

FULL ADAPTIVE OPTICS IMAGES OF ADS 9731 AND μ CASSIOPEIAE: ORBITS AND MASSES¹

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ABSTRACT

The double double ADS 9731 and the nearby Population II astrometric binary μ Cas have been imaged with full adaptive optics employed at the 1.5 m telescope belonging to the USAF Phillips Laboratory's Starfire Optical Range near Albuquerque, New Mexico. On 1994 May 4 at a wavelength of $0.85 \mu\text{m}$, the Δmag values and separations for ADS 9731 components AB were observed to be 2.19 and $1''.27$, 0.90 and $1''.55$ for CD, and 0.40 and $14''.8$ between A and C. Combining these observations with earlier ones, we have obtained circular, highly inclined, long-period (834 and 1230 yr) preliminary orbits for both pairs and derived their masses along with other astrophysical quantities. However, consideration of stellar evolutionary models leads to a disagreement in the derived distance to the system (108.1 pc) and/or masses, which will perhaps be resolved by subsequent orbital refinement. The first ever real-time images at short wavelengths ($< 1 \mu\text{m}$) of the faint companion to the astrometric binary μ Cas A component were easily made at $0.85 \mu\text{m}$ on 1994 August 28 and October 22, with movement noted between the two dates. The Δmag value was determined to be 4.9 ± 0.1 , and the separation on the two dates was $0''.73$ and $0''.66$. Combining our measurements with astrometric data from both Sproul and Allegheny Observatories, we make a simultaneous fit for a new master orbit and find masses of 0.742 ± 0.059 and $0.173 \pm 0.011 M_{\odot}$ for A and B, respectively. From further analysis we derive a presumably primordial helium abundance for μ Cas of $Y = 0.24 \pm 0.07$ for an assumed age of 10^{10} yr and the latest measured metallicity of $Z = 0.0021$.

Subject headings: astrometry — binaries: visual — infrared: stars — stars: evolution — stars: fundamental parameters — stars: individual (ADS 9731, μ Cassiopeiae)

1. INTRODUCTION

Three close binary star systems have been resolved with adaptive optics (AO) on the 1.5 m telescope at the USAF Phillips Laboratory's Starfire Optical Range (SOR), located on Kirtland AFB, New Mexico, near Albuquerque. Combining our measurements of ADS 9731 ($=\Sigma 1964 = \text{HD } 139691 = \text{SAO } 64821 = \text{BD } + 36^{\circ}2626$) made on 1994 May 4 with previous data from Washington Double Star (WDS) catalog, kindly supplied by C. Worley, we have made the first orbit determinations for both of the two pairs in this system and have made preliminary estimates of their masses. It appears that all four stars may have evolved off the main sequence. On 1994 August 28 and October 22 we resolved the large magnitude difference binary μ Cas. Simultaneously fitting Sproul Observatory's (Lippincott 1981) 29 astrometric data points of the primary with respect to the center of mass, Allegheny's (Russell & Gatewood 1984) 34 astrometric points, and the (now) 12 relative orbit position angles and separations between the primary and secondary, we have determined the latest orbit and individual masses for this nearby Population II binary.

2. OBSERVATIONS

Fugate et al. (1993, 1994) and Fugate (1994) describe the AO system employed at the SOR. Briefly, the system uses a tilt mirror controlled by a 4×4 avalanche photodiode array

operating in a centroid mode or as a quad cell to track the target and provide low-order (tip-tilt) atmospheric compensation. The pupil is reimaged onto a Shack-Hartman sensor which controls a 241 element deformable mirror and provides higher order compensation. For ADS 9731, the brightest A component of V magnitude 7.1 was too faint for high-bandwidth, higher order compensation, so an artificial star, a Rayleigh backscatter laser beacon, was created at a range of 10 km in the direction of the A component by a copper vapor laser. For μ Cas, the Rayleigh beacon was not necessary because the visual magnitude of the A component was 5.1 and of sufficient brightness to run the wave front sensor. Data recorded on a 512×512 Photometrics CCD were obtained at an effective wavelength of $0.846 \mu\text{m}$. Atmospheric characterization, as measured with ADS 9731, is reported by Christou et al. (1995).

Postobservation processing is critical in order to perform photometry on overlapping stars, even with the use of adaptive optics, so for both ADS 9731 and μ Cas we performed iterative blind deconvolution (IBD), as developed by Jefferies & Christou (1993), and Lucy-Richardson deconvolution as found in the IRAF package. Section 4.2 below describes some of the postprocessing for μ Cas, and Christou et al. (1995) describe, in greater detail, the postprocessing involved for both ADS 9731 and μ Cas.

