

# On the birth-place of the Sun and the places of formation of other nearby stars

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**Abstract.** The Sun has a [Fe/H] metallicity which is larger by  $+0.17 \pm 0.04$  dex than the average metallicity of nearby stars of solar age. This result is derived from an age-metallicity relation based on the very accurate data published by Edvardsson et al. (1993) for nearby F and G dwarfs. We adopt a radial galactic gradient in metallicity of  $\partial[\text{Fe}/\text{H}]/\partial R = -0.09 \pm 0.02$  dex/kpc, independent of the age of the stars. From the solar anomaly and this galactic gradient, we derive that the Sun has been formed at a galactocentric distance  $R_{i,\odot} = 6.6 \pm 0.9$  kpc, if we adopt  $R_0 = 8.5$  kpc for the present distance of the Sun from the galactic center. Hence the Sun has migrated from its birth-place in the inner part of the Galaxy outwards by  $1.9 \pm 0.9$  kpc during its lifetime of  $4.5 \cdot 10^9$  years. This amount is in good agreement with predictions on the diffusion of stellar orbits in space which are based on the observed relation between velocity dispersion and age of nearby stars (i.e. on the diffusion in velocity space). The accurate determination of metallicities, coupled with a galactic gradient in metallicity, allows us to investigate empirically the diffusion of stellar orbits in space, at least in galactocentric distance. A direct consequence of this diffusion, and hence a good confirmation of it, is the increase in the dispersion of metallicities of nearby stars with increasing age. From such a relation, we derive also that the initial dispersion of metallicities is rather small. This is favourable for deriving the birth-place of the Sun from its anomaly in metallicity.

**Key words:** Sun: abundances – stars: formation – Galaxy: evolution – Galaxy: kinematics and dynamics – Galaxy: abundances

## 1. Introduction

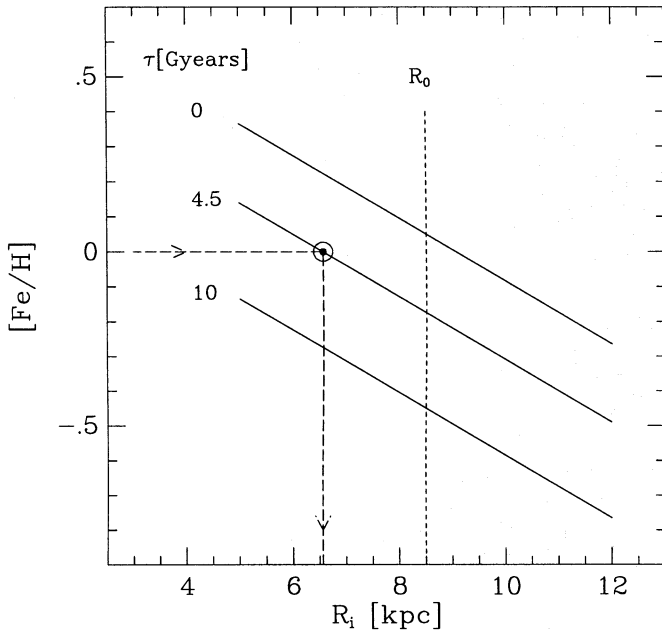
We try to answer, at least partially, the question: Where has the Sun been formed? This question can be understood qualitatively or quantitatively. On statistical grounds and using our knowledge about the formation of stars at present, we expect that the Sun has been formed on a nearly circular orbit, close to

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the galactic plane, in a spiral arm, and most probably in a stellar association, not in an open star cluster. The statistical probability for the Sun to be born in a cluster is only a few percent (Wielen 1971). Even then, the cluster would most probably have been dissolved long ago, since the typical lifetime of an open cluster is about  $2 \cdot 10^8$  years. Only if the Sun was a member of a very massive open cluster, could this stellar system have survived over many billion years. At present, the Sun is neither a member of a still existing old cluster (like M67 or NGC188) nor a member of a moving group, which would be the observable relict of an old cluster dissolved not too long (e.g.  $10^8$  years) ago.

A quantitative answer to the question of the solar formation place may be expected from calculating the galactic orbit of the Sun backwards in time. This method, however, is not feasible for various reasons: (1) Both the solar motion (i.e. the present ‘initial value’ of the solar orbit) and the gravitational field of our Galaxy are not known accurately enough in order to obtain a meaningful result for an orbital position  $4.5 \cdot 10^9$  years ago (the present age of the Sun). (2) The fluctuations (in space and time) of the galactic gravitational field introduce a high statistical uncertainty into the determination of orbital positions in the distant past or future. This diffusion of stellar orbits has been discussed quantitatively by Wielen (1977). His results show that it is completely hopeless to obtain the place of formation of the Sun from its orbit to better than a few kpc in galactocentric distance and to very many kpc in the direction of galactic rotation, even if the initial values of the solar orbit and the mean gravitational field of the Galaxy would be known perfectly.

There is, however, another possibility to determine at least the distance  $R_{i,\odot}$  of the Sun from the galactic center at the time of formation of the Sun (The index  $i$  of  $R_i$  indicates the *initial* value of  $R$ , at the time of birth of the star). The method requires the following scenario: (1) The mean metallicity of stars, formed at the same instant of time, shows a significant dependence on the galactocentric distance  $R_i$  (i.e., there is a sufficiently large radial gradient of metallicity in our Galaxy); (2) The initial dispersion in metallicity of stars born at a common time  $t$  and a common galactocentric distance  $R_i$  is small; (3) The age and the metallicity of the star, here of the Sun in



**Fig. 1.** Schematic explanation for the determination of the birth-place of the Sun. The metallicity  $[\text{Fe}/\text{H}]$  is shown as a function of the galactocentric distance  $R_i$  of a star at birth for various ages  $\tau$  of a star. The solid lines are based on Eq.(3).  $R_0$  is the present distance of the Sun from the galactic center. Using  $[\text{Fe}/\text{H}]_{\odot} = 0$  and  $\tau_{\odot} = 4.5 \cdot 10^9$  years as input data, we derive  $R_{i,\odot} = 6.6$  kpc.

particular, are known. The resulting determination of  $R_{i,\odot}$  is illustrated schematically in Fig. 1. We use as a typical metallicity the quantity  $[\text{Fe}/\text{H}]$ , i.e. the logarithm of the ratio of the number  $n$  of Fe atoms to H atoms in a stellar atmosphere divided by the same ratio for the Sun:  $[\text{Fe}/\text{H}] = \log((n_{\text{Fe}}/n_{\text{H}})/(n_{\text{Fe}}/n_{\text{H}})_{\odot})$ . From the measured values of  $[\text{Fe}/\text{H}]$  and of the age  $\tau$  of the Sun, we derive  $R_{i,\odot}$  from the function  $[\text{Fe}/\text{H}] = f(R_i, \tau)$ . The method works, of course, also for stars other than the Sun, if  $[\text{Fe}/\text{H}]$  and  $\tau$  are known for these stars. The application of this method to the Sun indicates that the Sun has been formed at a galactocentric distance  $R_{i,\odot} \sim 6.6 \pm 0.9$  kpc, if we use  $R_0 = 8.5$  kpc as the present value of  $R$  of the Sun. This would mean that the Sun has been formed significantly closer to the center of the Galaxy than the Sun is located today, and that the Sun has migrated outwards over its lifetime by nearly 2 kpc.

