

Telescope Image Modeling (TIM)

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ABSTRACT. The Telescope Image Modeling (TIM) software has been designed to simulate a polychromatic image of a large number of point sources with known colors, as recorded on a pixellated detector after reimaging through an optical system of known characteristics. Though developed primarily for images produced by the *Hubble Space Telescope* and its scientific instruments, it is general enough for wider application. The software is modular and produces monochromatic point-spread functions and modulation-transfer functions at intermediate stages. Details of known optical aberrations of the system, microroughness scatter from the telescope mirrors, telescope pupil aberrations, pointing jitter, photon noise and detector defects can all be included.

1. INTRODUCTION

Simulation of the point-spread function (PSF) and stellar images as viewed on the detector of a telescope is of considerable importance in a variety of astronomical applications such as in planning observations, analysis and understanding of data, and development of instrument calibration procedures. In this paper we describe the theoretical models which form the basis of the Telescope Image Modeling (TIM) software that we developed with the *Hubble Space Telescope* (*HST*) in mind but which is general enough to have much wider applications. Techniques devised during the course of program development are described at relevant stages of the paper.

An overview of the *HST* Optical Telescope Assembly (OTA) and science instruments is given in Sec. 2, since several references will be made to it in what follows. Some basic definitions are given in Sec. 3. The model used for generating the point-spread function (PSF) is described in Sec. 4. The effect of mirror irregularities on the PSF is examined in Sec. 5.2 and that of dust on the primary mirror in Sec. 5.3. Effects of spacecraft jitter, Poisson noise, detector defects, and sources of diffraction are discussed in Sec. 8. Finally, in Sec. 10 we show some simulations of the *HST* point-spread function and images and summarize in Sec. 11.

A copy of the program is available from either the User Support Branch, Space Telescope Science Institute or via ftp from the Space Telescope Electronic Information System (STEIS). TIM is a VAX/VMS specific software and cannot be compiled on a Unix machine in its present form.

2. OVERVIEW OF *HST* OPTICAL TELESCOPE ASSEMBLY

The *HST* Optical Telescope Assembly (OTA) is a 2.4-m $f/24$ Ritchey Chrétien Cassegrain consisting of an $f/2.3$ hyperbolic primary mirror separated from a hyperbolic secondary by 4.9 m (Schroeder 1987). The primary mirror is supported by three circular pads which are 15 cm in diameter, while the secondary is supported by a four-legged spider

mount. The secondary-mirror baffle obscures the central one-third of the beam. This obscuration, together with those caused by the primary-mirror support pads and the secondary-mirror spiders when superimposed on a clear aperture representing the primary mirror define the OTA entrance pupil.

The on-axis portion of the focal plane is used by the Wide Field Planetary Camera (WFPC) which has now been replaced by the second-generation camera, WFPC2. The science instruments and the corrective optics device, COSTAR, which corrects for the *HST* spherical aberration, are located in the axial position connected to the OTA focal-plane structure. The axial instruments consist of the Faint Object Camera (FOC), the Goddard High Resolution Spectrograph (GHRS), and the Faint Object Spectrograph (FOS). The High-Speed Photometer (HSP) has now been removed and replaced with COSTAR.

3. BASIC DEFINITIONS

When an optical system forms an image, degradation of the image takes place because of design limitations and other factors such as diffraction, atmospheric seeing, pointing jitter, etc. Image quality is characterized by changes in amplitude and frequency distribution and modulation of the detected image, from the corresponding quantities in the object. In this section mathematical definitions of image quality measures as used in TIM are described. (An excellent review by Wetherell 1980 gives in-depth discussions of the main theoretical models used in the present paper. Standard texts such as those by Born and Wolf 1980 and Schroeder 1987 may also be consulted.)

Pupil Function. For unaberrated light the pupil function is real and is synonymous with the aperture function which specifies the ratio of the amplitude of the transmitted light to that of the incident light. In the software, parts of the pupil that are uniformly illuminated by unaberrated light are represented by ones, while obscured regions are represented by zeros. Aberrations of the light beam at a point (r, θ) manifest

themselves in terms of the optical-path difference (OPD), $F(r, \theta)$, between the incoming beam and the chief ray. The complex pupil function, $P(r, \theta)$, which includes the effects of aberrations is defined as

$$P(r, \theta) = A(r)O(r, \theta + \omega) \exp[ikF(r, \theta)], \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number for light with wavelength λ , $O(r, \theta + \omega)$ is the aperture function which takes into account obscurations in the pupil, and $A(r)$ is the apodization function

$$A(r) = \exp(-N^2 r^2/4). \quad (2)$$

Here N parametrizes the apodization required. For *HST*, $N=0$. In TIM the aperture function, $O(r, \theta + \omega)$, consists of a superposition of ellipses and rectangles which describe the obscurations in the pupil. The angle, ω , gives an overall rotation of apertures with respect to detector coordinates (so that the aberrations in F are in detector coordinates).

Amplitude-Spread Function (ASF). The amplitude of the diffracted light is given by the amplitude-spread function which is the Fourier transform of the pupil function. In TIM, the ASF is available as an intermediate product.

Point-Spread Function (PSF). The intensity distribution of the image of a point source is given by the PSF which is computed as the modulus squared of the ASF. The PSF is the chief output product of TIM and is available in several forms (Sec. 5.4).

Optical-Transfer Function (OTF). The inverse transform of the PSF gives the OTF. The OTF represents the degree of attenuation by the optical system of spatial frequencies in an object, when it forms an image. In TIM, image degradation caused by scattering from microroughness of the *HST* primary mirror, spacecraft jitter, and atmospheric seeing is modeled by appropriately modifying the OTF.

Modulation-Transfer Function (MTF). The real part of the OTF gives the MTF. The MTF is a measure of the change in contrast in an image compared with that in the object. In TIM, the MTF may be output as an intermediate product.

4. POINT-SPREAD FUNCTION

4.1 Diffraction-Limited PSF

In the absence of aberrations, the image of a monochromatic point source in the focal plane corresponds to a normalized intensity distribution in the sky of (Schroeder 1987)

$$\frac{1}{I} \frac{dI}{d\Omega} = \frac{\pi R^2}{\lambda^2(1-\epsilon^2)} \left| \frac{2J_1(x)}{x} - \epsilon^2 \frac{2J_1(\epsilon x)}{\epsilon x} \right|^2, \quad (3)$$

where λ is the wavelength, $x = 2\pi R \theta/\lambda$, θ is the field angle from the source, R is the pupil radius, ϵR the radius of a central obscuration in the pupil, and $J_1(x)$ is the Bessel function of the first kind. This expression gives the light distribution, or the point-spread function, for an optical instrument with diffraction-limited performance. The PSF is spherically symmetric and Eq. (3) gives a one-dimensional radial profile. The zeros of the Bessel function $J_1(x)$ correspond to the diameters of the Airy ring pattern for an unobscured aperture.

