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# MAGNETIC FIELD AND ROTATION IN LOWER MAIN-SEQUENCE STARS: AN EMPIRICAL TIME-DEPENDENT MAGNETIC BODE'S RELATION?

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## ABSTRACT

We find a significant correlation between the magnetic and rotational moments for a sample of 112 lower main-sequence stars. The rotational moment is calculated from measurements of the rotation period in most of the stars (not from the projected rotational velocity inferred from Doppler broadening). The magnetic moment is computed from a database of homogeneous measurements of the mean level of Ca II H and K emission fluxes sampled for most of the stars over an interval of 25 yr. The slope connecting the logarithm of the magnetic moment and the logarithm of the rotational moment is about +0.5–0.6, with a Pearson correlation coefficient of about +0.9. The scatter of points from the mean relation has a component that is natural and caused by decade-long surface variability.

*Subject headings:* stars: magnetic fields — stars: rotation

## 1. INTRODUCTION

Blackett (1947) found a positive correlation between magnetic moment, $\mu$, and angular moment, $L$, for Earth, the Sun, and one star (78 Virginis, A2p), and proposed it as a fundamental physical law. The dependence of $\mu$ on $L$ seems to be a direct proportionality that is sometimes referred as the magnetic Bode's law (see, e.g., Russell 1978). An empirical relation between $\mu$ and $L$ may even extend from planetary through stellar to galactic magnetic fields (see, e.g., Arge, Mullan, & Dolginov 1995).

The idea of a physical concept relating magnetic field to rotation seems correct but is likely to remain controversial for some time. For example, Cain et al. (1995) were unable to establish a statistically significant relation between magnetism and rotation among the six magnetic planets of the solar system. One reason for the inability to find a relation may be that variability of regular, global planetary magnetic field is largely undersampled because of the rather long timescales involved (Cain et al. 1995). Although such criticisms may be valid for planets, they may not apply to stars.

The surface magnetic activity (for which the Ca II H [396.8 nm] and K [393.4 nm] emission fluxes are proxies) has been measured in a large sample of lower main-sequence stars for three decades. Some of those stars vary rather regularly on the order of 10 yr (see, e.g., Wilson 1978), similar to the magnetic sunspot cycle (see, e.g., Hoyt, Schatten, & Nesme-Ribes 1994). Those observations yield an opportunity to examine the relation between $\mu$ and $L$. The utility of the Wilson sample is its ability to reduce several possible sources of systematic error. For example, the measurements were made with the same instrument over an interval of time long enough to study physical variability in the average and can be analyzed in a straightforward way.

The correlation between magnetic moment and rotation per se is of general interest for magnetic activity in stars, independent of the existence of a universal magnetic Bode's law, because it would allow stellar rotation to be predicted simply from the average Ca II flux. We present here an empirical relation for lower main-sequence stars as well as a first glimpse into the critical question of the effect of time dependence on the relation.

## 2. EMPIRICAL MAGNETIC BODE'S RELATION FOR THE WILSON SAMPLE

At Mount Wilson Observatory, the Ca II H and K emission fluxes have been observed in ~100 stars on or near the lower main sequence since 1966. Surface magnetic activity is closely linked to the Ca II emission fluxes in the Sun (Skumanich, Smythe, & Frazier 1975; Schrijver et al. 1989) and presumably other lower main-sequence stars. The original intention of the monitoring program, which was conducted monthly by Wilson (1978), was to record fluctuations in magnetic activity on timescales of years. The program has continued at a higher rate of observations and for a larger sample of stars beginning in 1980, when measurements were scheduled at nearly nightly intervals.

These long, and more recently, densely sampled records contain information not only on the magnetic activity cycle but also on the period of axial rotation (Baliunas et al. 1995). The inferred rotation period is independent of the inclination of the rotation axis to our line of sight, unlike traditional spectroscopic determinations based on Doppler line broadening, which yield only the projected rotational velocity. We will discuss results from the 112 stars of the Wilson sample, which range in spectral type from early F to late K (including one M dwarf), and for which we have determined average level of Ca II flux and rotation. Rotation periods have been inferred in 80 stars from periodic fluctuations in the Ca II records; in the remaining 32, rotation has been computed from the close relation between Ca II flux and Rossby number for lower main-sequence stars (Noyes et al. 1984).

The measurement of the Ca II flux is made relative to the flux in the nearby, photospheric continuum. A correction must

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be introduced in order to compare the level of activity in stars of different masses (i.e., $B-V$ color). In addition, other corrections are usually made to provide an estimate of the part of the Ca II flux produced by magnetic heating. The quantity that we used here to represent the magnetic heating is called $R'_{\rm HK}$ (Noyes et al. 1984).

