PIETRO COSSALI AND THE PARMA OBSERVATORY BETWEEN THE END OF THE 18TH CENTURY AND THE BEGINNING OF THE 19TH CENTURY. MATHEMATICAL CONTRIBUTIONS TO THE DYNAMICS IN THE SOLAR SYSTEM

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ABSTRACT The Italian astronomical Observatories have been matter of historical studies about their origin and evolution for several years, however some of the smallest observatories escaped the investigation. Till now only few news have been published about the observatory built in Parma, first thanks to the Jesuits and then developed in association with the university born in this city. The observatory was particularly active under the management of the Theatin Father Pietro Cossali, established astronomy, meteorology and hydraulic professor by the Bourbon. Cossali revivified the observatory, renewed the instrumentation and amplified the activities, so that the observatory reached the apex of its history at the end of XVII century.
In his astronomical work Cossali gave an interesting theoretical contribution to the solar system dynamics with the elaboration of a mathematical model through which he tried to explain the meteoritical rains origin. In fact he assumed that the stones could be thrown from a lunar volcano toward the Earth

1. The birth of observatories

The structural changes that happened to the field of the astronomy in the XVIII century made essential also in Italy the passage from the private initiative, that dependeds in remarkable manner from the wish and from the capabilities of the individual astronomers, or of their religious brotherhoods, to public initiative, with the patronage of sovereigns and governments. This for a new plan of research that should have guaranteed stability of economic means in the course of the time, in order to face the increasing costs of most modern instrumental apparatuses now essential for the research, and moreover the presence of institutions that were not tied only to the fate of the single and to the maecenatism[1].

The first public observatory was that in Bologna[2,3,4], the fifth in Europe after Paris, Greenwich, Leida and Norimberga, and for a long time not only it would have been the unique in Italy, but also the most modern and less tied to the scientific interests of a "prince". His origins are linked to the figure of the earl Luigi Ferdinando Marsigli (1658-1730), who decided in 1702 to build an observatory in the tower of his palace, and he trusted the charge to Eustachio Manfredi (1674-1739) and to Vittorio Stancari (1678-1709). The second country to endow of a public observatory was the Grand Duchy of Tuscany with the Pisan one, even if in this case it was an initiative less lasting and remarkable[5]. The completion of the observatory was carried out, between 1735 and 1746, by Thomas Perelli (1704-1783) who was its first manager (1739). In Tuscany there were difficulties with the ridden events of the Florentine[6,7] observatory, of which completion Felice Fontana (1730-1805), a good technician of the tools, was entrusted.

Even more important was the completion of the Brera observatory[8], that had some warning signs in 1572, but it was only in 1765 that Milan had a real observatory, one of the most modern and best fitted of the time. The merits go to the Dalmatian Jesuit Ruggero Giuseppe Boscovich (1711-1787), that projected and directed the building[9,10] between 1764 and 1765, when he got the instrumentation commissioned to the French astronomer Lalande.

Among the greater contributions there was that furnished by Giuseppe Toaldo (1719-1787) founder of the Padova Observatory[11,12]. In Veneto the problem of weather forecasts was linked to the necessities of the agriculture, which had become more pressing once the predominance on the sea was missed. The professorship of astronomy and meteors had been founded in 1678 for Geminiano Montanari, and among the professors who occupied it afterward, from 1709 to 1715 there was also the marquis Giovanni Poleni (1683-1761).

In many Italian cities public Observatories were borne and among others also the little observatory that Giuseppe Casella (1755-1808) succeeded to obtain in Napoli[13] in 1791. On the other side the most important institution in Rome, represented at the end of the XVIII century from the Specula Caetani[14], had private character. Promoter of the initiative was, in fact, Francesco Caetani (1738-1810) duke of Sermoneta, that entrusted Giovanni Battista Audifreddi (1714-1794) in 1777, that is to say ten years before the creation of the Roman College Observatory[15].

The Palermo Observatory[16,17], linked to the name of Giuseppe Piazzi (1746-1826) one of the most famous astronomers of that epoch, and the Torino[18,19,20] and the Teramo[21] Observatory are the last, but not for importance, in this series.

Taking into consideration the abundant material of historic research of several Italian observatories I don't find instead, in the literature of the sector, practically any reference to the Parma Observatory, that was active in alternate phases since the second half of the XVIII century and that had a certain vitality under the direction of the Veronese astronomer Father Pietro Cossali.

2. Pietro Cossali and Parma observatory

The following historical researches have been based chiefly on the Cossali's manuscripts which is guarded at the Civic Library of Verona, and which is composed of the mathematical notes, letters, official documents and academic certificates. The contribution of the abbot Pietro Cossali to the astronomy, to the physics and to the math always seems more substantial, during the analysis of the great legacy of his manuscripts, of the scientific letters that he exchanged with the colleagues and of the printing works[22]. Among his merits we give acknowledge that he had main part in the reviving of the history of the Parma Observatory, as he had worked there from 1786 to 1805, and that besides to the university teaching he took care of the observatory instrumentation that allowed him to publish many works of astronomy, particularly about eclipses[23,24,25,26].

The first Parmesan observatory was borne from the wish of Iacopo Belgrado[27], a Friulian Jesuit (1704-1783) that in 1738 occupied the chair of math and physics in the Parmesan university which he held for about twelve years. In 1750 he became confessor and theologian of Philip duke of Borbone and mathematical of the Royal house. Although he had left the teaching, the protection of the duke allowed him to continue his scientific activity and in 1754 he could realize an astronomic observatory in the Jesuitic college of Parma. This fact gave him the chance to publish two little studies *Dell'azione del caso nelle invenzioni e dell'influsso degli astri ne' corpi terrestri* e *Observatio defectus Lunae habitae Parmae in novo observatorium patrum Societatis Iesu die 30 iulii 1757.*

In these years Belgrado produced many works and got appreciations from various academies, but in the time he missed the favour of the Borboni, probably because of religious politics. Despite the prestige achieved at the court in 1763 he was dismissed from the offices which he covered by the duke. The provision was probably suggested indirectly by the Theatin prioress Paolo Maria Paciaudi who pushed the growth of the Theatines to loss of the powerful Jesuits and who had become influential by the minister DuTillot[28]. The Jesuits were expelled from Parmesan state but Belgrado was preserved in his teaching.

In the period preceding their expulsion the Regular clerics of the state of Parma, Piacenza and Guastalla were 1.100, the suppressed cloisters 13, the religious expelled 400, of which 150 Jesuits. The incomes, 300,000 Parmesan lire roughly, were assigned to the Big hospital of the Mercy of Parma and a part of them was sold. The competencies of 300,000 lire were used for the pension of the expelled religious and part for the university located in the principal College of S. Rocco. The cloisters which remained in the city were those of the Nuns, Benedictines, Dominicans, Theatines, Carmelites, Vincentian, Capuchins, Cruciferi, Cistercians of S. Martin, Observants, Reformeds[29,30].

Some years later, in 1767, Ferdinando of Borbone expelled the Jesuits, from the Kingdom of the Two Sicilies, and closed the schools of each type that Jesuits managed. This gesture followed analogous provisions taken from the court of Portugal, France, Spain and Naples like reprisal to the monitory against Parma of Clement XIII. In this way the Bourbon government missed the principal cultural institutions and moreover it had to reorganize the sector of the public education and of consequence also that of the research. It was the moment in which the Theatines began to dominate the parmesan scene.