## 2. Stellar metallicities

### 2.1. The galactic gradient of metallicity

There is a large number of investigations which aim at determining and/or explaining the radial gradient of metallicities of stars and interstellar matter in our Galaxy and in other galaxies. Some reviews and relevant papers are Janes 1979, Pagel and Edmunds 1981, Shaver et al. 1983, Panagia and Tosi 1981, Cameron 1985, Gehren et al. 1985, Grenon 1987, Pagel 1987, Janes et al. 1988, Lewis and Freeman 1989, Neese and Yoss 1989, Matteucci 1991, Wilson and Matteucci 1992, Edvardsson et al. 1993, Francois and Matteucci 1993, Friel and Janes

1993, Carraro and Chiosi 1994, Kaufer et al. 1994, Kilian-Montenbruck et al. 1994, Köppen 1994, Maciel and Köppen 1994, Friel 1995, Joenck-Soerensen 1995.

While the results of the individual authors are sometimes conflicting, most of the investigations find in our Galaxy a radial gradient  $\alpha = \partial[\text{Fe}/\text{H}]/\partial R$  of the order of  $-0.1$  dex/kpc, which seems to be rather independent of the age  $\tau$  for disk stars. As a typical example, we quote the results of Friel and Janes (1993) and Friel (1995) for old open clusters:  $\alpha = -0.09 \pm 0.02$  dex/kpc. The mean age of these clusters,  $3.2 \cdot 10^9$  years, is not too different from the age of the Sun,  $4.5 \cdot 10^9$  years. For young clusters, Janes (1979) derived  $\alpha = -0.075 \pm 0.034$  dex/kpc, while Panagia and Tosi (1981) found  $\alpha = -0.095 \pm 0.034$  dex/kpc. Hence no dependence of  $\alpha$  on age is indicated. Therefore, we shall adopt in this paper a time-independent radial gradient of

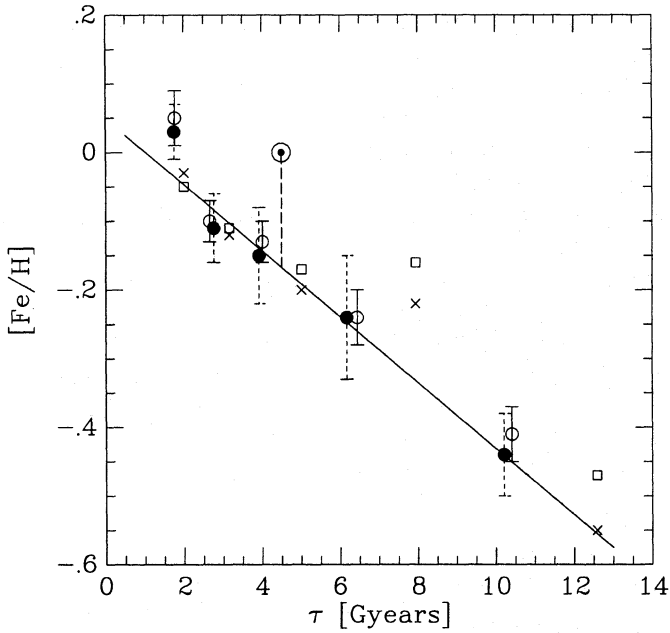
$$\alpha = \partial[\text{Fe}/\text{H}]/\partial R = -0.09 \pm 0.02 \text{ dex/kpc} \quad (1)$$

To a first approximation, the diffusion of stellar orbits in space does not affect such a radial gradient, if (1) the surface density of stars declines exponentially with  $R$  (exponential disk), (2)  $\alpha$  is constant with respect to  $R$  (linear decline of  $[\text{Fe}/\text{H}]$  with  $R$ ), and (3) the diffusion coefficient is also constant with respect to  $R$ . All these conditions are probably at least approximately fulfilled over the range in  $R$  under discussion.

### 2.2. The age-metallicity relation

The mean metallicity  $\langle [\text{Fe}/\text{H}] \rangle$  of stars at a given galactocentric distance  $R$  is a function of age  $\tau$  (see e.g. the following papers or reviews: Mayor 1974, Twarog 1980, Carlberg et al. 1985, Matteucci 1991, Meusinger et al. 1991, Rana 1991, Edvardsson et al. 1993, Francois and Matteucci 1993, Joenck-Soerensen 1995, Nissen 1995). For a new determination of this age-metallicity relation,  $\langle [\text{Fe}/\text{H}] \rangle (\tau)$ , we use the very precise data on stars in the solar neighbourhood, provided by Edvardsson, Andersen, Gustafsson, Lambert, Nissen, and Tomkin (1993). The quoted mean errors are  $\pm 0.05$  dex (corresponding to  $\pm 12\%$  in a linear scale) for the metallicities, and typically  $\pm 0.1$  in  $\log \tau$ , corresponding to  $\pm 30\%$  in age  $\tau$ .

In Fig. 2, we present the resulting age-metallicity relation of nearby stars based on these data. The open circles represent direct averages over  $[\text{Fe}/\text{H}]$  in five age groups. For the filled circles, each value of  $[\text{Fe}/\text{H}]$  has been weighted with  $|W + W_{\odot}|$ , where  $W$  is the component of the space velocity of the stars towards the galactic north pole, measured relative to the Sun, and  $W_{\odot} = +7$  km/s is the component of the solar motion with respect to the circular velocity at  $R_0$ . This procedure (e.g. Wielen 1977) gives results which are representative for a cylinder perpendicular to the galactic plane (instead of a sphere around the Sun), and hence avoids an overrepresentation of stars with small  $W$  velocities. It is unclear to what extent the stars in the sample of Edvardsson et al. (1993) are fully representative with respect to age and metallicity. Edvardsson et al. (1993) give in their Table 14 mean metallicities for a sample which was corrected for representing a volume-limited catalogue of nearby stars. These data are shown as open squares in Fig. 2. All the results shown



**Fig. 2.** The relation between the mean metallicity  $\langle [Fe/H] \rangle$  (in dex) and age  $\tau$  (in  $10^9$  years) for nearby stars, i.e. for stars which are now at ( $R \sim R_0$ ). The basic data are taken from Edvardsson et al. (1993). The open circles (○) represent the direct averages for stars in five age groups. The mean errors of these averages are obtained from the Poisson noise, based on the number of stars in each group. The filled circles (●) are averages in which  $|W+W_\odot|$  of a star is used as its ‘weight’, and are hence representative for stars in a cylinder perpendicular to the galactic plane. For comparison, we also plot the original data of Edvardsson et al., who have used a different binning. Crosses (×) show their direct results. The open squares (□) give their results for a sample which is corrected for representing a volume-limited sample. The fitting line (Eq.(2)) is mainly based on the filled circles. The solar anomaly  $(\Delta[Fe/H])_\odot = +0.17$  dex, i.e. the difference between  $[Fe/H]_\odot$  and the mean metallicity of now nearby stars of the same age  $\tau_\odot$ , is indicated by the dashed line.

in Fig. 2 agree quite well. A linear age-metallicity relation at  $R_0$  can be fitted to the data:

$$\langle [Fe/H] \rangle = +0.05 \text{ dex} - 0.048 \text{ dex} (\tau/10^9 \text{ years}) . \quad (2)$$

Combined with a constant radial gradient  $\alpha$  (Eq. (1)), the full relation between metallicity, age, and galactocentric distance follows as

$$\begin{aligned} [Fe/H] = & +0.05 \text{ dex} \\ & -0.09 \text{ dex} ((R_i - R_0)/\text{kpc}) \\ & -0.048 \text{ dex} (\tau/10^9 \text{ years}) . \end{aligned} \quad (3)$$

For each star for which we know  $\tau$  and  $[Fe/H]$ , we can derive the initial galactocentric radius  $R_i$  at birth from

$$(R_i - R_0)/\text{kpc} = -11 [Fe/H] - 0.53(\tau/10^9 \text{ years}) + 0.6 . \quad (4)$$

### 2.3. The initial spread of metallicities

The method for deriving birth-places of stars (i.e.  $R_i$ ) from metallicities, outlined above, would not be useful if the initial variation in metallicity for stars born at the same time and the same galactocentric distance would be too large with respect to  $\alpha(R_i - R_0)$ . From a discussion of the age-dependence of the dispersion of stellar metallicities, we find in Sect. 4.3 by an extrapolation to zero age that the initial dispersion in the measured metallicities  $[Fe/H]$  is probably as small as  $\pm 0.06$  dex, already largely explained by the measuring errors of  $[Fe/H]$  in the sample of stars used. Since this sample (Edvardsson et al. 1993) provides the most precise metallicities available, the larger estimates of the initial dispersion in metallicity given by other authors are probably due to higher measuring errors in their samples.