4.2 Aberrated PSF

The diffraction-limited PSF is modified by aberrations of the focal-plane image. For the *HST* these are caused by the OTA design specifications, spherical aberration of the primary mirror, secondary-mirror alignment errors, orbit-to-orbit variations, and mirror figure errors. These aberrations modify the wavefront and, for a circular aperture, manifest themselves as Zernike polynomial coefficients Z_i in an expansion of the optical path difference (OPD), $F(r, \theta)$, given at a point (r, θ) by (Born and Wolf 1980)

$$F(r, \theta) = \sum_i a_i Z_i(r, \theta). \quad (4)$$

The wavefront error at the exit pupil, together with the geometry thereof determined the PSF through the diffraction integral. Using the definitions in Sec. 3, the PSF is given by

$$\frac{1}{I} \frac{dI}{d\Omega} = |\mathcal{F}(O(r, \theta + \omega) \exp[2\pi i F(r, \theta)/\lambda])|^2, \quad (5)$$

where $\mathcal{F}$ represents the Fourier transform.

4.3 One-Dimensional PSF

Computation of a two-dimensional PSF is limited in TIM to grid sizes (1024×1024). For some applications, PSFs to larger radii are required. TIM offers the option of computing one-dimensional PSFs in which the only aberration that can be included is spherical. A one-dimensional PSF is computed as the modulus squared of the Hankel transform of the pupil function.

4.4 Asymptotic PSF

Using the asymptotic expression of the Bessel function, the asymptotic form of Eq. (3) becomes

$$\frac{1}{I} \frac{dI}{d\Omega} = \frac{4r^2}{\pi^3(1-\epsilon)\lambda^2 N^3}, \quad (6)$$

where $N = x/\pi = 2R\theta/\lambda$ is the number of Airy rings out to a given angle, θ .

In the presence of aberrations the normalized intensity distribution can be expressed asymptotically as

$$\begin{aligned} \frac{1}{I} \frac{dI}{d\Omega} = \frac{8R^2}{\lambda^2(1-\epsilon^2)x^3} & \left\{ \cos^2 \left[x - \frac{3\pi}{4} + F_A(1, \theta) \right] + \epsilon \cos^2 \left[\epsilon x - \frac{3\pi}{4} \right. \right. \\ & \left. \left. + F_A(\epsilon, \theta) \right] - 2\sqrt{\epsilon} \cos \left[x - \frac{3\pi}{4} + F_A(1, \theta) \right] \cos \left[\epsilon x - \frac{3\pi}{4} \right. \right. \\ & \left. \left. + F_A(\epsilon, \theta) \right] \cos[F_S(1, \theta) - F_S(\epsilon, \theta)] \right\}, \quad (7) \end{aligned}$$

where $F_{S(A)}$ are the symmetric and antisymmetric parts of the optical-path difference defined in Eq. (4). Because of the aberrations, the PSF is not spherically symmetric and Eq. (7) gives the radial profile along a direction θ .

4.5 Strehl Ratio

The intensity at the core of the PSF, in the presence of aberrations, is generally lower than the corresponding diffraction-limited value. In the plane of observation it is convenient to normalize the PSF with the diffraction-limited PSF [Eq. (3)] giving the Strehl intensity or Strehl ratio. This quantity is a useful “measure” of departure of the PSF from a diffraction-limited one.

5. IMAGE DEGRADATION

In addition to lens design aberrations represented by Zernike polynomials, further image degradation can occur from other causes such as scattering from irregularities or dust on the mirror surface, spacecraft jitter, detector limitations and, in the case of ground-based observations, atmospheric seeing. All these effects have been modeled in TIM.

5.1 Mirror Maps

Low-frequency wavefront errors caused by mirror-surface irregularities in the OTA primary and secondary mirrors are modeled in TIM as an additive component to the design aberrations. Surface figure maps extracted for the OTA primary mirror by Krist and Burrows (1994) are used for the purpose, while for the OTA secondary mirror, maps based on ground measurements have been provided by Hughes Danbury Optical Systems (HDOS). Primary-mirror maps produced by HDOS are coarsely sampled and do not give as good images as do the on orbit maps (Krist and Burrows 1994). For off-axis instruments, such as the FOC, the beam offset on the mirrors has been accounted for in TIM. When simulating non-*HST* images, the user may replace the OTA mirror maps with those specific to the telescope being modeled.

5.2 Effect of Mirror Microroughness

An effect which is more important in the UV than in the visible is the scattering caused by irregularities in the primary mirror. Despite the fact that the *HST* primary is of remarkably good optical quality with a total rms surface error of only $\lambda/60$ at 633 nm, microroughness can cause scattering of light into the wings of the PSF. This effect can be modeled in several different ways depending on the frequency range under consideration. We will discuss four different methods, all of which are available as options in the TIM software. The effect on the PSF of each model is shown in Fig. 2 for four different wavelengths.

5.2.1 Autocorrelation Method

Image degradation caused by scattering from mirror microroughness is treated as a random process and modeled as a modification of the OTF, which is multiplied by a frequency-dependent degradation function, $M(\nu)$. In the mid- and high-frequency range, a statistical analysis (O'Neill 1963) results in the following Gaussian form for $M(\nu)$:

$$M(\nu) = \exp\{-k^2\sigma^2[1 - C(\nu)]\}, \quad (8)$$

where ν is the frequency, k the wave number, σ the rms wavefront error and $C(\nu)$ is the mirror-surface autocorrelation function. We assume that $C(\nu)$ has the following form:

