightly longer "practicae" that present astrological advice and the times of new and full moons as well as any lunar or solar eclipses that were to occur during the year in question.[7] Quite typical for this popular printed genre, Angelus's calendars never claim to be innovative or "corrected". In the several cases we have examined, his lunar and solar (and occasional planetary) predictions remain strictly Alfonsine. Indeed, it seems likely that Angelus did not compute the lunar and solar positions for his early calendars, but rather derived these data from Regiomontanus's ephemerides for 1475–1506, first published in 1474.[8]

Although histories of astronomy and some bibliographies of incunabula have long attributed an *Ephemerides coelestium motuum usque ad annum 1500* (Vienna, 1494) to Angelus, such a work probably never existed.[9] Angelus's first published criticism of received planetary theory and his first attempt to modify that theory appear in his 1510 *Almanach novum atque correctum*. It is in this almanac, and in that for 1512, that we find an interest in mathematical astronomy that sets Angelus apart from his contemporary iatromathematicians. And it is to these two almanacs that we must turn if we are to reconstruct the now lost mathematical tables Angelus claimed to have used in preparing his "corrected" planetary positions.

2(i). *Evidence from the Prefaces*

In the prefaces to these almanacs (see Appendices 1 and 2), Angelus explicitly located his "corrections" within a pragmatic, astrological context. "[W]ith a view to the restoration of the most noble science and for the usefulness of prognostication", he sought to correct the common tables' predictions of planetary motions. Angelus called for critics who "could reveal light from darkness and truth from errors, and who, for the glory and utility of all of astronomy, could complete the unfinished labours of past correctors".[10] Peurbach and Regiomontanus, he noted, also had challenged the accuracy of the received astronomical tables. Had he lived longer, Regiomontanus would have published a "carefully corrected" ephemerides. In Vienna, Angelus himself, along with Stiborius, had observed planetary conjunctions that deviated from the predicted dates by several days. Angelus promised that his corrected ephemerides, whose longitudes, he claimed, often differed by one, two or even three degrees from the "common almanacs", would more accurately predict the planetary motions. To buttress this claim, he compared fifteen of his predictions with those of the previous almanacs, promising that his would prove to be truer (see Appendix 3). Not once in the prefaces did Angelus offer any physical, philosophical or methodological critiques of Ptolemaic or Alfonsine astronomy; his rhetoric emphasizes exclusively the need to improve the accuracy of predictions for the astrologers. That is, Angelus (at least in his prefaces) took a stridently instrumental approach to mathematical astronomy.

In the 1510 and 1512 prefaces, Angelus explicitly refused to disclose the details of his modified theory ("my secrets"), promising to do so only in a later, larger work. In 1510, Angelus merely stated that he himself had "composed new tables". Yet in a vitriolic, polemical exchange in the 1512 preface with an unnamed Cracovian astronomer (whom we have identified as Stanislaus Cracoviensis), Angelus offered a crucial disclosure about the larger project: it was Peurbach who had initiated the correction of the tables. According to Angelus, Peurbach in a new table of planetary equations, which "for a long time I always have had at hand", had introduced corrections in the equations of the argument "especially in the lower part of the epicycles", i.e., for Mercury and Venus around lower conjunctions with the Sun, and around oppositions for the upper planets. Had Peurbach lived longer, continued Angelus, he would have finished emending the equations of the argument for the upper part of the epicycles, and also would have corrected the equations of the centre.[11] It remained for Angelus to complete his predecessor's modifications of the planetary equations and then to employ the new tables to compute the almanacs for 1510 and 1512.

2(ii). *Evidence from the Ephemerides*

To seek the underlying structure of Angelus's now lost planetary tables, we compared his longitudes against longitudes of our computer-generated "Alfonsine ephemeris", which essentially duplicates the procedures of the 1483 published edition of the Alfonsine Tables (see Appendix 4). To explore how closely fifteenth-century known Alfonsine computations match our "Alfonsine ephemeris", we also examined Stöffler's and Pflaum's longitudes for the same years, 1510 and 1512. In both cases, we compared the longitudes at five-day intervals (two-day-intervals for Mercury), after having corrected some easily recognizable typographical errors in both almanacs.[12]

Our search has been limited to tables for planetary longitudes. In the 1510 preface, Angelus stated that although he possessed modified solar and lunar theories he would not use them in this almanac since their predictions "differ but little" from those of the standard (i.e., Alfonsine) tables. He promised, however, in "future years" to use these modified tables for the major luminaries. Angelus's 1510 positions for the Sun, Moon and lunar nodes do exactly match those in Stöffler and Pflaum. In 1512, Angelus's lunar nodes again correspond exactly to Stöffler and Pflaum. His 1512 solar and lunar longitudes were computed for a different meridian than Stöffler's and Pflaum's (see below), but vary on the average by less than 0;10° from Alfonsine positions. Hence, even if Angelus did employ modified solar and lunar tables in 1512, nothing thereof can be discerned from the longitudes for those bodies listed in the 1512 almanac. Furthermore, in 1512 Angelus included not only planetary longitudes but also latitudes, at 10-day intervals. His latitudes for the outer planets agree exactly with those in Stöffler and Pflaum. For the inner planets, Angelus's latitudes deviate randomly from Stöffler and Pflaum by up to ± 0;10°. Yet in the 1512 preface, Angelus did not include latitudes in the comparisons he offered between his "corrected" positions and those of the "common almanac". As with the Sun and Moon for 1512, even if Angelus had used modified tables for planetary latitudes, these would be impossible to reconstruct solely from the evidence in the almanacs — and no other evidence is at hand.

Stöffler and Pflaum specified Ulm as the meridian of their almanac, giving it the "zero" longitude in the list of geographical longitudes for 65 localities they appended to their ephemerides. Angelus, whose table of 24 localities (including neither Ulm nor Toledo) matches exactly the geographical longitudes in Stöffler and Pflaum, did not identify his 1510 meridian. Yet since his 1510 solar and lunar longitudes correspond to those in Stöffler and Pflaum, it seems reasonable to assume that Angelus computed his 1510 positions for what he understood to be Stöffler's and Pflaum's meridian. By comparing lunar longitudes from the almanacs against our computer-generated Alfonsine positions, we have determined that both 1510 almanacs were calculated for a meridian of 16.4° E of Toledo (the Alfonsine meridian).[13] For his 1512 almanac, Angelus claimed to have employed "the same meridian that Master Johannes Regiomontanus used in his calculations".[14] Regiomontanus, whose table of geographical longitudes differs considerably from Stöffler's and Pflaum's, had listed Ulm and Nuremberg as his meridional cities, and had computed his ephemerides for a longitude of 21.1° E of Toledo (exactly matching the Toledo–Nuremberg meridional difference in his table). Angelus's 1512 lunar longitudes, however, best match a meridian of 19.1° E of Toledo.[15] For all our comparisons, we have used a meridian of 16.4° E for Stöffler and Pflaum and for Angelus's 1510 almanac, and a meridian of 19.1° E for Angelus's 1512 almanac.

Our "Alfonsine ephemeris" matches Stöffler's and Pflaum's positions for all the planets but Mars to ±0;05° of longitude or less. For Mars, the deviations can rise to slightly more than ±0;10° (see Figure 3). Similar evaluations of several other contemporary daily ephemerides yield similar results; hence, on empirical grounds we conclude that the "noise" in late fifteenth-century Alfonsine computation of planetary longitudes was about 0;10°.[16] Yet Angelus's longitudes for 1510 and 1512 deviate by much larger amounts from the Alfonsine computed positions: by nearly 3° for Mars and Jupiter, by nearly 2° for Venus, and by more than 1° for Saturn and Mercury (see Figures 5(a)–(e)). Clearly, the magnitude of the deviations in Angelus's longitudes far exceeds the usual noise in Alfonsine computation. Furthermore, the deviations appear to be regular rather than random. Hence these comparisons corroborate Angelus's claim that he computed his ephemerides from a modified set of Alfonsine planetary tables and not from tables based on an entirely new set of predictive models.

The instrumentalist rhetoric of Angelus's prefaces also prompted us to compare his longitudes and those of Stöffler and Pflaum against a modern ephemeris, the accuracy of which is better than 0;01° for sixteenth-century dates.[17] As might be expected, the modifications introduced by Angelus did not substantially improve his predictions. Indeed, if anything, Angelus's "new and corrected" almanacs would have predicted the motions of the planets less accurately than did Stöffler's and Pflaum's Alfonsine-based almanac (see Appendix 3 for an analysis of the specific comparisons Angelus presented in his prefaces).[18] Nonetheless, even if Angelus would not have provided improved longitudes for sixteenth-century readers, his pragmatic emphasis on predictive accuracy is unmistakable and links him to Regiomontanus who also hoped to improve the exactness of astronomical tables.

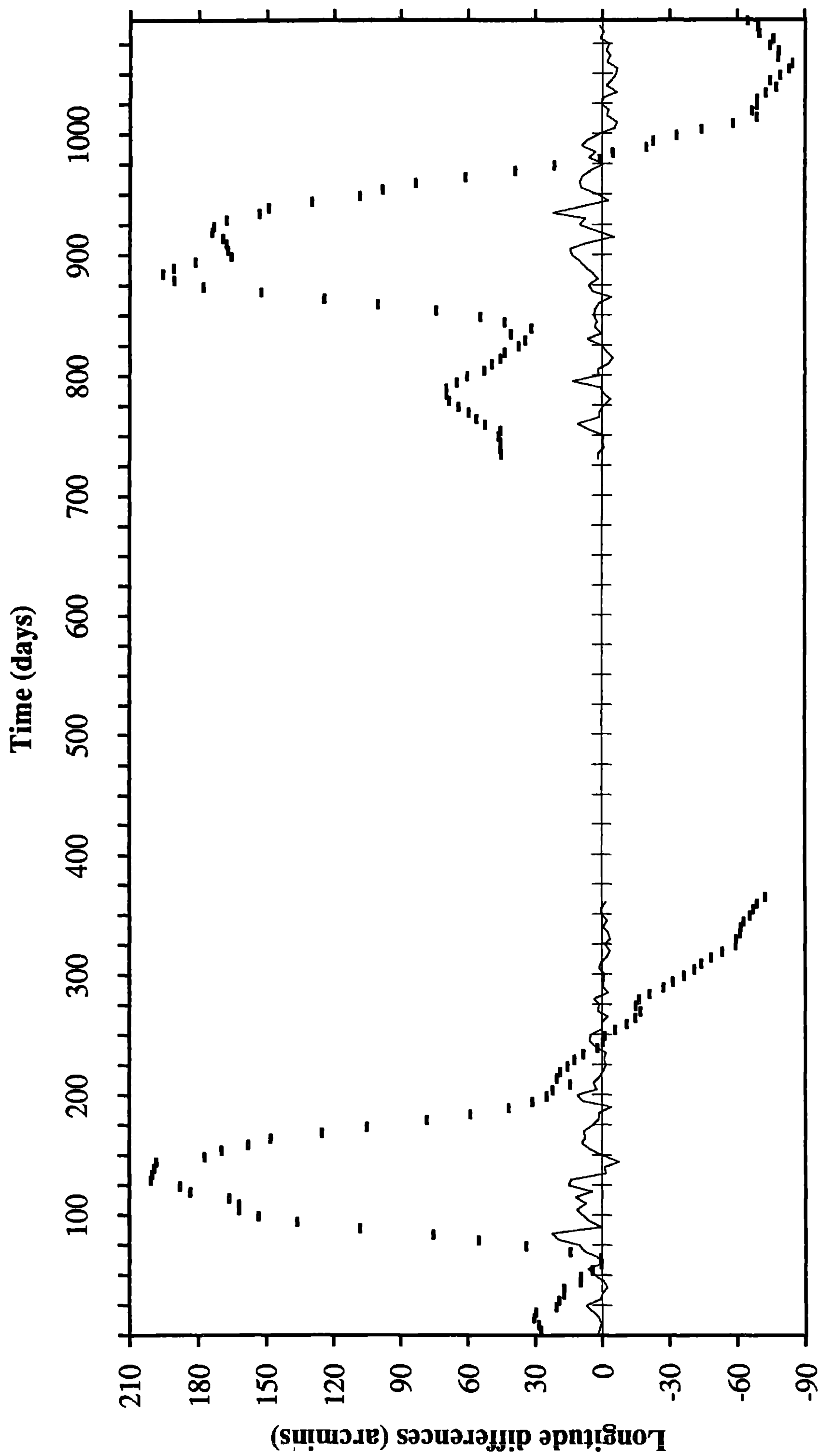

FIG. 3. Alfonsine comparisons. Mars, longitudes for 1510–12, at five-day intervals. Angelus minus 1483 *Alfonsine tables* (bold points), Stöffler and Pflaum minus 1483 *Alfonsine tables* (solid line).

2(iii). *The Mathematical Algorithms Underlying Angelus's Lost Planetary Tables*

Being unable to find any relevant manuscripts, we must attempt a reconstruction of Angelus's tables solely from the evidence in the prefaces and the ephemerides. We have sought mathematical algorithms that:

(i) reproduce Angelus's longitudes within the noise level of fifteenth-century Alfonsine computation, i.e., to ± 0;10° of arc;
(ii) operate within the basic structure and procedures of the Alfonsine Tables, i.e., employ single-entry rather than double-entry tables, and emend the equations of the argument and the centre as described by Angelus in the 1512 preface;
(iii) require as few additional mechanisms as possible, and offer common mechanisms for as many planets as possible.

That is, we have assumed that, despite the bold rhetoric of the prefaces, Angelus (and Peurbach) would have remained mostly traditional and would have tinkered with existing models rather than inventing radically new predictive theory.

As is well known, the most widespread and straightforward among medieval attempts to 'correct' Ptolemy involved simple modifications of the parameters within otherwise unaltered Ptolemaic models. Drawing on their own observations, Islamic *zīj*-makers such as al-Battānī or Ibn Yūnus reset the positions of the radices, the eccentricities, the radii of the epicycles, and some of the mean motions of the *Almagest*. Levi ben Gerson, whose lunar models replace Ptolemaic epicycles with a radically innovative mechanism Bernard R. Goldstein has called "reflected angles", nonetheless apparently accepted Ptolemy's planetary models and sought by means of his own observations only to improve their radices. Likewise Regiomontanus, in his most sustained criticism of received planetary theory, compared Alfonsine predictions against his own observations and concluded that the Alfonsine parameters "have not been found entirely accurately". As Otto Neugebauer has noted, "every attempt at a revision of the foundations of [Ptolemaic] planetary theory must have appeared, rightly, as a gigantic task, not lightly to be undertaken in view of the consistency of the structure erected in the *Almagest*".[19] Hence we initially suspected that Angelus (and Peurbach) would have "corrected" the tables merely by modifying the Alfonsine parameters.

Yet our explorations soon convinced us that Angelus, although retaining the structures of Ptolemaic theory, did not simply adjust its parameters. For example, Angelus's deviations from Alfonsine longitudes for Mercury reveal a clear dependency on the true argument. A slight decrease in the radius of Mercury's epicycle produces a model that matches half of Angelus's peaks; yet the additional peaks indicate a second modification in the Ptolemaic machinery, also dependent on the true argument. Likewise for Jupiter, a decrease in the Alfonsine eccentricity yields a model that reproduces some features of Angelus's curve but not others. Revising Alfonsine parameters does not enable us to meet our

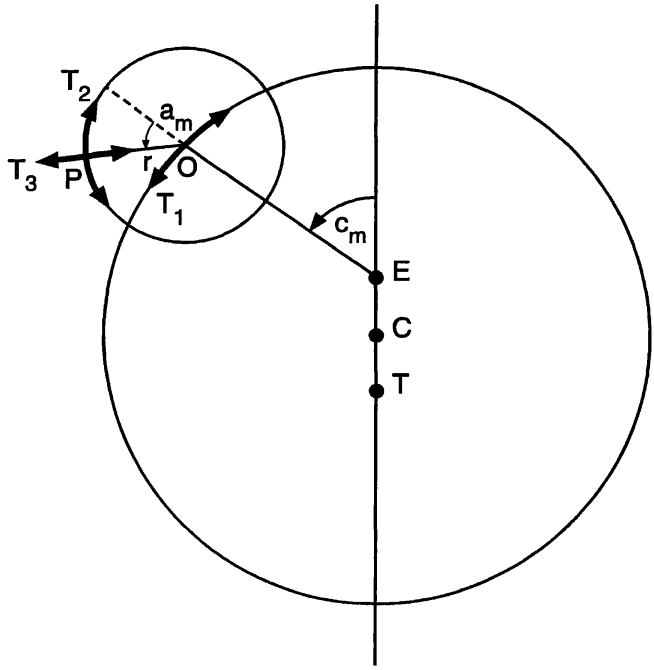

FIG. 4. Harmonic mechanisms added to the Ptolemaic model, not to scale. Maximal amplitudes of oscillation given by T_1, T_2 and T_3, respectively.

first criterion in reconstructing Angelus's lost planetary tables.

Likewise, we cannot reproduce Angelus's longitudes by modifying the geometry of Ptolemy's models in obvious ways, i.e., by adding an epicycle, by moving the equant point to the centre of the deferent, or by placing the equant at some distance not equal to the eccentricity from the centre of the deferent.[20] Only by adding several mechanisms of harmonic motion to otherwise unmodified Ptolemaic geometry and Alfonsine parameters have we been able to generate longitudes that match those in Angelus's ephemerides. Apparently, Angelus formulated three such mechanisms, only two of which he applied to any given planet. In what follows, the primed values are those altered by the harmonic operators.[21]

The first two mechanisms add angular harmonic motion to the mean centre (c_m) and the mean argument (a_m) (see Figure 4). With T_1 and T_2 as the maximal amplitudes (in radians) for the variations of the mean centre and the mean argument, respectively, the modified model requires the following replacements in the Ptolemaic algorithms:

$$c'_m = c_m + T_1 f(c_m), \quad [1]$$

and

$$a'_m = a_m + T_2 \sin(a_m). \quad [2]$$

Angelus employed the mechanism on the mean centre only for the superior planets. In [1], $f(c_m) = \sin(c_m)$ for Mars and Jupiter; for Saturn, $f(c_m) = \cos(2c_m)$. $T_1 f(c_m)$ must also be subtracted from a superior planet's mean anomaly, since

Ptolemaic theory requires that $a_m = c_m(\text{Sun}) - c_m(\text{planet})$. The third mechanism adds linear harmonic motion to the length of the radius of the epicycle (r), with T_3 as the maximal amplitude (in units of length where the deferent radius $R = 1$):

$$r' = r + T_3\cos(a_m). \qquad [3]$$

These three mechanisms, deployed two at a time, can easily be implemented within the computational procedures of the Alfonsine Tables, especially since some simplifying approximations may be made that alter the predicted longitudes by less than 0;10°. As described in the 1512 preface, Angelus "corrected and emended" the columns for the equations of the argument and the centre in his now lost tables. The additional columns required to implement our harmonic mechanisms modify either the arguments used to enter equations of the argument and centre, or those equations themselves.