Fig. 1 An incision of 1821 that represents the abbot Pietro Cossali, drawn out from the written Praise by Giuseppe Avanzini to commemorate his death, published on the *Memories of the Italian Society of the Sciences* (Mathematical Series), Vol. 19 part first, p. CXI

They are the years in which the Theatines became also a cultural power with which it was possible to collate in Parma, where besides to the Paciaudi an other Theatin father worked, Giovanni Maria Carminati professor of Physics in the royal university of Parma since 1776. He had acquired a certain notoriety for his works and had gained a certain reputation, as we learn from a letter that Sebastiano Canterzani[31] sent him on 22 September 1776, with the congratulation for the promotion to the university chair. Canterzani had correspondence of mathematical matter also with the Cossali, and this correspondence was intense and based on several problems such as the dynamics, the integration systems[32,33], with the methods of Eulero, D'Alembert, Bernoulli, Frisi[34,35]. In fact in those years the interests of the math concentrated on the differential equations and the techniques of resolution, just like in the years 1784-85 when also the influential Father Paciaudi was mentioned.

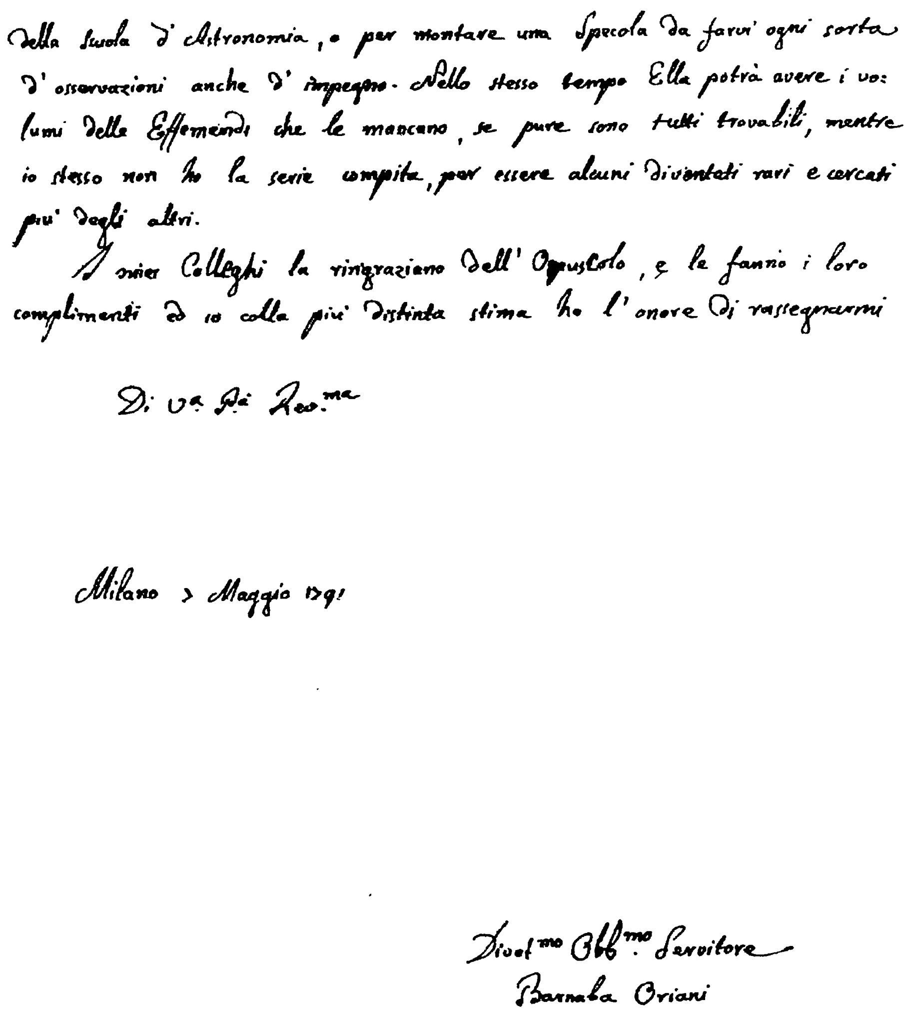
He had the commission by the Borbone to organize the growing university as regards the material and formal aspects, the place, the rules, the choice of the professors. The university was situated in the cloister of S. Rocco of the Jesuits, where the observatory realized by Belgrado had been risen and then renewed by Cossali. Paciaudi quickly arranged that three theaters were built, for the experimental physics, the anatomy and the chemistry and this to stimulate the university center, also thanks to the convoking of the Cossali through duke Ferdinando.

Probably also his epistolary acquaintance with the Carminati abbot influenced this decision. Since 1782 they had been working to the solution of equations higher of the second degree, both for distinctive cases and for the general case which was treatised facing the problem of the reducibility of an equation through the Cardano's method, author much studied by the Cossali that rediscovered his works, particularly difficult because of the idiom used. Another sector on which they had focused their attention was the solution of the differential equations. The correspondence continued up to 1785 and later they could meet.

Fig. 2 The duke Ferdinando of Borbone in a portrait of the German painter Zoffany, preserved in the National Gallery of Parma.

By now the stature of mathematician continued to grow with two works (*Controversia analitica* and *Confronti a pubblico lume*) recognized as excellent from the Italian and French mathematicians, so that his notoriety diffused also to the foreign countries. At certain point the ambassador in Paris of Ferdinando duke of Parma informed his ruler of the qualities of this Veronese mathematician. After this message, and probably driven by the Parmesan Theatines, in 1787 the duke Ferdinando was persuaded to offer him the chair of theoretical physics at the university of Parma. Some years later, in November 1790, we learn from a letter of Cesare Ventura[36,37] sent to Cossali, that Cossali had been elected professor of Astronomy, meteorology and hydraulic to the university of Parma, nomination that will be official in 1791.

The Parmesan cultural flowering[38] of the second part of '700 had interested the university too, and the scientific teaching had been rendered much more alive by the presence of Iacopo Belgrado, that was however conscious of the limitations of the tools of the observatory, which were still rather labile and handicraft. So the first care of Cossali was to strengthen immediately the means of the observatory with the acquisition of modern and numerically more substantial instruments.



della Scuola d'Astronomia, e per montare una Specola da farvi ogni sorta
d'osservazioni anche d'impegno. Nello stesso tempo Ella potrà avere i vo:
lumi delle Effemeridi che le mancano, se pure sono tutti trovabili, mentre
io stesso non ho la serie compita, per essere alcuni diventati rari e cercati
più degli altri.
I miei Colleghi la ringraziano dell'Opuscolo, e le fanno i loro
complimenti ed io colla più distinta stima ho l'onore di rassegnarmi

Di V.a S.a Rev.ma

Milano 7 Maggio 1791

Divot.mo Obb.mo Servitore
Barnaba Oriani

Fig. 3 The last page of the letter that Barnaba Oriani from the Observatory of Brera sends to Pietro Cossali to Parma in 1791, where the observatory which had to be strengthened is mentioned.

A letter received from Gio Battista Nelli[39], 10 February 1789, informs him about the problems relating to the construction of the pendulum-clocks and it demonstrates that already before the definitive commission Cossali had begun the work of updating of the observatory. One of his letter of 1791 to Barnaba Oriani shows the hurry that he had in this direction and the promptness of the Oriani who answers him from Milan on 7 May. Oriani invited Cossali to see the new quadrant wall of Ramsden in the Brera observatory[40] he promised him in that occasion some information about the London builders of astronomic tools. "Nella stessa occasione le darò quelle notizie ch'ella brama intorno agli artisti di Londra ed agli stromenti d'Astronomia, non sapendo io ora su quale piede voglia provvedersi gli stromenti, cioè se pel solo uso della scuola d'astronomia, o per montare una specola da farvi ogni sorta d'osservazioni anche d'impegno"[41].

Cuore del Lione detto *Regolo*.
Di grandezza prima.