We list in Table 1 the stars in our sample, including the Sun (col. [1]), $(B-V)$ color index (col. [2]), log $\langle R'_{\rm HK}\rangle$ (col. [3]), and mean rotation period, $\langle P_{\rm rot}\rangle$ (col. [4], either measured or computed). The angular momentum is calculated using $L = MR^2/P_{\rm rot}$, where $M$ and $R$ are the stellar mass and radius, respectively. The values of stellar radius and mass were computed from reference values given by Lang (1992). We approximate the magnetic moment by $\mu = \langle R'_{\rm HK}\rangle R^3$, since direct detections of surface magnetic fluxes for our sample of stars using the Zeeman broadening of a magnetically sensitive line are still scarce. [A study, not presented, defining the magnetic moment alternatively as $\mu = B_s R^3$ and using available data on photospheric magnetic field strength, $B_s$, for a group of 13 stars common to Table 1 collected by Montesinos & Jordan 1993 suggests a slope of +0.54 in a log $(\mu)$–log $(L)$ relation, similar to that obtained below.]

The magnetic moment of the 112 stars is plotted against angular momentum on a log-log scale in Figure 1; both quantities are normalized by the corresponding solar values. There is a significant, positive slope of +0.60 and correlation coefficient of 0.90. Using different definitions of the rotational and magnetic moments from the literature does not significantly alter the statistical significance of the correlation for our sample of stars. For example, if we recompute the rotational moment using the scaling adopted by Cain et al. (1995), $L \sim R^5/P_{\rm rot}$, we obtain a slope of +0.50 and the Pearson correlation coefficient of +0.88. Furthermore, the observed correlation coefficient and slope in Figure 1 cannot be fitted by either random uncorrelated sets of synthetic $(B-V$, $\langle R'_{\rm HK}\rangle$, $P_{\rm rot})$ data or by sets of data with random $(B-V)$ but actual observed relations between $\langle R'_{\rm HK}\rangle$ and $P_{\rm rot}$. Therefore, we conclude that the log $(\mu)$–log $(L)$ relation presented in Figure 1 is robust. [These same two tests invalidate the statistical significance of the log $(\mu)$–log $(L)$ relation for the six magnetic planets in the solar system; e.g., see Cain et al. 1995.]

One possible explanation for a scaling of $\mu \sim L^{1/2}$ for lower main-sequence stars is based on a suggestion by Curtis & Ness (1986) for planetary dynamos, despite the subsequent negative finding by Cain et al. of such a relation for planets. Curtis & Ness presented their explanation in terms of the balance between the Coriolis and Lorentz forces appropriately imposed at the dynamo core:

$$2\boldsymbol{\Omega} \times \boldsymbol{v}_c \sim \boldsymbol{J} \times \boldsymbol{B}/\rho, \tag{1}$$

where $\boldsymbol{\Omega}$ is the angular velocity, $\boldsymbol{v}_c$ is the convective velocity, $\boldsymbol{J} \sim$ curl $\boldsymbol{B}$ is the current, $\boldsymbol{B}$ is the magnetic field, and $\rho$ is the mean plasma density. The interiors of lower main-sequence stars seem more closely related than those of the six magnetic planets. As a result, a magnetostrophic-balance argument (see, e.g., Eltayeb & Roberts 1970) may be more relevant to stellar dynamos than to planetary magnetic fields. However, many challenging theoretical and observational details need to be resolved before the concept of magnetostrophic balance can be better explored. For example, many theoretical ideas of back-reaction of magnetic field on fluid motion are unconstrained by observations. Another example is the log $(\mu)$–log $(L)$ relation in Figure 1. While the relation appears to be an essential observational step, the existence of time dependence in the relation surely adds yet another fundamental dimension to the complexity involved.

## 3. DISCUSSION

Many correlations exist between magnetic and rotational properties of the Wilson sample as well as similar observations of other lower main-sequence stars, such as the activity–Rossby number relation (see, e.g., Noyes et al. 1984; Stępień 1994). The relation between magnetic and rotational moments presented here is an alternative correlation that is statistically significant and warrants further investigation. This correlation is the simplest parameterization available for comparing measured rotation and average surface activity in lower main-sequence stars that differ in mass and age.

Previous studies have assumed that, of several possible parameterizations of the rotation-activity relation, the one with the least scatter must represent the physics. That assumption does not account for the existence of stellar variability. On the other hand, our formulation allows for the presence of physical scatter arising from variability in time. In Figure 1 we have indicated the long-term magnetic behavior of the stars that have relatively well defined classes within the 25 yr records such as periodic cycles or low inactive state (i.e., rms variability of less than 2% over the interval of observation). A pattern emerges in Figure 1: the inactive stars border the lower bound of the log $(\mu)$–log $(L)$ relation. Also visible is the tendency of those stars that have two distinct periods (see Baliunas et al. 1995) to concentrate near the mean of the relation. Stars with periodic cycles are not confined to any region but instead extend the full range of the relation.