Furthermore, we would like to emphasize that the initial dispersion of metallicities should not be confused with the observed dispersions in metallicities of stars of higher ages. The higher dispersions of metallicities of stars of higher ages mainly reflect the diffusion of stars in space, which, coupled with a radial gradient in metallicity, rapidly increases the dispersion in metallicity with increasing age, as discussed in Sect. 4.3.

## 3. Application to the Sun

### 3.1. The solar anomaly in metallicity

The metallicity  $[Fe/H]_\odot$  of the Sun is zero by definition. There are no indications that the indirect reference of the stellar metallicities to that of the Sun, given by Edvardsson et al. (1993) and based on a rather elaborate observational and theoretical work, have produced any systematic error (i.e. an off-set) with respect to the solar metallicity.

Fig. 2 shows an anomaly in the solar  $[Fe/H]$  metallicity: The Sun has a metallicity  $[Fe/H]$  which is by 0.17 dex larger than the mean metallicity of nearby stars of the same age,  $\tau = 4.5 \cdot 10^9$  years. This corresponds to an overabundance of Fe in the Sun by about 50 % with respect to presently nearby stars of the same age. The accuracy of the fitting line for the mean metallicity at  $\tau = \tau_\odot$  in Fig. 2 is about  $\pm 0.04$  dex. If there are no systematic errors in the stellar metallicities with respect to the Sun, this should also be the mean error of the solar anomaly in  $[Fe/H]$ :  $+0.17 \pm 0.04$  dex.

The solar anomaly of +0.17 dex lies rather well within the observed dispersion of  $[Fe/H]$  metallicities of stars of solar age, which we have determined from the Edvardsson et al. sample to be about  $\pm 0.24$  dex. This does not mean, however, that the solar anomaly is not remarkable. It implies only that the behaviour of the Sun with respect to its migration in galactocentric distance  $R$  over  $4.5 \cdot 10^9$  years is rather typical for stars of this age.

Finally, we shall also take into account the initial dispersion in metallicities discussed in Sect. 2.3. If the initial dispersion is zero in reality and its formally deduced value is completely due to the measuring errors in the individual  $[Fe/H]$  values, then these errors are already included into the uncertainty of the mean metallicities. If, however, the initial dispersion of  $\pm 0.06$  dex is

a real physical phenomenon, then the initial dispersion has to be added quadratically to the measuring error of the solar anomaly. In this case, the solar anomaly is  $(\Delta[\text{Fe}/\text{H}])_{\odot} = +0.17 \pm 0.07$  dex.

Up to now, we have used only  $[\text{Fe}/\text{H}]$  as an indicator of metallicity. Does the Sun show anomalies also in other metallicities, e.g. in  $[\text{O}/\text{H}]$ ,  $[\alpha \text{ elements (Mg, Si, Ca, Ti)}/\text{H}]$ ,  $[\text{O}/\text{Fe}]$ ,  $[\alpha/\text{Fe}]$ ? In our scenario, the solar anomalies in different metallicities should be proportional to the radial gradient  $\alpha$  for the corresponding metallicity. Using again the stellar sample of Edvardsson et al. (1993), we find the following solar anomalies: for  $[\text{O}/\text{H}] +0.17$  dex, for  $[\text{O}/\text{Fe}] -0.02$  dex, for  $[\alpha/\text{H}] +0.09$  dex, and for  $[\alpha/\text{Fe}] -0.08$  dex (The quantities  $[\text{O}/\text{H}] - [\text{O}/\text{Fe}]$  and  $[\alpha/\text{H}] - [\alpha/\text{Fe}]$  can differ from  $[\text{Fe}/\text{H}]$ , because of uncertainties in the fitting procedure and because the samples of stars used are slightly different, since  $[\alpha/\text{H}]$  is not available for all the stars). Within the error limits, these anomalies are in agreement with the solar anomaly in  $[\text{Fe}/\text{H}]$  of  $+0.17$  dex, if we adopt the measured or estimated radial gradients of these metallicities. We prefer to use  $[\text{Fe}/\text{H}]$  as an indicator for metallicity, because it is commonly used, well-determined, and its radial gradient is large.

### 3.2. The birth-place of the Sun

From the solar anomaly  $(\Delta[\text{Fe}/\text{H}])_{\odot}$  in metallicity and from the adopted radial gradient  $\alpha$  in  $[\text{Fe}/\text{H}]$ , we can derive the difference between the galactocentric distance  $R_{i,\odot}$  of the Sun at birth and its present galactocentric distance  $R_0$ :

$$R_{i,\odot} - R_0 = (\Delta[\text{Fe}/\text{H}])_{\odot} / \alpha . \quad (5)$$

We adopt  $\alpha = -0.09 \pm 0.02$  dex/kpc (Eq.(1)), and the IAU (1985) value of  $R_0 = 8.5$  kpc.

Using the determined solar anomaly of  $(\Delta[\text{Fe}/\text{H}])_{\odot} = +0.17 \pm 0.07$  dex, we derive

$$R_{i,\odot} - R_0 = -1.9 \pm 0.9 \text{ kpc} , \quad (6)$$

or

$$R_{i,\odot} = 6.6 \pm 0.9 \text{ kpc} . \quad (7)$$

The result that the Sun was born in the inner parts of our Galaxy is certainly interesting, even if its statistical significance is obviously limited. If we assume a gaussian distribution of  $R_{i,\odot} - R_0$  with a dispersion of 0.9 kpc around  $-1.9$  kpc, we find the following probabilities for the Sun being born within a galactocentric distance  $R$  (i.e. that  $R_{i,\odot} < R$ ): 98 % for  $R = R_0 = 8.5$  kpc, 94 % for  $R = 8.0$  kpc, 84 % for 7.5 kpc, 67 % for 7.0 kpc, 50 % for 6.6 kpc, 46 % for 6.5 kpc, 25 % for 6.0 kpc, and 11 % for 5.5 kpc.

## 4. The diffusion of stellar orbits in space

### 4.1. The solar migration

In Sect. 3.2, we have derived that the Sun has moved outwards by about 1.9 kpc (in net) within  $4.5 \cdot 10^9$  years. Is this value

acceptable in view of other considerations? The theory of the diffusion of stellar orbits provides a positive answer. Already in our first paper on this topic (Wielen 1977, Tables 2, 3, and 4), we have predicted for a star of age  $\tau = 4.5 \cdot 10^9$  years an rms value for the migration in galactocentric distance,  $\Delta R$ , of about 2 kpc. From this point of view, the Sun is perfectly within the expectation.

The Sun has, besides its anomaly in metallicity, another anomaly, namely its untypically low space velocity. The total space velocity of the Sun with respect to the local circular velocity (i.e. the peculiar solar motion) is  $v_{\odot} = 17$  km/s (Delhaye 1965). Nearby stars of the age of the Sun have a total velocity dispersion  $\sigma_v$  of about 50 km/s, three times the value of  $v_{\odot}$ . This is most probably a chance effect during the diffusion of the vector  $\mathbf{v}_{\odot}$  in velocity space. A discussion of the probable fate of  $\mathbf{v}_{\odot}(t)$  has been given by Fuchs and Wielen (1987), who concluded that, during most of the past,  $\mathbf{v}_{\odot}(t)$  was significantly larger than both the initial value and the present value of  $\mathbf{v}_{\odot}$ . This investigation should be repeated in view of the new additional restriction of  $R_{i,\odot} - R_0 = -1.9$  kpc. We expect even higher values of  $\mathbf{v}_{\odot}(t)$  in the past, since the Sun has to migrate a significant distance in  $R$  inspite of its presently low velocity.