G. 1 del Mese	Leva O.	Leva M.	Culmina O.	Culmina M.	Tramonta O.	Tramonta M.
Gennajo	8 S.	14	3 M.	9	10 M.	4
Febbrajo	6	3	0	58	7	53
Marzo	4	10	11 S.	5	6	0
Aprile	2	17	9	12	4	7
Maggio	0	26	7	21	2	16
Giugno	10 M.	24	5	19	0	14
Luglio	8	20	3	15	10 S.	10
Agosto	6	15	1	10	8	5
Settembre	4	20	11 M.	15	6	10
Ottobre	2	32	9	27	4	22
Novembre	0	36	7	31	2	26
Decembre	10 S.	32	5	27	0	22

Culminando dista dallo Zenit ad Ostro
Gradi 31 minuti 47.

7

Spiga nella mano sinistra, od Australe della Vergine.
Di grandezza prima, seconda.

G. 1 del Mese	Leva O.	Leva M.	Culmina O.	Culmina M.	Tramonta O.	Tramonta M.
Gennajo	1 M.	4	6 M.	26	11 M.	48
Febbrajo	10 S.	52	4	14	9	36
Marzo	9	4	2	26	7	48
Aprile	7	10	0	32	5	54
Maggio	5	15	10 S.	3-	3	59
Giugno	3	13	8	35	1	57
Luglio	1	9	6	31	11 S.	53
Agosto	11 M.	5	4	27	9	49
Settembre	9	9	2	31	7	53
Ottobre	7	21	0	43	6	5
Novembre	5	25	10 M.	47	4	9
Decembre	3	21	8	43	2	5

Culminando dista dallo Zenit ad Ostro
Gradi 14 minuti 49.

Stella sul fiocco della Coda del Lione
di grandezza seconda.

G. 1 del Mese	Leva O.	Leva M.	Culmina O.	Culmina M.	Tramonta O.	Tramonta M.
Gennajo	9 S.	43	4 M.	50	11 M.	57
Febbrajo	7	32	2	39	9	46
Marzo	5	43	0	50	7	57
Aprile	3	46	10 S.	53	6	0
Maggio	1	55	9	2	4	9
Giugno	11 M.	53	7	0	2	7
Luglio	9	49	4	56	0	3
Agosto	7	44	2	51	9 S.	58
Settembre	5	47	0	55	8	2
Ottobre	4	1	11 M.	8	6	15
Novembre	2	5	9	12	4	19
Decembre	0	1	7	8	2	15

Culminando dista dallo Zenit ad Ostro
Gradi 29 minuti 1.

Cuore dello Scorpione, detto *Antares*.
Di grandezza prima.

G. 1 del Mese	Leva O.	Leva M.	Culmina O.	Culmina M.	Tramonta O.	Tramonta M.
Gennajo	5 M.	20	9 M.	28	1 S.	36
Febbrajo	3	8	7	16	11 M.	24
Marzo	1	19	5	27	9	35
Aprile	11 S.	26	3	34	7	42
Maggio	9	35	1	43	5	51
Giugno	7	29	11 S.	37	3	45
Luglio	5	25	9	33	1	41
Agosto	3	21	7	29	11 S.	37
Settembre	1	25	5	33	9	41
Ottobre	11 M.	37	3	45	7	53
Novembre	9	41	1	49	5	57
Decembre	7	37	11 M.	45	3	53

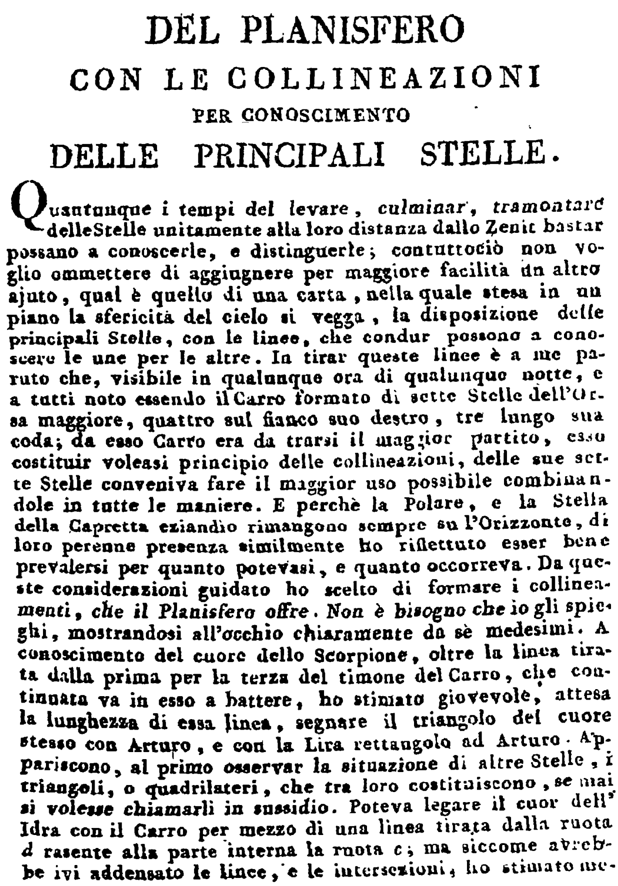
Culminando dista dallo Zenit ad Ostro
Gradi 70 minuti 42.

Fig. 4 Charts with the data of forecast about the observation of some stars, printed in appendix to a work about the eclipses.

His researches were then embodied in seven volumes of annual nautical almanac from 1791 to 1804, determining the time of the apparitions and of the duration of the various phenomena (eclipses, stellar coordinates, occultations) with introductory essays[42,43,44] about themes of general astronomy[45,46,47,48,49]. His interest was particularly centered on the eclipses and to control each of their phases he had a net of correspondences located across Italy. For example we know from some letters that he was in touch with Giangiacomo Barattieri[50,51] (student of Father Belgrado with whom he had done some observations in the S. Rocco observatory) after having received the publication of *Apparenze del solare eclisse*....by Cossali, he answered to him announcing an error in the latitude of Parma because he had measured it personally for Piacenza, *che l'ha simile a Parma* obtaining 45° 3' 54,5".

Cossali exposed, compared and discussed the hypotheses and the calculuses of Piazzi and Olbers to determine the orbits and the brightness of the new planets Cerere and

Pallade especially in the nautical almanac of 1803. He also obtained from the government a financial help for observations of planets that he effected in Milan, in Brera observatory. Besides he received the financings and he followed the installation of a meteorological observatory in the Parmesan house of the Theatines. In it he effected pluriannual systematic recordings that he won't ever succeed to publish, despite the intent and despite his correspondents, such as Alessandro Barca, sent to him constantly recordings of the temperatures of different places[52,53,54].



DEL PLANISFERO

CON LE COLLINEAZIONI

PER CONOSCIMENTO

DELLE PRINCIPALI STELLE.

Quantunque i tempi del levare, *culminar*, *tramontare* delleStelle unitamente alla loro distanza dallo Zenit bastar possano a conoscerle, e distinguerle; contuttociò non voglio ommettere di aggiugnere per maggiore facilità un altro ajuto, qual è quello di una carta, nella quale stesa in un piano la sfericità del cielo si vegga, la disposizione delle principali *Stelle*, con le linee, che condur possono a conoscere le une per le altre. In tirar queste linee è a me paruto che, visibile in qualunque ora di qualunque notte, e a tutti noto essendo il Carro formato di sette Stelle dell'Orsa maggiore, quattro sul fianco suo destro, tre lungo sua coda; da esso Carro era da trarsi il maggior partito, esso costituir voleasi principio delle collineazioni, delle sue sette Stelle conveniva fare il maggior uso possibile combinandole in tutte le maniere. E perchè la Polare, e la Stella della Capretta eziandio rimangono sempre su l'Orizzonte, di loro perenne presenza similmente ho riflettuto esser bene prevalersi per quanto potevasi, e quanto occorreva. Da queste considerazioni guidato ho scelto di formare i collineamenti, *che il Planisfero offre*. *Non è bisogno che* io gli spieghi, mostrandosi all'occhio chiaramente da sè medesimi. A conoscimento del cuore dello Scorpione, oltre la linea tirata dalla prima per la terza del timone del Carro, che continuata va in esso a battere, ho stimato giovevole, attesa la lunghezza di essa linea, segnare il triangolo del cuore stesso con Arturo, e con la Lira rettangolo ad Arturo. Appariscono, al primo osservar la situazione di altre Stelle, i triangoli, o quadrilateri, che tra loro costituiscono, se mai *si volesse chiamarli* in sussidio. Poteva legare il cuor dell' Idra con il Carro per mezzo di una linea tirata dalla ruota *d* rasente alla parte interna la ruota *c*; ma siccome avrebbe ivi addensato le linee, e le intersezioni, ho stimato me-

Fig. 5 Page of a volume of nautical almanac, in which Cossali developed various matters, to instruct the readers on a variety of astronomical themes.