The mechanism of Equation [1], used only for the superior planets, can be computed with an additional column (that we shall call "delta centre") giving $T_1 f(c_m)$. By means of Equation [1], one determines the modified mean centre (c'_m), which becomes the argument for looking up the equation of the centre [$q(c'_m)$]. The modified true centre (c') then becomes:

$$c' = c'_m + q(c'_m), \qquad [4]$$

and the modified true argument (a'_v) becomes:

$$a'_v = a_m - T_1 f(c_m) - q(c'_m). \qquad [5]$$

For Saturn and Jupiter, where Angelus used the mechanisms of Equations [1] and [3], the latter is implemented by modifying the provisional equation of the argument [$p'_0(a'_v)$]. In the Alfonsine case, the provisional equation of the argument, determined for a value of the mean centre where the centre of the epicycle is at its mean distance from the Earth, is given by:

$$p_0(a_v) = \tan^{-1}\left(\frac{r\sin(a_v)}{r\cos(a_v) + 1}\right). \qquad [6]$$

In the modified case:

$$p'_0(a'_v) = \tan^{-1}\left(\frac{r'\sin(a'_v)}{r'\cos(a'_v) + 1}\right), \qquad [7]$$

where a'_v is defined in Equation [5]. To preserve Equation [7] as a function of a single variable (i.e., requiring a single- rather than double-entry table), r' in Equation [3] can be made a function of a'_v. Although this approximation makes Equation [3] non-harmonic, the final longitudes differ by less than 0;10° when computing the provisional equation of the argument with $r'(a'_v)$ instead of $r'(a_m)$, for values of T_3 used by Angelus. Because of the geometries involved, the proportional parts in the Alfonsine procedure can be determined as in the unmodified case, using c' from Equation [4] as the argument for looking up the proportional minutes, and a'_v from Equation [5] for looking up the *diversitas diametri*.[22] For Saturn and Jupiter, then, the second emended column, which we

TABLE 1. Modified Alfonsine planetary equations for Jupiter.[23]

Common Numbers		**Delta Centre**	Equation Centre	Mins Prop	Longitude longior	**Delta Argument**	Equation Argument	Longitude propior
10	350	**0;23**	0;59	60 l.	0;03	**0;02**	1;37	0;03
20	340	**0;46**	1;57	57	0;07	**0;06**	3;11	0;08
30	330	**1;07**	2;51	53	0;10	**0;08**	4;42	0;11
40	320	**1;26**	3;41	47	0;13	**0;09**	6;09	0;14
50	310	**1;43**	4;24	39	0;17	**0;09**	7;28	0;19
60	300	**1;56**	5;01	30	0;20	**0;09**	8;37	0;22
70	290	**2;06**	5;31	19	0;23	**0;07**	9;36	0;25
80	280	**2;12**	5;49	9	0;26	**0;04**	10;21	0;29
90	270	**2;14**	5;57	2 pr.	0;27	**0;00**	10;51	0;30
100	260	**2;12**	5;55	11	0;29	**-0;05-**	11;03	0;32
110	250	**2;06**	5;41	21	0;30	**-0;10-**	10;55	0;33
120	240	**1;56**	5;19	30	0;29	**-0;12-**	10;23	0;33
130	230	**1;43**	4;43	40	0;28	**-0;16-**	9;30	0;31
140	220	**1;26**	3;59	47	0;25	**-0;17-**	8;13	0;28
150	210	**1;07**	3;06	52	0;21	**-0;17-**	6;34	0;22
160	200	**0;46**	2;08	56	0;15	**-0;13-**	4;35	0;16
170	190	**0;23**	1;05	59	0;08	**-0;07-**	2;21	0;09
180	180	**0;00**	0;00	60	0;00	**0;00**	0;00	0;00

call "delta argument", gives $p_0'(a_v') - p_0(a_v')$. Table 1 indicates how Equations [1] and [3] can be implemented for Jupiter, using the coefficients listed in Table 2.

For Mars, the only planet for which Angelus used the mechanisms of Equations [1] and [2], the "delta argument" column is not required. Rather, a column we might call "delta mean argument" is needed to look up the harmonic term in Equation [2]. In this case, Equation [5] becomes:

$$a_v' = a_m - T_1\sin(c_m) + T_2\sin(a_m) - q(c_m'). \qquad [8]$$

The modified true argument (a_v') is then used to determine the equation of the argument [$p(a_v')$], itself not modified since the mechanism of Equation [3] is not used for Mars.

For the inner planets where Angelus used the mechanisms of Equations [2] and [3], the "delta mean argument" column is used to compute the modified true argument:

$$a_v' = a_m + T_2\sin(a_m) - q(c_m). \qquad [9]$$

Determining the equation of the argument then proceeds as in the cases of Saturn and Jupiter.

To fit Angelus's longitudes with the three mechanisms of harmonic motion implemented via these Alfonsine procedures, we determined the values of the coefficients (T_1, T_2, and T_3) by reducing the sums of the differences squared to a minimum, sampling at five-day intervals (two-day for Mercury) over the two years. Each of the "delta" columns described above was computed at 1-degree intervals, with intermediate arguments interpolated linearly (as in all Alfonsine procedures). To make visible the deviations from the standard Alfonsine longitudes, we compared both Angelus and our modified algorithms against the

TABLE 2. Reproducing Angelus's planetary tables.

Planet	T_1 (rads)	T_2 (rads)	T_3 (R=1)	RRMS (arcmins)
Saturn	0.022	—	0.0024	4.6
Jupiter	0.039	—	0.0076	3.2
Mars	0.022	0.049	—	26.0
Venus	—	0.057	0.0024	13.0
Mercury	—	0.056	0.0056	9.0

Alfonsine positions (see Figures 5(a)–(e)). Table 2 lists the values of the coefficients set by these curve-fitting procedures and the measures of 'goodness of fit', based on the sum of the squares of the differences between the longitudes of Angelus's almanacs and our algorithms (the residual root mean squares).

Since the same harmonic mechanisms and coefficients allow us to match Angelus's longitudes for both years, it seems obvious that Angelus did not alter his "corrected" planetary equations between 1510 and 1512. Yet to produce the curves of Figures 5(a)–(b), we have placed *ad hoc* increments in the modified longitudes of Saturn and Jupiter for 1512. For the first fifty-five days of 1512, we have incremented our longitudes for Saturn and Jupiter by 0;27° and 0;15°, respectively. For the remainder of that year, longitudes for both planets have been incremented by 0;33°.[24] A shift in the radices of mean motion would increment longitudes by a constant amount. Might Angelus have inadvertently altered these radices through computational error? Yet a single arithmetic mistake would produce an abrupt jump in the daily longitudes, something not observed in Angelus's ephemerides where the changes in the radices after fifty-five days of 1512 are gradually phased into the longitudes over several weeks. Perhaps therefore Angelus in his quest to improve Alfonsine predictions not only employed "corrected" planetary tables but also experimented by shifting values for some of the radices, a conjecture strengthened by the strident rhetoric of empiricism in the prefaces.[25]

From Figures 5(a)–(e) and Table 2 it can be seen that our modified algorithms best reproduce Angelus's positions for Jupiter and Saturn. For these planets, the differences between Angelus's and our longitudes remain consistently less than 0;10°. For the inner planets, our algorithms are not quite as successful. Here, the mean differences are somewhat larger than those for Jupiter and Saturn, and at the stationary points the differences may exceed 0;30°. Only for Mars do our algorithms deviate significantly from Angelus (see below), with the differences occasionally approaching 1;00° at the stationary points and during retrogradation. Obviously, only by discovering a manuscript containing Angelus's lost tables or their description could we confirm the algorithms by which he generated longitudes for the two almanacs. Yet the three harmonic mechanisms, deployed as listed in Table 2, do yield results that correspond generally to the periods and contours of Angelus's curves.

Might computational artifacts produce some of the residual differences not

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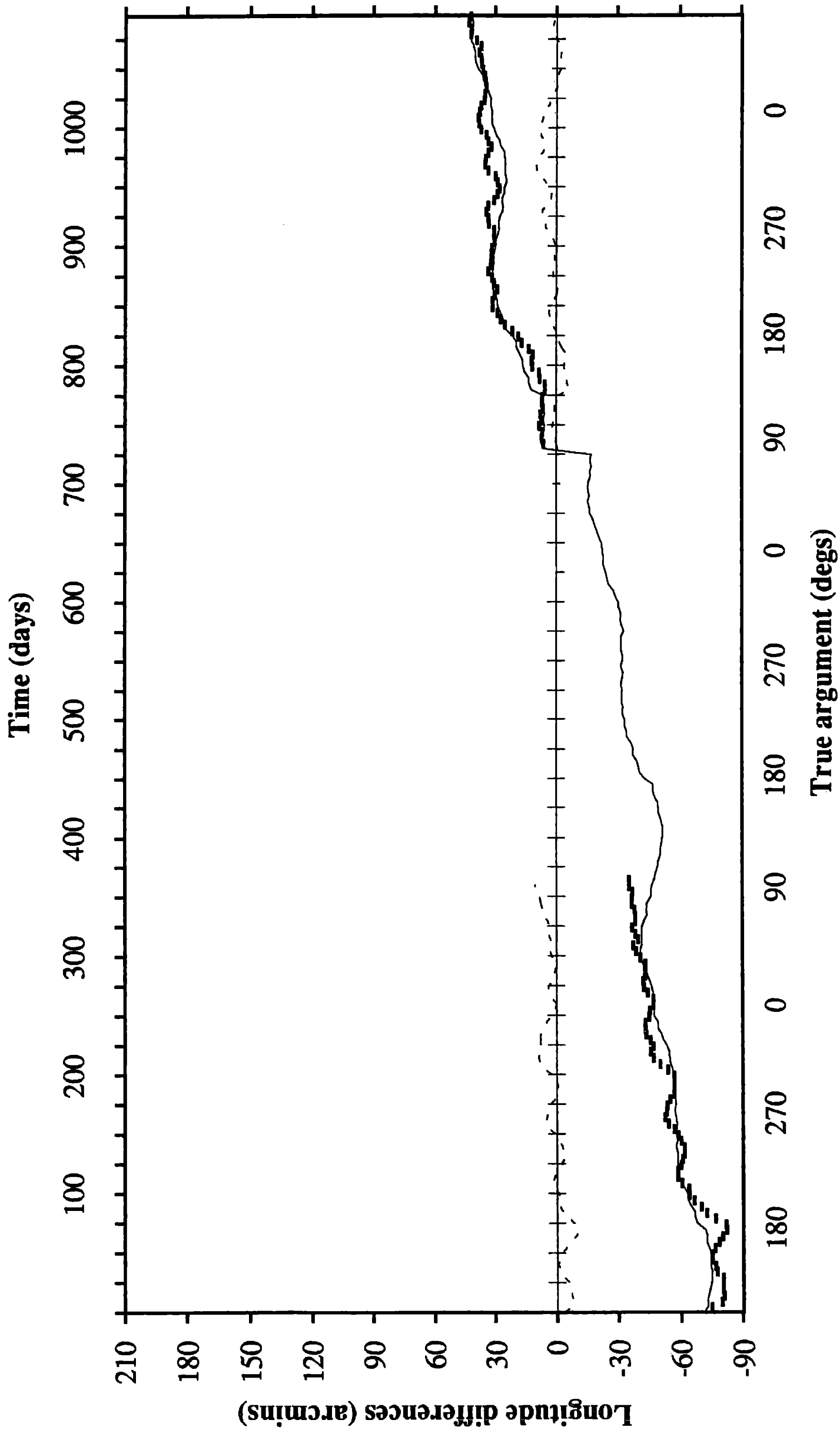

FIG. 5(a). Angelus and the modified algorithms compared. Saturn, longitudes for 1510–12, at five-day intervals. Angelus minus 1483 *Alfonsine tables* (bold points), modified algorithms minus 1483 *Alfonsine tables* (solid line), Angelus minus modified algorithms (dotted line).

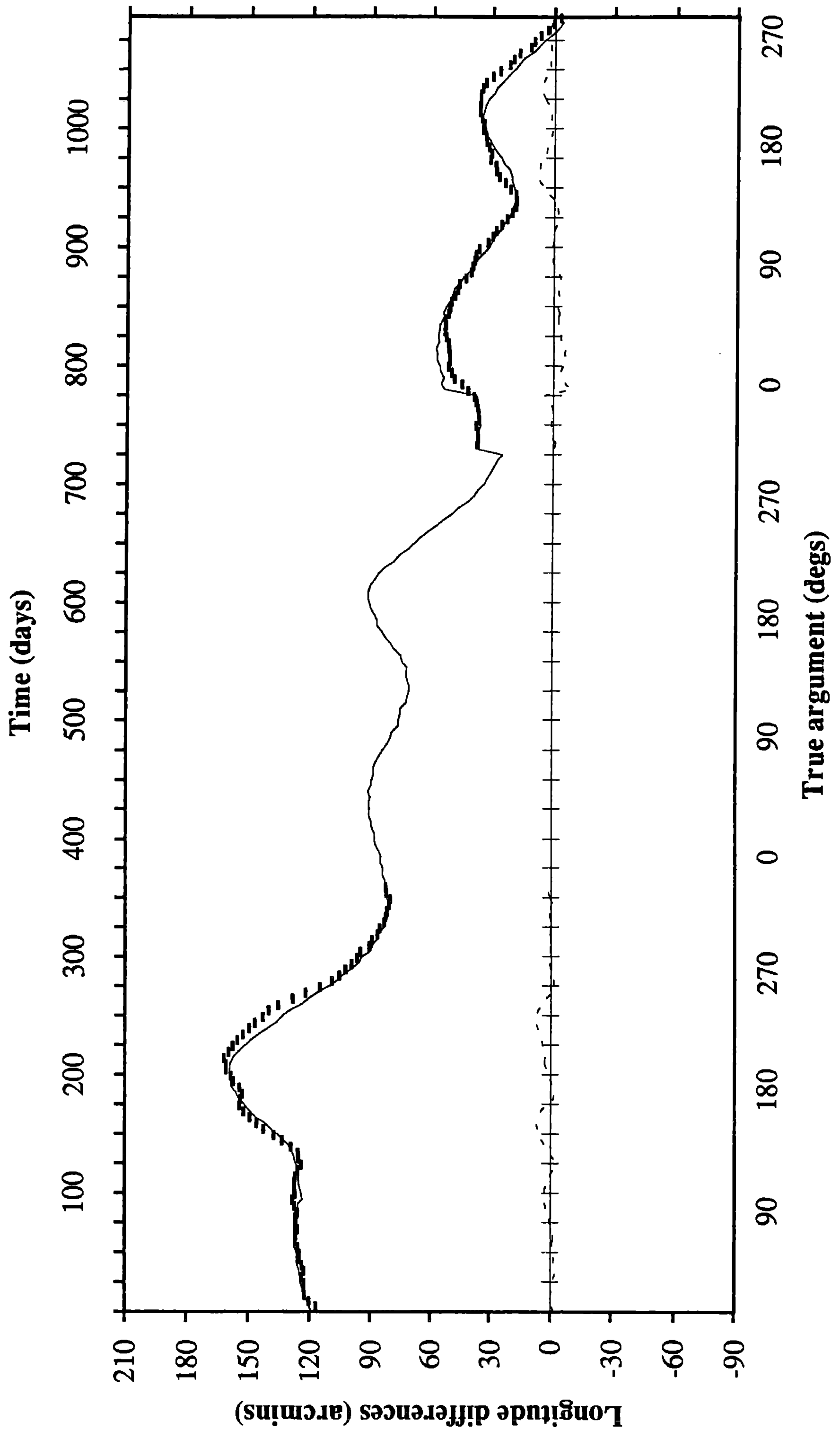

FIG. 5(b). The same for Jupiter.

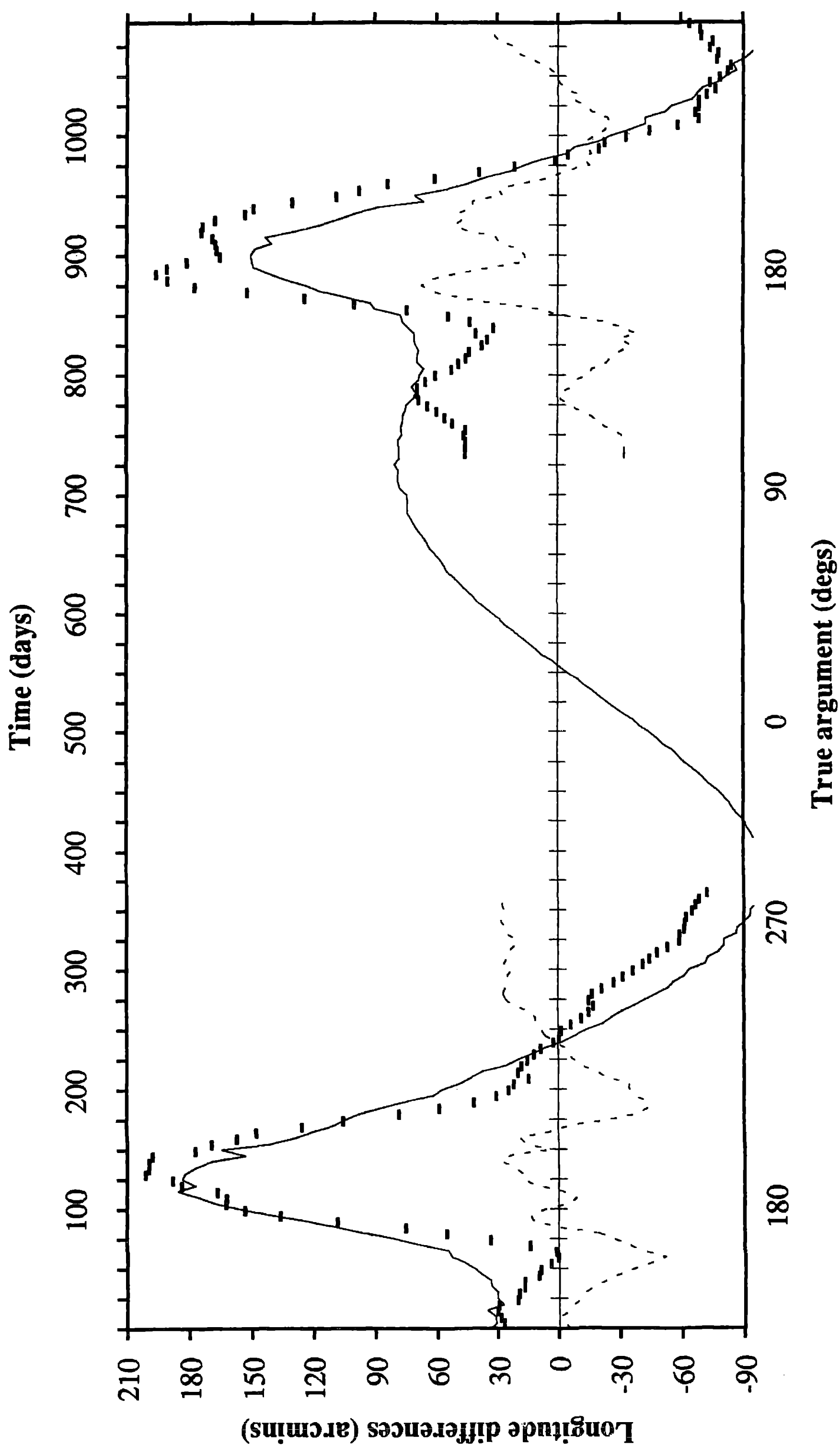

FIG. 5(c). The same for Mars.