A relationship between magnetic moment and rotation for lower main-sequence stars may not be surprising because rotation is the fundamental ingredient that drives the large-scale magnetic field of the dynamo. What is indeed surprising

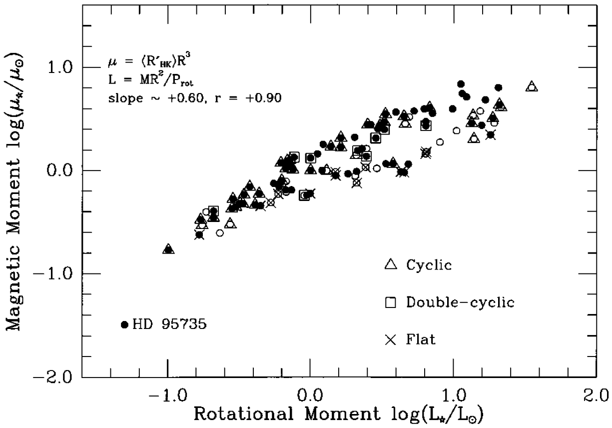

FIG. 1.—Magnetic moment vs. rotational moment normalized to the Sun's values for the Wilson sample of 112 lower main-sequence stars, including the Sun. Filled circles denote stars with measured rotational moments; open circles denote stars with rotational moments calculated using the rotation period estimated from the Ca II activity–Rossby number relation (Noyes et al. 1984). The slope for the log $(\mu)$–log $(L)$ relation is $d \sim +0.60$, and the Pearson correlation coefficient is about +0.90. The three classes of variability in Ca II based on 25 yr records discussed in Baliunas et al. (1995) are also indicated. Circles within triangles denote stars with a periodic cycle; circles within squares denote stars with two distinct periods ("double cyclic"); circles with superimposed crosses denote stars with inactive (i.e., rms variability less than ~2% over 25 yr) long-term Ca II variability.