Naturally his wealth of mathematician was put in the astronomy and the problems tied to the solution of the differential equations forced him to maintain contacts with most famous Italian and foreign colleagues as Gregorio Fontana, Sebastiano Canterzani, Giordano Riccati, Paolo Frisi, Charles Messier[55].

His nautical almanac was well known to a vast scientific and unskilled audience, as the vice-president of Italian Republic Francesco Melzi D'Eril[56], because he sent regularly his works not only to the colleagues but also to friends and acquaitances. From a letter written in Parma on 16 January 1794, addressed to the Veronese Girolamo Murari della Corte, we learn that he had sent copy of a nautical almanac also for the professor Serafino Volta[57], for Borsa, and for the abbots Andres and Pinazzo. At the same time he promised them also a work, in that moment in copying, for the Memories of the Academy of the XL called also Italian Society of the Sciences of which he was real partner, and constituted by the Veronese Lorgna.

PIANETI.

Gior. del M.	MAGGIO Si alzano (O. M.)	MAGGIO Passano per il Merid. (O. M.)	MAGGIO Distanti dallo Zenit (G. M.)	MAGGIO Cadono (O. M.)	GIUGNO Si alzano (O. M.)	GIUGNO Passano per il Merid. (O. M.)	GIUGNO Distanti dallo Zenit (G. M.)	GIUGNO Cadono (O. M.)	Gior. del M.
	Herschel				**Herschel**				
1	0 S.12	7 S.13	30 39	2 M.14	10 M 11	5 S.12	30 49	0 M 13	1
11	11 M.33	6 34	41	1 35	9 32	4 32	55	11 S.32	11
21	10 54	5 55	43	0 56	8 52	3 52	31 3	10 52	21
	Saturno				**Saturno**				
1	5 M.22	0 S.28	29 33	7 S.34	3 M 31	10 M.42	28 30	5 S.53	1
11	4 47	11 M.55	13	7 3	2 54	10 6	13	5 18	11
21	4 11	11 20	28 53	6 29	2 16	9 29	27 57	4 42	21
	Giove				**Giove**				
1	11 S.22	3 M.42	67 49	8 M. 5	8 S. 8	1 M.30	67 55	5 M.53	1
11	10 39	3 2	51	7 25	8 21	0 43	56	5 5	11
21	9 57	2 19	53	6 41	7 35	11 S.57	57	4 19	21
	Marte				**Marte**				
1	6 S. 4	11 S.21	55 55	4 M.38	3 S.30	8 S.52	54 43	2 M.14	1
11	5 9	10 29	9	3 49	2 52	8 12	55 9	1 32	11
21	4 18	9 40	54 43	3 2	2 19	7 36	56 0	0 53	21
	Venere Vespertina				**Venere Vespertina**				
1	5 M.24	0 S.44	26 34	8 S. 4	5 M.31	1 S.23	20 14	9 S.15	1
6	5 38	0 50	24 53	8 18	5 38	1 30	13	9 22	6
11	5 20	0 56	23 26	8 32	5 46	1 36	32	9 26	11
16	5 20	1 2	22 14	8 44	5 54	1 41	21 8	9 28	16
21	5 23	1 9	21 18	8 55	6 3	1 46	22 0	9 29	21
26	5 26	1 16	20 38	9 6	6 14	1 51	23 9	9 28	26
	Mercurio				**Mercurio**				
1	4 M.10	10 M.26	41 32	4 S.42	3 M.53	11 M.18	25 32	6 S.43	1
6	4 3	10 24	40 9	4 45	4 4	11 43	22 41	7 22	6
11	3 57	10 26	38 7	4 55	4 18	0 S. 9	20 20	8 0	11
16	3 53	10 34	35 31	5 15	4 39	0 35	19 38	8 31	16
21	3 50	10 43	32 31	5 36	5 7	1 0	20 4	8 53	21
26	3 50	10 57	29 18	6 4	5 30	1 16	21 20	9 2	26

Fig. 6 A copy of the planetary nautical almanac, prepared by Cossali from 1791 to 1804.

In 1802 Ferdinando of Borbone died, maybe for a suspicion poisoning[58], and the govern was entrusted temporarily[59] to the administrator Moreau de Saint-Mery[60], that wanted Cossali to stay on his chair which he had hold for 15 years, and in the attempt of keeping a famous character, in November 1804 he wrote him from Milan sending the renovation of the nomination to emeritus professor of Astronomy meteorology and hydraulic.

Maybe already in this period Cossali began to mature the decision to leave Parma, also because the administration of the Saint-Mery, although it had revealed itself open to culture and wise, it was faulty because of a serious disorganization, and probably a long period of impermanent health that tormented him in alternate phases contributed to this decision. His health had considerable problems after 1800 for a probable gout, intestinal and nervouse indispositions, in fact Antonio Cagnoli in the February of 1801 writes to him his cheerfulness for the news that has arrived him of his recovery[61,62], not certain a definitive one. But the health gave him further sufferings as the academic senate of Parma in the June 1802 assigned to Cossali the professor Luigi Pazzoni[63] who helped him in the calculus of the nautical almanac. He felt the absence of a governor with whom he had established a very friendly relation and in 1805, perhaps also because of the wars in course, that had political consequences in Parma, he decided to return to Verona[64]. Here he taught for a brief period in the local high school while later the school autorithies gave him the chair of sublime calculus[65] by the Padova university.

Fig. 7 The administrator Elia Moreau de Saint-Mery, governor of Parma from 1802 to 1805, year in which he was deposed by Napoleon.

In his stay in Parma Cossali had the opportunity to frequent various surroundings linked to the cultural world and of elevated social class, and in fact because of his notable celebrity he was called to many ambients. For example, at the Royal Ducal College of the Noble of Parma (the college of the Noble was managed by the Jesuits and after their expulsion was entrusted to the Piarists) he was invited to hold a discourse in defence of the mathematics[66]. In his surrounding he was well known and many letters were chiefly centered on astronomic questions, physics or maths, about which he was consulted or he entertained discussions. Giangiacomo Barattieri in the May 1791 asked him a question about the method of construction of a vertical sundial[67], while with Petronio Matteucci he maintained a correspondence on varied astronomical questions[68,69] for many years. His abundant work at the observatory, part edited and part not published, such as astronomical memories, and the calculuses to foresee the position of the Easter during each year, is testified by the numerous letters of thanks for the received publications, as those belonging to Francesco Soave[70] and dating May 1791 and April 1802[71].

His diligence in writing about astronomic matters perhaps was not equal to the very rich production of mathematical character but in the Parmesan period he had probably his best occasion to write abundant notes about astronomic thematic among which we remember the works almost completed about the Lunar motion[72,73] respect to the stars, about the cometary orbits[74], about the visual appearance of the comets or about the Sun, like the *De Solis Planetarum Lune, Camerarum natura: Dissertatio de Solis Maculis* [75,76] or the *Osservazioni e modelli sulle macchie solari*[77], as well as about fixed stars with *Articulus II De stellis fixis seu solibus* e *Sulle stelle fisse e loro posizioni, loro origine, elenco di costellazioni*[78], *De iis quibus sit ut astra diverso loco appareant ac sunt ou De refractione et parallaxi in seno scholto*. Within the solar system he left us works such as *Sulla rivoluzione attorno al sole nel sistema planetario, dimensioni apparenti dei pianeti, parallassi planetarie*[79,80,81],*Astri in sphera mundana locus guatonis vel eclipses ope determinatus, Giorno 14 marzo 1804. Aspetto del sole con delineazione delle Macule, Nebulosità, e facole nello spettro*[82,83].