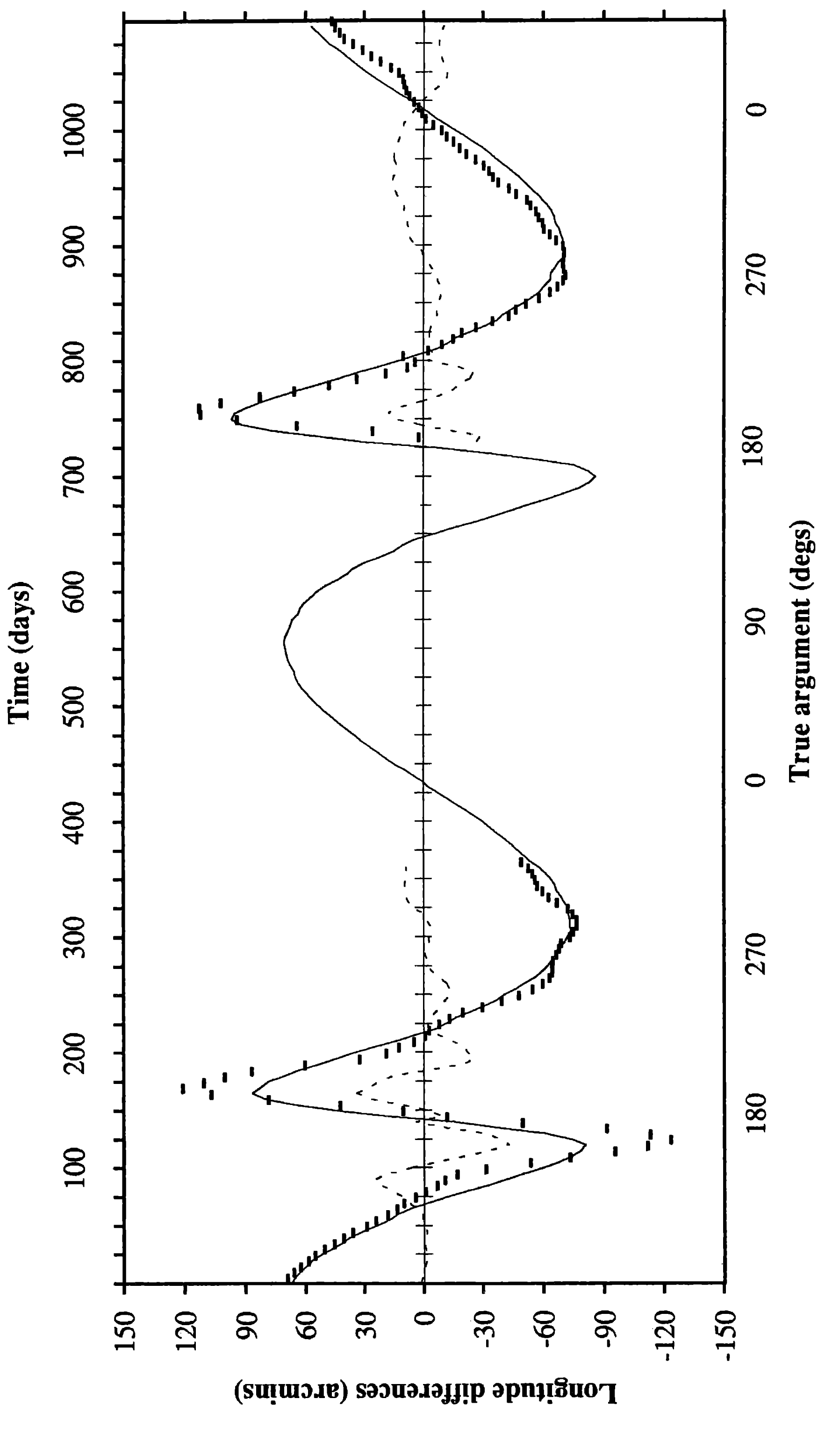

FIG. 5(d). The same for Venus.

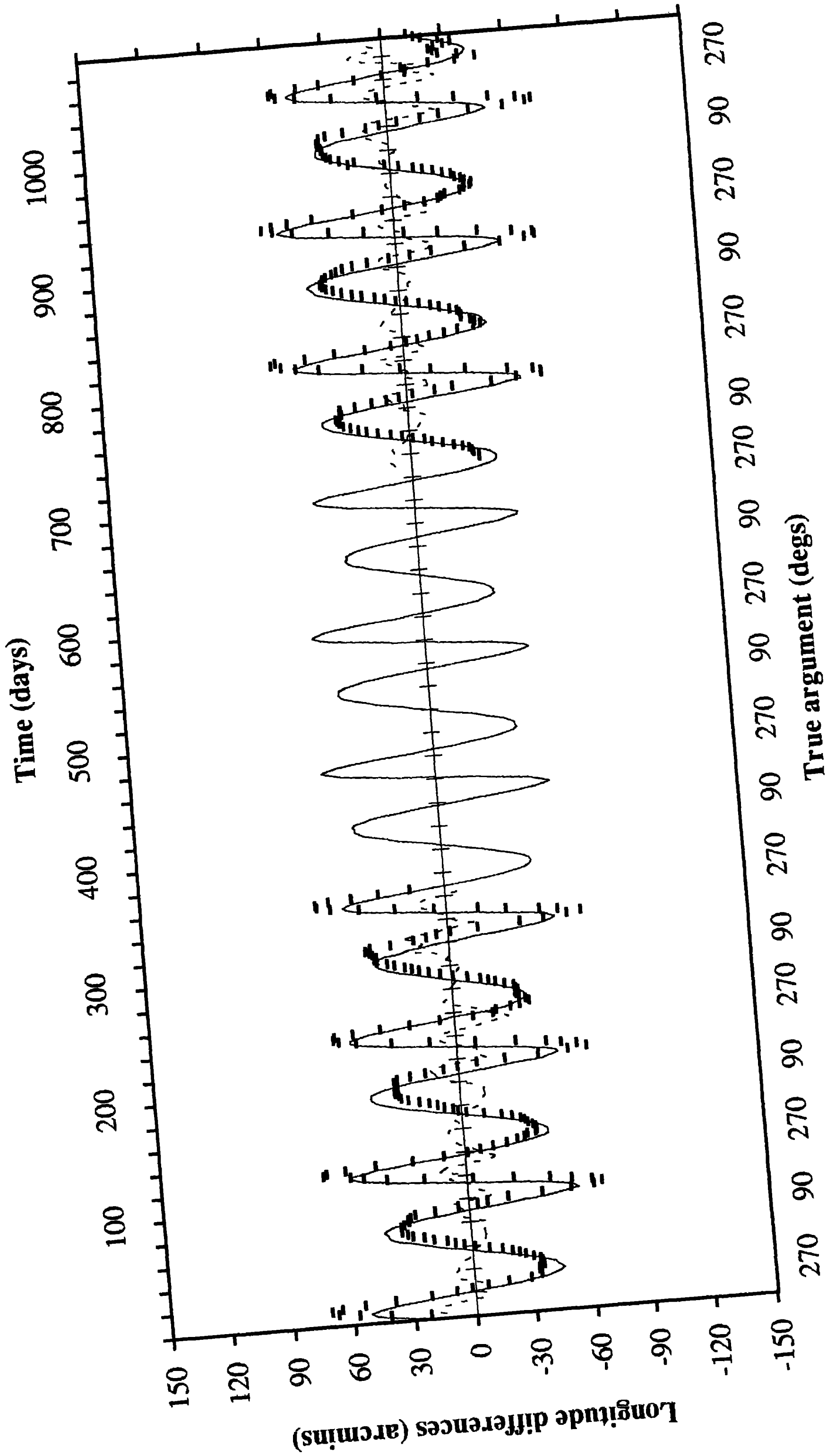

FIG. 5(e). The same for Mercury, but at two-day intervals.

accounted for by our algorithms? From Figures 5(a)–(e) it can be seen that distinctly periodic patterns appear in the differences between Angelus and the modified algorithms for each of the planets. We do not know how Angelus rounded his intermediate values; whether he computed longitudes for each day or interpolated between positions determined for epochs longer than one day;[26] or for what intervals his tables of planetary equations were computed (we have found fifteenth-century tables computed for 1°-, 3°- and 6°-intervals of the argument). Likewise, we do not know the intervals for which Angelus computed the modified columns in his tables (i.e., the "delta" columns described above). If we assume, for example, that his Alfonsine Tables were computed to one-degree intervals, that his "delta" columns were computed at only ten-degree intervals, and that he interpolated linearly, then some of the periodic features of the differences, especially for Mars in 1510 and for Jupiter, can be reduced. Yet any attempt to deduce Angelus's computational procedures from the residual differences in Figures 5(a)–(e) would be, at best, *ad hoc*. Further study is needed of fifteenth-century practices for computing almanacs before we can determine whether or which computational effects might explain the periodic features of the differences between Angelus and our algorithms.

Might some of the unexplained residuals result from the fact that two separate authors apparently constructed the lost tables we are seeking to reconstruct? In the 1512 preface, Angelus indicated that Peurbach had completed the tabular corrections for the "lower" parts of the epicycles, and that Angelus himself had corrected the "upper" parts. Might the residual differences therefore show different features for the "upper" ($270° < a_v < 90°$) and "lower" ($90° < a_v < 270°$) epicycles, suggesting that Angelus had implemented the "corrections" somewhat differently than his predecessor, or at least had employed different computational techniques? For Saturn and Jupiter, no visible 'splice' appears in Angelus's longitudes at the upper/lower boundaries of the epicycle. Unfortunately for Mars we have longitudes only from the lower parts of the epicycles, where Peurbach rather than Angelus corrected the equations of the argument. For Venus and Mercury, however, a splice is clearly visible where $a_v = 90°$ or 270°. And the differences between Angelus and our algorithms for Jupiter, Venus and Mercury assume a different pattern for the upper and lower parts of the epicycle. In each case, the differences are flatter with fewer periodic features in the upper part of the epicycle (where Angelus rather than Peurbach had made the correction). Nevertheless, these effects are small and we can find no definitive evidence that multiple authorship of Angelus's modified tables might account for some of the residual differences not explained by our algorithms.

Finally, we must consider why the harmonic mechanisms match Angelus's Martian longitudes so poorly, relative to the fits for the other planets. Given this planet's large eccentricity and epicycle, linear interpolation can introduce considerably more random scatter in the computed longitudes for Mars than for the other planets, especially if the tabular arguments are spaced at intervals larger than one degree. This may well explain the fact, noted above, that the random variation in the longitude differences between Stöffler and Pflaum and the 1483 Alfonsine Tables is significantly larger for Mars than for the other planets (see Figure 3). Yet interpolation cannot account for the large residual differences in Figure 5(c). In their study of the Alfonsine Tables, Emmanuel Poulle and Owen Gingerich found more variation among the various manuscripts in the values for Mars's equations than for the other planets. And in our examination of fifteenth-century almanacs, we consistently found that Mars's longitudes deviate more from the 1483 Alfonsine positions than do longitudes of the other planets.[27] Perhaps, therefore, blemishes in the 1483 Alfonsine equations for Mars might contribute to the residual differences in Figure 5(c). Yet an exact geometrical solution to our algorithms for Mars, rather than a tabular solution, leaves nearly identical residual differences. It may well be, therefore, that an additional algorithm is required to match Angelus's longitudes for Mars. From other sources we know that Peurbach and Regiomontanus had long concluded that the motion of Mars, especially, was poorly predicted by the Alfonsine Tables.[28] Perhaps they developed additional algorithms for this planet which we have been unable to deduce. Nonetheless, the degree to which we have been able to fit Angelus's curve indicates that, at the least, the new algorithms for Mars are not unrelated to those deployed for the other planets.

3. PEURBACH'S PHYSICAL ASTRONOMY AND THE MODIFIED MODELS

Our reconstruction of Angelus's "new and corrected" astronomical tables in Section 2(iii) has not presupposed any geometrical or physical mechanisms that may have been used to generate the non-Ptolemaic harmonic motions. The instrumentalist rhetoric of the prefaces displays no interest in any geometrical, physical or philosophical issues, and we might conclude that Angelus would have been willing to intervene anywhere in the Ptolemaic models simply to save the phenomena. That is, Angelus might well have been ready to insert trigonometric functions into Alfonsine computations without visualizing the rotation of points around circles. Yet it was probably not Angelus but Peurbach who designed the models underlying the "corrected" tables, even if he had not completely transformed these models into tabular form. For the values of the coefficients listed in Table 2, the revised equations of the argument shift the final longitudes of the superior planets by less than one quarter of the total deviation from the Alfonsine positions. Had Peurbach limited his modifications only to the equations of the argument, as Angelus stated in the 1512 preface, the revised models would not have produced longitudinal deviations of the magnitude ostensibly required by the observations of Peurbach and Regiomontanus (up to several degrees). Hence, it seems likely that Peurbach formulated the new models, even if it was Angelus who completed the new tables from which he computed the "new and corrected" almanacs. And it seems quite unlikely that

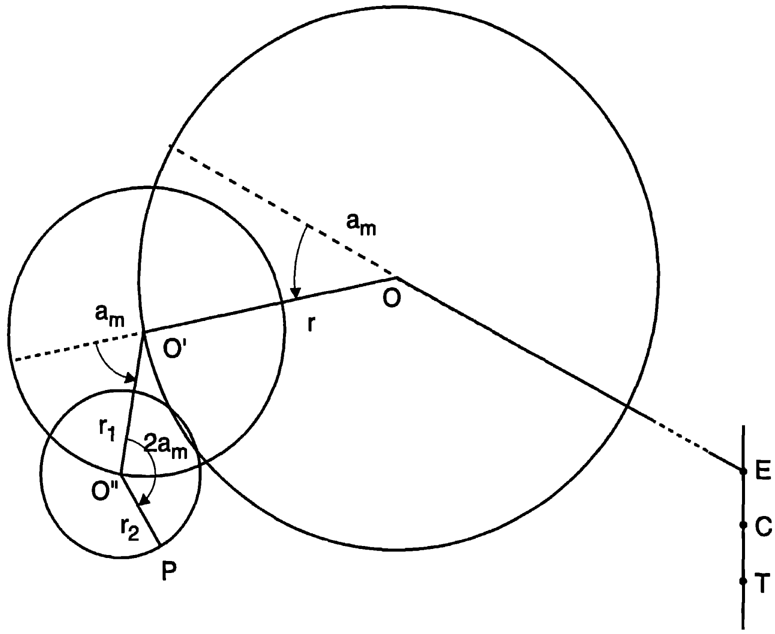

FIG. 6. Double-epicycle to reproduce the harmonic mechanisms for an inferior planet, not to scale.

Peurbach, whose *Theoricae novae planetarum* emphasizes an astronomy based on physically realizable spheres and spherical shells, would have introduced sine functions into planetary models without visualizing the motions of spheres.[29]

3(i). *Physical Models for the Harmonic Mechanisms*

It is certainly not difficult to construct geometrical models to generate the harmonic algorithms upon which we think Angelus based his "corrected" planetary tables. Without claiming to have exhausted all the possibilities, we have found several different arrangements of uniformly rotating spheres, each of which can produce the harmonic motions of the algorithms of Section 2 and can be easily solved by Alfonsine computations.

For the inner planets, the mechanisms of Equations [2] and [3] can be represented, to accuracies within 0;10°, by at least two different physical models.[30] One such model employs the double epicycles of Ibn al-Shāṭir and Copernicus.[31] Although they used the two additional epicycles to replace the Ptolemaic equant (i.e., for the first anomaly), such circles in a slightly altered arrangement can also be deployed to alter the equation of the argument (i.e., for the second anomaly). As seen in Figure 6, the first extra epicycle of radius r_1 is centred at O′, rotates in the same direction and with the same angular velocity (a_m) as the Ptolemaic epicycle, and carries the second extra epicycle. This second epicycle of radius r_2 is centred at *O″* and rotates in the opposite direction and at twice the angular velocity as the Ptolemaic epicycle. The planet is then placed at *P*. From the positions of these epicycles at the quadrants, it can be seen that to reproduce the mechanisms of Equations [2] and [3], $r_1 + r_2 = rT_1$ and $r_1 - r_2 = T_2$. The double epicycle thus produces an ellipse around O', with the minor axis along OO' and a semimajor axis of rT_1 and semiminor axis of T_2.[32]

Alternatively, the motions of Equations [2] and [3] can be produced via Ṭūsī couples.[33] The epicycle radius (r) of Equation [3] can be varied directly with a rectilinear Ṭūsī couple (recently labelled more exactly by Mario di Bono, the "spherical version with parallel axes and radii in the ratio of 1:2"), the well-known device used by Ṭūsī for his lunar and planetary (first anomaly) models of longitude and by Copernicus in the *Commentariolus* for Mercury's longitude. For the variation in the true argument, a second rectilinear Ṭūsī couple can be deployed, orthogonal to the first, or more elegantly the curvilinear Ṭūsī couple might be used (di Bono's "spherical version with oblique axes and equal radii"), such as Ṭūsī and Copernicus employed for planetary latitudes and for the variation of the motion of the eighth sphere and of the obliquity of the ecliptic.

For the superior planets, the harmonic modifications also can be represented by several different arrangements of spheres. As above, the variation of the length r in the epicycle can be produced by a linear Ṭūsī couple. The variation in the mean centre can be achieved by the double epicycles of Ibn al-Shāṭir and Copernicus. For Mars and Jupiter, the double epicycles are arranged exactly as in al-Shāṭir's model, but with $r_1 - r_2 = e$ and $r_1 + r_2 = 2e - T_1$. To realize the mechanism of Equation [1] for Saturn (where the variation of c_m is a function of $2c_m$), a third epicycle (with $r_3 = T_1$, and as in al-Shāṭir's model $r_1 + r_2 = 2e$, and $r_1 - r_2 = e$) is required, rotating in the same direction as, but 90° behind, r_2. And of course the Ṭūsī couple — either linear or curvilinear — can also be employed to vary the mean centre.

The angular modifications of Equations [1] and [2] might also be generated by means of a small circle placed orthogonal to the plane of the deferent (for varying the mean centre) or the epicycle (for varying the mean argument), similar to the circles used by medieval astronomers in models of trepidation.[34] Both Peurbach in the *Theoricae novae* and Regiomontanus in a 1464 letter to Bianchini had extensively discussed various models for the motions of the eighth sphere, and Peurbach certainly would have realized that a 'trepidation circle' could harmonically vary motion around the deferent.[35] Likewise, the small circles Ptolemy employed in his latitude models to rock the planes of the epicycles would have provided another mechanism for generating angular harmonic motions, as would the so-called polar epicycles on the surface of a sphere in al-Biṭrūjī's planetary models.[36] Yet such circles have the distinct disadvantage of producing simultaneously two orthogonal harmonic motions.[37] In the *Theoricae* Peurbach noted that trepidation circles vary not only the motion of the eighth sphere but also the obliquity of the ecliptic. Regiomontanus later discussed the ridiculous consequences such a model would entail for solar declinations, and copied into his "Viennese computation book" a standard criticism of al-Biṭrūjī's planetary theories, namely, that polar epicycles cannot simultaneously produce correct longitudes and latitudes.[38] Given these texts, it seem unlikely that Peurbach would have used 'trepidation circles' to produce the angular modifications to the mean centre and argument.

Might he have employed homocentric spheres? In a 1460 letter to Johannes Vitéz, the bishop of Nagyvárad to whom Peurbach had dedicated his *Tabulae eclypsium*, Regiomontanus proposed homocentric theories of the Sun and Moon that deploy small circles on the surface of a sphere to generate the inequalities of the motions. By placing the planets not directly on the small circles but on the ends of arcs 90° away from the circles, Regiomontanus managed to restrict, albeit with some rather arbitrary mechanical assumptions, the harmonic motions in these mechanisms to a single direction; that is, he solved a crucial problem that had made al-Biṭrūjī's planetary theories unworkable.[39] Such quadrant arcs and small circles can approximate angular harmonic motions, but not the linear harmonic motions of Equation [3]. That is, Regiomontanus's homocentric machinery alone could not generate the algorithms we have reconstructed from Angelus's almanacs.

However, might the entire planetary models (and not just the modifications) that Angelus inherited from Peurbach have been homocentric? In the letter to Vitéz, Regiomontanus promised to "complete a new work in four treatises" which would "set out a theory of concentric orbs by which all inequalities of the motions can be saved". Yet Regiomontanus is not known to have completed this homocentric project, and we think it highly unlikely that he could have constructed workable planetary models with the mechanisms of that letter.[40] Furthermore, Angelus in his prefaces clearly wrote of epicycles and of equations of the centre and argument, mechanisms that are eliminated in homocentric astronomy. It seems improbable, therefore, that Peurbach's innovation, completed by Angelus, would have entailed a completely homocentric configuration or would have used small circles placed orthogonal to the planes of the deferent or epicycle. We are left then with the distinct possibility that Peurbach employed some version of the Marāgha techniques in his modified models.