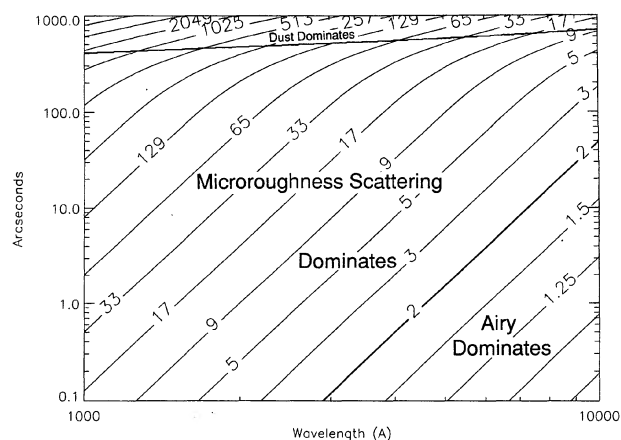


FIG. 1—Contour plots showing the variation of the ratio of the *HST* PSF to the diffraction-limited PSF as a function of wavelength and distance from the center of the PSF. Regions where mirror microroughness scattering and dust scattering dominate over Airy diffraction are demarcated by thick lines. Microroughness scattering dominates at shorter wavelengths for all angles, while for longer wavelengths it is important only in the wings of the PSF. The effect of dust is to increase the overall intensity in the extreme wings beyond ~ 400 arcsec.

tion function. We assume that $C(\nu)$ has the following form:

$$C(\nu) = \begin{cases} 1 - (\nu/\nu_{\text{cut}})^{1.5} & \text{if } \nu < \nu_{\text{cut}} \\ 0 & \text{if } \nu > \nu_{\text{cut}} \end{cases}, \quad (9)$$

where ν_{cut} is the cutoff frequency. For mid frequencies $\nu_{\text{cut}} = 1/18$, while for high frequencies $\nu_{\text{cut}} = 1/360$. In TIM there is an option to model mid, high, or both frequency ranges.

This approach works well for small angles but fails for large angles. The form chosen for the autocorrelation function contains a discontinuity in its derivatives, and this leads to unphysical oscillations in the corresponding power spectrum at high frequencies. These are reflected as ripples at large distances in the PSF. Indeed, as the associated power spectrum is not constrained to be positive definite, and contains negative components, the PSF in the far wings can even appear negative.

We report this method here because of its simplicity but do not recommend its use. Other methods which better model the mirror microroughness are described below.

5.2.2 Modified Autocorrelation Method

The autocorrelation function, $C(\nu)$, has been modeled by Kenyon and Deurig in the following form (Burrows 1990):

$$C(\nu) = \cos(b\nu D) \exp(-c\nu D), \quad (10)$$

where b and c are parameters and D the diameter of the primary mirror.

The advantage of this method is that it can be analytically transformed to obtain the power spectrum. The parameters b and c can then be obtained from metrology data, which is available for *HST* in three bandpasses. The PSF obtained here is positive. At very large angles the method breaks down. It also fails to give a full description if the bandpasses

of the different measurements do not overlap.

5.2.3 Microstructure Mirror Modeling

Yet another model which works well at large angles but not so well at small ones is the power spectrum method (Brown and Burrows 1990), in which a power law is fitted to the measured power spectrum. In this approach the normalized intensity, $1/I \, dI/d\omega$, is approximated by

$$\frac{1}{I} \frac{dI}{d\omega} = \exp(-k^2 \sigma^2) \text{PSF}_0 + \frac{16\pi^2}{\lambda^4} G(2\pi\theta/\lambda), \quad (11)$$

where PSF_0 is the PSF without microroughness effects, λ is the wavelength and $k=2\pi/\lambda$ the wave number of the light, σ the rms wavefront error, and $G(K)$ the power spectral density for the mirror microstructure surface errors (not wavefront errors). It is represented as

$$G(K) = \alpha K^{-\beta}, \quad (12)$$

where β is the power law exponent and α is related to the rms microroughness δ_e^2 through the relation

$$\delta_e^2 = \frac{\alpha}{2\pi} \int_{K_1}^{K_2} K^{-\beta+1} dK, \quad (13)$$

or,

$$\alpha = \frac{(2\pi)^{\beta-1}(\beta-2)\delta_e^2}{d_1^{\beta-2} - d_2^{\beta-2}}, \quad (14)$$

where $d_i \equiv 2\pi/K_i$. The total integrated scattering (TIS) is then given by

$$\text{TIS} = \frac{8\pi\alpha}{\lambda^2(\beta-2)} \frac{2\pi}{d_{\text{lim}}^{2-\beta}}, \quad (15)$$

where d_{lim} corresponds to the smallest scattering angle θ_{min} used in the simulation. The new PSF is given by

$$(\text{PSF})_{\text{new}} = (1 - \text{TIS})\text{PSF}_0 + \left| \frac{1}{I} \frac{dI}{d\omega} \right|_{\theta > \theta_{\text{min}}}. \quad (16)$$

The parameters β and δ_e have been obtained by an on-orbit test to measure large-angle microroughness scattering (Hasan and Burrows 1993). We obtain $\beta=2.5$ and $\delta_e=0.0096$ waves at 633 nm in the bandpass $K_1 > 0.5 \text{ cm}^{-1}$ or, $d < 120$ mm corresponding to scattering angles beyond the 20th ring. The details of the scattering beyond the 1000th ring or so are uncertain.

This model is well behaved at large angles. However, within the first 20 Airy rings, the fit to the power spectrum breaks down. It does not correspond to any particular function in transform space because of a singularity at the origin.

5.2.4 Fractal Surface Finish Mirror Modeling

The central portion of the PSF can be modeled with better accuracy using the autocorrelation method (Secs. 5.2.1 and 5.2.2) while large angles are well modeled by the power spectrum method (Sec. 5.2.3). The two approaches are neatly combined by treating the mirror as composed of fractals (Church 1988) and comparing structure functions (SF) which

are defined for normal and fractal surfaces. For normal surfaces a structure function, $F(\nu D)$, is defined as

$$F = \frac{1}{2} \sigma^2 [1 - C(\nu)]. \quad (17)$$

For fractal surfaces, $F(\nu D)$ can be expressed in terms of the topothesy, T (which is the distance over which the chord between two surface points has an rms slope of unity) by the formula

$$F = T^2 |\nu D/T|^{n-1}. \quad (18)$$

The parameters n and T can be related to the microroughness parameters α and β by comparing the expression of the power spectrum, $G(K)$, in terms of topothesy

$$G(K) = -\frac{\Gamma[(n+1)/2]}{2\Gamma(1/2)\Gamma(n/2)} \frac{2\Gamma(n)\cos(n\pi/2)}{(2\pi)^n K^{-(n+1)}}, \quad (19)$$

with its power-law form [Eq. (12)]. The resulting relationships are

$$n = \beta - 1, \quad (20)$$

and

$$T^{4-\beta} = \frac{-\Gamma(1/2)\Gamma[(\beta-1)/2]\alpha}{2\pi\Gamma(\beta-1)\Gamma(\beta/2)\cos[(\beta-1)\pi/2]}. \quad (21)$$

Multiplying the MTF by the degradation function $M(\nu)$, given by

$$M(\nu) = \exp[-2k^2 F(\nu D)], \quad (22)$$

and taking its inverse Fourier transform then gives the modified PSF.