His works were much waited by his colleagues, as it is pointed out by the anxious waiting of the nautical almanac by Antonio Cagnoli, prepared by Cossali for 1804 in which he hoped to find solution to the problem of the residual light in the eclipse that had cited in his "Prenuncio ristretto"[84].

Although he had gone sway from Parma in 1804, ten years after, 2nd of October 1814, he was elected honorary professor of the university of Parma, nomination that is kept in his manuscripts collection. It is a printed edict with decree of the Count Philip Magawly-Cerati De Calry[85], minister of the dukedom of Parma Piacenza Guastalla[86].

3. "Sull'opinione delle piogge de' sassi dai vulcani lunari" (About the opinion of the stone rains from Lunar volcanoes)

Cossali gave an interesting contribution to the spatial dynamics with a job[87] published in 1807 in the 13° volume of the Memories of the Italian Society of the Sciences with the title *Sull'opinione delle piogge de' sassi dai vulcani lunari.*. It was a theoretical research completed in 1806 and so probably elaborated in 1805 considering the length and the complexity of the calculuses, that is to say while he was still to Parma. The hypothesis of work concerns the capability that some stones erupted from a lunar volcano could be thrown with a sufficient force to arrive on the Earth, thesis that had demonstrated one of the possible origins of the meteoritic rains and that furnishes an original contribution to the problematic of a Moon-Earth trip, that was faced by the NASA after 150 years for the return of Apollo 11.

Cossali thought in fact that *la Luna nostro satellite, oltre a quegli influssi, che segretamente, a pretesa di alcuni, esercita sulle piante delle nostre campagne, su gli animali di nostro servigio, e su i corpi nostri medesimi, abbi eziandio la potestà di flagellarci con piogge, o piuttosto grandini di sassi!* The Vaquelin colleague hypothesized that the meteorites had lunar origin *l'opinione, che li fa venir dalla Luna, comunque strana ella sembri, è forse la meno irragionevole, e se non possono darsene prove dirette, nemmeno se le può opporre un ben fondato ragionamento.* (If we think that the Moon has some influences on plants and animals, it could have also the power to lash us with stone rains, probably an assumption that could seem strange but not even unreasonable)

Naturally Cossali had at his disposition only a quadrant wall, professional tool in that time but insufficient to observe the details of the lunar surface. If he had had a simple tool, today at disposition of an amateur astronomer, like a reflection telescope of 60 cm of diameter[88] and a CCD Sbig ST6, he could have seen well that the lunar volcanoes were not active.

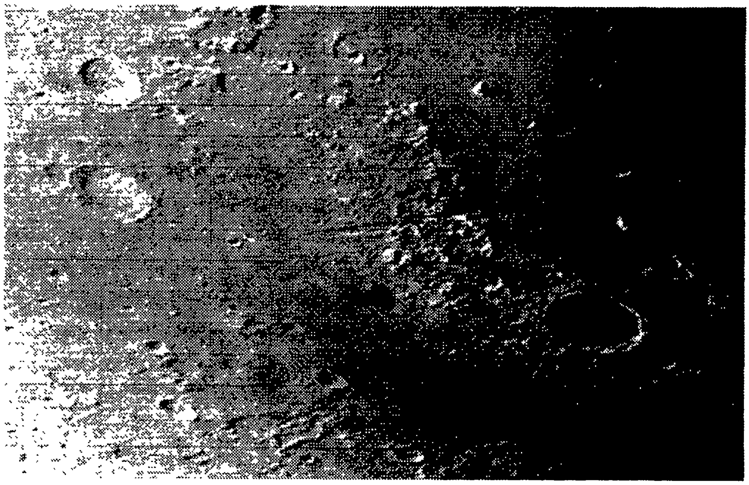


Fig. 8 An image of the Moon recovered by the Veronese amateur astronomer with a CCD ST6 applied on a reflection telescope with a mirror of 60 cm of diameter.

But the hypothesis could have been not completely absurd if we consider the energy used in the volcanic phenomena and that was necessary for achieving the escape velocity to leave the Moon. We evaluate the energetic hypothesis tied to the eruptive power of a volcano. A terrestrial volcano has a latent energy[89] (among thermal, kinetics, etc.) roughly 10^{20} J. The kinetic energy developed in the expulsion of the materials has been valued of the order of 10^{17} J but one must consider that part of the energy develops in the lava castings, part in the issue of gas, part in the throwing of solid material. However we go down of another order of quantity so that it remains an energy of 10^{14} J. If a lunar volcano had an eruptive power comparable to that of a terrestrial volcano (but we reduce it of a further dimensional factor linked lunar mass up to 10^{12} J) we could appraise the kinetic energy of a stone whose mass is 10 kg and that goes off from the Moon with an escape velocity of around 2.500 m/s. The energy is in the order of 10^8 - 10^9 J, in appearance sufficient to the purpose, as it would be a of four orders of quantity within the previously calculated energy. However during an eruption a stones quantity equal to hundred of tons would be ejected if a meteoritic rain should arrive on the Earth. Moreover what the stone misses is a propulsion system able to drive it until to a certain quota maintaining a velocity superior to the escape velocity.

We begin with a first analysis of the data that is revealed as interesting also to verify which quantities were fairly precise at the beginning of 18th century.

The average ray of the Earth was valued R=3,269,511 "tese" (from Lalande). Considering the "tesa" around 1,825 m long, we obtain R≈5,966.857 Km against an average real ray R=6,371 Km[90].

The velocity acquired by a falling body after a second was measured equal to v=15.0515 Parisian feet/s, and considering the Parisian foot≈0.304 m we calculate a velocity of 9.4 m/s.

The average ray of the Moon had been valued equal to r=(3/11) R=1,627.324 Km. The ratio of r/R=3/11 was correct within the fourth decimal digit but the average ray of the Moon is r≈1,738 Km.

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SULL' OPINIONE

DELLE PIOGGIE DE' SASSI DAI VULCANI LUNARI

DISQUISIZIONE MATEMATICA

DEL P. D. PIETRO COSSALI

Ricevuta il dì 6 Maggio 1806.

Che la luna nostro satellite, oltre a quegli influssi, che segretamente, a pretesa di alcuni, esercita sulle piante delle nostre campagne, su gli animali di nostro servigio, e sui corpi nostri medesimi, abbia eziandio la podestà di flagellarci con pioggie, o piuttosto grandini di sassi! Così è, così è si echeggia tra Alemagna, e Francia, ed anche in Italia. Ed il celebratissimo Fisico, e Chimico Vauquelin, tra gli altri, dopo avere esaminate le estrinseche qualità, e con diligente analisi ricercati i componenti dei sassi della sì decantata pioggia di sassi nei contorni dell' Aquila in Francia, pronuncia, che *l' opinione, che li fa venir dalla Luna, comunque strana ella sembri, è forse la meno irragionevole, e se non possono darsene prove dirette, nemmeno se le può opporre un ben fondato ragionamento*. Trascrivo le parole della versione italiana del Chiarissimo nostro Collega Sig. Ab. Amoretti nel Tomo XXII degli *Opuscoli scelti* stampati in Milano, non avendo il *Journal des Mines*, nel numero 76 del quale egli fa sapere essere inscrita la Memoria del Sig. Vauquelin. Importa molto il veder con chiarezza, se l' ideare de' sassi da lunari vulcani lanciati in terra, cosa in realtà ella si sia, cui, giusta l' asserto del Vauquelin, fondato ragionamento oppor non si possa. La disquisizione ha due parti: matematica l' una, fisica l' altra, ed è d' uopo, che dalla matematica incominci, dalla quale la fisica deve trarre appoggio, e lume. Mi accingo io qui pertanto alla parte matematica, e premetto i seguenti

Da-

Fig. 9 Introducing page of the article entitled *Sull'opinione delle piogge de' sassi dai vulcani lunari*, published in the 13° volume of the Memories of the Italian Society of the Sciences, in 1807.