3(ii). *Peurbach and Astronomical Reform*

Unlike Regiomontanus, who outlined his vision for removing "the rust from the heavenly spheres" in a series of extant texts,[41] Peurbach is not known to have written any specific programmatic statements about the state of astronomy or its need for reform. Yet given his close collaboration with Regiomontanus, it seems plausible that Peurbach would have shared at least some of his younger colleague's concerns. In his most sustained criticism of received planetary theory, Regiomontanus argued, primarily on the basis of comparisons of observations with Alfonsine predictions, that the numerical parameters (eccentricities, radii of epicycles, mean motions and radices of the mean motions) "have not been found entirely accurately".[42] Likewise, some years later Tanstetter, in reporting

Peurbach's view that Alfonsine predictions for Mars could err by 1;30°, suggested that the size of this planet's epicycle must be incorrectly known (an explanation he may have received from Angelus, with whom Tanstetter undoubtedly was acquainted). Such remarks suggest a program of astronomical reform restricted to correcting the values of static parameters, similar to the efforts of *zīj*-makers such as Ibn Yūnus or al-Battānī.[43] Yet these remarks do not foreclose the possibility that some of the incorrectly known parameters might vary periodically over time. At the least, therefore, Regiomontanus's programmatic statements do not contradict our findings about the structure of Angelus's lost tables.

The possibility that the new harmonic algorithms were generated by some combination of Marāgha techniques is strengthened by independent evidence that suggests that Peurbach may have known about at least some sections of Ṭūsī's *Tadhkira*, the key Marāgha text in which both versions of the Ṭūsī couple (but not al-Shāṭir's double epicycle models) are presented. In the *Theoricae novae planetarum*, Peurbach described the three-fold motions (deviation, inclination and slant) in Ptolemy's latitude theory for the inner planets, but did not discuss any physical models of circles or spheres by which these motions could be produced. Instead, Peurbach merely concluded: "On account of these inclinations and slants of the epicycles, some assume small orbs with enclosed epicycles, in consequence of whose motions the said [inclinations and slants] occur."[44] In his 1987 translation of this text, E. J. Aiton suggested that Peurbach was referring to Ṭūsī's description of Ibn al-Haytham's latitude model. In the *Tadhkirah*, II.11[16–18], Ṭūsī wrote that al-Haytham had enclosed the epicycles of the outer planets in two concentric spheres, rotating in opposite directions about axes slightly inclined to each other, and had placed the epicycles of the inner planets in four concentric spheres. Ṭūsī then noted that a slightly different arrangement of these spheres could achieve the same effects, as could the addition of a third sphere to each pair in a grouping that comprises the curvilinear version of his couple.[45] As Aiton noted, al-Haytham's latitude model is not found in his well-known treatise, *On the configuration of the world*, a work translated into Latin in the thirteenth century and widely believed to have provided the basis for Peurbach's solid-sphere representation of planetary models in the *Theoricae novae*.[46] Instead al-Haytham's model appears in his *Treatise on the motion of iltifāf*, a text no longer extant in Arabic and not known to have been translated into Latin.[47] Hence, if the *Theoricae novae* does refer to al-Haytham's or Ṭūsī's latitude models, we might well conclude that Peurbach was acquainted with Ṭūsī's *Tadhkira* (also never known to have been translated into Latin), with some later commentary on that work, or with some other Marāgha text.[48]

More recently, J. L. Mancha has suggested that not only Peurbach's reference, cited above, but also similar comments on spheres enclosing epicycles by Henry of Hesse (in 1364), Magister Julmann (in 1377) and Albert of Brudzewo (in 1483) might all refer to Ibn al-Haytham's latitude model.[49] Mancha does not speculate on how these Latin authors may have learned about Ibn al-Haytham's homocentric epicycles, or whether they were acquainted with the *Tadhkira*. But his findings do indicate a wider knowledge, among fourteenth- and fifteenth-century Latin authors, of what F. J. Ragep has called the "*haya'* tradition" of Islamic astronomy, i.e., of the quest to integrate physics into mathematical astronomy.[50]

We know of no extant text by Peurbach or Regiomontanus in which the Ptolemaic models are criticized explicitly on the grounds that they violate uniform, circular motion. This was, of course, a critical difficulty in Ptolemaic astronomy, which had prompted the Marāgha mathematicians to develop their innovative physical models and Copernicus to write the *Commentariolus*.[51] Yet unlike the Marāghans (or Copernicus, for the planets), Peurbach would have been using the Ṭūsī couples or double epicycles not merely to reproduce the predicted longitudes of Ptolemy's models but to modify and "correct" the predictions. Indeed, the goal of the project, initiated by Peurbach and Angelus but left incomplete by both, was closer to that of the fourteenth-century astronomer, Levi ben Gerson, than it was to the Marāghans. As expounded by Bernard R. Goldstein, Levi's two lunar models not only replace the Ptolemaic epicycle with a "reflected angle" mechanism but also yield longitudes that vary distinctly from Ptolemy's. And these variations, both in magnitude and regularity, do not look unlike the variations in Angelus's longitudes (see Figure 7).[52] It may well be, therefore, that Peurbach was pursuing a Gersonian goal by means of Marāgha mathematics.

How Peurbach might have learned of the *Tadhkira* or Marāgha techniques, if he did employ them in his modified models, remains unclear. But that he could have learned about them seems not impossible, given the fact that Copernicus in the *Commentariolus* (before 1514) and Johann Werner in *De motu octavae sphaerae* (before 1522) used similar techniques.[53] Indeed, discussions of Ṭūsī-like mechanisms (in their linear version) can be found much earlier in the Latin West, in scattered settings such as Nicole Oresme's *Questiones de spera* or in a fourteenth-century Hebrew mathematical text partially copied in the mid-fifteenth century by Mordekhai Finzi, a Mantuan Jew known to have had many contacts with Christian scholars.[54] Although specific routes of transmission have not yet been identified, by early in the sixteenth century, Latin astronomers from Cracow to Nuremberg to Vienna had discovered that linear or curvilinear harmonic motions could be produced by combining the motions of two spheres rotating uniformly in opposite directions. Hence, if our reconstruction of Angelus's astronomical tables is plausible, then Noel Swerdlow's speculation in 1973 that there may exist "in Italy a Latin treatise from the late fifteenth century describing these [Marāgha] models and catalogued, if at all, under the uninformative title *Theorica planetarum*", may require revision.[55] We may be looking for a means of transmission both more fragmentary and widespread than a single treatise, and at least one of the Marāgha sources must have been available to the Latin West before 1461, the year of Peurbach's death.

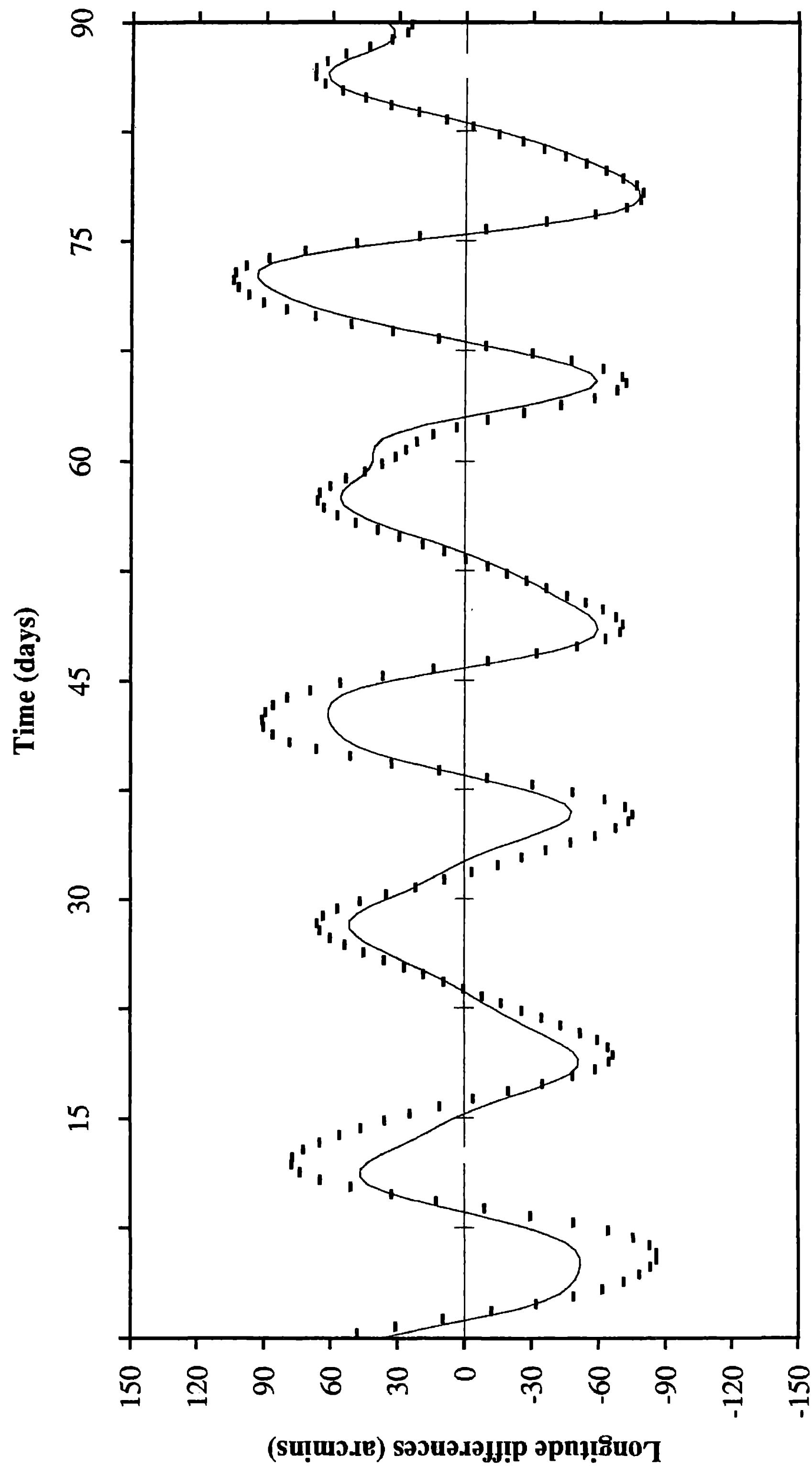

FIG. 7. Levi ben Gerson's and Ptolemy's lunar models compared. Levi's final model minus Ptolemy (bold points), Levi's preliminary model minus Ptolemy (solid line), over 90 days at half-day intervals. Maximal deviations occur near the octants.

ACKNOWLEDGEMENTS

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APPENDIX 1: Angelus, *Almanach 1510*, "Prefatio", sigs. a1^{v}–a2^{r}.[56]

Motus planetarum in tabulis Alfonsi et Blanchini sunt falsi, uti manus propria magistri Georgij Peurbachij astrorum observatoris perspicacissimi Vienne in monasterio fratrum Predicatorum hodie attestatur. Magister quoque Joannes de Monte Regio falsitatem motuum cuidem mathematico Erfordensi litteris significavit atque meo posthac preceptori fidelissimo sepe commonstravit. Praeterea ego Joannes Angelus cum magistro Andrea Stiborio cumque nonnullis aliis mathematicis Vienne in turri Collegij Ducalis prenominatores errores in motibus planetarum sepius conspeximus. Quando coniunctio superiorum planetarum in communi almanach ad certum diem annotabatur, duobus vel tribus diebus ante vel aliquando post exquisita observatione comperiebatur. Quo circa multis iam transactis annis cogitavi et studui: quo id quod clarissimorum astronomorum Georgij Peurbachij et Joannis de Monte Regio immatura morte interceptum esset, mea diligenti observatione ac calculatione in lucem veniret, ac denique iste communis error certe non insensibilis — qui se ad duos et nonunquam ad tres gradus extendit — castigaretur. Et ob nobilissime scientie restaurationem atque prognosticantium utilitatem novas tabulas composui. Ex quibus pro anno etc. 1510 Almanach novum ab alijs communibus aliquando 2 vel 3 gradibus diversitatem habens calculando designavi, prout in hac prefatiuncula certissimis quibusdam exemplis servens quisque huius scientie amator eo anno observare et veritatem huius nove calculationis investigare poterit.

Saturnus enim iuxta antiquum almanach 14 die Aprilis reperitur in 2 gradu minuto 57 Libre, latitudinem habens septemtrionalem duorum graduum 48 minutorum: ubi fere ponitur stella fixa de tercia magnitudine in ala sinistra Virginis. Huic itaque stelle Saturnus in prememorata die secundum antiquum almanach coniunctus. Secundum meas tabulas et secundum istud novum Almanach, secundum denique ipsam veritatem, que intuentibus patebit, die 29 Martij in eadem fere latitudine se coniungebat [*sic*]. Consimiliter videbitur Saturnus 20 die Julij per suam directionem coniunctus prenominate stelle et non 8 die eiusdem mensis ut ponit antiquum almanach.

Falsitas in motu Jovis hoc anno non ita sensibiliter percepietur, nisi per diligentiorem observationem: eo quod in signo suo pauce reperiuntur stelle nisi in modica quantitate, tamen circa finem Augusti post occasum solis in medio celi ipse Jupiter videbitur accedere secundum antiquum almanach aliquas ex prefatis parvis stellis per retrogradationem, quas secundum veritatem et hoc novum Almanach nunquam pro tempore attinget.

Martis motus multipliciter hoc anno falsus deprehenditur in antiquis almanach brevitatis tamen amore pauca et magis sensibilia exempla proponam. Die 22 Maij Mars sua retrogradatione secundum antiquum almanach preteribit quandam stellam parvam de 4 magnitudine in 21 gradu 27 minuto Libre in latitudine unius gradus 30 minutorum ubi secundum novam hanc calculationem per tres gradus eam non attinget, sed dirigetur ab ea recedendo. Item 22 die Aprilis in antiquo almanach signatur coniunctio Martis et Lune ad horam 10 post meridiem, que secundum veritatem et hoc novum Almanach erit infra 14 et 15 horas post meridiem eiusdem diei. Et iterum coniunctio Jovis et Martis signata in communi almanach ad 28 diem Octobris, secundum veritatem 31 die eiusdem mensis apparebit.

Haud dissimiliter falsitas motuum Veneris atque Mercurij deprehenditur, presertim illis in maxima elongatione a sole existentibus. Motus vero Solis et Lune, quia valde parum sunt diversi a motibus antiqui almanach, presenti anno non posui. Deo tamen annuente futuris annis pari ratione calculabo.

Quantum insuper prenominati errores deceperint practicantes, brevibus libuit perstringere verbis. Astrologus enim videns Saturnum ex communi almanach in aliquo gradu et minuto in quo sine notabili latitudine ponitur quedam stella fixa notabilis, statim prognosticat mutationem aurae. Et quia secundum veritatem ut supra ostensum est, idem Saturnus ab illo gradu et stella 2 vel 3 gradibus distat, ipse fallitur fere ad spatium unius mensis. Sic in Jove ad spacium medii mensis, in Marte, Venere et Mercurio in 2 vel tribus hoc modo decipietur diebus. Et iterum astrologus dirigens alchocoden vel alium significatorem determinat certum annum imminentis periculi: in quo iuxta falsitatem motuum pretactam, errabit ad 3 vel 4 annos. Hec singula exempla breviori modo quo potui deduxi, tum quod sideralis scientie amatoribus occasionem observandi prebeam, tum etiam quod ea observatione veritas ipsa in lucem prodeat.

TRANSLATION

The motions of the planets in the tables of Alfonso and Bianchini are false,[57] as the very own hand of Master Georg Peurbach, the most acute observer of the stars, confirms today in the Dominican Convent in Vienna.[58] Also, Master Johannes Regiomontanus in a letter to a certain mathematician of Erfurt[59] indicated the falsity of the motions and thereafter often demonstrated [it] to my most faithful teacher.[60] Moreover I, Johannes Angelus with Master Andreas Stiborius[61] and with several other mathematicians, often observed the aforementioned errors in the motions of the planets from the tower of the Ducal College in Vienna.[62] When the conjunction of the superior planets was marked in a common almanac[63] for a certain day, it was discovered by accurate observation to be two or three days before or sometimes later. For many years now I have thought and studied about all this, so that my diligent observation and calculation brought to light that which was interrupted by the untimely death of the renowned astronomers Georg Peurbach and Regiomontanus, and thus this certainly not insensible common error, which extends to two and sometimes to three degrees, finally was corrected. And with a view to the restoration of the most noble science[64] and for the usefulness of prognosticators I have composed new tables. From them I calculated for the year 1510 a new almanac that differs by some two or three degrees from other common almanacs, so that any devotee of this science thanks to the most certain examples will be able to observe this year and investigate the truth of this new calculation.

Indeed according to the old almanac [Case 1[65]], on 14 April Saturn is found at 2 degrees 57 minutes of Libra, having a northern latitude of 2 degrees 48 minutes, very nearly where the fixed third-magnitude star in the left wing of Virgo [γ Vir] is located. And thus Saturn, according to the old almanac, conjuncts this star on the aforementioned day. But according to my tables and this new almanac, in a word according to truth itself which will be obvious to those who consider the matter, the conjunction took place on the twenty-ninth of March in exactly the same latitude. Similarly [Case 2], one will see Saturn in direct motion in conjunction with the said star on 20 July and not the eighth day of the same month, as the old almanac has it.

The falsity in the motion of Jupiter is not perceived so sensibly this year except by diligent observation, because few stars are found in its sign, if not too few. Around the end of August [Case 3], however, after sunset, according to the old almanac Jupiter near the meridian will be seen to approach some of the aforementioned little stars [43, ρ^1, υ Sgr] in retrograde, which according to the truth and this new almanac in no way happens at that time.

The motion of Mars in the old almanac repeatedly is found to be false this year. For the sake of brevity, however, I will advance a few of the more striking examples. On 22 May according to the old almanac [Case 4], Mars in retrogradation will pass a little star of the fourth magnitude at 21 degrees 27 minutes in Libra [86 Vir], at a latitude of 1 degree 30 minutes; where according to this new calculation Mars gets no closer than three degrees but moves receding from it. Also on 22 April [Case 5], the old almanac indicates a conjunction of Mars and the Moon at the tenth hour after noon, which according to the truth and this new almanac will occur between the fourteenth and fifteenth hours after noon on the same day. And again the conjunction of Jupiter and Mars [Case 6] indicated in the common almanac on 28 October in truth will appear on the thirty-first of the same month.

Similarly the falsity of motions of Venus and Mercury is discovered, especially when they are in maximal elongation from the Sun [Case 7]. I have not, however, presented the [modified] motions of the Sun and Moon in the current year since they differ but little from the motions in the old almanac. But God willing, I will calculate them for future years using the same rules.[66]

The extent to which the aforementioned errors have deceived astrologers may be touched upon with a few words. The astrologer, seeing Saturn by the common almanac in some degree and minute where a notable fixed star is situated without notable latitude, firmly prognosticates a change of the weather. And since according to the truth, as shown above, Saturn is removed from that degree and star by 2 or 3 degrees, the astrologer errs by up to one month. Thus in Jupiter he will be deceived by half a month; in Mars, Venus and Mercury by 2 or 3 days. And again the astrologer who is arranging the alchocoden[67] or another signifier determines the certain year of imminent danger; because of the aforementioned falsity of motions, he will err by 3 or 4 years. I have worked out these specific examples for the sake of brevity, in part that I might present lovers of astronomy the opportunity of observing, in part also that by that very observation the truth itself comes to light.

APPENDIX 2: Angelus, *Almanach 1512*, "Prefatio", sigs. [ii]^r-[iv]^r.

Almanach ex veris atque correctis meis tabulis calculatum ab aliis sepe numero vel uno duobus aut tribus gradibus discrepans. Cuius veritatem hoc anno currente Millesimo quingentesimo duodecimo diligens astrorum observator manifesta ut sequitur indagine deprehendet.

Jovi in primis anno presenti secundum vetus almanach Venus 16 die Aprilis coniungi putatur, que tamen iuxta almanach novum et verificatum eidem Jovi die 18 eiusdem mensis coniungetur. Hoc esse verum observator manifestę videbit, si celum non nubilum tunc fuerit, mane ante solis ortum in parte orientis.