The PSF obtained by this technique has the accuracy of the autocorrelation method for small angles and of the power-spectrum method at large angles. It is, therefore, the default technique offered in TIM for microstructure mirror modeling.

5.3 Effect of Dust on Primary

As part of the *HST* integration testing, photographs of the primary were taken and scanned to indicate the dust contamination. The mirror is cleaner than expected with about 1% coverage by particles bigger than $a_{\text{min}} = 10 \text{ }\mu\text{m}$ radius (Burrows 1990). A conservative upper limit based on an extrapolation of the observed power law to the full mirror is $a_{\text{max}} = 500 \text{ }\mu\text{m}$. The particles have a size distribution that falls as $a^{-4.5}$. Each dust particle scatters to large angles (several degrees) or absorbs all light that falls on it. This affects the throughput of the telescope but is negligible in its effect on the PSF. More significant is the diffractive scattering caused by the many tiny obscurations in the pupil. The scattering amplitude is essentially constant out to the half-width of the diffraction pattern for the largest particle. Measured in Airy rings this is half the ratio of the primary-mirror radius to that of the largest particle. In other words the scattering amplitude is essentially constant out to about the 2400th Airy ring. For one particle the proportion of the incident flux scattered is $\pi a^2/A$ for the entrance pupil of area A . The Bessel function $J_1(x)$ tends to $x/2$ for small x . Hence integrating over all sizes with $n(a)da$ per unit area

$$\frac{1}{I} \frac{dI}{d\Omega} = \int_{a_{\min}}^{a_{\max}} \frac{\pi a^2}{A} \frac{\pi a^2}{\lambda^2} A n(a) da. \quad (23)$$

The fractional area covered serves to normalize $n(a)$. Choosing a power law, $n(a) = q a^{-\xi}$, to describe the dust distribution, and evaluating Eq. (23), the PSF becomes

$$\left(\frac{1}{I} \frac{dI}{d\Omega} \right)_{\text{dust}} = (1 - .02f) \left(\frac{1}{I} \frac{dI}{d\Omega} \right) + P_0, \quad (24)$$

where f is the percentage covering factor and P_0 is given by

$$P_0 = \frac{\pi f}{\lambda^2} \frac{3 - \xi}{5 - \xi} \frac{a_{\max}^{5-\xi} - a_{\min}^{5-\xi}}{a_{\max}^{3-\xi} - a_{\min}^{3-\xi}}. \quad (25)$$

Here ξ is the dust power-law exponent, $a_{\max}$ is the radius of the largest dust particle, and $a_{\min}$ is the radius of the smallest dust particle. The effect of dust is significant in the far wings of the PSF, but is small elsewhere.

5.4 Asymptotic PSF with Mirror and Dust Effects

The effect of mirror microroughness and dust on a diffraction limited PSF can be expressed asymptotically by putting together the results of the previous sections [Eqs. (6), (11), and (23)]. For *HST*, we get

$$\frac{1}{I} \frac{dI}{d\Omega} = \frac{6.52}{\lambda^2 N^3} + \frac{0.0735}{\lambda^4 N^{2.5}} + \frac{1.3510^{-9}}{\lambda^2}, \quad (26)$$

for λ in μm . The three terms are equal at $\lambda = 1.35 \mu\text{m}$ and Airy ring number $N = 2R\theta/\lambda = 1690$ corresponding to $\theta = 194$ arcsec. Figure 1 shows the regions of the (λ, θ) plane in which each term dominates, together with indications of the range of applicability of the above formula.

5.5 Jitter

If observations occurred instantaneously or the spacecraft were rigidly fixed in space the resulting image would be maximally sharp. However, this is not the case, and some account needs to be taken of the movement of the optical axis. The only jitter considered is rigid movements of the field of view, caused, for example, by movement of the spacecraft as a whole. Image distortions caused by vibrations of the primary for example will be very small. The jitter is expected to have Gaussian statistics, and estimates of its magnitude are available. Degradation of the OTF by jitter is modeled as

$$(\text{OTF})_{\text{new}} = (\text{OTF})_{\text{old}} \exp[-2(\pi f \sigma_{\text{jit}})^2], \quad (27)$$

where f is the focal length in meters, σ_{jit} is the rms jitter about one axis in radians, and ν is the spatial frequency (ranging from 0 to $1/\lambda F$). This OTF is then used to compute the new PSF and/or MTF.

5.6 Seeing Modeling

An option to model atmospheric seeing is offered in TIM, though it has no relevance to *HST* observations. A statistical model used and the effect on the OTF is as follows:

$$(\text{OTF})_{\text{new}} = (\text{OTF})_{\text{old}} \exp[-3.44(Dr/\rho)^{5/3}], \quad (28)$$

where

$$\rho = \rho_0 (\lambda/\lambda_{\text{ref}})^{6/5}, \quad (29)$$

λ_{ref} being a reference wavelength and ρ_0 the Fried parameter. This is the same functional form (with a different slope) as the mirror power-spectrum model.

6. RESAMPLING AND PIXEL INTEGRATION

The discussion so far deals with the simulation of a PSF critically sampled at the Nyquist frequency. In other words, the spacing between the grid points in the 2D PSF array is $F\lambda/2$. The normalization of the PSF is such that we obtain the power per square arcsec. The TIM user may wish to simulate the PSF on a finer grid in order to study its properties on a finer scale, or to integrate it over detector-sized pixels to produce a realistic PSF. Options are provided within TIM to achieve either of these objectives.