The mass of the m Moon=M/58,6, with M (=7,348x10^{22} kg) Earth mass. The lunar mass wasn't furnished from Cossali but according to the evaluation of Laplace it was m=0,0171 in relation to the M mass, while we know today that m/M= 0,0123, and so m=1,25x10^{21} kg.

Cossali gave an average distance between the Earth-Moon centers d=60R=357.960 Km. The ratio in terrestrial rays was correct but d=384.399 Km.

The average density of the Moon ρ_L=0,8412337 in relation to the terrestrial density ρ_T. Taking into consideration that ρ_L=3,342x10^3 kg/cm^3 and ρ_T=5,515x10^3 kg/cm^3 we have that ρ_L/ρ_T=0,606.

The acceleration of terrestrial gravity was for Cossali g_T=9,18 m/s^2 while the acceleration of gravity of the Moon g_L=0,229427* g_T=2,106 m/s^2, against an areal acceleration g_L=1,623 m/s^2.

Cossali starts the works calculating the position of the libration point where the forces of attraction of the Earth and of the Moon are in balance along their conjunction line; equating the attraction effects of the Earth and of the Moon in a point the distance of which is yR from the center of the Moon and (k-y)R from the center of the Earth, the following equation is obtained,

$$\frac{G}{m(yR)^2} = \frac{G}{m(k-y)^2R^2} \qquad (1)$$

that resettled becomes the equation

$$y^2 + \frac{2ky}{(m-1)} - \frac{k^2}{(m-1)} = 0 \qquad (2)$$

If we resolve the equation we obtain y=6,932358 and then k-y=53,067642.

Now that the fundamental data are decided Cossali goes on to the calculuses. *Posto, che da un vulcano della Luna situato ad una data distanza dal suo centro sia scagliato in direzione nella direzione della linea, che da quel centro stesso viene al centro della Terra, un sasso con la velocità v, determinare la espressione della velocità del sasso medesimo a qualunque punto del viaggio lungo la detta linea, considerando il continuo conflitto delle due opposte attrazioni, quinci della Terra, di là della Luna, l'una continuamente crescente, l'altra continuamente decrescente in ragione duplicata inversa della distanza pel viaggio diminuita verso il centro della Terra, ed accresciuta al centro della Luna.* (Cossali wants to calculate the expression of velocity with which the stone is thrown from the Moon toward the Earth along the straight which links the center of two planetary bodies and which travels into their gravitational field)

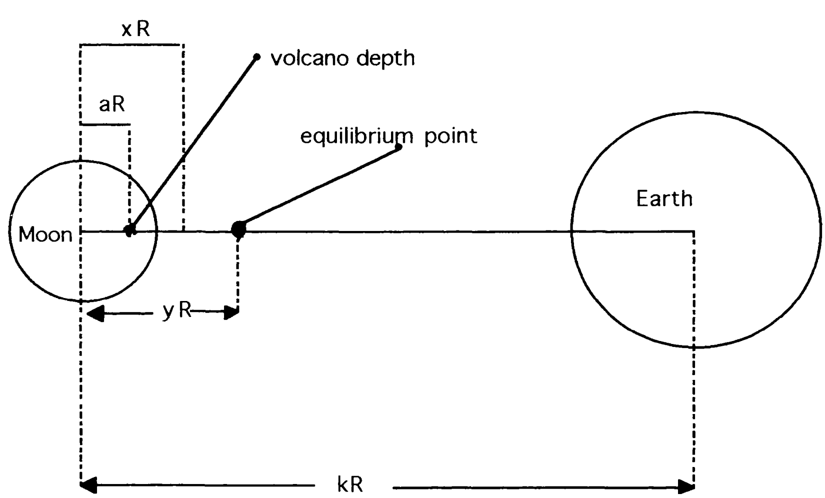


Fig. 10 geometric Diagram of the Moon-Earth trajectory proposed in the work of Cossali.

The model is much simplified and doesn't take into consideration the dynamics and the mutual influences of the two planetary bodies.

Cossali defines aR the distance of the volcano or of the ejection point from the center of the Moon, xR the distance already covered from the stone, (x+a)R the distance at a certain time from the center of the Moon. In this instant the lunar attraction will be

$$g_L = \frac{GR^2}{m(a+x)^2R^2} \qquad (3)$$

and that of the earth

$$g_T = \frac{GR^2}{(k-a-x)^2R^2} \qquad (4)$$

for this reason the stress on the stone will be

$$F = \frac{G}{(k-a-x)^2} - \frac{G}{m(a+x)^2} \qquad (5)$$

If we characterize the velocity at a certain instant with V, and told d(xR) the increase of the distance covered with the increase of the velocity dV, the stone will have in this instant

$$VdV = \frac{GRdx}{(k-a-x)^2} - \frac{GRdx}{m(a+x)^2} \qquad (6)$$

from which, integrating, Cossali obtained

$$\frac{1}{2}V^2 = \frac{GR}{(k-a-x)} - \frac{GR}{m(a+x)} + C \qquad (7)$$

The constant C is determinated by the condition that the stone has been thrown from the lunar volcano with the initial velocity v, moreover when the stone is on the lunar surface x=0 and so V= v, from which we extract

$$C = \frac{1}{2}v^2 - \frac{GR}{k-a} - \frac{GR}{am} \qquad (8)$$

from which the complete formulation of the stone velocity is

$$V = \sqrt{v^2 + \frac{2GR}{k-a} \bullet \frac{x}{k-a-x} - \frac{2GR}{am} \bullet \frac{x}{v+x}} \qquad (9)$$

If we consider v_f the escape velocity from the Moon $v_f = \sqrt{2gm/r} \approx 8.546$ km/h, with $g=6,67 \times 10^{-11}$ and the point of departure of the stone coinciding with the Moon surface, that is a=R/r, we have an idea of the progress of the velocity in each instant in function of the x distance from the Moon.

However the problem for Cossali was the inverse, that is to say he had determined v, the initial velocity of the stone so that it was able to arrive to the point of equilibrium. Here V=0 and so it is possible to draw out v, that is

$$v = \sqrt{2GR\left[\frac{h-a}{amh} - \frac{h-a}{(k-a)H}\right]} \qquad (10)$$

where h=y=6,932358 and H=k-y=53,067642.