TABLE 1

TIME-AVERAGED Ca II FLUXES AND ROTATION PERIODS FOR WILSON SURVEY STARS

| HD | $B-V$ | $\log\langle R'_{\rm HK}\rangle$ | $\langle P_{rot}\rangle$ (days)* | HD | $B-V$ | $\log\langle R'_{\rm HK}\rangle$ | $\langle P_{rot}\rangle$ (days)* | HD | $B-V$ | $\log\langle R'_{\rm HK}\rangle$ | $\langle P_{rot}\rangle$ (days)* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun | 0.66 | -4.901 | 25 | 75332 | 0.49 | -4.464 | 4 | 149661 | 0.82 | -4.583 | 21 |
| 1835 | 0.66 | -4.433 | 8 | 76151 | 0.67 | -4.659 | 15 | 152391 | 0.76 | -4.448 | 11 |
| 2454 | 0.43 | -4.792 | 3 | 76572 | 0.43 | -4.924 | (4) | 154417 | 0.57 | -4.533 | 8 |
| 3229 | 0.44 | -4.583 | 2 | 78366 | 0.60 | -4.608 | 10 | 155885 | 0.86 | -4.559 | 21 |
| 3443 | 0.72 | -4.903 | (30) | 81809 | 0.64 | -4.921 | 41 | 155886 | 0.86 | -4.570 | 21 |
| 3651 | 0.85 | -4.991 | 44 | 82443 | 0.77 | -4.211 | 6 | 156026 | 1.16 | -4.662 | 21 |
| 3795 | 0.70 | -5.035 | (32) | 82885 | 0.77 | -4.638 | 18 | 157856 | 0.46 | -4.674 | (4) |
| 4628 | 0.88 | -4.852 | 39 | 88355 | 0.46 | -4.819 | (5) | 158614 | 0.72 | -5.028 | (34) |
| 6920 | 0.60 | -4.793 | 14 | 88737 | 0.56 | -4.622 | (8) | 159332 | 0.48 | -5.006 | (7) |
| 9562 | 0.64 | -5.177 | 29 | 89744 | 0.54 | -5.120 | 9 | 160346 | 0.96 | -4.795 | 37 |
| 10476 | 0.84 | -4.912 | 35 | 95735 | 1.51 | -5.451 | 53 | 161239 | 0.65 | -5.164 | (29) |
| 10700 | 0.72 | -4.958 | 34 | 97334 | 0.61 | -4.422 | 8 | 165341A | 0.86 | -4.548 | 20 |
| 10780 | 0.81 | -4.681 | 23 | 100180 | 0.57 | -4.922 | 14 | 165341B | 1.16 | -4.710 | (34) |
| 12235 | 0.62 | -4.983 | 14 | 100563 | 0.46 | -4.674 | (4) | 166620 | 0.87 | -4.955 | 43 |
| 13421 | 0.56 | -5.195 | (17) | 101501 | 0.72 | -4.546 | 17 | 176051 | 0.59 | -4.874 | (16) |
| 16160 | 0.98 | -4.958 | 48 | 103095 | 0.75 | -4.896 | 31 | 176095 | 0.46 | -4.671 | (4) |
| 16673 | 0.52 | -4.664 | 7 | 106516 | 0.46 | -4.651 | 7 | 178428 | 0.70 | -5.048 | 22 |
| 17925 | 0.87 | -4.311 | 7 | 107213 | 0.50 | -5.103 | 9 | 182101 | 0.44 | -4.608 | (2) |
| 18256 | 0.43 | -4.722 | 3 | 111456 | 0.46 | -4.402 | (1) | 182572 | 0.77 | -5.099 | 41 |
| 20630 | 0.68 | -4.420 | 9 | 114378 | 0.45 | -4.530 | 3 | 185144 | 0.80 | -4.832 | 27 |
| 22049 | 0.88 | -4.455 | 12 | 114710 | 0.57 | -4.745 | 12 | 187013 | 0.47 | -4.922 | (6) |
| 22072 | 0.89 | -5.205 | (55) | 115043 | 0.60 | -4.428 | 6 | 187691 | 0.55 | -5.026 | 10 |
| 23249 | 0.92 | -5.184 | 71 | 115383 | 0.58 | -4.443 | 3 | 188512 | 0.86 | -5.173 | (52) |
| 25998 | 0.46 | -4.401 | 2 | 115404 | 0.93 | -4.480 | 18 | 190007 | 1.17 | -4.692 | 29 |
| 26913 | 0.70 | -4.391 | 7 | 115617 | 0.71 | -5.001 | 29 | 190360 | 0.73 | -5.102 | (38) |
| 26923 | 0.59 | -4.503 | (7) | 120136 | 0.48 | -4.731 | 4 | 190406 | 0.61 | -4.797 | 14 |
| 26965 | 0.82 | -4.872 | 43 | 124570 | 0.54 | -5.156 | 26 | 194012 | 0.51 | -4.720 | 7 |
| 29645 | 0.57 | -5.110 | (17) | 124850 | 0.52 | -4.682 | (7) | 201091 | 1.18 | -4.764 | 35 |
| 30495 | 0.63 | -4.511 | 11 | 126053 | 0.63 | -4.957 | (22) | 201092 | 1.37 | -4.891 | 38 |
| 32147 | 1.06 | -4.948 | (47) | 129333 | 0.61 | -4.152 | 3 | 206860 | 0.59 | -4.416 | 5 |
| 33608 | 0.46 | -4.628 | (3) | 131156A | 0.76 | -4.363 | 6 | 207978 | 0.42 | -4.890 | 3 |
| 35296 | 0.53 | -4.378 | 4 | 131156B | 1.17 | -4.424 | 11 | 212754 | 0.52 | -5.073 | 12 |
| 37394 | 0.84 | -4.454 | 11 | 136202 | 0.54 | -5.088 | (14) | 216385 | 0.48 | -5.025 | (7) |
| 39587 | 0.59 | -4.426 | 5 | 137107AB | 0.58 | -4.828 | (14) | 217014 | 0.67 | -5.074 | 37 |
| 43587 | 0.61 | -5.001 | (20) | 141004 | 0.60 | -5.004 | 26 | 219834A | 0.80 | -5.066 | 42 |
| 45067 | 0.56 | -5.094 | 8 | 142373 | 0.56 | -5.042 | (15) | 219834B | 0.91 | -4.944 | 43 |
| 61421 | 0.42 | -4.777 | (3) | 143761 | 0.60 | -5.039 | 17 | 224930 | 0.67 | -4.875 | 33 |
| 72905 | 0.62 | -4.375 | 5 |  |  |  |  |  |  |  |  |

*All rotation periods are measured except for those indicated with parentheses; in those cases the rotation periods are calculated using the Ca II activity vs. Rossby number relation (Noyes et al. 1984).

is that the observational data would so faithfully follow the scaling of +0.5 implied by magnetostrophic dynamos. However, any successful theoretical explanation of the relation must also account for the time-dependent characteristics in Figure 1.

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