In Marte vero novum hoc almanach verificatur. Mars 13 die Aprilis mane ante solis ortum iungi videbitur cuidam stelle fixe magnitudinis tertie in 28 gradu 41 minuto Sagittarij circa medium celi, cui iuxta vetus almanach 15 die euisdem mensis coniungi deberet. Et eidem stelle iterum coniungi ad diem 28 Maij ubi secundum almanach novum primo circa diem quintum Junii coniungetur. Itaque istorum duorum almanach in motu Martis differentia erit trium graduum. Sic manifeste peritus mathematicus sereno celo visurus est quodnam illorum almanach verius sit.

Observatio vero talis fieri debet post noctis ad ipsum celi medium propter enim latitudinem meridionalem Mars fallere posset spectantem. Nisi quis habeat instrumenta ad hoc apta per que veraciter planeta possit observari veluti instrumentum armillarum. Poteris tamen idipsum hoc pacto considerare. Accipe spheram materialem. Cuius axem ad meridianam lineam colloces per campastum correctum et ad elevationem poli arctici regionis tue affigendo quadrantem axi suo. Quo facto: figas circulum per medium divisum polis zodiaci per quem prospicias Martem et stellam fixam prenominatam quotto tunc die simul Mars et stella fixa videbuntur, erit dies coniunctionis huiusmodi siderum. Et quodcunque duorum almanach vetus scilicet vel novum magis cum tali die concordaverit verius erit procul dubio, addendo planete si directus fuerit aut ab eo subtrahendo si retrogradus existit semper minuta quibus movetur a meridie ad horam observationis.

Mars etiam prima die Januarij mane ante solis ortum ad celi medium secundum vetus almanach iungi putatur cuidam stelle fixe secundae magnitudinis in 7 gradu 41 minuto Scorpionis ubi secundum nostrum almanach in veritate eandem stellam tunc precessit pene per 1 g. et coniunctus fuit eidem stelle ultima Decembris anni precendentis.

Item Mars in sua prima statione retrogradus fieri incipiet 13 mensis Maij, attamen iuxta vetus almanach retrogradari ponitur quinta die eiusdem Maij ubi potissimum tunc subtilissimi mathematici Geber dictum circa principium primi sui tractatus verificabitur.

In Venere quam astronomus stellarum fixarum notitiam callens veritatem manifeste deprehendet, utrum vetus an novum almanach concordet cum stellis fixis minoribus ad medium mensis Januarij ante solis exortum, ubi differenter inter sese almanach discrepant in motu Veneris per unum gradum et dimidium, magis tamen illud percipi potest ad Decembris medium anni undecimi ex certis stellis fixis circa 27 gradum Capricorni, ubi tunc Venus retrogradabitur in 26 grad. 58 minuto Capricorni die 16 Decembris.

Motus vero Mercurij sepe numero differt inter hec duo almanach in uno gradu et ultra maxime in suis stationibus. Licet tunc pauci observare possint propter raram eius a sole absentiam. Perspicax tamen observator dignoscere poterit ad finem mensis Octobris post undecim milium Virginum festa ubi cum quibusdam stellis fixis invenietur. Ex quibus veritatem elicere possumus.

Plurima sunt huius aperta rerum testimonia ex quibus luce meridiana clarius est in planetarum motibus errorem esse quam plurimum. Sed hisce iam dictis studiosus siderum observator contentus sit, donec reliqua in lucem feramus et hasce certissimas ad oculum demonstrationes interea suscipiat. Rationes quam efficacissimas domino auxiliante hoc ipsum demonstrantes in alio opusculo divulgabo. Non enim in propatulo eas ipsas rationes meas super falsitate motuum planetarum hactenus in vulgum edere volui quoad laborem ampliorem iamdiu per me ceptum in corrigendis motibus adusque finem perducerem, quas quidem rationes cum consulto in alium mihi locum reservarem nec dum aperte posuerim. Excanduit in me nuper quidam Cracoviensis studij audaculus et semieruditus homunculo fortassis percupidus secretorum meorum quibus clam erudiri vellet. Qui si per me palam doceri non erubuerit de quibus meis multo iam tempore congestis laboribus mihi satisfacere velit ut errorem et falsitatem in planetarum motibus indubiam rationibus quidem demonstrantivis et solidissimis ostendam, quibus vel risum cachinumque barbarum sedabo suum. Utque proprius tue me fronti obiectem quisquis es qui risu tam facili et petulanti cachinno preclara mathematices et philosophantium studia ridere te iactas. Cum enim reprehendere me putas tu rerum omnium incautissimus viros insignes in astronomia excellentissimos carpis quos tamen una cum alijs quibusdam in almanach isto tuo lacero atque caduco extollis. In primis certe nostrum Georgium Peuerbachium virum ex confesso preclarum Viennensis gymnasij magistrum insignem qui novas edidit tabulas equationum planetarium quas multo iam tempore ad manus semper habeo. In quibus quidem tabellis partim correxit ac emendavit tabulas equationum argumentorum potissimum in epiciclorum parte inferiori et immatura sublatus morte partem epiciclorum superiorem reliquit et centrorum equationes. Quod si superi viro tanto dies paulo amplificassent procul omni dubio ad finem optatum laborem tam necessarium duxisset omniaque emendasset. In cuius quidem viri labores ingenuos licet incompletos ipse ego Omnipotentis auxilio subintravi Deoque auspice complevi. Tu igitur qui me carpere conaris, tuos imo et meos maiores turpiter interis commaculare.

Johannes quoque de Regio Monte germanus vir insignis memorati Viennensis studij alumnus hoc idem posteris memorandum reliquit videlicet contingere in planetarum motibus errorem graduum duorum. Cuius rationes atque probationes sui tempore domino volente patefacere conabor. Ipse quem ego hijs auribus audivi cum in Herbipoli forem ab ore familiarissimi auditoris memorati Joannis de Regio Monte post suum ex Italia reditum equationum planetarum tabellas eum emendare voluisse ex armillarum instrumento abipso confecto. Voluitque vir ille clarissimus proposito certo completis almanach prioribus de novo emendatiori limo correctis planetarum motibus in publicum impressorum manibus (si vita duraret) suum almanach tradidisse. Audi igitur emule invidule qui tuo isti cerebello confisus tantos viros non erubescis ridere cum me arguere putas. Sed quid opus est multis. Cum alios te longe sapientiores contaminare contendis te ipsum omnibus manifesta inscitia non modo sciolum ut me calumniaris verum inscium et erroneum pandis.

Tuumque manifestum errorem ausculta in motu quippe Lune aliorumque planetarum inniteris fundamento erroneo. Uterisque radice falsissima quod sic patefaciam. Lune motus in tuo almanach discrepabat alijs almanach super civitate Ulmensi calculata 40 minutis aliquando plus aliquando minus que si recte peritus astronomus calculaverit inveniet longitudinis differentiam inter Cracoviam et Ulmam 18 esse gradum temporis. Quorum graduum unicuique miliaria 16 germanica communiter attribui solent sicque resultarent ex toto miliaria alemanica

288 quam rem falsissimam esse omnes quibus ambe iste urbes cognite sunt manifeste dignoscunt. Quo circa si tui maiores in planetarum motibus hac tua radice seu longitudine usi sunt que circa 45 erit graduum veluti ex motibus tuis diiudico veraciter ausim affirmare rarum vel nullum in tua Cracovia ad unguem peritum astronomum fuisse quos tamen pro tuo ingenio supra reliquorum gymnasiorum mathematicos inpudenter extollere conaris. Existimas preterea te ad veram longitudinem calculare motus tuos quam longitudinem ab occidente vero et non ab habitato sumpsisse te iactitas ac si forsan alius nemo veram cuiusvis civitatis longitudinem noverit inuenire, cum tamen ingeniosissimus ille virorum magister Jo. de Regio Monte cui plusquam omnibus tuis Cracoviensibus astronomis in hac re fidei ubique gentium tribuitur, suos calcula[v]erit motus ad alicuius loci veram longitudinem, paucis astronomis notam, mihi quidem optime cognitam, a quo quidem loco adusque Cracoviam iuxta motum Lune tui almanach essent miliaria 240 quod adeo falsum est ut in 170 miliaribus error sit. Tu igitur tecum ipse cogita quam acceperis longitudinem sive radicem in motibus tuis que quidem longitudo discrepat tantum a longitudine viri tam insigniter docti, scilicet magistri Jo. de Regio Monte. Putasne etiam virum tam clarum novisse veram alicuius loci longitudinem elicere. Ne dubita verius hunc idipsum calluisse quam ingens astronomorum cetus reliquorum quemadmodum sui dicta comprobant.

Teneo quam atque iamdiu agnovi eam longitudinem quam Georgius ipse Peurbachius noster assignavit ab urbe scilicet Bavarie capitalis Monaco a qua quidem urbe Monacensi iuxta radicem tuam ad Cracoviam essent miliaria 260 que res etiam illiteratis hominibus super te et risum et stomachum movebit illis potissimum quibus cognite sunt urbes ambe. Adde etiam quod a Monaco ad Budam usque que ferme per directum situatur in eadem quasi longitudine Cracoviensium non centum miliaria numerentur. Imo certissimum habeo Erfordienses uti radicibus planetarum 27 graduum longitudinis et ad Nurmbergam ipse Jo. de Regio Monte usus; fuit longitudine 28 gradum que longitudines ab ea qua tu uteris longitudine 17 et 18 gradibus distant de quibus gradibus si unicuique 16 alemanica miliari dederis resultabunt secundum errorem tuum ab Erfordia ad Cracoviam ipsam 288 miliaria germanica quod adeo falsum et erroneum est ut tui te mercatores hinc atque illinc vagantes manifeste reprehendant.

Resipiscas igitur audacule tuamque domi reconde qua me culpas inscitiam. Nec prius alios docere atque reprehendere debueras quam tu teipsum erudivisse et plane ab erratis manifestis emendasse. Sed tu qui forsan ad pauca spectasti facile falsus atque deceptus es et qui te nemini extra paternum nidum comparasti alios facile superbo et arroganti spiritu contemnis. Tuosque idcirco cunctis aliis quos tu ignoras anteponere non erubescis. Qui si forsitan veritatis amicus magis quam calumniator esse volueris plurimas errorum mendas quibus astronomie pars magna corrupta est ostenderem. Unde etiam fit ut evo iam nostro hec eadem illustris atque excellentissima mathematices et astronomie scientia a pluribus rideatur et vilipendatur que tamen ab antiquissimis seculis semper inclita venerandaque habita est.

Vix enim caecutientibus oculis paucula coeli sidera contemplantes iam totam astronomie peritiam audacissime nobis arrogare presumimus cum innumera vero vel nobis incognita vel temporis iniuria mutata atque etiam depravata preterimus. Hec plurima correcta et emandata iam nostra tempora apertis oculis viderent si modo vir ille clarissimus Joannes de Regio Monte vitam longius in terris egisset. Is enim aperta permissione pollicitus est in scriptis suis que mecum versantur emendare voluisse libros nonnullos precipue vetulam illam traductionem cosmographie Ptholomei iam vulgo habitam reliquosque libros quos illic enumerat. Nec mirum cuiquam videri debet si vel Blankinus atque Alfonsus in suis tabellis arguant cum Geber ipse et Ptholomeum in locis pluribus multosque alios reprehenderit. Quo modo enim per Campanum decepti fuerint, longam scribendi moram exigeret. Mallem igitur viri climatores huiusmodi errorum quoniam plurimi resurgerent qui ex tenebris lucem, ex erratis veritatem elucidarent. Et priorum castigatorum ingressi laborem semicompletum cum ingenti totius astronomie gloria et utilitate ad finem optatum perducerent. Que omnia si tu calumniator recte animaduertis ses nequaquam me veterum virorum represensorem et quidem falso in simulasses. Non enim maiorum pios auferre vel carpere labores animus est. Sed vel tenebrosis addere lucem. Errataque iterum in veritatis orbitam recto rationis limite traducere.

Fistula nunc tenues tibi dat mea zoile flatus
Maioresque dabit buccina mota sonos.

Motus planetarum huius almanach calculati sunt ad eam longitudinem ad quam magister Joannes de Regio Monte calculavit ut tabella sua civitatum in addendo et subtrahendo ad diversa loca firma maneat. Hore etiam aspectuum Lune ad planetas computari debent more astronomorum a meridie ad meridiem per 24 horas.

Qui autem observant horalogium medium teneant horas a meridie more astronomorum ad medium noctis et a media nocte post 12 horas in circulo extremali. Notent alias 12 horas ponendo super 1/13 supra 2/14 et sic de singulis horis ad meridiem etiam habebunt suas horas. Illi autem qui horalogium suum inchoant ab occasu solis depingant sibi rotulam cum horis ac minutiis suis per 24 divisam et aliam cartham minorem quam centro priori affigant et signaturam vigesime quarte hore ponant ad occasum solis habebunt tempus aspectuum sub horis et minutis in almanach positis.

TRANSLATION

An almanac calculated from my true and corrected tables, differing repeatedly from other tables by one, two or three degrees, the truth of which a careful observer of stars in this current year 1512 may grasp thanks to the following clear tests.

First, according to the old almanac [Case 8], Venus this year is supposed to be in conjunction with Jupiter on 16 April; however, according to the new, verified almanac, her conjunction with the said Jupiter takes place on the eighteenth of the same month. That this is true an observer will clearly see, if the sky is unclouded, in the morning in the east before sunrise.

Next, this new almanac is confirmed in Mars. On 13 April in the morning before sunrise [Case 9], Mars will be seen to join with a certain fixed star of third magnitude [λ Sgr] in 28 degrees 41 minutes of Sagittarius, near the meridian. By the old almanac the conjunction with this star should happen on the fifteenth of the same month, and again with the same star on the twenty-eighth day of May [Case 10], whereas according to the new almanac it first will be in conjunction around the fifth day of June. Thus the difference between the two almanacs as regards the motion of Mars will be three degrees.

Given a clear sky, a skilled mathematician will see obviously which of those almanacs is closer to the truth. Such an observation must be done after midnight near the meridian, because the southern latitude of Mars could deceive the observer unless one has instruments suited to the purpose by which the planet can be observed truthfully, such as an armillary instrument. You will be able to consider this by the following rules: Use a material sphere, whose axis you should orient to the meridian line using a corrected compass, and to the elevation of the north pole of your region by placing a quadrant along its axis. Having done this, you should take a bearing with the latitude circle divided in half by the zodiacal poles, with which you will see Mars and the aforementioned fixed star. On which day then will Mars and the fixed star be seen together? This will be the day of such a conjunction of the stars. And whichever of the two almanacs, the old or the new, will agree with that day will without a doubt be the truer, having added to the planet the [arc]minutes by which it moves from noon to the hour of observation (if it is direct motion), or subtracted the same (if it is retrograde).[68]

Also, according to the old almanac [Case 11], on the first day of January in the morning before sunrise Mars close to the meridian is thought to be in conjunction with a certain star of the second magnitude [α_2 Lib] at 7 degrees 41 minutes of Scorpio, whereas in truth according to our almanac it precedes the same star by 1 degree and the conjunction was on the last day of December of the preceding year.

Similarly [Case 12] Mars in its first station begins to retrograde on the thirteenth of May, yet the old almanac puts its retrograde motion on the fifth day of the same May. It is here that the saying of the truly subtle mathematician Geber, near the beginning of his first treatise, will best be confirmed.[69]

As for Venus, an astronomer experienced in knowledge of the fixed stars clearly recognizes in truth whether the old almanac or the new agree with the faint fixed stars [ρ^1, υ Sgr] toward the middle of the month of January [Case 13] before sunrise, where the almanacs differ from each other in the motion of Venus by one degree and a half. This can be perceived even more in the middle of December of 1511 [Case 14], from certain stars near 27 degrees of Capricorn

[α_1, β Cap], where Venus will be retrograding in 26 degrees 58 minutes of Capricorn on the sixteenth day of December.

Indeed, in these two almanacs the motion of Mercury repeatedly differs by one degree and more, especially in its stations. Although few may observe it at that time on account of its infrequent separation from the Sun, the acute observer nevertheless will be able to recognize it at the end of the month of October [Case 15], after the Feast of the Eleven Thousand Virgins [21 October], when it will be found with certain fixed stars [ν, ω_1 Sco]; these facts will enable us to elucidate the truth.

There are many clear evidences of these things from which it is clearer than midday light that there are many errors in planetary motions. But what has already been said should satisfy the assiduous observer of stars until we bring the rest to light, and in the meantime he may grasp these certain and evident demonstrations. Lord willing, I will reveal in another work the most effective reasons which demonstrate this very matter, as I did not want to bring out openly in public my own reasons for the false motion of the planets until I might bring to conclusion a more extensive work begun a long time ago on correcting the motions — reasons I deliberately have reserved for another place and do not now reveal.

Not long ago a brash and semi-erudite homunculus from the University of Cracow burst into anger at me, probably desiring my secrets which he wanted to learn covertly.[70] Since he did not succeed in being taught openly by my labours accumulated for such a long time, he now wants to please me so that I would show the error and undoubted falsity in the planetary motions by demonstrative and most solid reasons and would put an end to his laughter and barbarous jeering. It is more fitting that I confront you, whoever you are, you who are so quick to mock by cheap laughter and impudent ridicule the distinguished studies of mathematics and philosophy. While you expect to reprehend me, you — reckless in all things — snipe at the most excellent astronomers; yet in that mangling and groundless almanac of yours you exalt these with some others — in the first place, surely, Georg Peurbach, a man distinguished beyond doubt, a notable teacher of the University of Vienna who brought out new tables of planetary equations which for a long time I always have had at hand.[71] It is in these tables that he partly corrected and emended the tables of the equations of argument, especially in the lower part of the epicycles. Taken away by an untimely death, he left [uncorrected] the upper part of the epicycles and the equations of centre, which if a higher power had extended his days a little longer, he undoubtedly would have completed and would have emended all. With the help of the Almighty, I have taken up the ingenious if incomplete labours of this man and under God's auspices I have completed it. Thus you who attempt to tear me to pieces, you shamelessly defile your predecessors and mine.

Also Johannes Regiomontanus, the distinguished German, student of the famous school of Vienna, bequeathed to posterity this same message, namely that there occurs an error of two degrees in planetary motions, the reasons and proofs of which I will attempt to reveal at the appropriate time, Lord willing.[72] When I was in Würzburg I heard with my own ears from the mouth of a student close to the celebrated Johannes Regiomontanus that after his return from Italy he had wanted to correct the tables of planetary equations with the aid of an armillary instrument he had made.[73] This illustrious man, having completed earlier almanacs following a definite plan, wanted again to print his almanac (if his life had lasted) with the planetary motions carefully corrected. Listen, then, jealous imitator, you who, trusting in this little brain of yours, are not ashamed to deride such great men while thinking you are arguing with me. But this is of wider importance: when you strive to debase others far wiser than yourself, with an ignorance evident to all, you show yourself to be a genuine ignoramus and vagabond and not merely a half-wise man, when you disgrace me falsely.

And listen carefully to your obvious error. In the motion of the Moon and the other planets, you have built on an erroneous foundation and used a false radix, which I will explain as follows. In your almanac, the motion of the Moon for the city of Ulm differed from other almanacs by more or less 40 minutes. If a skilled astronomer had computed it correctly, he would discover that the difference in longitude between Cracow and Ulm is 18 degrees of time, each of which is commonly accepted to be 16 German miles, and thus would yield 288 German miles. Those who know both cities obviously know that this is completely false. In this regard, if your predecessors had used your radix or longitude (about 45 degrees) which follows from your values, I would dare to affirm that in all of your Cracow there is rarely or never an experienced astronomer, whom you nevertheless impudently try to extol. In addition you think that you calculate your motions with respect to the true longitude, which publicly you declare to refer to the western limit of the inhabited world and not from the usual one, as if perhaps no one else would know how to find the true longitude of any given city. Now, however, that most ingenious of men, Master Johannes Regiomontanus, on whom people rely more than on all of your Cracow astronomers, has calculated his motions to the true longitude known to few astronomers but exceptionally well known to me,[74] from which place to Cracow, according to the motion of the Moon in your almanac, there would be 240 miles, which is false to such an extent that the error is 170 miles. Think to yourself, therefore, how you can adopt in your motions a longitude or radix that differs so much from the longitude of such a learned man as Johannes Regiomontanus. Do you not think that such a brilliant man knew how to produce the true longitude of a given place? Do not doubt that he himself has truly revealed this more so than is shown by the enormous group of other astronomers who accept his words as correct.