Now the only quantity that it is to be defined is the depth of the volcano, that is to say a, that Cossali considers a= 3/ 11, that is an equal depth to the lunar ray. *A determinare v bisogna assegnare un valore ad a [...] il punto dello scagliamento del sasso dista dal centro della Luna. E' quistione tra i Naturalisti intorno al sito dei focolari de' vulcani. Hamilton sostiene essersi a partito ingannati quelli, che pretesero generalmente stabilire, che ardano alla sommità, o pure al mezzo per lo meno delle montagne; e dalle sue osservazioni sul Vesuvio, su tutta la estensione dei Campi Flegrei, sui tremuoti della Calabria, su l'Etna, sul vulcano di Stromboli, sul sorgimento delle isole Eoliche accumula prove per dimostrare, che i focolari vulcanici si trovano piuttosto profondi sottoterra, e persino sotto il fondo del mare. Prendendo un medio, non si potrebbe condannare il porre i focolari vulcanici terrestri alla superficie media della terra. E similmente si concederà il porre per un medio i focolari vulcanici lunari alla superficie media della Luna, e quindi alla distanza (3/11)R dal centro della Luna medesima.* (Cossali explains the theories about volcanoes stoking. Hamilton thought that the source at high temperature was very deep, under the bottom of the sea, hypothesis that he had extrapolated from his observations about volcanoes Vesuvio, Etna and Stromboli. However Cossali considers that the volcanic furnaces are near the lunar surface)

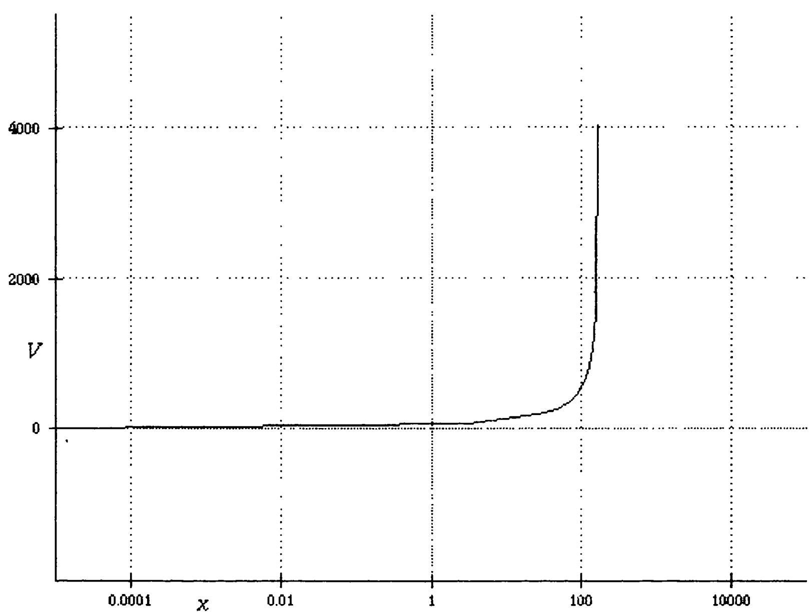


Fig. 11 Progress of the velocity in each instant versus the x distance from the Moon, according to Cossali's formula.

Now we can consider the principal error made by Cossali in the physical-mathematical model, because he considered a system with two fixed points in the space for the departure and the arrival of the stone. He didn't take into consideration the orbits movements of the Earth and of the Moon that during the trip of the stone move from the mutual and absolute positions, altering the gravitational influence both in modulus and in direction.

However with this value Cossali finds v=2.520.9849 m/s, and if we consider the real escape velocity seen before $v_f \approx 2.373$ m/s, we note that it is not very discordant from that calculated two centuries ago. Cossali observes that the calculated velocity was about a fifth of the one of a ball of 24 pounds shoot from a gun loaded with 12 pounds of gun powder. In this sense there is an anticipating echo of the trip From the Earth to the Moon, by Jules Verne, even if it is exactly to the contrary, where we can see the volcanic chimney like a kind of gun that shoots a stone toward the Earth[91].

Naturally the velocity calculated from Cossali brings the stone to the point of equilibrium, but if we want to exceed it the initial velocity has to be lightly greater than the velocity calculated now. Then Cossali defined V= v+ u, with u the necessary velocity excess and he replaced it in the expression (9), and simplifying obtained $V = \sqrt{2vu + u^2}$.

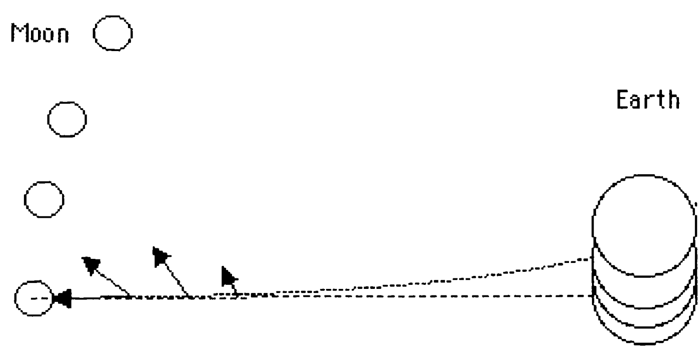


Fig. 12 Cossali in his calculuses hadn't taken into consideration the change of position of the Moon and of the Earth in the time, and so of the change in module and direction of the gravitational force of the Moon and of the Earth.

With a series of considerations Cossali found that increasing the velocity of only $4{,}9248 \times 10^{-3}$ m/s (then V=2.520,9899 m/s) when the stone reaches the equilibrium point it still preserved a velocity V=4,982 m/ s.

After having arrived to these conclusions Cossali had the sufficient velocity for winning the lunar gravity and to fall in the earthly one, and the velocity with which the stone would have arrived on the Earth would have been V=10.356,489 m/s.

We note that all preceding calculuses present some problems and scarceness. The increasing attraction of the Earth from the lunar surface to the point of equilibrium contributes to decelerate the stone for V=479,7 m/s. With such a modification the stone arrives on the Earth with a velocity V=9.899,29 m/s. Moreover we have to consider that the lunar attraction decreases but it acts also in the space between the equilibrium point and the Earth, and this one acts also in the space between lunar surface and attraction point, with a contribution of V=45,9 m/s.

In the modern theory about the transfers orbit necessary in the spatial trips the problem has been resolved and examined abundantly in different ways. We can assimilate the

Cossali problem to a ballistic flight in which the spatial object is launched with only one impulse.

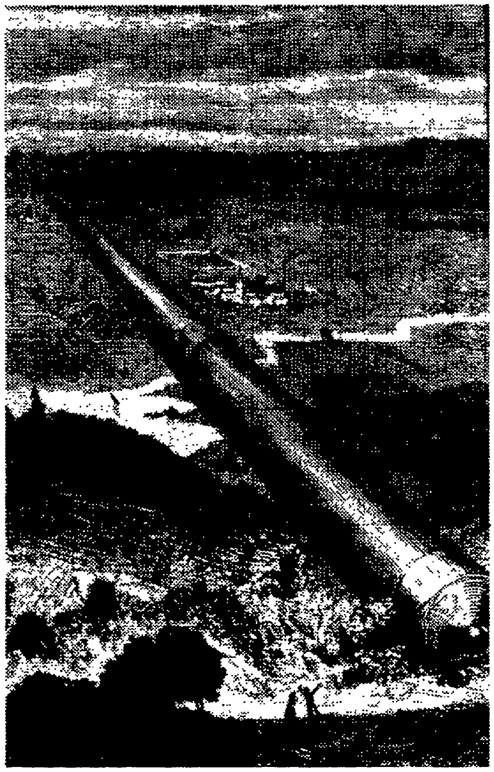


Fig. 13 The Columbiad gun in an illustration of the book by Jules Verne From the Earth to the Moon.

However for a field of forces dependent from the reverse of the square of the distance and for general limit conditions, it is unknown how many impulses are necessary and not even if the solution is impulsive. However Neustadt[92], Potter and Stern[93] demonstrated that the minimum energetic transfer could be obtained through a number of impulses equal to the number of the variables of specified condition in the final orbit. Today the transfer of a probe is resolved with an optimization that foresees the route of a series of conical arcs. The bielliptical case and optimal bielliptical case are represented in the figure.

While this manoeuvre rather complex is the only one that really minimizes, it could be well approximated by a finite number of impulses and by arcs of coasting. This is guaranteed by a theorem of approximation of Neustadt[94] that demonstrated that each solution with an energetic minimum of the problem of Lawden[95,96] could be approximated arbitrarily and with only a finite number of impulses. This result includes the substitution of a same individual arc with impulsive subarcs and coasting.