6.1 Resampling

Since the PSF is a band-limited function, the sampling theorem may be used to resample it at desired grid spacings. Let (x', y') be the coordinate system of the resampled PSF, $P'(x', y')$, centered at (x'_c, y'_c) on a $(M1 \times M2)$ grid with the points spaced by $(\Delta x', \Delta y')$. The origin of the coordinate systems is at the center of the lower left-hand-corner pixel of the corresponding grid. Denoting the critically sampled PSF by $P(x, y)$ and its center on an $(N1 \times N2)$ grid by (x_0, y_0) , we get the interpolation formula

$$P'(x'_k, y'_l) = \sum_{ij} P(x_i, y_j) \text{sinc} \{ \pi [y_j - (y_0 - y_{c0}) - (y'_l - y'_c)] / \Delta y \} \text{sinc} \{ \pi [x_i - (x_0 - x_{c0}) - (x'_k - x'_c)] / \Delta x \}. \quad (30)$$

Here (x_{c0}, y_{c0}) are the coordinates of the center of the resampled PSF with respect to (x_0, y_0) , and $\Delta x, \Delta y$ are Nyquist intervals in the units of (x, y) . The subscripts (i, j) and (l, m) refer to grid points in the $(N1 \times N2)$ and $(M1 \times M2)$ grid, respectively. A trivial case is obtained by taking $x_{c0} = y_{c0} = 0$, $\Delta x = \Delta x'$, $\Delta y = \Delta y'$, $M1 = N1$, $M2 = N2$. In this case the primed coordinates are the same as the unprimed and the original PSF is reproduced.

6.2 Pixel Integration

A realistic PSF on detector-sized pixels is obtained by integrating the intensity over the area of each pixel. Three integration options are offered in TIM, each of which are outlined below. The *sinc interpolation method* is the most accurate, but takes more computation time. Techniques for speeding up the integration have been developed and are described. The *OTF convolution* method is less accurate but faster. This method when used with bilinear interpolation is faster, though less accurate.

6.2.1 Sinc Interpolation Method

The default method provided in TIM performs a numerical integration over each detector pixel, computing the integrand with the sinc interpolation, Eq. (30). Thus, if the pixel has dimensions D_x, D_y along x and y , respectively, the integrated PSF is given by

$$P_{\text{int}}(x'_k, y'_l) = \int_{-D_x/2}^{D_x/2} \int_{-D_y/2}^{D_y/2} P'[(x'_k + x'), (y'_l + y')] dx' dy', \quad (31)$$

As before, the separation of the output grid points are denoted by $\Delta x', \Delta y'$. By setting $\Delta x' = D_x, \Delta y' = D_y$, values of the integrated PSF at the center of each detector pixel are obtained. A finer sampling, e.g., on a $(p \times q)$ grid on a single pixel, is obtained by setting $\Delta x' = D_x/p, \Delta y' = D_y/q$.

The pixel integration, Eq. (31), involves two integrations and two summations. The computation time is considerably reduced by employing the following technique. First, the integration and summation orders are changed. Second, the limits of the summation over i, j in Eq. (30) are reduced by exploiting the fact that both the sinc function and the PSF are rapidly decaying peaked functions thus causing rapid convergence of the summation. For each summation, first the position of the peak in the sinc function is identified. A summation block of n units is then defined with the peak position as the center of the block. The value of the sum within this block is then compared with the sum in adjacent blocks on each side till convergence is obtained. A convergence factor is defined as the ratio of the current sum to the total so far. When it attains the desired accuracy the summation is stopped. The block size and convergence factor determine both the accuracy and speed of the computation. For most applications, a block size of 6 and convergence factor of 0.01 has been found to be adequate.

6.2.2 OTF Convolution Method

The effect of pixel integration on the PSF is incorporated as a modification of the system OTF. Each pixel is considered ideal in the sense that it is independent of the neighboring pixels. The pixel OTF is given by

$$(\text{OTF})_{\text{pixel}}(x_i, y_i) = \text{sinc}(2\pi\nu_x D_x / F\lambda), \quad (32)$$

ν_x, ν_y being frequencies along x and y , respectively. The product of the system OTF and the pixel OTF is used to get a critically sampled PSF integrated over detector pixels. The spacing of the integrated PSF can be changed by using either a *sinc interpolation* or a *bilinear interpolation*.

7. DETECTOR INTEGRATION

The image of a point source on an ideal pixellated detector is obtained by summing contributions from different wavelengths after these have been normalized for the stellar magnitude, observation time, optical throughput, and detector efficiency. Each star is treated separately in this manner, the contributions selected for each pixel depending suitably on the registration of the star center with respect to the pixel boundaries.

The intensity map, P_{map} , for a single star is given in terms of monochromatic integrated PSFs, $P_{\text{int}}(\lambda)$, as

$$P_{\text{map}} = T_{\text{ref}} \sum_{\lambda} P_{\text{int}}(\lambda) K_{\lambda} 10^{-0.4M} W_{\lambda}, \quad (33)$$

where T_{ref} is a reference observation time, M is the magnitude of the star, W_{λ} models the flat-field variations of the detector chip and K_{λ} are the number of counts per second for an $M=0$ star for a wavelength set $d\lambda$ centered at λ .

The value of K_{λ} depends on the star type and the throughput at wavelength λ of the filter used. An interpolated value is computed for K_{λ} , using the following procedure. First, the color-magnitude difference of the star in the star field is compared with those of a chosen set of star types (we choose representative O, B, A, F, G, K, M type stars from the Bruzual Atlas and the two between which it falls are identified). By specifying the wavelength and the filter, the throughput, $K_{\lambda 1}$ and $K_{\lambda 2}$, for these two limiting stars is obtained using some of the subroutines from XCAL, the Synthetic Photometry Subroutine Library developed by the STScI's Calibration Data Group. Let M, M_1, M_2 be the magnitudes of the given star and the two limiting stars respectively, and $\Delta M_0, \Delta M_1, \Delta M_2$, their color-magnitude differences. The value of K_{λ} is computed as

$$K_{\lambda} = \frac{K_{\lambda 1}(A_0 - A_2) + K_{\lambda 2}(A_1 - A_0)}{A_1 - A_2}, \quad (34)$$

$$A_i = 10^{-0.4\Delta M_i}. \quad (35)$$

This value of K_{λ} is used in Eq. (33).