I maintain and also have long known the longitude assigned by our Georg Peurbach to Munich, the Bavarian capital city, from which town to Cracow, according to your radix, would be 260 miles. Now this will move even uneducated men to wild laughter, especially those who know both cities. Add also that from

Munich to Buda, which certainly lies near the meridian of Cracow, there are not one hundred miles. Indeed, I hold as most certain that the Erfurters use radices of planets for the longitude of 27 degrees; as for Nuremberg, Regiomontanus used a longitude of 28 degrees, which differ from that you use by 17 and 18 degrees. If you gave each degree the value of 16 German miles, there would be according to your error 288 German miles from Erfurt to Cracow, which is so evidently wrong that your merchants who travel there and back would criticize you.

Come to your senses, then, you brash fellow, and hide at home the ignorance of which you accused me. You ought not to teach and criticize others before instructing yourself and clearly healing yourself of obvious errors. But you, perhaps having seen few things, are easily mistaken and deceived; you who have measured yourself against no one outside the paternal nest easily condemn others with a proud and arrogant spirit. And therefore you feel no shame in putting yours [i.e., compatriots] before those unknown to you. If perhaps you had wanted to be a friend of truth rather than a vilifier, I would show you the sense of the many errors that have corrupted a large part of astronomy. Whence it also happens that even in our time many ridicule and vilify this excellent science of mathematics and astronomy, which from most ancient times has always been famous and an object of veneration.

We who observe a few stars with weakening eyes, we already presume audaciously to have mastered the whole of astronomy while we pass by countless [parts] unknown to us or changed and even deformed by the ravages of time. Our times would see many of these things corrected and emended if only that most excellent Johannes Regiomontanus had lived longer. In his writings that remain with me, he openly declared that he wanted to correct some books, especially the generally known old translation of Ptolemy's *Cosmography*, and other books that he enumerates here.[75] Let no one wonder if Bianchini and Alfonso should find fault with [Ptolemy] in their tables since even Geber has censured Ptolemy, as he does many others, in many places.[76] In what way they were deceived by Campanus would require a long time to write.[77] I would prefer therefore that many critics of such errors would come to life, who could reveal light from darkness and truth from errors and who, for the glory and utility of all of astronomy, could complete the unfinished labours of past correctors. Which, if you had paid attention, you would never have falsely accused me of being a critic of ancient men. Indeed, the intention is not to push aside or attack the efforts of our predecessors but to bring light into darkness and to transform errors into the orbit of truth by the rules of reason.

My flute gives you small sounds now
But the playing of the trumpet will provide greater sounds.[78]

The motions of planets in this almanac have been calculated for the same longitude that Master Johannes Regiomontanus used in his calculations, so that his table of towns for adding and subtracting to various fixed locations is preserved.[79] The times of lunar aspects have to be computed according to the astronomical custom, in twenty-four hours from noon to noon.

And those who use the mean clock should keep the hours from noon to midnight and twelve hours from midnight, following the astronomers. They designate the latter by putting 13 over 1 and 14 over 2, and so on for each hour until noon. In this way, they will have their reckoning. And those who start their clock from sunset should make themselves a circle with hours and minutes divided into 24 parts and another smaller card to be put at the centre of the first, and by putting the signature of the 24th hour at the sunset, they will have the time of aspects in hours and minutes as they are laid out in the almanac.

APPENDIX 3: Angelus's Comparisons of the "New" and "Old" Almanacs.

One might ask how a sixteenth-century reader, following Angelus's admonitions in his two prefaces to compare the competing predictions against the heavens for selected dates, might have judged the "new and corrected" almanacs.[80] In the prefaces, Angelus described fifteen specific comparisons, usually of the times of conjunction, which meant that his readers could make the observations without using angle-measuring instruments. He also set his comparisons for times when his predictions generally varied by maximal amounts from standard Alfonsine predictions (see Table 3). For example, for all four of the Mars comparisons (Cases 4, 5, 10, and 12 — all conjunctions) Angelus's predictions differ by more than 2° from those of Stöffler and Pflaum. Only for the inner planets, whose "false motions" (wrote Angelus in 1510) are discovered near maximal elongation, did Angelus propose what would have been an observationally meaningless comparison for a sixteenth-century reader (Case 7). For the nine maximal elongations in 1510, five of Angelus's predictions vary by less than 0;10° from the Alfonsine longitudes; the remaining four predictions vary by less than 0;30°. And to measure elongations accurately, observers would have needed an armillary sphere or some other angle-measuring instrument. Clearly for Case 7 Angelus either misunderstood how his modified model for the inner planets deviated from Alfonsine positions or he overestimated the observational capabilities of his contemporaries.[81]

As seen from Table 3, Angelus was overly optimistic in touting the improved accuracy of his new tables. Of the fifteen comparisons he suggested, the standard Alfonsine prediction would have been more accurate in eleven cases. Comparisons at maximal elongations would have been indecisive. Only in three of the cases described by Angelus (1, 6, 11) would the "new and corrected" almanacs have been more accurate, and only by small amounts.

APPENDIX 4: Producing "Alfonsine" Longitudes *circa* 1500.

The so-called Alfonsine Tables circulated widely from the fourteenth-century onward and completely dominated the making of ephemerides in the Latin West

TABLE 3. Angelus's comparisons of the almanacs.[82]

Case		Almanac Predictions "Common"	"New and Corrected"	Modern Configurations
1510 Preface				
[1]	Sat–γ Vir conjunction [in mid-retrograde] [γ Vi = 182;55]	Apr 14 Sat = 182;57	**Mar 29** [Sat = 182;56]	Conjunction on Apr 5 Mar 29, Sat = 183;50 Apr 14, Sat = 182;44 γ Vir = 183;23
[2]	As above [in direct motion]	**Jul 8** [Sat = 182;59]	Jul 20 [Sat = 183;00]	Conjunction on Jul 13 Jul 8, Sat = 182;57 Jul 20, Sat = 183;52
[3]	Jup among "little stars" [2d stationary pt = S2] [43 Sgr = 281;05] [ρ^1 Sgr = 282;05] [υ Sgr = 282;35]	**End of Aug** [Jup = 281;48, at S2 on Aug 26]	Never [Jup = 284;17, at S2 on Aug 29]	Jup at S2, 282;06 on Aug 28 43 Sgr = 281;31 ρ^1 Sgr = 282;38 υ Sgr = 282;54
[4]	Mars–[86 Vir] conjunction [at S2] [86 Vir = 201;25]	**May 22** Mars = 201;27[83]	Mars "3° from star" [Mars = 204;27, at S2 May 22]	Mars at S2 on May 22, at 201;45 86 Vir = 202;12
[5]	Moon–Mars conjunction [in mid-retrograde]	**Apr 22, 10 p.m.** [Mars = 206;47]	Apr 23, 2:30 a.m. [Mars = 209;16]	Apr 22, 8:30 p.m. Mars = 207;28
[6]	Jup–Mars conjunction [both in direct motion]	Oct 28[84]	**Oct 31**	Conjunction on Oct 30
[7]	Max elongations of inferior planets [all times at noon, on the same days]	Venus distance from Sun [Mar 15, **−45;50**] [Aug 9, 45;16] Mercury distance from Sun [Jan 21, 27;55] [Apr 4, −20;45] [May 17, 22;49] [Jul 28, −26;32] [Sep 12, 16;13] [Nov 19, −19;20] [Dec 31, 25;54]	 [−45;56] **[45;18]** [27;55] **[−20;44]** **[22;53]** **[−26;49]** **[16;37]** **[−19;32]** **[25;37]**	 −45;45 45;41 24;21 −16;51 23;50 −26;42 16;50 −20;29 23;39
1512 Preface				
[8]	Jup–Ven conjunction [both in direct motion]	**Apr 16**[85]	Apr 18	Conjunction on Apr 17 at 11 a.m.
[9]	Mars–[λ Sgr] conjunction [in direct motion] [λ Sgr = 268;45]	**Apr 15** [Mars = 268;42]	Apr 13 Mars = 268;41	Closest approach on May 3 Apr 13 at 6 a.m. = 266;20 Apr 15 at 6 a.m. = 266;45 λ Sgr = 269;31
[10]	Mars–[λ Sgr] conjunction [in mid-retrograde]	**May 28** [268;45]	Jun 5 [269;34][86]	May 28 at 8 p.m. = 264;36 Jun 5 at 8 p.m. = 262;12 Jun 7 at 8 p.m. = 261;34
[11]	Mars–[α_2 Lib] conjunction [in direct motion] [α_2 Lib = 217;45]	Jan 1 at 7 a.m. 217;41	**Dec 31**	Conjunction on Dec 31 Jan 1 at 7 a.m. = 218;41 α_2 Lib = 218;17
[12]	Mars begins retrograde [at S1]	**May 5**	May 13	May 5
[13]	Ven in mid-Jan close to [ρ^1, υ Sgr] [near S2]	**Jan 19**[87] [284;35]	Jan 19 [286;05]	Jan 19 at 6 a.m. = 285;02
[14]	Ven near [α_1, β Cap] [near S1] [α_1 Cap = 297;05] [β Cap = 297;05]	**Dec 16, 1511** [298;31]	Dec 16, 1511 296;58[88]	Dec 16 at 6 p.m. = 299;49 α_1 Cap = 296;59 β Cap = 297;14
[15]	Mer near [ν, ω_1 Sco] [near S1] [ν Sco = 237;05] [ω_1 Sco = 237;05]	**Oct 21** [237;03]	Oct 21 [236;43]	Oct 21 = 240;10 ν Sco = 237;51 ω_1 Sco = 236;52

until the publication of Erasmus Reinhold's Copernican *Prutenicae tabulae* in 1551.[89] Although several versions of the Alfonsine canons (by John of Saxony, John of Lignères, John of Murs) and several arrangements of the columns and computing procedures were formulated (the Oxford Tables, John of Gmunden's tables, Giovanni Bianchini's tables, the *Tabulae resolutae*), the underlying planetary models and parameters remained unchanged. To date, no critical edition of any manuscript version of the Alfonsine Tables has been prepared. Yet Emmanuel Poulle, Owen Gingerich and John D. North have noted some of the numerical inconsistencies (beyond simple copying errors) that can be found in many Alfonsine manuscripts.[90] For example, in many tables Jupiter's equation of the argument appears to be computed for an eccentricity of 2^p;45, while its equation of the centre uses a value of 3^p;07. Likewise, different rounding procedures in the intermediate computations may shift the computed longitudes by several minutes of arc. To produce fifteenth-century "Alfonsine" positions, therefore, one cannot simply insert Alfonsine parameters into a mathematically consistent algorithm based on Ptolemaic geometry and the Alfonsine implementation of the motion of the eighth sphere.

Hence, we derive our "Alfonsine" longitudes from the first published set of *Alfonsine tables* (Venice, 1483), as edited by Poulle, 1984. Using the 1483 parameters for the mean motions, radices, and the motion of the eighth sphere, we first compute the aux, mean centre and the mean argument. Our computer program then employs the 1483 tables of planetary equations, with all their blemishes uncorrected, to 'look up' the equations of the centre and argument, the *diversitas diametri* and the proportional minutes, with linear interpolation between the one-degree tabular intervals. We do not round intermediate results.

To investigate the significance of internal inconsistencies and other errors in the 1483 planetary tables, we have compared longitudes computed by our '1483 generator' with those from algorithms consistently employing the Alfonsine parameters. For Saturn, the longitudes for 1510–12 determined by these two procedures vary by up to ± 0;01°; for Jupiter, by ± 0;04°; for Mars, by ± 0;10°; for Venus, by ± 0;12°; and for Mercury, by ± 0;02°. Likewise, we have found that our '1483 generator' generally matches to within 0;10° the planetary longitudes in a variety of mid- to late fifteenth-century ephemerides (for which we have no information concerning the version of the Alfonsine Tables used in their preparation). Thus, despite the possibilities for divergent rounding procedures and inconsistencies in the tables themselves, it seems possible for dates early in the early sixteenth century to identify reliably a set of 'Alfonsine' longitudes within a 'noise' band of ± 0;10°.

Although our '1483 generator' does not require the use of planetary eccentricities or epicycle radii, we do need the latter values to compute our modified models. For the epicycle radii, we employ the values found by North to be used consistently in five sets of ancient and medieval tables, including the *Alfonsine tables*.[91]

REFERENCES

1. Johannes Angelus, *Almanach novum atque correctum ... calculatum super anno domini 1510* (Vienna, [1510]); *idem*, *Almanach novum atque correctum ... calculatum super anno domini 1512* (Vienna, 1512). Both almanacs are extremely rare: the 1510 is extant in Gniezno, Cathedral Chapter Library, Inc. 43e; Vienna, University Library, I. 233605; Munich, Bavarian State Library, Res. 4° Eph. astr. 155/2 (incomplete); the 1512 in Vienna, Austrian National Library, 72.J.114(3); Graz, University Library, I 4070 (incomplete); Munich, Res. 4° Eph. astr. 155/3. Both almanacs contain a preface, an annual ephemeris in twelve folios, the canon from Johannes Regiomontanus's ephemerides published in 1474, and a three-folio abridgment of some rules for astrological prognostication that Erhard Ratdolt had appended to his 1484 edition of Regiomontanus's ephemerides. Angelus's 1510 work presents only planetary longitudes as had Regiomontanus; the 1512 almanac, however, also lists planetary latitudes at 10-day intervals, following the pattern established by Johann Stöffler and Jacob Pflaum, *Almanach nova plurimis annis venturis inservientia* (Ulm, 1499). For typographical descriptions of Angelus's almanacs, see Walther Dolsch, *Bibliographie der österreichischen Drucke des XV. und XVI. Jahrhunderts*, i/1: *Trient-Wien-Schrattenthal* (Vienna, 1913), 69–70, 73–74; *Verzeichnis der im deutschen Sprachbereich erschienenen Drucke des XVI. Jahrhunderts* (Stuttgart, 1983–), E1196–7.
2. Appendices 1 and 2, below.
3. Stanislaus Aurifaber, *Ephemerides anni Christi MDCII* (Cracow, 1512), sigs. ai^v-aii^v; Georg Tanstetter Collimitius (ed.), *Tabulae eclypsium magistri Georgij Peurbachij. Tabula primi mobilis Joannis de Monteregio* (Vienna, 1514), sig. aa6^r; Georg Joachim Rheticus, *De libris revolutionum Copernici narratio prima* (Gdańsk, 1540), sig. Ii^r. None of the recent translators of the *Narratio prima* explored what Rheticus or Copernicus might have known about Angelus's work. See Georg Joachim Rheticus, *Erster Bericht über die 6 Bücher des Kopernikus von den Kreisbewegungen der Himmelsbahnen*, transl. by Karl Zeller (Munich, 1943), 184; Edward Rosen, *Three Copernican treatises*, 2nd edn (New York, 1959), 192; Georg Joachim Rheticus, *Narratio prima: Edition critique, traduction française et commentaire*, ed. by Henri Hugonnard-Roche and Jean-Pierre Verdet (*Studia Copernicana*, xx; Wrocław, 1982), 192.
4. For recent reviews of the secondary literature on Angelus's life and work, see Bernhard D. Haage (ed.), *Das Heidelberger Schicksalbuch: Das 'Astrolabium planum' deutsch aus CPG 832 der Universitätsbibliothek Heidelberg, Kommentar* (Frankfurt, 1981), 34–59; Eberhard Knobloch, "Astrologie als astronomische Ingenieurkunst des Hochmittelalters: Zum Leben und Wirken des Iatromathematikers und Astronomen Johannes Engel (vor 1472–1512)", *Sudhoffs Archiv*, lxvii (1983), 129–44; Helmuth Grössing, "Angelus, Johannes", *Archiv der Geschichte der Naturwissenschaften*, xvi (1986), 789–92; and especially Christoph Schöner, *Mathematik und Astronomie an der Universität Ingolstadt im 15. und 16. Jahrhundert* (Berlin, 1994), 195–200, 223–32, 466–73, who offers a new and detailed account of Angelus's years in Ingolstadt. For the role of mathematics in a sixteenth-century medical career, see Robert S. Westman, "Humanism and scientific roles in the sixteenth century", in *Humanismus und Naturwissenschaften*, ed. by Rudolf Schmitz and Fritz Krafft (Boppard, 1980), 83–99.
5. For the Celtis circle, see Franz Stuhlhofer, "Georg Tanstetter (Collimitius): Astronom, Astrologe und Leibarzt bei Maximilian I. und Ferdinand I.", *Jahrbuch des Vereins für Geschichte der Stadt Wien*, xxxvii (1981), 7–49; Helmuth Grössing, *Humanistische Naturwissenschaft: Zur Geschichte der Wiener mathematischen Schulen des 15. und 16. Jahrhunderts* (Baden-Baden, 1983), 170–92; Schöner, *Mathematik und Astronomie* (ref. 4), 202–15, 257–80.
6. The only autograph of Angelus we have found is a signed receipt for five medical books that he deposited with the Ingolstadt faculty in 1483 as security for a loan. Munich, Ludwig-Maximilian-University Archive, O-V-I, 34a^r, transcribed in Paul Ruf, *Mittelalterliche Bibliothekskataloge Deutschlands und der Schweiz*, iii/2: *Bistum Augsburg* (Munich, 1932), 231–2 (see Figure 2).
7. See Grössing, "Angelus" (ref. 4), 790–2, for a bibliography of Angelus's published works. Josef Seethaler, "Das Wiener Kalenderwesen von seinen Anfängen bis zum Ende des 17. Jahrhunderts", Ph.D. dissertation, University of Vienna, 1982, pp. 748–9, attributes to Angelus two single-sheet calendars for 1512 and 1513, printed in Vienna and recently found at the Hungarian Academy of Sciences in Budapest. We have been unable to examine these calendars,

cited by neither Grössing nor standard bibliographies of publishing or astronomical history.