3. ADS 9731

3.1. Orbits

Components (AB) $\times$ (CD) and CD were discovered by Struve (1837) in 1829 ($\Sigma 1964$), who measured a separation of $15''.34$ between (AB) and (CD), and $1''.38$ between C and D. Components A and B (Hu 1167) were not resolved until 1905

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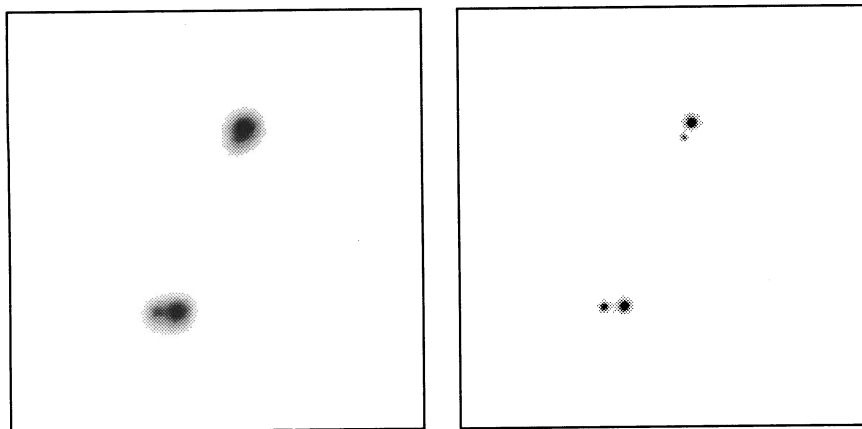


FIG. 1.—Open (*left*) and closed (*right*) loop images of ADS 9731 at $\lambda = 0.85 \mu\text{m}$. Tracking, which removes tip-tilt, was on for both. See Fig. 2 for scale and designations.

by Hussey (1905) when they were $0''.85$ apart. Figure 1 shows our image of the four stars both with and without the higher order adaptive optics turned on (“closed” and “open” loop). In both cases, full aperture tilt compensation was accomplished by tracking on component A. Figure 2 shows a blowup of the fully compensated, closed loop image. Table 1 gives the parameters derived from measuring 12 such 30 s integrations. Orientation and scales were determined from position angles and separations of the binaries ϵ Boo (P.A. = 321° ; separation = $2''.8$) and α CVn (229° ; $19''.4$), and from a known

low-level fixed pattern produced by the deformable mirror. The binaries used for calibration are wide, fixed, or slow-moving pairs, chosen from Hirschfeld & Sinnott (1985). We use predicted positions if an orbit (of very long period) is available. Since the optical setup generally permits Nyquist sampling, the distance between the calibration components on the CCD encompasses many pixels (46 and 309 for ϵ Boo and α CVn, respectively), and the scale is easily and accurately determined. Their position angles provide a zero point for the rotating field of our altitude-azimuth telescope. Calibration quantities (scale