For WFPC, flat-field variations are modeled as a combination of low- and high-frequency effects as follows for a wavelength λ close to a reference λ_0 :

$$W_{\lambda}(i, j) = \left[1 + L(i, j) * \left(\frac{\lambda}{\lambda_0} \right)^a \right] * \left[1 + H(i, j) * \left(\frac{\lambda}{\lambda_0} \right)^b \right], \quad (36)$$

where $L(i, j)$ and $H(i, j)$ model the low- and high-frequency effects, respectively, for a pixel (i, j) and the exponents a and b are parameters. The effects are separately switchable and determined as

$$L(i, j) = a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2, \quad (37)$$

where x and y scale the coordinate on the CCD to be in the range $[-1, +1]$. This is a deterministic function that models primarily the effects of varying chip thickness. The default is $a_1 = a_2 = a_4 = 0$ and $a_3 = a_5 = -0.05$. This gives a 5% downturn at the edge centers, increasing quadratically with distance from the CCD center. The effect is assumed to get worse linearly with wavelength hence the default value for the exponent a is 1.

$H(i, j)$ is assumed uncorrelated and Gaussian distributed from pixel to pixel. It thus depends on the seeds used to start the random number generator, the mean M and the standard deviation S . Having a possibility to input M nonzero allows us to introduce simply variations in throughput from nominal as a function of wavelength. However, the default values will be $M=0.0$ and $S=0.01$, i.e., 1% rms random fluctuations. The effect is assumed predominantly wavelength insensitive and therefore the default value for the exponent b is 0.

If the star is located exactly at one of the $(n \times m)$ points at which the integrated PSF has been computed and if n, m are even, the value of P_{map} at the center of each detector pixel is easily computed from Eq. (33) by a direct mapping of the grid of integrated PSFs onto the detector pixels. However, if the star is off-centered, the value of P_{int} used in Eq. (33) is a linear interpolation of the two values closest to the center of each detector pixel. However, the choice of the PSF grid size ($M1 \times M2$) and the step sizes $\Delta x', \Delta y'$, is limited by the fact that the maximum array size the program can take is (1024×1024) . For computing efficiency it is desirable to keep array sizes to a minimum. We choose the values for $M1, M2, \Delta x', \Delta y'$ corresponding to the brightest star observed.

For fainter stars only a subset ($J1 \times J2$) of the $(M1 \times M2)$ values of P_{int} need be used. The following criterion is used to compute $J1, J2$. For each wavelength a search is made over the grid of PSFs and $J1(M, \lambda), J2(M, \lambda)$ are taken to be the coordinates of the PSF corresponding to the largest value of r at which the PSF is greater than

$$P_{\text{min}}(m, \lambda) = C_{\text{cut}} / (10^{-0.4M} TK_{\lambda}). \quad (38)$$

This value of P_{min} would give C_{cut} counts for an $M=0$ star. The value K_{λ}^{ref} for an F type star (from the representative set) is used in Eq. (38) to generate a pixel table for a specified set of M values for each wavelength. For a given star its “effective” M, M_{eff} , is computed as the minimum of the “effective” M values of the two stars it falls between in the representative set. The “effective” M value for each star is defined as

$$M_{\text{eff}}^i = M_i - [\log_{10}(K_{\lambda i} / K_{\lambda}^{\text{ref}})] / 0.4, \quad (39)$$

where $i=1,2$. The subarray dimensions $J1, J2$ are taken to be the maximum of the values $J1(M_{\text{eff}}, \lambda), J2(M_{\text{eff}}, \lambda)$ respectively.

One more question needs to be addressed and that is how to choose the set of wavelengths. The range of values is dictated by the choice of the filter. The present version of the software limits us to twenty-six values of wavelengths. One may choose them to be equally spaced though this may not necessarily be the ideal choice.

8. DETECTOR CHARACTERISTICS

The detector can suffer from various defects such as saturation, charge-transfer problems, wavelength-dependent flat-field variations, and so on. Representative types of CCDs modeled in TIM are the WFPC CCDs which suffer from a number of these defects (Griffiths 1990). TIM can be modified to model other CCDs. For WFPC, the average CCD readout noise, in the absence of signal shot noise or interference, is 13 electrons rms. A cosmic-ray background of ~ 2 events s^{-1} per (1600×1600) frame causes random streaks in the image. The CCDs in the WFPC suffer from a low-level nonlinearity called “deferred charge” which is basically caused by a low-level charge being poorly transferred, resulting in tails behind faint images. Blooming up and down the CCD column occurs when more than about 80,000 electrons

collect in any pixel. When the adjacent pixels up and down the column are full, the charge will flow into the next pixels along the column, and so on.

The following effects are modeled in TIM in the order that they are documented here.

8.1 Field-Angle Dependence

Field-angle dependence may be incorporated by starting with a set of simulated images at specified field angles and then combining these images so that in the final image each pixel becomes a linear combination of the pixel values from each field position according to some interpolation scheme.

Three options are available.

(1) No field-angle dependence.

(2) An azimuthally symmetric option is activated when the field angles are all non-negative and increasing with the field-angle number for the first (x) component, and zero for the second (y) component. Then a spline interpolation is used to provide the image at any field angle from the center of the image.

(3) The third option is for the field angles to be arbitrary coordinates in the image plane, in which case the weight assigned to each field position in the interpolation is inversely proportional to the cube of its distance from the target-field point.

8.2 Background

The background B is computed as

$$B = 10^{-0.4M_{\text{back}}} T D_x D_y \sum_{\lambda} K_{\lambda}^{\text{back}} W_{\lambda_{\text{eff}}}, \quad (40)$$

where $K_{\lambda}^{\text{back}}$ are the counts per second for an $M=0$ star of the M type, M_{back} is the magnitude of the background, T the observation time and $W_{\lambda_{\text{eff}}}$ the flat-field variations at an “effective” wavelength λ_{eff} which depends on the filter used.

The PSF including the background is given as

$$\text{PSP} = P_{\text{map}} \frac{T}{T_{\text{ref}}} + B. \quad (41)$$

8.3 Dark Current

Dark current is assumed constant over the entire CCD. The effects of hot pixels are treated separately. Thus a single dark count rate C_{dark} is multiplied by the integration time in seconds and added to all pixels:

$$\text{PSF} = P_{\text{map}} \frac{T}{T_{\text{ref}}} + B + C_{\text{dark}} T. \quad (42)$$

8.4 Cosmic Rays

The cosmic-ray background is simulated as a series of cosmic-ray events occurring at random pixels on the detector. The cosmic-ray particles deposit energy as a constant fraction of path length as they pass through each pixel which is modeled as a box. The incoming particle is given a random velocity and position on a pixel. The transit time and distance traveled through the pixel and the position of the next

pixel hit is computed and the energy deposited is deduced. We use the software of J. Mac Kenty to model the cosmic rays. Pixel dimensions and energy-loss rate are provided as user inputs to the program.