How does the present solar orbit behave in other respects? To a first order, the regular orbit of a disk star (such as the Sun) in the mean gravitational field of our Galaxy can be described as an epicyclic orbit. The star moves on an elliptic epicycle, the center of which moves on a circular orbit around the center of the Galaxy. We call the center of the epicycle the guiding center, its galactocentric distance  $R_m$ , and the semi-axis of the epicycle in the  $R$  direction  $a_R$ . If we know for a star its present value of  $R$  and its space velocity (with respect to the circular velocity at its position), we can derive both  $R_m$  and  $a_R$ . For the Sun, Wielen (1982) found  $R_{m,\odot} - R_0 = +0.6$  kpc and  $a_{R,\odot} = 0.7$  kpc. Since the Sun was probably born on a nearly circular orbit, the galactocentric distance of its birth-place,  $R_{i,\odot}$ , corresponds rather closely to its initial value of  $R_m$ , which we call  $R_{m,i,\odot}$ . The difference between the initial value of  $R_m$  and its present value for the Sun is

$$R_{m,\odot} - R_{m,i,\odot} = +2.5 \text{ kpc} . \quad (8)$$

Using the model of the constant isotropic diffusion coefficient (Wielen 1977), the expectation values for stars of solar age are  $\langle (\Delta R_m)^2 \rangle^{1/2} = 1.5$  kpc and  $a_R = 1.8$  kpc (based on Eqs.(A5) and (A8), derived below). Hence the change in  $R_m$  of the Sun is larger than on average, while  $a_{R,\odot}$  is much smaller than expected. The smallness of  $a_{R,\odot}$  is essentially the same phenomenon as the unexpectedly small peculiar motion of the Sun. The large change in the galactocentric distance of the guiding center of the solar orbit, about 2.5 kpc, is most probably also mainly a chance effect. The probability for such a deviation in  $R_m$  from the expectation (1.5 kpc) is still 10 % in a gaussian distribution.

In summary, the derived migration of the Sun in space is by no means unusually large; it is within the limits of expectation.

#### 4.2. Places of formation of other stars

Similar to the birth-place of the Sun, we can derive the initial galactocentric distance  $R_i$  also for other stars, especially for those in the Edvardsson et al. sample. Only the discussion of such a reasonably large sample of stars can show that the migration of the Sun, as one individual object, is really acceptable and typical. The results are shown in Fig. 3 in a somewhat indirect way. We basically plot the individual metallicities  $[\text{Fe}/\text{H}]$  of the stars as a function of their age  $\tau$  for  $\log \tau[\text{years}] \leq 10.1$ . In addition to the values for the individual stars (open squares), we have plotted in Fig. 3 also the mean metallicities  $\langle [\text{Fe}/\text{H}] \rangle$  from Fig. 2 (filled circles, weighted with  $|W+W_\odot|$ ). The error bars of  $\langle [\text{Fe}/\text{H}] \rangle(\tau)$ , shown in Fig. 3, do not, however, indicate the mean errors of this quantity (as in Fig. 2), but the dispersion  $\sigma_{[\text{Fe}/\text{H}]}(\tau)$  of the individual metallicities of the stars in the corresponding age group, relative to the mean of this group, also weighted with  $|W+W_\odot|$ . We admit that it is sometimes difficult to reconcile the visual impression of the spread of the individual points in Fig. 3 with the correctly calculated values of  $\sigma_{[\text{Fe}/\text{H}]}$ . This is partially an effect of the proper weight  $|W+W_\odot|$ . Using Eq.(3), we then add in Fig. 3 the lines of constant  $R_i$ . Hence Fig. 3 indicates graphically  $R_i$  for each star. Some rather implausible values, e.g. young stars with large  $|R_i - R_0|$ , are probably mainly due to errors in the determined ages  $\tau$ , but may be partially also due to errors in metallicity. In general, however, the results confirm the expectations from the diffusion scenario: Older stars reach us from larger distances than younger objects. This can be seen quantitatively by inspecting the ‘error bars’ in Fig. 3, which show that  $\sigma_{[\text{Fe}/\text{H}]}$  grows, on the average, with increasing age  $\tau$ . Since we have

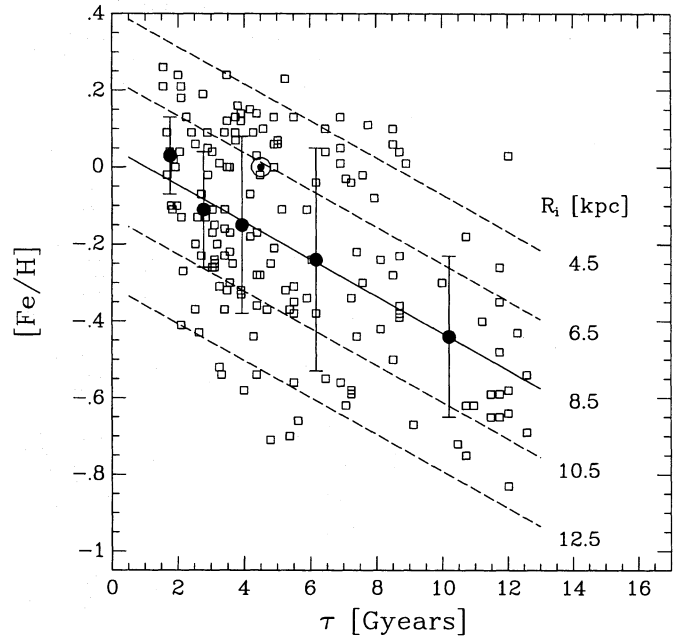
$$\sigma_{\Delta R} = \langle (R_i - R_0)^2 \rangle^{1/2} = \frac{1}{|\alpha|} \sigma_{[\text{Fe}/\text{H}]} , \quad (9)$$

it follows that  $\sigma_{\Delta R}$  also increases with  $\tau$ .

#### 4.3. The age-dependence of the dispersion in metallicity

In Fig. 4, we show the observed dispersion  $\sigma_{[\text{Fe}/\text{H}]}$  of the stellar metallicities as a function of age. The input data are the same as used for deriving the age-metallicity relation (Sect. 2.2). The dispersion is measured always with respect to the mean value of the corresponding age group of stars. Fig. 4 indicates that  $\sigma_{[\text{Fe}/\text{H}]}$  increases significantly with age  $\tau$  (except for the oldest group, which may deviate by chance from the general trend).

In our interpretation, the increase of  $\sigma_{[\text{Fe}/\text{H}]}$  with  $\tau$  is due to the diffusion of stellar orbits in space: With increasing age, more and more stars arrive from larger distances (in  $R_i$ ) at a given position  $R$ , say at  $R_0$ , where we observe now. Due to the galactic gradient  $\alpha$  in metallicity, these older stars have a larger spread in  $[\text{Fe}/\text{H}]$  than younger stars at  $R_0$ , because of their larger spread in  $R_i$ . This explanation for the increase of  $\sigma_{[\text{Fe}/\text{H}]}$  with  $\tau$  has first been published by Francois and Matteucci (1993), following the basic work by Grenon (1987), and has also been put forward by us (Fuchs, Dettbarn, and Wielen 1994, 1995).