8. In Angelus's 1484 German calendar (*Gesamtkatalog der Wiegendrucke* (Leipzig, 1925–), 1892; hereafter *GW*), the times for new and full moons exceed those in Regiomontanus's ephemerides by 0;02° in all but two cases, and a conjunction of Saturn and Jupiter, predicted by Angelus at 233;44° on 25 November at 6:52 p.m., is set by Regiomontanus for 7:00 p.m. on the same day at 233;43°. Angelus's 1489 Latin calendar (*GW* 1896) presents verbatim Regiomontanus's times for the new and full moons, and for a lunar eclipse on 7 December. Angelus's 1490 German calendar (*GW* 1899) also exactly matches Regimontanus's times for the new and full moons in all but two cases. And the 1497 German practica (*GW* 1904) increases Regiomontanus's times for the new and full moons by 0;02° in all but two cases. To construct these calendars, Angelus undoubtedly employed Regiomontanus's ephemerides, occasionally shifting the lunar positions to a different meridian.
9. According to Knobloch, "Astrologie" (ref. 4), 136, Claude François Milliet Dechales, *Cursus seu mundus mathematicus*, 2nd edn (Lyon, 1690), 1:88, first attributed a 1494 ephemerides to Angelus. Johann Friedrich Weidler, *Historia astronomiae* (Wittenberg, 1741), 327, continued this attribution, and a "1494 ephemerides" remained firmly entrenched in the bibliographical literature through J. C. Houzeau and A. Lancaster, *Bibliographie générale de l'astronomie* (Brussels, 1882–89), #14509. Yet neither *GW* (ref. 8), ii:267, nor Ernst Zinner, *Geschichte und Bibliographie der astronomischen Literatur in Deutschland zur Zeit der Renaissance*, 2nd edn (Stuttgart, 1964), 541, could confirm the existence of a such a treatise, and after considerable searching we have concluded that Dechales probably misread Tanstetter, *Tabulae eclypsium* (ref. 3), and created a ghost.
10. Appendices 1 and 2, below.
11. Appendices 1 and 2, below.
12. To sample longitudes at five-day intervals starting with 5 January 1510 and at two-day intervals for Mercury starting with 1 January 1510, we corrected 13 typographical errors in Angelus and 7 in Stöffler and Pflaum. Each data set (ignoring the lunar nodes) includes a total of 1442 longitudes; hence, the rate of obvious typographical errors is below one percent in both almanacs.
13. In their table of geographical longitudes, Stöffler and Pflaum list Toledo as 20.5°W of Ulm. For sixteenth-century readers' complaints about inaccuracies in this table, see Richard L. Kremer and Jerzy Dobrzycki, "Disputationes inter Viennensem et Cracoviensem, II" (forthcoming).
14. Appendix 2, below.
15. According to Stöffler's and Pflaum's chart of geographical longitudes, Vienna is 5;30°E of Ulm. Angelus apparently decided to displace the meridian for his 1512 almanac from Ulm to Vienna, but erred by a factor of 2 (perhaps confusing minutes of time with minutes of lunar motion in longitude, both of which were included in Stöffler's and Pflaum's table of geographical longitudes). See Kremer and Dobrzycki, *op. cit.* (ref. 13).
16. Leipzig, 5-day (Sun, Moon, Mercury), 15-day (Venus, Mars), and 30-day (Jupiter, Saturn) ephemerides for 1440–69, although filled with egregious scribal errors, is Alfonsine to ± 0;15° (Nuremberg, City Library, Cent VI 16, 149r–95r; *cf.* Ernst Zinner, *Leben und Wirken des Joh. Müller von Königsberg genannt Regiomontanus*, 2nd rev. edn (Osnabrück, 1968), 14–15); Regensburg, daily ephemerides for 1463, Alfonsine to ± 0;10° for all planets except Mars (±0;20°) (Munich, Bavarian State Library, Clm 14504, 140r–5v); Leipzig and Vienna, Regiomontanus's daily ephemerides for 1448, 1451, 1453–63, Alfonsine to ± 0;10° for all planets except Mars (± 0;20° when near the second stationary point) (Vienna, Austrian National Library, lat. 4988, 1r–188nr); Bamberg, daily ephemerides for 1464–84, Alfonsine to ± 0;10° (*ibid.*, 189r–314v); Nuremberg, Regiomontanus's daily ephemerides for 1472, Alfonsine to ± 0;10° (13 handwritten folios bound with Regiomontanus, *Ephemerides* (ref. 1), Munich, Bavarian State Library, Rar. 299a); Nuremberg, Regiomontanus's daily ephemerides for 1475–1506, Alfonsine to ± 0;10° (Regiomontanus, *Ephemerides* (ref. 1)). We do not know which versions of the Alfonsine Tables were used to compute these various ephemerides; yet in all cases, they generally match longitudes generated from Alfonso X, *Tabule astronomice* (Venice, 1483) to ±0;10°.

17. To compute planetary longitudes according to modern theory, we have used computer programs kindly provided by Owen Gingerich, which follow the procedures of Bryant Tuckerman, *Planetary, lunar, and solar positions, A.D. 2 to A.D. 1649* (Philadelphia, 1964), but include additional subroutines written by Peter Huber to implement perturbation theory more exactly.
18. Over the two years in question, Angelus's longitudes for Saturn deviate by up to 1° from the modern positions, Stöffler's and Pflaum's by only half that amount. For Jupiter, the respective maximal deviations are about 2° and 1°; for Mars, nearly 7° and 4°; for Venus, 2.5° and 1°; and for Mercury, both almanacs differ from modern positions by a maximum of 14° near the stationary points.
19. Bernard R. Goldstein, "Levi ben Gerson's lunar model", *Centaurus*, xvi (1972), 257–84; *idem*, "Levi ben Gerson's preliminary lunar model", *Centaurus*, xviii (1974), 275–88; *idem*, "A new set of fourteenth century planetary observations", *Proceedings of the American Philosophical Society*, cxxxii (1988), 371–99; Noel M. Swerdlow, "Regiomontanus on the critical problems of astronomy", in *Nature, experiment, and the sciences*, ed. by T. H. Levere and W. R. Shea (Dordrecht, 1990), 165–95, pp. 172–3; O. Neugebauer, *A history of ancient mathematical astronomy* (Berlin, 1975), 145.
20. Levi ben Gerson explored this latter possibility, an arrangement Bernard R. Goldstein has called the "skew equant model" in *The astronomy of Levi ben Gerson* (New York, 1985), 114–29, 192–7.
21. We employ the notation of Olaf Pedersen, *A survey of the Almagest* (Odense, 1974). For useful discussions of Alfonsine procedures, see Emmanuel Poulle and Owen Gingerich, "Les positions des planètes au moyen âge: Application du calcul életronique aux tables alphonsines", *Académie des Inscriptions & Belles-Lettres, Comptes rendus*, 1968, 531–48; J. D. North (ed.), *Richard of Wallingford: An edition of his writings, with introductions, English translation and commentary* (3 vols, Oxford, 1976), iii, 168–200; and Emmanuel Poulle (ed.), *Les tables alphonsines avec les canons de Jean de Saxe: Édition, traduction et commentaire* (Paris, 1984).
22. An exact tabular solution for the mechanisms of Equations [2] and [3] would require modifying the *diversitas diametri*. Yet for the values of the cofficients determined below, such modifications would not shift final longitudes by more than 0;10°. Furthermore Angelus in the prefaces did not mention altering any Alfonsine columns other than the equations of the centre and argument. It seems quite likely, therefore, that he employed unmodified equations for the *diversitas diametri*, as we have done for our computations.
23. The two columns in bold type implement the harmonic mechanisms of Equations [1] and [3]; the remaining columns are taken unchanged from the Alfonsine Tables.
24. In such cases, the true argument must be further modified, so that Equation [5] becomes: $a'_v = a_m - T_1 f(c_m) - q(c'_m) - \Delta\lambda$.
25. Adjusting radices in planetary tables was not uncommon. A copy of John of Lignères's table of mean planetary motions, written *c.* 1470 at the St Egidien Monastery in Nuremberg, bears the following annotations in the hand of the copyist (Frater Laurentius): secundum magistrum Iohannem 3 gradus addendi 1473 [for Saturn]; uno gradu subtrahendo secundum Ioh Kunig [for Jupiter]; addendi 4 gradus secundum magistrum Iohannem Kunics [for Mars] (Munich, Ludwig-Maximilian-University Library, 2° Cod. ms. 593, 12^r, 14^v, 17^r). Apparently the scribe thought that Regiomontanus in 1473 had introduced several constant modifications into the radices of planetary tables, alterations which do not appear, however, in Regiomontanus's extant ephemerides (Zinner, *Regiomontanus* (ref. 16), 198; Gerhard Schott, *Die Handschriften der Universitätsbibliothek München*, iii/2, *Die lateinischen mittelalterlichen Handschriften* (Wiesbaden, 1979), 102–5). Likewise, a mid-fifteenth-century Jewish scholar in Mantua, Mordekhai Finzi, suggested changing the Alfonsine lunar radices (Y. Tzvi Langermann, "The scientific writings of Mordekhai Finzi", *Italia: Studi e ricerche sulla storia, la cultura e la letteratura degli ebrei d'Italia*, vii (1988), 8–44, pp. 15–20).
26. Examination of the daily longitudinal increments in both almanacs suggests that Stöffler and Pflaum often computed positions at 10-day (Mars), 5-day (Venus) or 3-day (Mercury) intervals, and linearly interpolated between these epochs. Angelus's daily increments, however, vary more smoothly and reveal no obvious computational epochs longer than one day.

27. Poulle and Gingerich, *Les positions des planètes* (ref. 21), 542, 544; see above, ref. 16.
28. Already in 1456, Peurbach and Regiomontanus were computing ephemerides together, using Giovanni Bianchini's tables. By 1457, they began systematically observing the motion of Mars, noting that its longitude might deviate by up to 1;00° from the Alfonsine predictions. In 1458, Regiomontanus found that his own computed longitudes for Mars differed dramatically from those in an almanac calculated by one Master Purkhard Nestler of Salzburg, and wrote into his almanac for that year: "Corrige Martem." At some point, Peurbach apparently asked Nestler to observe Mars, noting that the predicted locations for that planet could err by 1;30°. See Peurbach to Johann Nihil of Bohemia, n.d. [1456], in Albin Czerny, "Aus dem Briefwechsel des grossen Astronomen Georg von Peurbach", *Archiv für österreichische Geschichte*, lxxii (1888), 281–304, p. 302; Regiomontanus's unpublished ephemerides for 1457–58, in Vienna, Austrian National Library, lat. 4988, 90^{v}, 116^{v}, 117^{v}, 118^{r}, 118^{v}, 119^{r}, 120^{r}; Jacob Ziegler, *In C. Plinii De naturali historia librum secundum commentarius* (Basel, 1531), 446; Zinner, *Regiomontanus* (ref. 16), 44, 56. Extant sources record 19 planetary observations by Regiomontanus from which longitudes can be determined; 13 of these were of Mars. Bernard Walther, the Nuremberg merchant who continued Regiomontanus's observational activity starting in 1475, devoted half of the nearly 100 planetary observations he made through 1479 to Mars. See Felix Schmeidler (ed.), *Joannis Regiomontani Opera collectanea* (Osnabrück, 1972), 645–71.
29. See E. J. Aiton, "Peurbach's *Theoricae novae planetarum*: A translation with commentary", *Osiris*, 2nd ser., iii (1987), 5–43, p. 8.
30. We shall ignore more esoteric geometries for the inner planets, such as placing an equant point and eccentric within the epicycle. Although such an arrangement can produce the ellipse drawn out by Equations [2] and [3], we know of no ancient or medieval astronomer who sought to generate the second anomaly with Ptolemaic techniques developed for the first anomaly.
31. See Victor Roberts, "The solar and lunar theory of Ibn ash-Shāṭir: A pre-Copernican Copernican model", *Isis*, xlviii (1957), 428–32; *idem*, "The planetary theory of Ibn al-Shāṭir: Latitudes of the planets", *Isis*, lvii (1966), 208–19; E. S. Kennedy and Victor Roberts, "The planetary theory of Ibn al-Shāṭir", *Isis*, l (1959), 227–35; Fuad Abbud, "The planetary theory of Ibn al-Shāṭir: Reduction of the geometric models to numerical tables", *Isis*, liii (1962), 492–9; Noel M. Swerdlow, "The derivation and first draft of Copernicus's planetary theory: A translation of the *Commentariolus* with commentary", *Proceedings of the American Philosophical Society*, cxvii (1973), 423–512.
32. See Swerdlow, "*Commentariolus*" (ref. 31), 470, for a derivation of this property of the double epicycle model.
33. See F. J. Ragep, "The two versions of the Ṭūsī Couple", in *From deferent to equant: A volume of studies in the history of science in the ancient and medieval Near East in honor of E. S. Kennedy*, ed. by David A. King and George Saliba (New York, 1987), 329–56, pp. 332–40; *idem*, *Nasir al-Din al-Ṭūsī's Memoir on astronomy* (New York, 1993), 194–212; Swerdlow, "*Commentariolus*" (ref. 31), 499–509; *idem* and O. Neugebauer, *Mathematical astronomy in Copernicus's De revolutionibus* (New York, 1984), 46–47; George Saliba, "The role of the *Almagest* commentaries in medieval Arabic astronomy: A preliminary survey of Ṭūsī's redaction of Ptolemy's *Almagest*", *Archives internationales d'histoire des sciences*, xxxvii (1987), 3–20; Mario di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's device: Observations on the use and transmission of a model", *Journal for the history of astronomy*, xxvi (1995), 133–54.
34. The literature on medieval theories of precession and trepidation is vast; for useful surveys, see Jerzy Dobrzycki, "Teoria precesji w astronomii średniowiecznej", *Studia i materiały z dziejów nauki polskiej*, ser. c, xi (1965), 3–47, and most recently, Bernard R. Goldstein, "Historical perspectives on Copernicus's account of precession", *Journal for the history of astronomy*, xxv (1994), 187–97.
35. Aiton, "Peurbach's *Theoricae*" (ref. 29), 36–43; Swerdlow, "Regiomontanus" (ref. 19).
36. G. J. Toomer, *Ptolemy's Almagest* (London, 1984), 599–601; R. C. Riddell, "The latitudes of Venus and Mercury in the *Almagest*", *Archive for history of exact sciences*, xix (1978), 95–111; Bernard R. Goldstein, *al-Biṭrūjī on the principles of astronomy* (2 vols, New Haven, 1971).
37. This same objection can be made against another means of producing angular harmonic motion by

uniformly rotating spheres, viz., the combination of two concentric spheres rotating in opposite directions on non-parallel axes which Ragep, *al-Ṭūsī's Memoir* (ref. 33), 451–2, felicitously has labelled the "Eudoxan couple".

38. Aiton, "Peurbach's *Theoricae*" (ref. 29), 38; Regiomontanus to Speier, 1465, in Maximilian Curtze, "Der Briefwechsel Regiomontan's mit Giovanni Bianchini, Jacob von Speier und Christian Roder", *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, xii (1902), 187–336, p. 303; Swerdlow, "Regiomontanus" (ref. 19), 180–1; Michael H. Shank, "The 'Notes on al-Biṭrūjī' attributed to Regiomontanus: Second thoughts", *Journal for the history of astronomy*, xxiii (1992), 15–30.
39. Florence, Bib. Naz. Cen., MS Magl. XI 144, 16^{r}–17^{v}. This text, discovered by Ernst Zinner in 1953, has not been published. See Shank, "The 'Notes on al-Biṭrūjī'" (ref. 38), 19–22 and note 32, and Michael H. Shank and Richard L. Kremer, "Regiomontanus's homocentric astronomy" (forthcoming). We thank Menso Folkerts for allowing us to use his microfilm of this manuscript, and Noel Swerdlow for sending us his working transcription and translation of the text.
40. MS Magl. XI 144 (ref. 39), 16^{r}, transl. by Shank, "The 'Notes on al-Biṭrūjī'" (ref. 38), 20–21. See Shank and Kremer, "Regiomontanus's homocentric astronomy" (ref. 38).
41. Swerdlow, "Regiomontanus" (ref. 19), 174. For Regiomontanus's program of reform, see Zinner, *Regiomontanus* (ref. 16), 279–80; Paul Lawrence Rose, *The Italian Renaissance of mathematics: Studies on humanists and mathematicians from Petrarch to Galileo* (Geneva, 1975), 90–117; Swerdlow and Neugebauer, *Mathematical astronomy* (ref. 33), 52–54.
42. Swerdlow, "Regiomontanus" (ref. 19), 172–3. All of Regiomontanus's extant almanacs, however, are strictly Alfonsine (see ref. 16), as is Peurbach's *Tabulae eclypsium* (completed 1459, published in Tanstetter, *Tabulae eclypsium* (ref. 3)) and an anonymous perpetual almanac for 1455–1540 (copies in Munich, Bavarian State Library, Clm 3001, 2–52; Clm 19550, 93–148; Clm 18778, 79–97), attributed by Zinner, *Regiomontanus* (ref. 16), 35, to Peurbach.
43. See E. S. Kennedy, "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society*, n.s., xlvi (1956), 121–77.
44. Aiton, "Peurbach's *Theoricae*" (ref. 29), 36. "Propter dictas epicyclorum inclinationes atque reflexiones orbes parvi epicyclos intra se locantes a quibusdam ponuntur ad quorum motum eaedem contingunt" (Vienna, Austrian National Library, lat. 5203, 21^{r}; Schmeidler (ed.), *Opera* (ref. 28), 787). None of the early commentators on the *Theoricae* remarks on this passage. See L. A. Birkenmajer (ed.), *Albertus de Brudzewo: Commentariolum super theoricas novas planetarum Georgii Purbachii [1482]* (Cracow, 1900), 143–5; Franciscus Capuani, *Theoricarum novarum textus Georgij Purbachij cum utli ac preclarissima expositione* (Paris, 1515), 52^{r}–53^{r}; Erasmus Reinhold, *Theoricae novae planetarum Georgij Peurbacchii* (Wittenberg, 1542), sig. dv^{r}.
45. See Ragep, *al-Ṭūsī's Memoir* (ref. 33), 214–16, 450–4.
46. See C. Doris Hellman and Noel M. Swerdlow, "Peurbach, Georg", in *Dictionary of scientific biography* (New York, 1970–80), xv, 473–9, p. 475; Y. Tzvi Langermann, *Ibn al-Haytham's On the configuration of the world* (New York, 1990).
47. Ragep, *al-Ṭūsī's Memoir* (ref. 33), 450. Another work by al-Haytham, replying to a critic of his text on latitude theory, is extant and has been edited by A. I. Sabra, "Ibn al-Haytham's treatise: Solutions of difficulties concerning the movement of *Iltifāf*", *Journal for the history of Arabic science*, iii (1979), 388–422. But apparently this latter text also was never translated into Latin.
48. Ragep, *al-Ṭūsī's Memoir* (ref. 33), 58–62, lists eight Arabic commentaries on the *Tadhkira* written before 1460, none of which is known to have been translated into Latin.
49. J. L. Mancha, "Ibn al-Haytham's homocentric epicycles in Latin astronomical texts of the XIVth and XVth centuries", *Centaurus*, xxxiii (1990), 70–89.
50. Ragep, *al-Ṭūsī's Memoir* (ref. 33), 58.
51. *Cf.* Ragep, *al-Ṭūsī's Memoir* (ref. 33), 48–51; Swerdlow, "*Commentariolus*" (ref. 31), 433–5. See George Saliba, "The astronomical tradition of Marāgha: A historical survey and prospects for future research", *Arabic sciences and philosophy*, i (1991), 67–99, pp. 74–81; and Di Bono, "Copernicus, Amico, Fracastoro" (ref. 33), for surveys of research on links between the Marāgha School and Copernicus.