8.5 Bad Pixels

There are three types of bad pixels, each type maintained as a separate bad pixel list and individually controlled. Each pixel is stored as coordinates (i, j) together with a parameter $(C_{\text{hot}}, C_{\text{cold}}, C_{\text{cte}})$ explained below.

Hot pixels have a high dark rate. For each, we have a spurious extra count rate C_{hot} per unit time and coordinates (i, j) . For each *hot* pixel we have

$$\text{PSF} = P_{\text{map}} = \frac{T}{T_{\text{ref}}} + B + (C_{\text{dark}} + C_{\text{hot}})T. \quad (43)$$

Cold pixels or dropouts have a very low QE relative to the expected level (it may be zero). For each *cold* pixel we have

$$\text{PSF} = \left(P_{\text{map}} \frac{T}{T_{\text{ref}}} + B + C_{\text{dark}}T \right) C_{\text{cold}}. \quad (44)$$

Some pixels transfer charge very badly (maybe not at all). Then all pixels above them are effectively degraded. The PSF for all j bigger than or equal to the y coordinate of the bad pixel is given by

$$\text{PSF} = \left(P_{\text{map}} \frac{T}{T_{\text{ref}}} + B + C_{\text{dark}}T \right) C_{\text{cte}}. \quad (45)$$

8.6 Pixel Saturation

If any pixel (i, j) is greater than a saturation level S , the difference is allocated to its four nearest neighbors.

8.6.1 Charge-Transfer Efficiency

The charge-transfer efficiency E_{cte} is a single parameter that defaults to 0.99999. It reflects the inefficiency of transferring charge down columns. The readout process is to be modeled in detail, so the effect when each row is read out will be to add $(1 - E_{\text{cte}})$ times each remaining pixel in the CCD to its immediate neighbor up the column, while multiplying it by CTE. We start at the top of the CCD in order to avoid transferring transferred charge.

8.7 2×2 On-Chip Summation

If this option is chosen, the array is resummed in 2×2 blocks, thus halving its dimensions.

8.8 Poisson Noise

The expected number of electrons going to the ADU is not an integer. It is converted to a statistical number by multiplying the PSF with a Poisson distribution function.

8.9 Bias

Bias is added to all the pixels as an additive constant.

8.10 Readout Noise

The readout noise is possibly different for odd and even column numbers. It is modeled as additive integer Gaussian noise.

8.11 Analog-to-Digital Conversion

Input is a number of detected photons (electrons) for each pixel, $I(i, j)$. Output is the corresponding A/D integer output, which is obtained by taking the nearest integer to $I(i, j)/G$ where G is the gain.

8.12 Analog-to-Digital Errors

The Wide Field and Planetary Camera CCDs suffer from a problem in the analog-to-digital converters. We use the routine of T. R. Lauer to simulate the effects of A/D conversion errors in data obtained with the WFPC (Lauer 1989). The routine will work for any CCD that suffers from this problem. The WFPC2 detectors are free from this defect. The basic algorithm is a direct software duplication of the A/D conversion process. The pixels input are assumed to be analog signals having a smooth continuous distribution. The A/D converter translates these into output integer DN by successively comparing the input signal with a decremting power of two series of voltages. At any stage if the input signal is greater than the comparing voltage, the appropriate bit is set in the output DN, and the reference voltage is subtracted from the signal before the next stage. The WFPC produces 12 bit DN, so the first bit tested is the 2048 bit. The 1024 bit is then tested and so on.

The errors in the WFPC digitization process appear to occur during the reference voltage comparison, but not during the subsequent subtraction of the reference from the signal once the (possibly erroneous) bit has been set. More specifically, the error model assumes that a bit dependent error in the reference voltage occurs during the comparison that will cause improper setting of the corresponding bit in the output DN for signals within “error value” of the reference voltage. Once the bit decision is made, however, the actual voltage subtracted from the signal for subsequent bit tests is free from error (even though the decision to subtract it or not may be erroneous).

Errors in the reference voltages appear to be constant for a given WFPC CCD at a given electronic bay temperature.

9. TIM AND HST IMAGES

The TIM software has general applicability and can be used to simulate PSFs for any optical telescope the design aberrations of which are known in terms of Zernike coefficients, and the pupil map of which can be described in a single plane as clear and obscured apertures. There are several ways of determining Zernike coefficients. For example, they may be obtained by ray tracing, as is done in commercial software packages such as CODEV; they can be extracted directly from observed images by fitting them to model PSFs (e.g., Krist and Burrows 1994; Hasan et al. 1993). For estimating the effects of specific aberrations on the PSF, values of the Zernike coefficients corresponding to

the aberrations can be computed analytically and used as inputs to TIM. Other image-degradation effects can be modeled as desired by the user.

For *HST* there are some additional known effects which are modeled in TIM. For convenience of the *HST* user, known aberrations for each of the *HST* science instruments are available as defaults. The effect of OTA desorption on focus as a function of time is included (Hasan et al. 1993). As mentioned in Sec. 5.1 low-frequency aberrations caused by mirror-surface irregularities are available as mirror maps and can be included if desired. For WFPC and WFPC2, there are additional obscurations which cause a field dependence of the PSF. The diffraction effects of these obscurations have been treated as though they are in the same plane as the OTA obscurations. For *HST*, multiplane diffraction effects are small.

The regions of dominance of microroughness scattering and dust on the OTA primary have been estimated and are plotted in Fig. 1. A contour plot of the ratio of *HST* PSF and diffraction-limited PSF is plotted as a function of wavelength and angle. Airy diffraction dominates for longer wavelengths and smaller angles, the effect of microroughness scattering and dust being small. For shorter wavelengths microroughness scattering is the dominant effect till ~ 400 arcsec, after which dust takes over. The effect of dust scattering on the PSF is to fill in the Airy rings and cause an overall rise in intensity in the wings, while at the same time removing power from the center of the PSF. When planning observations, such as searching for planets or observing circumstellar disks, the effect of microroughness scattering should be taken into account. Dust is not a problem for most applications. A judicious choice of wavelengths and filters is essential for the success of such studies.

10. SIMULATION OF *HST* IMAGES

As a demonstration of the TIM software package and its applicability to *HST* imaging simulations are performed which show the effect of mirror microroughness scattering, spacecraft jitter, and polychromatic effects on the PSF.