**Fig. 3.** Places of formation of now nearby stars. We have plotted the metallicity  $[\text{Fe}/\text{H}]$  of stars of the sample of Edvardsson et al. (1993) as a function of their age  $\tau$  (open squares  $\square$ ). The Sun has been added. Inserted are lines of constant initial galactocentric distance  $R_i$  which are based on Eq.(3). Hence the place of formation (i.e.  $R_i$ ) can be read-off for each star (including the Sun). The mean metallicity of the stars as a function of age is shown again (as in Fig. 2) as filled circles ( $\bullet$ ) in order to indicate the observational basis of the adopted slope of the lines of constant  $R_i$ .

Due to the diffusion of stellar orbits in space,  $\langle (\Delta R)^2 \rangle^{1/2}$ , and due to the galactic gradient  $\alpha$  in metallicity,  $\sigma_{[\text{Fe}/\text{H}]}$  increases as

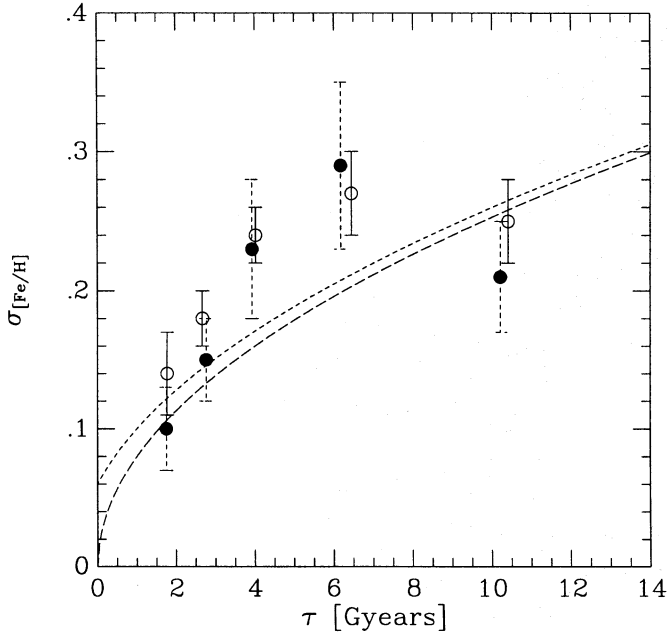
$$\sigma_{[\text{Fe}/\text{H}]}(\tau) = |\alpha| \langle (\Delta R)^2(\tau) \rangle^{1/2} . \quad (10)$$

Adopting the rms value of the displacement of stars in  $R$  given by Eq.(A1) and  $\alpha = -0.09$  dex/kpc, we derive the following prediction

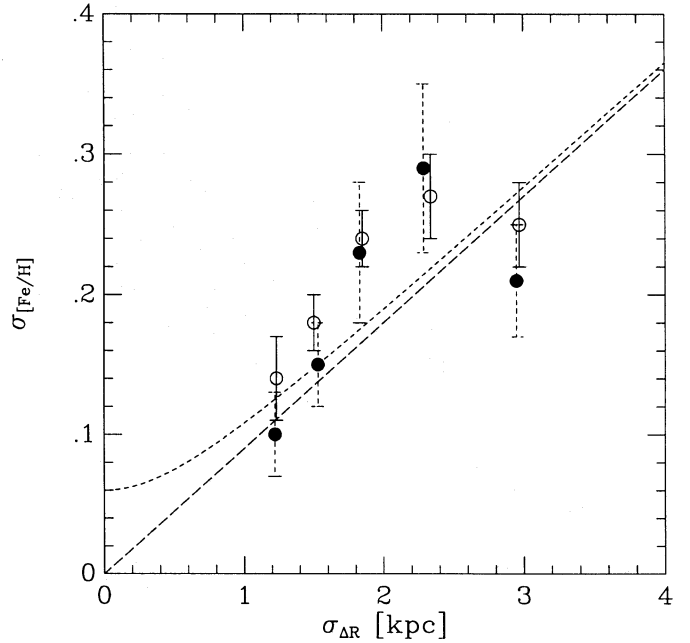
$$\sigma_{[\text{Fe}/\text{H}]}(\tau) = 0.08 \text{ dex } (\tau/10^9 \text{ years})^{1/2} , \quad (11)$$

which is shown as the dashed curve in Fig. 4. The agreement between the observed and predicted values of  $\sigma_{[\text{Fe}/\text{H}]}$  is remarkably good.

Instead of plotting  $\sigma_{[\text{Fe}/\text{H}]}$  as a function of age  $\tau$ , it is also interesting to plot  $\sigma_{[\text{Fe}/\text{H}]}$  as a function of  $\langle (\Delta R)^2 \rangle^{1/2}$  which is predicted using the age  $\tau$  of the corresponding group (Fig. 5). The advantage of such a presentation is that we expect a linear relation between both quantities, according to Eq.(10). The predicted relation is shown in Fig. 5 by the dashed line. From a comparison between the observed and predicted values of  $\sigma_{[\text{Fe}/\text{H}]}$ , we get an estimate for the initial value of  $\sigma_{[\text{Fe}/\text{H}]}$  at  $\tau = 0$ . For  $\sigma_{[\text{Fe}/\text{H}]}(0) = 0.06$  dex, we derive the dotted curve by adding this value quadratically to our original prediction (dashed line). We do not think that much higher values for  $\sigma_{[\text{Fe}/\text{H}]}(0)$  are acceptable in Fig. 5. The proposed value of 0.06 dex is essentially



**Fig. 4.** The relation between the dispersion of stellar metallicities (around the mean value),  $\sigma_{[\text{Fe}/\text{H}]}$ , and age  $\tau$ . The basic data are taken from the sample of Edvardsson et al. (1993). As in Fig. 2, the open circles ( $\circ$ ) are the direct results for the chosen age groups, while for the filled circles ( $\bullet$ ) we have used the weights  $|W + W_{\odot}|$ . The error bars indicate the statistically estimated uncertainty of the dispersions. The dashed curve is the theoretical prediction based on Eq. (11). The dotted curve includes an initial dispersion in  $[\text{Fe}/\text{H}]$  of 0.06 dex.



**Fig. 5.** The relation between the observed dispersion of stellar metallicities,  $\sigma_{[\text{Fe}/\text{H}]}$ , and the predicted dispersion in galactocentric distance,  $\sigma_{\Delta R} = \langle (\Delta R)^2 \rangle^{1/2}$ , due to the diffusion of stellar orbits in space.  $\sigma_{\Delta R}$  is obtained from the mean age of the group of stars using Eq. (A1). The symbols (open circles, filled circles) have the same meaning as in Fig. 4. The dashed line represents the prediction based on Eq. (10) and  $\alpha = -0.09$  dex/kpc. The dotted curve includes an initial dispersion in metallicity of 0.06 dex.

identical to the measuring errors in the individual metallicities, estimated by Edvardsson et al. (1993) as 0.05 dex. Hence there is not much room for a real dispersion of metallicities of stars born at the same time and at the same galactocentric distance  $R_i$ . As already mentioned, higher values of  $\sigma_{[\text{Fe}/\text{H}]}(0)$  claimed by other authors, may be due to less accurate measurements of metallicities.