52. Figure 7 is produced according to Levi's lunar models and parameters, as decribed by Goldstein, "Lunar model" (ref. 19); *idem*, "Preliminary lunar model" (ref. 19). Apparently, Levi never completed the planetary modified models. See *idem*, "Planetary observations" (ref. 19).
53. For Werner's highly original model of precession which attracted no followers, see Dobrzycki, "Teoria precesji" (ref. 34), 29–32; *idem*, "Astronomical aspects of the calendar reform", in *Gregorian reform of the calendar*, ed. by G. V. Coyne, M. A. Hoskin, and O. Pedersen (Vatican City, 1983), 117–26, p. 122. Werner allotted the trepidational motion of "Thabit's" and Peurbach's models to the solstitial points of two concentric spheres. Two circles of trepidation, of equal radii and centred on the solstitial points of the next higher sphere, rotate in opposite directions so that trepidational variations in longitude do not introduce shifts in the obliquity of the ecliptic. Werner thus managed to generate linear harmonic motion by the uniform motions of two circles.
54. See Claudia Kren, "The rolling device of Nasir al-Din al-Ṭūsī in the *De spera* of Nicole Oresme?", *Isis*, lxii (1971), 490–8; Alfonso [Abner of Burgos], *Meyashsher 'Aqov*, fac. edn with Russian transl. by G. M. Gluskina and commentary by Gluskina, S. Y. Lurie and B. A. Rosenfeld (Moscow, 1983), 85–86, 107; Langermann, "Mordekhai Finzi" (ref. 25), 38; *idem*, "Medieval Hebrew texts on the quadrature of the lune", *Historia mathematica*, xxiii (1996), 31–53. We thank Y. Tzvi Langermann for providing a preprint of this latter article, and for drawing our attention to Alfonso's text.
55. Swerdlow, "*Commentariolus*" (ref. 31), 424; see Saliba, "Marāgha" (ref. 51), 98–99. Furthermore, our reconstruction of Angelus's tables weakens Di Bono's recent claim, in "Copernicus, Amico, Fracastoro" (ref. 33), that Copernicus reinvented the Ṭūsī couple rather than borrowing it from Arabic traditions via unknown sources.
56. Punctuation and spelling have been standardized, and the text has been divided into paragraphs.
57. For the *Alfonsine tables*, see Appendix 4. The *Tabulae astronomicae* of Giovanni Bianchini (d. *c.* 1469), written around 1440 and published in 1495 in Venice, include a large set of planetary tables based on Alfonsine parameters and theory, but arranged in a double-entry format for easier use. See Louis Birkenmajer, "*Flores Almagesti*: Ein angeblich verloren gegangener Traktat Giovanni Bianchini's", *Bulletin international de l'Académie des Sciences de Cracovie, Cl. des sciences mathématiques et naturelles*, sér. A, 1911, 268–78; Swerdlow, "Regiomontanus" (ref. 19), notes 6–7.
58. Hans Dorn (*c.* 1430–1509), a Viennese Dominican and student of Peurbach's, might have brought one of his teacher's manuscripts into the cloister library (Grössing, *Humanistische Naturwissenschaft* (ref. 5), 145, 278). For relations between the cloister and the Viennese university in the fifteenth century, see Isnard Wilhelm Frank, *Hausstudium und Universitätsstudium der Wiener Dominikaner bis 1500* (Vienna, 1968). According to a 1513 catalogue of the Dominican library, several manuscripts were then extant which may have contained the remarks Angelus here attributed to Peurbach: "Theoricam planetarum, textus cum tabulis faciendi tabulas" (shelfmark I47); "Almanach Purkardi ad meridianum Salczeburgensem" (O19); "Theorice planetarum textus", which included "Tabula Alphonsij cum canonibus" (S5); "Tabula de motibus planetarum" (S7); and "Canones tabulares magistri Iohannis de Lyneriis primi et secundi mobilis et alia plura ad astronomiam deserviencia" (S9). Of these manuscripts, only S7 and S9 can be found currently in the Convent library (Cod. 141/111 and Cod. 189/155), neither of which contains any comments about the reliability of tables (Theodor Gottlieb, *Mittelalterliche Bibliothekskataloge Österreichs*, i: *Niederösterreich* (1915; fac. reprint edn, Aalen, 1974), 357, 384, 400; Felix Czeike, *Verzeichnis der Handschriften des Dominikanerkonvents in Wien bis zum Ende des 16. Jahrhunderts* (Vienna, 1952), 128, 170). In observational records published in 1544, and in 1512 apparently still in Nuremberg, Peurbach and Regiomontanus criticized the accuracy of the Alfonsine Tables after finding that the predicted times for eclipses and planetary conjunctions did not match the observed times (Schmeidler (ed.), *Opera* (ref. 28), 645–60; Czerny, "Aus dem Briefwechsel" (ref. 28), 302; Zinner, *Regiomontanus* (ref. 16), 354).
59. Regiomontanus in a 1471 letter to Christian Roder (d. 1478), rector of the university in Erfurt, chastized astronomers who used the Alfonsine Tables as a "gift of heaven" without considering that these tables did not explain how the celestial sphere could have a double motion (precession

and trepidation); did not reflect the fact that the solar and other eccentricities had changed since Antiquity; and yielded predictions that agreed neither with ancient nor modern observations (Curtze, "Der Briefwechsel Regiomontan's" (ref. 38), 326–7; *cf.* Menso Folkerts, "Conrad Landvogt, ein bisher unbekannter Algebraiker um 1500", in *Amphora: Festschrift für Hans Wussing zu seinem 65. Geburtstag*, ed. by Sergei S. Demidov *et al.* (Basel, 1992), 229–59, pp. 233–4). In earlier letters to Bianchini and Jacob Speier, Regiomontanus had harshly criticized the Alfonsine Tables for employing a false model for the motion of the eighth sphere; yielding Martian longitudes that deviated by varying amounts (up to 2°) from the observed positions; predicting longitudes for Venus that exceeded by 0;45° the observed motion, and giving false latitudes for this planet; and incorrectly predicting the times and extents of lunar eclipses (Curtze, "Der Briefwechsel Regiomontan's" (ref. 38), 263–6, 303–4; *cf.* Swerdlow, "Regiomontanus" (ref. 19)). Angelus probably had access to all these letters, since Regiomontanus himself had aggregated his correspondence with Bianchini, Speier and Roder, and the letters (currently Nuremberg, City Library, Nür Cent app. 56c) remained in Nuremberg in the library of Bernard Walther through at least 1523 (Curtze, "Der Briefwechsel Regiomontan's" (ref. 38), 187–9; Zinner, *Regiomontanus* (ref. 16), 259–60, 324).

60. Probably Johannes Tolhopf (d. 1503), a mathematician who lectured at the university in Ingolstadt from 1472 until 1475 when he went to Rome to work on calendar reform for Pope Sixtus IV and probably met Regiomontanus (Lynn Thorndike, *Science and thought in the fifteenth century* (New York, 1929), 298–300). Thereafter, Tolhopf returned only briefly to Ingolstadt in 1479 and in 1482, when he was prevented from rejoining the Arts Faculty by a group of young masters that included Angelus. Citing this latter incident, Schöner, *Mathematik und Astronomie* (ref. 4), 162–89, argued that Tolhopf could not have been close to Angelus. Yet the only other Ingolstadt professor who could have introduced Angelus to astrology and astronomy, the physician Erhard Windsberger, had first acquired his degree in Paris *c.* 1475, and hardly could have known Regiomontanus.

61. Stiborius (d. 1515) received his Master of Arts degree in 1484 in Ingolstadt, joined the circle of mathematicians around Conrad Celtis, and followed the latter to Vienna in 1497, where he held a chair for mathematics at the university from 1501 to 1503 and also lectured on mathematics in Celtis's Collegium poetarum et mathematicorum. With Georg Tanstetter and Stephanus Rosinus, Stiborius founded the so-called "second Viennese mathematical school". He wrote at least sixteen texts on astronomical instruments, cartographic projection and spherical trigonometry. Even before moving to Vienna, Angelus had become acquainted with Stiborius. See Angelus to Celtis, 29 March 1498, in Hans Rupprich (ed.), *Der Briefwechsel des Konrad Celtis* (Munich, 1934), 322; Conradin Bonorand, *Joachim Vadian und der Humanismus im Bereich des Erzbistums Salzburg* (St Gallen, 1980), 213–14; Grössing, *Humanistische Naturwissenschaft* (ref. 5), 174–81, 197–8.

62. John of Gmunden may have observed from this well-known tower, and Stiborius mentioned it in a text on spherical trigonometry, in Munich, Bavarian State Library, Clm 24103, 5^v, 6^v. For a fourteenth-century illustration of the tower, see Grössing, *Humanistische Naturwissenschaft* (ref. 5), 325.

63. Probably either Regiomontanus, *Ephemerides* (ref. 1), or Stöffler and Pflaum, *Almanach nova* (ref. 1). Several editions of both ephemerides had been published before 1510.

64. This humanist rhetoric about the need to restore science after its long period of decline frequently appears in Regiomontanus's prose, less so in Peurbach's. See Curtze, "Der Briefwechsel Regiomontan's" (ref. 38), 327; Rose, *The Italian Renaissance* (ref. 41), 90–117; Olaf Pedersen, "The decline and fall of the *Theorica planetarum*", in *Science and history: Studies in honor of Edward Rosen*, ed. by Erna Hilfstein, Paweł Czartoryski, and Frank D. Grande (Wrocław, 1978), 157–85; Grössing, *Humanistische Naturwissenschaft* (ref. 5), 84–91, 117–21.

65. Appendix 3 provides details concerning Angelus's comparisons (Cases 1 to 15) of the 'new' and 'old' almanacs.

66. In both the 1510 and 1512 almanacs, Angelus appears to have computed the solar and lunar positions with unmodified Alfonsine theory. See p. 192 above.

67. The 'ruler' of a horoscope, or that celestial body whose influence is most pronounced on the chart.

68. For the use of the armillary sphere, an unwieldy and not widely used observational instrument of pre-modern astronomy, see Richard L. Kremer, "Bernard Walther's astronomical observations", *Journal for the history of astronomy*, xi (1980), 174–91; Jarosław Włodarczyk, "Observing with the armillary sphere", *Journal for the history of astronomy*, xviii (1987), 173–95.

69. "And he [Ptolemy] has erred in the determination of the station of the planet and the time of its retrogradation, so that it is possible that he is wrong in determining the time of retrogration of Mars by almost 18 days, and in the retrogradation of Venus by almost two and a half days, and this always by an excess of time." Peter Apian, *Instrumentum primi mobilis: Accedunt ijs Gebri filii Affla Hispalensis ... libri IX de astronomia ... per Giriardum Cremonensem latinitate donati* (Nuremberg, 1534), sig. aa2r. Jābir ibn Aflaḥ's *Correction of the Almagest* (early twelfth century), best known for its critique of the classical ordering of the planets, had been translated into Latin by Gerard of Cremona. Regiomontanus owned and annotated a copy of this treatise. Zinner, *Regiomontanus* (ref. 16), 76, 310–11; Richard P. Lorch, "The astronomy of Jābir ibn Aflaḥ ", *Centaurus*, xix (1975), 85–107.

70. Stanislaus Cracoviensis, called Aurifaber, was a member of the Faculty of Arts in Cracow, and by 1517 would become a Fellow of the Collegium Maius of the Cracow Academy. While still a young member of the faculty, he obtained the privilege to prepare almanacs for the university. His extant works include two printed almanacs for 1511 and 1512, almanacs in manuscript for 1513 and 1514, and *Judicia* for 1512 and 1513 (Mieczysław Markowski, *Astronomica et astrologica Cracoviensia ante annum 1550* (Florence, 1990), 189–91). Aurifaber, *Ephemerides 1512* (ref. 3), sigs. aiv–aiiv, contains a preface that sharply attacks Angelus's 1510 almanac albeit without naming its author, stresses the competence of Alfonso and Bianchini as authors of astronomical tables, and defends Aurifaber's value for the geographical longitude of Cracow. For an extended analysis of the dispute between Aurifaber and Angelus, see Kremer and Dobrzycki, "Disputationes" (ref. 13).

71. Tanstetter, *Tabulae eclypsium* (ref. 3), sig. aa4r, listed among Peurbach's works a "Tabulae aequationum motuum planetarum novae, nondum perfectae et ultimum completae". This text, undoubtedly identical to that described here, has not been found. If Angelus had these tables "for a long time", he may have acquired them before moving to Vienna, perhaps from the materials Regiomontanus left in Nuremberg after his death (see Zinner, *Regiomontanus* (ref. 16), 245–65). Hellman and Swerdlow, "Peurbach" (ref. 46), 478, translated a note by Regiomontanus, written in his copy of the *Almagest* below the tables of planetary equations, asserting that Peurbach "had made more accurate equations" (Nuremberg, City Library, Nür Cent III 25, 80r, actually: "Correctiones invenies in libro magistri Georgij"), and speculated that Peurbach's "tables" to which Tanstetter alluded were simply "recomputations of the planetary equations at 0;10° intervals using Alphonsine parameters". A comparison of the tables of planetary equations in the copies of the *Almagest* owned by Peurbach (Vienna, Austrian National Library, lat. 4799, 65v–67v) and Regiomontanus (Nür Cent V 62, 219v–22v; Nür Cent III 25, 79v–80v), and Toomer, *Almagest* (ref. 36), indicates that all three manuscripts are filled with scribal errors, but that Peurbach's has considerably fewer than does Nür Cent III 25. It seems likely, therefore, that Regiomontanus's remark in Nür Cent III 25 refers to Peurbach's *Almagest* in Vienna lat. 4799 (as suggested by Zinner, *Regiomontanus* (ref. 16), 310) and not to some other independent set of tables. More recently, Grössing, *Humanistische Naturwissenschaft* (ref. 5), 108, suggested that Peurbach's incomplete tables might be in Vienna, Dominican Convent, Cod. 141/111, 113v–47v. Yet this MS contains an incomplete set of the Oxford Tables which do not appear to be modified in any way.

72. In notes acquired by Johann Schöner and published in 1544, Regiomontanus occasionally compared the longitudes of planets he observed (positions set essentially by the locations he assumed for the reference stars against which he measured the planets) with Alfonsine predictions. In six cases of explicit comparisons between 1461–71, the Alfonsine errors ranged from 0;40° to 3;00°. However Angelus probably is referring here to Regiomontanus's 1464 letter to Bianchini: "Mars was seen to differ in the heavens and in computation by two degrees in relation to the fixed stars and other observations" (transl. in Swerdlow, "Regiomontanus" (ref. 19), 172).

73. In 1497, Angelus probably spent some time in Würzburg. Zinner, *Regiomontanus* (ref. 16), 228, suggested that this "student" of Regiomontanus may have been the Würzburg jurist Friedrich

Brogel, the Ingolstadt mathematician Johann Tolhopf, or the physician Eberhard Schleusinger. *Cf.* Knobloch, "Astrologie" (ref. 4), 132–3.

74. Regiomontanus had calculated his printed ephemerides for what he considered to be the meridian of Nuremberg (21.1°E of Toledo). See p. 193 above.

75. Regiomontanus, *Disputationes contra Cremonensia in planetarum theoricas deliramenta* (Nuremberg, *c.* 1474) criticized the early fifteenth-century translation of Ptolemy's *Geography* by Jacobus Angelus. In the 1474 inventory of works he wanted to publish in Nuremberg, Regiomontanus placed a new translation of the *Geography* third on the list. See Ernst Zinner, "Die wissenschaftlichen Bestrebungen Regiomontans", *Beiträge zur Inkunabelkunde*, N.F., i (1935), 89–103; Pedersen, "The decline and fall" (ref. 64), 179. Michael H. Shank, who is preparing an edition of the *Disputationes*, kindly let us consult his translation of this text.

76. The Alfonsine Tables replaced Ptolemy's uniform precession with a scheme in which the motion of the eighth sphere is composed of secular and periodic changes. For Jābir ibn Aflaḥ's criticisms of Ptolemy, see Lorch, "Jābir ibn Aflaḥ" (ref. 69).

77. Apparently Angelus thought that "Alfonso" (d. 1284) and Jābir ibn Aflaḥ (*fl.* first half of twelfth century) were deceived by Campanus (*fl.* thirteenth century), even though the latter's *Theorica planetarum* once explicitly referred to Jābir ibn Aflaḥ. Angelus does not specify Campanus's alleged deceptions, but may be referring to the latter's misunderstanding of "Thabit's" model for trepidation. See Francis S. Benjamin, Jr, and G. J. Toomer, *Campanus of Novara and medieval planetary theory: Theorica planetarum* (Madison, 1971), 35, 144–5, 332–3, 378–9.

78. Paraphrasing a ten-line verse in Aurifaber, *Ephemerides 1512* (ref. 3), sig. Aii^{v} (Nunc mea nunc tenues reddit tibi fistula flatus. / Et sonat exiguos parua cicuta sonos. / ... / Et ubi belligeros recinent mea classica cantus: / Edicentque graues tympana tacta vices.), these lines imply that Angelus wrote his preface after having read Aurifaber's text. See Kremer and Dobrzycki, *op. cit.* (ref. 13).

79. See above, pp. 192–3.

80. Beyond the comments at ref. 3 above, we have found few responses to Angelus's almanacs by contemporary readers. A sixteenth-century hand has lightly annotated the 1512 almanac now in Vienna (ref. 1), noting the geographical longitudes for several cities in Angelus's diatribe against Aurifaber and adding at the very end of the treatise a list of rules for astrological prognostication. Angelus's claims to have improved Alfonsine predictions, however, are passed over in silence. Lorenz Beheim (*c.* 1457–1521), the Bamberg canon well-known for his interest in astrology, wrote Willibald Pirckheimer in October of 1511 that he had seen a "new almanac and it seems good enough". Emil Reicke (ed.), *Willibald Pirckheimers Briefwechsel* (2 vols, Munich, 1940–56), ii, 114–16, suggests Beheim was referring to Angelus's 1512 almanac. By the end of the century, Angelus's almanacs had become scarce. See Thaddeus Hagecius to Hieronymous Munnos, 22 July 1574, in *Tychonis Brahe Dani Opera omnia*, ed. by J. L. E. Dreyer (15 vols, Copenhagen, 1913–29), vii, 399–400.

81. In the 1512 preface, Angelus described how an armillary sphere might be employed to measure planetary longitudes. Apparently he had used this expensive, cumbersome instrument that would not have been available to most of his contemporary readers. Bernard Walther, probably the most skilled fifteenth-century astronomical observer, achieved a precision of about 0;05° with the armillary sphere but only after many years of practice. See Kremer, "Bernard Walther's observations" (ref. 68); Włodarczyk, "Observing with the armillary sphere" (ref. 68).

82. Bold type indicates prediction closest to modern configuration. All data in columns 1–3 not explicitly given in the prefaces are placed in square brackets. "Common" values are taken from Stöffler's and Pflaum's *Almanach nova*, unless otherwise noted. To identify Angelus's unnamed stars and his longitudes for them (column 1), coordinates from the star catalogue printed in the 1483 *Alfonsine tables* (but not normally considered part of the Alfonsine corpus; see Poulle, *Les tables alphonsines* (ref. 21), 224; Paul Kunitzsch, "The star catalogue commonly appended to the Alfonsine Tables", *Journal for the history of astronomy*, xvii (1986), 89–98) were reduced to the epoch of the almanachs using Alfonsine motions of the eighth sphere, a procedure that Angelus followed, as can be seen explicitly in Case 1.

83. According to Stöffler and Pflaum, on May 22 Mars was at 201;05°, at its second stationary point.

84. According to Stöffler and Pflaum, the conjunction occurs on October 27 at 10 p.m.
85. According to Stöffler and Pflaum, the conjunction occurs on April 17 at 1 a.m.
86. Angelus probably means June 7, when his almanac places Mars at 268;52°.
87. The maximal difference between the almanacs for Venus occurs on January 19.
88. This value, not in the 1512 almanac, apparently was computed by Angelus especially for the preface.
89. See Poulle and Gingerich, "Les positions des planètes" (ref. 21); J. D. North, "The Alfonsine Tables in England", in *Prismata ... Festschrift für Willy Hartner*, ed. by Y. Maeyama and W. G. Satzer (Wiesbaden, 1977), 269–301; Emmanuel Poulle, "Jean de Murs et les tables alphonsines", *Archives d'histoire doctrinale et littéraire du Moyen Age*, lv (1980), 241–71; *idem*, "The Alfonsine Tables and Alfonso X of Castille", *Journal for the history of astronomy*, xix (1988), 97–113; Owen Gingerich, "The Alfonsine Tables in the age of printing", in *De astronomia Alphonsi Regis*, ed. by Mercè Comes, Roser Puig and Julio Samsó (Barcelona, 1987), 89–95; Jerzy Dobrzycki, "The 'Tabulae resolutae'", *ibid.*, 71–77.
90. Poulle and Gingerich, "Les positions des planètes" (ref. 21), 541–4; North, *Richard of Wallingford* (ref. 21), iii, 195–7.
91. North, *Richard of Wallingford* (ref. 21), iii, 197.

Added in proof: The 1513 single-sheet calendar, first cited by Seethaler (ref. 7), bears the incipit "Doctor Joannes angelus fabricator" and was printed by Johannes Winterburger in Vienna. Apparently uniquely extant as pastedown in a volume (RM III.65) at the Library of the Hungarian Academy of Sciences, this calendar does not refer to "new theory" and offers daily lunar longitudes and times of new and full moons that are strictly Alfonsine. We thank Dora F. Csanak of the Hungarian Academy for providing a microfilm of this calendar.