However Moyer[97] verified that in the transfer between a circular orbit and one described by coplanar hyperbole, if the probe cannot go under the circular original orbit, the escape with an individual impulse is already optimal, also if it there is the practical difficulty of travel in the field of massive objects. In this way if Cossali had approximated to an hyperbole the trip of his stone he would not have fallen in a so irrealistic position, but however he had demonstrated a certain "raw intuition" about problems come into play a century and a half later.

Today the problem is faced for some aspects with a better simplicity: the specification of flightime between two points characterizes the ellipse of connection of the free flight, so the terminal impulsive velocity changes and the two points are univocally definite. Once the available energy is known and the coordinates of the point of departure and of arrive are established, the old Keplero's problem is solved with the solution of the Lambert theorem[98,99,100,101,102] from which the flightime of the body in question is obtained, with the advantage that we take no interest in the type of conical that comes into play.

Returning to the stone investigated by Cossali and travelling toward the Earth, once known the minimum velocity to leave the field of lunar influence it was necessary to calculate the Moon-Earth flightime. If we define xR the route already accomplished, V the velocity in that point of the route, T the departed time, and d(xR)=Rdx as the infinitesimal space of the Moon-Earth distance, dT the infinitesimal increase of necessary time to cross Rdx, from which we have

$$dT = \frac{Rdx}{V} = \frac{Rdx}{\sqrt{v^2 + \frac{\omega x}{N - x} - \frac{\varphi x}{a + x}}} \qquad (11)$$

with ω=2GR/(k-a), φ=2GR/am, N=k-a. Integrating we obtain

$$T = R\int \frac{dx}{\sqrt{v^2 + \frac{\omega x}{N - x} - \frac{\varphi x}{a + x}}} \qquad (12)$$

This is an irrational function that Cossali had resolved with approximation using development in series rather complex (incidentally he filled more than twenty pages of calculuses) that required also the demonstration of a certain number of theorems and of the convergence of the series, calculuses that we don't quote here, being a specific mathematical problem, however out-of-date because other techniques of integration are used today. The result, obtained using the first five termini of the series, was T= 64^h 14^m 26^s. A period of almost three days during which the relative shifts Earth Moon would have brought the stone on a trajectory surely more complex than a simple straight.

A curious case calculated by Cossali was the one of the flightime in the case of a contingent fall of the Moon on the Earth when missing the action of the centrifuge force.

Cossali defines z any point of the trip, so that in a fraction of time dT the Moon crosses a space d(zR) from which

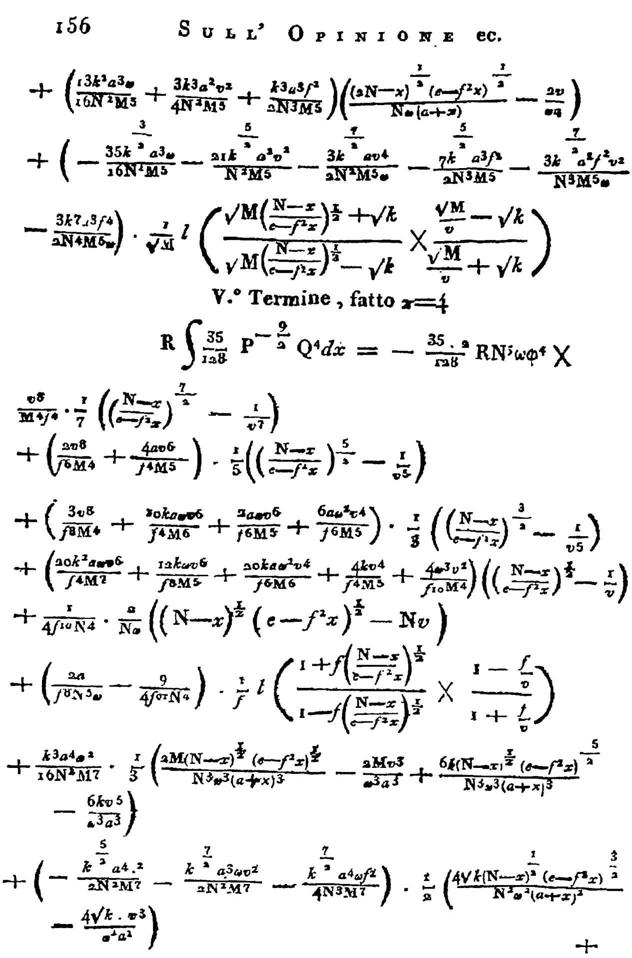
$$dT = \frac{d(zR)}{V} = \frac{Rdz}{V} = \frac{Rdz}{\sqrt{\frac{2GR}{k}\frac{z}{k-z}}} = \frac{R}{\sqrt{\frac{2GR}{k}}}\frac{dz\sqrt{k-z}}{\sqrt{z}} = \frac{R}{\sqrt{\frac{2GR}{k}}}\frac{dz(k-z)}{\sqrt{kz-z^2}} =$$

$$= \frac{R}{\sqrt{\frac{2GR}{k}}}\left[\frac{dz(\frac{k}{2} - z)}{\sqrt{kz - z^2}} + \frac{\frac{kdz}{2}}{\sqrt{kz - z^2}}\right] \qquad (13)$$

that has to be integrated and furnishes T

$$T = \frac{R}{\sqrt{\frac{2GR}{k}}}\left[\sqrt{kz - z^2} + \frac{1}{2}kR\arccos\frac{(\frac{k}{2} - z)R}{\frac{kR}{2}}\right] \qquad (14)$$

In this case the center of the Moon achieves the center of the Earth with a semicircle of ray kR/2 with its center at the half of the straight Lunar center-Earth center. Replacing the relative values we obtain T=115^h 25^m 56^s, always in the hypothesis that the Earth is considered like a fixed point in the space. It is not yet a Homan trajectory, because a correct dynamics resolving the Keplero's equation conducts to conical or cubic that are more complex.


156 Sull' Opinione ec.

V.° Termine, fatto x=4

Fig. 14 A page with the calculuses of the developments in series of the article in question.

A last annotation about the motion of minerals in the space. Some works of Brian O'Leary[103,104] and of Gerard O'Neill[105,106] going back to the years seventies recall the capability to make materials travel in the space. They proposed a linear electromagnetic accelerator for the transport (called mass driver), or better the launch, toward the Earth of some materials to be manipulated, by a human community installed on our satellite. It was necessary to convert electromagnetic energy in kinetic energy accelerating minerals up to mass of 10 kg. Any container in superconducting material had to glide in a guide without attrition exploiting the magnetic levitation, and accelerated by a magnetic field, with an efficiency of the 100% and a feeding of the system due to the solar energy. The study remained at theoretical level, but some small experimental models of sleighs of this type had produced accelerations of 30 g.

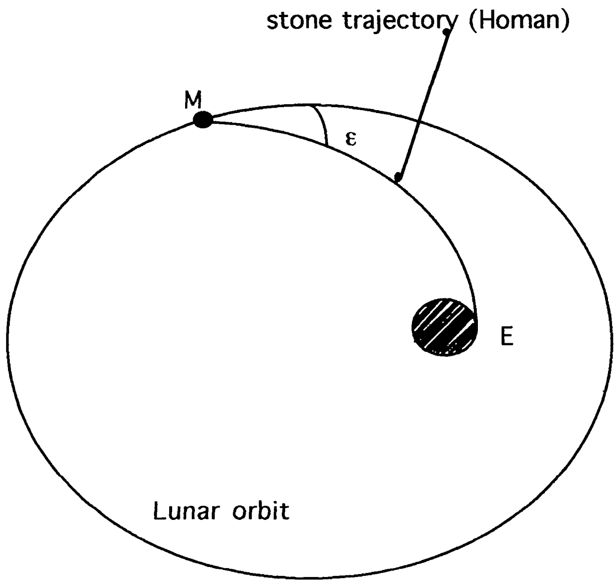


Fig. 15 Realistic Homan transfer trajectory for a body that travels from the Moon up to the terrestrial surface.

Aknowledgements

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