In the vast majority of models for the diffusion of stellar orbits, we obtain that the diffusion of stellar orbits in *space* is proportional to that in *velocity*. It is only the factor of proportionality which depends on the detailed mechanism of diffusion. Hence we expect from Eq. (10):

$$\sigma_{[\text{Fe}/\text{H}]} = |\alpha| f_U \sigma_U = |\alpha| f_v \sigma_v, \quad (12)$$

where  $\sigma_U$  and  $\sigma_v$  are the velocity dispersions in  $U$  (towards the galactic center) and in  $v$  (total peculiar velocity), and  $f_U$  and  $f_v$  are factors to be determined. In Fig. 6, we present  $\sigma_{[\text{Fe}/\text{H}]}$  as a function of  $\sigma_v$ , using again the Edvardsson et al. sample. The data can be well fitted by a linear relation implied by Eq. (12). From Eq. (A15), we predict

$$f_v = 3.8 \text{ kpc}/(100 \text{ km/s}), \quad (13)$$

and, using  $|\alpha| = 0.09$  dex/kpc,

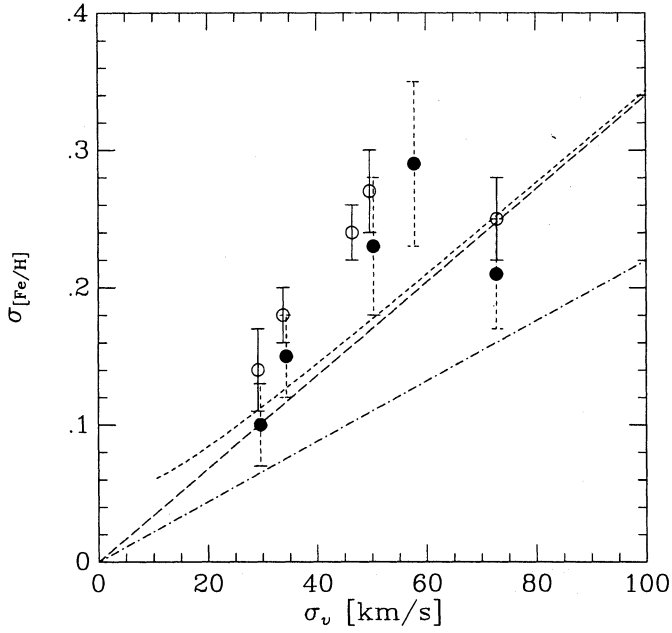
$$|\alpha| f_v = 0.34 \text{ dex}/(100 \text{ km/s}). \quad (14)$$

The factor  $f_v$  in Eq. (A15) has been checked by numerical simulations of the perturbations of stellar orbits by passing massive black holes. The agreement is extremely good. According to the simulations, the same factor  $f_v$  seems even to be valid for perturbations by giant molecular clouds, while a theoretical treatment predicts higher values for  $f_v$  for the Spitzer-Schwarzschild mechanism (Fuchs et al. 1994). The dashed line in Fig. 6 represents the prediction based on Eqs. (12) and (14). This prediction is independent of the assumed value for the diffusion coefficient  $D$ , since we are using here the directly observed values of  $\sigma_v$ . The agreement with the observational data is fair. As from Fig. 5, we infer also from Fig. 6 that the initial dispersion  $\sigma_{[\text{Fe}/\text{H}]}(0)$  is rather small.

Even if one would deny the diffusion of stellar orbits completely, the galactocentric distance  $R$  of a star would vary with time  $t$  because of the motion of the star on its epicycle. Since the size of the epicycle,  $a_R$ , is known from the observed peculiar velocity of the star, the minimal value of  $f_U$ , due to the epicyclic motion alone, is known:

$$f_{U,\min} = \frac{1}{\kappa}, \quad (15)$$

where  $\kappa$  denotes the epicyclic frequency. The ratio  $f_v/f_U$  is empirically fixed by the observed axial ratios of the velocity ellipsoid of the disk stars. Since the assumption of an isotropic



**Fig. 6.** The relation between the dispersion of stellar metallicities,  $\sigma_{[\text{Fe}/\text{H}]}$ , and the observed velocity dispersion,  $\sigma_v$ .  $v$  is the total peculiar velocity of a star. The meaning of the symbols (open circles, filled circles) is the same as in Fig. 4. The dashed line represents the theoretical prediction based on Eqs.(12) and (14). The dotted curve is based on Eq.(A18) and includes an initial dispersion in metallicity (0.06 dex) and in velocity (10 km/s). The dash-dotted line, based on Eq.(18), represents the lower limit due to the epicyclic motion alone.

diffusion coefficient  $D$  fits these observed axial ratios quite well (Wielen 1977), we may use the theoretical ratio:

$$\frac{f_U}{f_v} = \left( \frac{3 + \left(\frac{\kappa}{2B}\right)^2 + \left(\frac{2B}{\kappa}\right)^2}{1 + \left(\frac{\kappa}{2B}\right)^2} \right)^{1/2} = 1.3 . \quad (16)$$

This gives

$$f_{v,\min} = \frac{1}{1.3 \kappa} = 2.4 \text{ kpc}/(100 \text{ km/s}) , \quad (17)$$

or

$$|\alpha| f_{v,\min} = 0.22 \text{ dex}/(100 \text{ km/s}) . \quad (18)$$

In Fig. 6, we show this lower limit by the dash-dotted line. The actual effect is larger because the diffusion of stellar orbits does not only increase  $a_R$ , but changes also  $R_m$  stochastically, thereby increasing  $\langle (\Delta R)^2 \rangle^{1/2}$  significantly, as shown in the appendix.

#### 4.4. A possible correction

The diffusion of stellar orbits itself may affect some of the basic data we have used to derive the birth-places of the Sun and of other nearby stars. While the galactic gradient of metallicity should be rather unaffected by the diffusion of stellar orbits in space, the age-metallicity relation at  $R_0$  may be somewhat

changed by the diffusion. The basic reason is the following: The surface density of stars is decreasing with increasing  $R$  (e.g. as in an exponential disk). Hence, even if the diffusion coefficient  $D$  would not be decreasing with  $R$  (as it is probably the case in reality), absolutely more stars from the inner parts (with  $R_i < R_0$ ) shall arrive now at  $R = R_0$  by diffusion than stars do from the outer parts of our Galaxy (with  $R_i > R_0$ ). Coupled with the galactic gradient in metallicity, this increases the mean metallicity  $\langle [\text{Fe}/\text{H}] \rangle$  of stars now observed at  $R = R_0$  with respect to those stars which were born  $\tau$  years ago at  $R_i = R_0$ .

A very simplified treatment predicts a correction of about  $-0.02$  dex ( $\tau/10^9$  years) to be added to the now observed age-metallicity relation for deriving the age-metallicity relation of stars at birth. In Eq. (3), this would lead to a coefficient of ( $\tau/10^9$  years) of  $-0.07$  dex instead of the used value  $-0.048$  dex. The resulting solar anomaly in  $[\text{Fe}/\text{H}]$  metallicity would be  $+0.26$  dex (instead of  $+0.17$  dex used). The birth-places of the Sun would be shifted to  $R_{i,\odot} = 5.6$  kpc (instead of 6.6 kpc), and  $R_{i,\odot} - R_0 = -2.9$  kpc (instead of  $-1.9$  kpc).

The correction, however, is uncertain quantitatively and very probably too high, because it neglects the self-gravity of the disk and the probability that some collective effects in the disk itself (e.g. pieces of spiral arms or other wavelets) may be responsible for the diffusion of stellar orbits. Hence we prefer not to apply such a correction. It should also be mentioned that a correction of the described amount would produce implausible results in Sect. 4.2. On the other hand, it can be shown that the described effect of diffusion should not significantly change the dispersion  $\sigma_{[\text{Fe}/\text{H}]}(\tau)$ . Hence Sect. 4.3 does not require a correction for the effect under discussion.

## 5. Conclusions

From the determined solar anomaly in the  $[\text{Fe}/\text{H}]$  metallicity and an adopted galactic gradient in metallicity, we have derived that the Sun was probably born in the inner part of our Galaxy, perhaps 1.9 kpc closer to the galactic center than the Sun is presently located.

The derived value of  $R_{i,\odot} - R_0 = -1.9$  kpc is well explained by the diffusion of stellar orbits in space, predicted already by Wielen (1977). The same is true for the birth-places of 159 nearby stars in the sample of Edvardsson et al. (1993). This is shown quantitatively by the good agreement between the predicted and observed dispersions  $\sigma_{[\text{Fe}/\text{H}]}$  of the  $[\text{Fe}/\text{H}]$  metallicity as a function of age  $\tau$  or of velocity dispersion  $\sigma_v$ .