10.1 Mirror Microroughness Scattering

The main effect of the microroughness scattering is to smooth out the diffraction rings in the PSF and transfer power into the wings of the PSF. The effect is stronger in the UV than in the red. Four models were discussed in Sec. 5.2.3 which would simulate the effect of mid- and high-frequency errors caused by mirror-surface irregularities. The contribution of the scattering to the OTA PSF predicted by each model at four different wavelengths is shown in Fig. 2, where a one-dimensional cut of the PSF is plotted. The first line in the figure shows a diffracted-limited PSF with a well-defined Airy ring pattern. Fractal-mirror modeling, which is applicable in all angular regions, causes a filling in of the Airy rings. The effect is most pronounced at 100 nm, where the rings disappear. The power-law model does not affect the inner regions of the PSF, but gives a good representation of the scattering at larger angles. The two autocorrelation models do not fill in the Airy pattern to the extent that the fractal

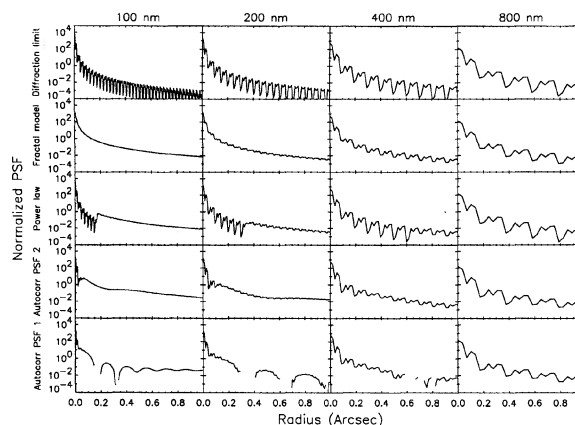


FIG. 2—Montage showing the effect of mirror microroughness scattering on the *HST* PSF predicted by the four models discussed in the text for four different wavelengths. Horizontal cuts of the PSF are plotted. The effect of the scattering is to fill in the Airy rings and smoothen the PSF. For all models, the effect is more pronounced at shorter wavelengths. The fractal model gives a good representation of the scattering over all regions of the PSF, while the power law is good only in the wings. The simple autocorrelation method (1) does not give a good description of the scattering and can lead to unphysical negative values. The modified autocorrelation method (2) is a better approximation, though not as good as the fractal method since it gives a discontinuity between the inner and outer regions of the PSF, particularly at short wavelengths.

and power-law models do, though the modified autocorrelation model gives smoother PSFs than the simple autocorrelation model.

The power-law model has been used to analyze an observatory level test conducted with *HST* to measure the microroughness scattering at large angles (Hasan and Burrows 1993). Values of the parameters α and β obtained from the test showed that microroughness scattering from the OTA primary mirror is smaller than estimated from ground measurements (Brown and Burrows 1990).

10.2 Spacecraft Jitter

Spacecraft pointing errors cause a blurring of the image, which again causes a smoothing of the PSF. Simulations are

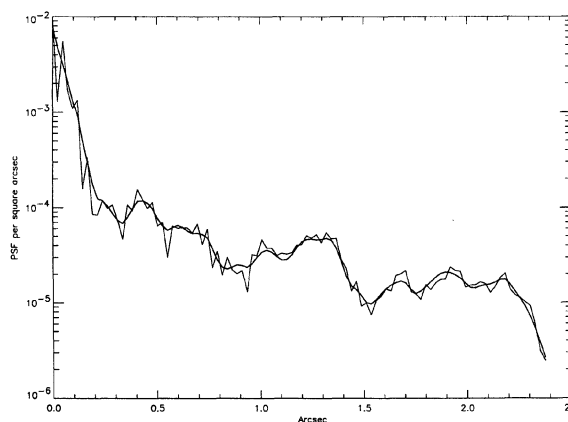


FIG. 3—Horizontal cuts of a FOC F/96 PSF at 300 nm without (thin solid line) and with (thick solid line) 20 mas jitter. The effect of jitter is to blur the PSF by filling in the minima and smoothing the PSF.

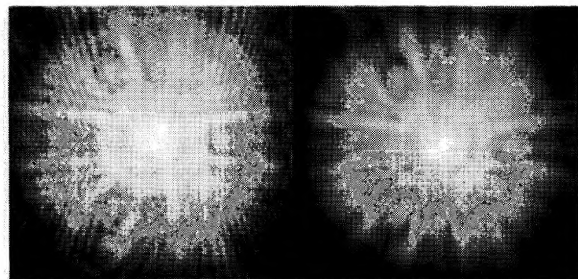


FIG. 4—Simulated PC6 PSFs showing the blurring effect of the overlap of wavelengths passed through filter F555W. The image at the left is a monochromatic image at 555 nm while the image at the right is a polychromatic image.

made of FOC *F*/96 images without and with 20 mas jitter. Prerefurbishment wavefront errors as well as mirror maps are included in the simulation. A horizontal cut of the PSF plotted in Fig. 3, clearly shows the smoothing effect of jitter.

10.3 Polychromatic Effect

Another interesting effect which is well demonstrated by simulations is that of the overlap of different wavelengths that pass through a filter. Simulations are performed of a PC6 image in the center of a chip through the filter F555W. Prerefurbishment wavefront errors, mirror maps, fractal-mirror modeling, and 7 mas of jitter are included in the simulation. Images of a monochromatic PSF at 555 nm and the polychromatic PSF are shown in Fig. 4. The polychromatic PSF is not as sharp as the monochromatic one and some of the ring features blend into each other.

11. SUMMARY

The Telescope Image Modeling (TIM) software is useful for image analysis and simulation of PSFs. The software has

been extensively used both in the early analysis of the OTA optics after launch of the telescope in 1990 April, and subsequent optical monitoring and PSF simulations. In particular, simulated PSFs have proved very useful in deconvolution of images from the aberrated telescope. The software has been periodically upgraded to include current knowledge of *HST* optics. New mirror maps obtained after the refurbishment mission of 1993 December have made a significant improvement in the agreement of observed and simulated PSFs.

The software is also valuable for planning observations with *HST* cameras and spectrographs. TIM is being used in the designing of new generations of *HST* science instruments such as NICMOS and the Advanced Camera. The software is versatile enough to have applications beyond *HST*, and may be used to model PSFs and images for a variety of telescopes.

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