We conclude that the derived birth-places of the Sun and of other nearby stars, and the quantitative explanation of the observed relation between the dispersion of the stellar metallicity and age, are a nice confirmation of the predicted diffusion of stellar orbits in space. Formerly, the diffusion was empirically only indicated by the relation between the velocity dispersion (e.g.  $\sigma_v$ ) and age  $\tau$  of nearby stars. Hence the diffusion was observed only in velocity space. Now, due to much more accurate measurements of stellar metallicities, the diffusion of stellar orbits is also observable in space.

Why was the quantitative prediction of the diffusion of stellar orbits in space rather successful? It is still true that the basic mechanism for the gravitational perturbations of stellar orbits is not accurately known. Proposed perturbers include giant molecular clouds, massive black holes from the dark corona of our Galaxy, pieces of spiral arms, other wavelets in the galactic disk, even incoming small galaxies. In view of this basic uncertainty, one may be inclined to doubt the predictions on the diffusion of stellar orbits in space. However, the prediction is based on a diffusion coefficient  $D$  which is determined *empirically* by fitting the observed age-velocity relation. Thereby, the largest uncertainty, namely the overall strength of the diffusion mechanism, is obviously overcome. Of course, finer details certainly depend on the detailed mechanism of diffusion. But the existence and the amount of the diffusion of stellar orbits seems to be confirmed now both in velocity space *and in positional space*.

If the Edvardsson et al. sample is biased with respect to metallicities and/or ages of the stars, then our quantitative results presented here would clearly be affected by such a bias. Nevertheless, the method of determining the birth-place of the Sun from its metallicity would remain valid in principle. The crucial ingredient of our method is a significant radial gradient of a suitably chosen metallicity in our Galaxy. If this gradient were too small or even zero, then our method would not be applicable.

The most important implication of the derived birth-place of Sun, at  $R_{i,\odot} \sim 6.6$  kpc, may be the higher cosmic-ray intensity for the Earth in earlier times, since the rate of supernovae events increases with decreasing  $R$ . Furthermore, the Sun should have encountered more molecular clouds in earlier times, since they are more frequent in the inner parts of the Galaxy. These encounters could have sent more comets in earlier times from the outer Oort comet cloud into the inner parts of the solar system than we do observe now.

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## Appendix A

In this appendix, we shall derive some formulae which are needed in the former sections. We use the same notation as Wielen (1977), and adopt always an isotropic, constant diffusion coefficient  $D$ . For consistency, we also use the same values for  $D$  ( $2 \cdot 10^{-7}$  (km/s)<sup>2</sup>/year) and for Oort's constants  $A$  (+15(km/s)/kpc) and  $B$ (−10(km/s)/kpc) as in our earlier papers. More recently determined values would not change our results significantly.

From Eq.(42) of Wielen (1977), it follows that

$$\begin{aligned} \langle (\Delta R)^2 \rangle &= \frac{1}{2} \left( \frac{1}{\kappa^2} + \frac{3}{(2B)^2} \right) D\tau \\ &= (0.92 \text{ kpc})^2 (\tau/10^9 \text{ years}) . \end{aligned} \quad (\text{A1})$$

The diffusion in  $R$  is, as already mentioned in Sect. 4.1, partially due to changes in the galactocentric distance  $R_m$  of the guiding center. For nearby stars at  $R = R_0$ ,  $R_m$  is given by

$$R_m = R_0 + \frac{V}{(-2B)} , \quad (\text{A2})$$

(Wielen 1982). A velocity perturbation of amount  $\delta U_i, \delta V_i, \delta W_i$  at time  $t_i$  leads to a change  $\Delta R_m$  in  $R_m$  of

$$\Delta R_m = \frac{1}{(-2B)} \delta V_i . \quad (\text{A3})$$

From this and Eqs.(36) and (39) of Wielen (1977), we obtain

$$\langle \Delta R_m \rangle = 0 , \quad (\text{A4})$$

and

$$\langle (\Delta R_m)^2 \rangle = \frac{1}{(2B)^2} D\tau = (0.71 \text{ kpc})^2 (\tau/10^9 \text{ years}) . \quad (\text{A5})$$

The semi-axis  $a_R$  of a stellar epicycle in the  $R$  direction is given by

$$a_R^2 = \left( \frac{U}{\kappa} \right)^2 + \left( \frac{V}{2B} \right)^2 \quad (\text{A6})$$

(Wielen 1982). A velocity perturbation, as described above, changes  $a_R^2$  by

$$\begin{aligned} \Delta(a_R)^2 &= \left( \frac{U + \delta U_i}{\kappa} \right)^2 + \left( \frac{V + \delta V_i}{2B} \right)^2 \\ &\quad - \left( \frac{U}{\kappa} \right)^2 - \left( \frac{V}{2B} \right)^2 . \end{aligned} \quad (\text{A7})$$

From Eq.(A7) and Eqs.(36)–(39) of Wielen (1977), we derive

$$\begin{aligned} \langle \Delta(a_R)^2 \rangle &= \left( \frac{1}{\kappa^2} + \frac{1}{(2B)^2} \right) D\tau \\ &= (0.84 \text{ kpc})^2 (\tau/10^9 \text{ years}) \end{aligned} \quad (\text{A8})$$

Since the rms distance of a star on its epicycle from its guiding center in  $R$  is

$$\langle (R - R_m)^2 \rangle = \langle a_R^2 \sin^2 \kappa(t - t_0) \rangle = \frac{1}{2} a_R^2 , \quad (\text{A9})$$

the total diffusion in  $R$  is given by

$$\langle (\Delta R)^2 \rangle = \langle (\Delta R_m)^2 \rangle + \frac{1}{2} \langle \Delta(a_R^2) \rangle . \quad (\text{A10})$$

Inserting Eqs.(A5) and (A8) into Eq.(A10) leads to the same result for  $\langle (\Delta R)^2 \rangle$  as given by Eq.(A1). From Eqs.(A5) and (A8), we obtain

$$\frac{\langle (\Delta R_m)^2 \rangle}{\frac{1}{2} \langle \Delta(a_R^2) \rangle} = \frac{1}{2B^2 \left( \frac{1}{\kappa^2} + \frac{1}{(2B)^2} \right)} = 1.43 \quad (\text{A11})$$



This and Eq.(A10) show that the changes in  $R_m$  are more important for the total diffusion in  $R$  than the changes in  $a_R$ .

We are now investigating the relation between the diffusion in space and in velocity. Eqs.(45)–(47) or (55), (56), (50) of Wielen (1977) give

$$\begin{aligned} \langle (\Delta U)^2 \rangle &= \frac{1}{2} \left( 1 + \left( \frac{\kappa}{2B} \right)^2 \right) D\tau \\ &= \sigma_U^2(\tau) - \sigma_U^2(0) \end{aligned} \quad (\text{A12})$$

and

$$\begin{aligned} \langle (\Delta v)^2 \rangle &= \frac{1}{2} \left( 3 + \left( \frac{\kappa}{2B} \right)^2 + \left( \frac{2B}{\kappa} \right)^2 \right) D\tau \\ &= \sigma_v^2(\tau) - \sigma_v^2(0) \end{aligned} \quad (\text{A13})$$

From Eqs.(A1) and (A12), the ratio  $f_U$  between  $\langle (\Delta R)^2 \rangle^{1/2}$  and  $\langle (\Delta U)^2 \rangle^{1/2}$  follow as

$$\begin{aligned} f_U &= \frac{\langle (\Delta R)^2 \rangle^{1/2}}{\langle (\Delta U)^2 \rangle^{1/2}} = \left( \frac{1 + 3\left(\frac{\kappa}{2B}\right)^2}{1 + \left(\frac{\kappa}{2B}\right)^2} \right)^{1/2} \frac{1}{\kappa} \\ &= 1.56 \frac{1}{\kappa} = 4.9 \text{ kpc}/(100 \text{ km/s}) \end{aligned} \quad (\text{A14})$$

The ratio  $f_v$  between  $\langle (\Delta R)^2 \rangle^{1/2}$  and  $\langle (\Delta v)^2 \rangle^{1/2}$  is given by Eqs.(A1) and (A13) as

$$\begin{aligned} f_v &= \frac{\langle (\Delta R)^2 \rangle^{1/2}}{\langle (\Delta v)^2 \rangle^{1/2}} = \left( \frac{1 + 3\left(\frac{\kappa}{2B}\right)^2}{3 + \left(\frac{\kappa}{2B}\right)^2 + \left(\frac{2B}{\kappa}\right)^2} \right)^{1/2} \frac{1}{\kappa} \\ &= 1.20 \frac{1}{\kappa} = 3.8 \text{ kpc}/(100 \text{ km/s}) \end{aligned} \quad (\text{A15})$$

What is the strict relation between  $\sigma_{[\text{Fe}/\text{H}]}$  and  $\sigma_U$ ,  $\sigma_v$ , or  $\langle (\Delta R)^2 \rangle^{1/2}$ ? Eq. (10) describes the contribution of the diffusion of stellar orbits only. In addition, we have to add quadratically the initial dispersion  $\sigma_{[\text{Fe}/\text{H}]}^{(\text{mix})}(0)$  in metallicity of the stars at birth (due to an incomplete mixing of the interstellar medium at a given  $R_i$ ), but also the contribution  $\sigma_{[\text{Fe}/\text{H}]}^{(\text{epi})}(0)$  of the initial epicyclic motions (due to the initial velocity dispersion of the stars at birth). The latter is given by

$$\sigma_{[\text{Fe}/\text{H}]}^{(\text{epi})}(0) = \frac{|\alpha|}{\kappa} \sigma_U(0) = \frac{f_v}{f_U} \frac{|\alpha|}{\kappa} \sigma_v(0) = 0.022 \text{ dex} \quad , \quad (\text{A16})$$

using  $\sigma_v(0) = 10 \text{ km/s}$ . Even if  $\sigma_{[\text{Fe}/\text{H}]}^{(\text{mix})}(0)$  were zero, the total effective initial metallicity dispersion  $\sigma_{[\text{Fe}/\text{H}]}(0)$ , including the measuring error  $\sigma_{[\text{Fe}/\text{H}]}^{(\text{meas})}$  of  $[\text{Fe}/\text{H}]$  of 0.05 dex, would be already

$$\sigma_{[\text{Fe}/\text{H}]}(0) = 0.055 \text{ dex} \quad . \quad (\text{A17})$$

A complete relation would be therefore

$$\begin{aligned} &\left( \sigma_{[\text{Fe}/\text{H}]}^2(\tau) \right. \\ &\left. - \left( \sigma_{[\text{Fe}/\text{H}]}^{(\text{mix})}(0) \right)^2 - \left( \sigma_{[\text{Fe}/\text{H}]}^{(\text{epi})}(0) \right)^2 - \left( \sigma_{[\text{Fe}/\text{H}]}^{(\text{meas})} \right)^2 \right)^{1/2} \end{aligned}$$

$$\begin{aligned} &= |\alpha| f_U \left( \sigma_U^2(\tau) - \sigma_U^2(0) \right)^{1/2} \\ &= |\alpha| f_v \left( \sigma_v^2(\tau) - \sigma_v^2(0) \right)^{1/2} \end{aligned} \quad (\text{A18})$$

Hence Eqs.(9)–(12) are only approximately valid. However, the errors are small, because all the stars in the Edvardsson et al. sample are older than  $1 \cdot 10^9$  years.

Up to now, we have identified the expectation values (e.g.  $\langle (\Delta R)^2 \rangle$ ) of an individual star with the ensemble average or even the average over a specially selected sample, such as stars now at  $R = R_0$ . This is allowed only if the properties of orbital diffusion are the same for all the stars, which is not strictly true in general. For example, already the radial dependence of the surface density of stars, of the diffusion coefficient  $D$ , and of the quantities  $A$ ,  $B$ ,  $\kappa$  introduce deviations from our simplified picture. One of such corrections has already been discussed in Sect. 4.4. The selection of a sample, such as choosing only stars which are now at  $R = R_0$ , introduces often certain biases into the results. Take the following unrealistic, but instructive example: Let us assume that all the stars have been born in a ‘star burst’ at the same galactocentric distance  $R_{i,\text{burst}}$ . Then we would observe at  $R = R_0$  only stars with a fixed value of  $R_i - R_0 = R_{i,\text{burst}} - R_0$ , which in general differs completely from the expectation value of  $\langle (\Delta R)^2 \rangle^{1/2}$  given by Eq.(A1). Also the dispersion  $\sigma_{[\text{Fe}/\text{H}]}$  would not follow Eq.(10), but would be simply equal to  $\sigma_{[\text{Fe}/\text{H}]}(0)$ . In general, an accurate prediction of  $\sigma_{[\text{Fe}/\text{H}]}$  needs a very detailed treatment which is beyond the scope of this paper. However, the results presented here are in most cases rather good approximations.

We would like to emphasize that the galactocentric distance  $R_m$  of the guiding center of an epicyclic orbit is strongly changed by the diffusion of stellar orbits. While  $\langle \Delta R_m \rangle$  is zero (Eq.(A4)), the dispersion  $\langle (\Delta R_m)^2 \rangle^{1/2}$  is rather large (Eq.(A5)). Grenon (1987) has assumed that  $R_m$  is approximately constant in time, and many authors (e.g. Edvardsson et al. 1993, 1994) have made strong use of this assumption. We do not see a justification for  $R_m$  being constant in time, if the diffusion of stellar orbits operates (see also Fuchs, Dettbarn, and Wielen 1996). On the contrary, our Eqs.(A5), (A8), and (A10) show that the effect of the diffusion is even stronger in  $R_m$  than in the epicyclic motion (characterized by  $\langle \frac{1}{2}(\Delta(a_R)^2) \rangle^{1/2}$ ). The example of a diffusion orbit, shown in Fig. 6 of Wielen (1977) and cited by Grenon (1987), is certainly not in conflict with our expectation values given by Eqs.(A5) and (A8). Can the present guiding center distance  $R_m(\tau)$  be used as a bias-free estimate of the initial galactocentric distance  $R_i$ , if nothing else (e.g. the metallicity of the star) is known?  $R_m$  can be used as an estimate of  $R_i$ , because of  $\langle \Delta R_m \rangle = 0$ , only if the selection of the sample does not introduce a bias. Such a bias occurs, however, already if we select nearby stars. It is already qualitatively clear that e.g. stars with  $R_i \ll R_0$  reach now  $R = R_0$  with a much higher probability, if their present values of  $R_m$  are larger than  $R_i$ . Hence the presently observed quantities of  $R_m(\tau)$  are strongly biased towards  $R_0$ . A rough calculation gives that  $\langle R_m - R_0 \rangle = 0.4 \langle R_i - R_0 \rangle$ , independent of  $\tau$ . Hence it is not allowed to identify an observed gradient  $\alpha' = \partial[\text{Fe}/\text{H}]/\partial R_m$  in the metallicity as a function of  $R_m$  of nearby

stars with the true radial gradient  $\alpha$ . In principle, however, it is possible to reconstruct the true gradient  $\alpha$  from  $\alpha'$ , e.g. by  $\alpha \sim 0.4 \alpha'$ . We have preferred not to use this method (e.g. by using the Edvardsson et al. sample which would indeed provide a value for  $\alpha'$ ), but to rely on the more directly determined values of  $\alpha$ , discussed in Sect. 2.1.

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