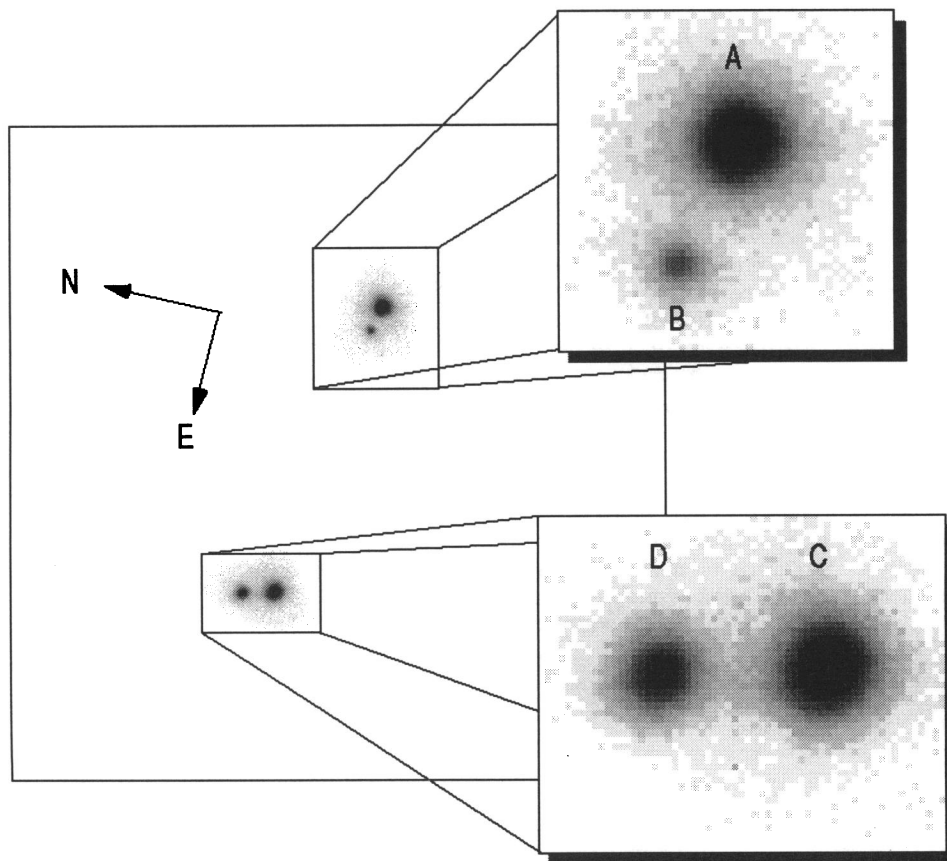


FIG. 2.—ADS 9731, closed loop at $\lambda = 0.85 \mu\text{m}$. Position angles, separations, and Δmag values are $80^\circ.2$, $1''.27$, and 2.19 for AB; $85^\circ.8$, $14''.8$, and 0.40 for AC; and $18^\circ.4$, $1''.55$, and 0.90 for CD.

TABLE 1
ADS 9731 1994 MAY 4, 9.5 UT

Components	Position Angle	Separation	Δmag (0.85 μm)
AB.....	$80^\circ 2 \pm 1^\circ 3$	$1''.27 \pm 0''.02$	2.19 ± 0.09
AC.....	85.8 ± 1.3	14.8 ± 0.24	0.40 ± 0.01
CD.....	18.4 ± 1.3	1.55 ± 0.03	0.90 ± 0.02

and orientation zero points) are means determined from several measurements of more than one binary, and the standard deviation of the mean is propagated through to the final position angle and separation of the objects. The scale adopted for ADS 9731 was $0''.0618 \text{ pixel}^{-1}$ (± 0.0010). Because the laser was pointed at A, atmospheric correction was more effective, and therefore resolution was better, near AB (Gaussian FWHM after postprocessing of $0''.07$) than near CD ($0''.20$). See Christou et al. (1995). Component C has been reported to have a closer companion than D, but we see no signs down to this limit of $0''.20$.

Combining these data points with those in the WDS catalog, and giving every observation equal weight, we have derived the orbital elements given in Table 2 with a nonlinear least-squares program. Following Worley & Heintz (1983; see their Introduction) the Theile-Innes constants are not given since the entire solution, with errors, is obtained at once. We were only able to obtain convergence in the least-squares fit by assuming that the orbits were circular (eccentricity and argument of periastron were zero), and for AB we were forced to ignore the three observations of Walker (1966; 1969) in the mid-1960s in order to obtain a solution. The standard error of fit for the 12 observations of AB is $0''.042$ and $0''.110$ for the 34 observations of CD. Using the definitions for orbit quality classes as given by Worley & Heintz (1983), both orbits should perhaps be considered class 4 (preliminary) because of the short arc of coverage.

Figures 3 and 4 show the computed orbits and observations connected to their predicted positions on the orbit. For comparison to Figures 1 and 2, Figure 5 shows both orbits to scale. Note that both secondaries appear to be near apastron; this is the best time to study the components separately. According to Table 2, AB should show a maximum separation of $1''.25$ within 92 yr of 2022, while CD will only have decreased to $1''.54$ by the year 2000 from its apastron of $1''.58$ in 1955 (± 21 yr).

3.2. Masses

From the orbital periods and semimajor axes of Table 2, we can find the sum of the masses for each pair, provided the distance is known so that the angular semimajor axes, a_r , can be converted to linear semimajor axes, a , in astronomical units. In the absence of a trigonometric parallax, we employ a dynamical parallax, but with the assumption that both pairs are at the same distance. Although there is consider-

TABLE 2
ADS 9731 ORBITAL ELEMENTS

Elements	AB	CD
P (yr).....	834 ± 272	1230 ± 261
T_0	2021.8 ± 91.9	1955.3 ± 21.3
a_r	$1''.249 \pm 0''.100$	$1''.582 \pm 0''.027$
i	$97^\circ 0 \pm 2^\circ 6$	$78^\circ 4 \pm 3^\circ 8$
Ω	$80^\circ 5 \pm 2^\circ 6$	$15^\circ 9 \pm 1^\circ 6$

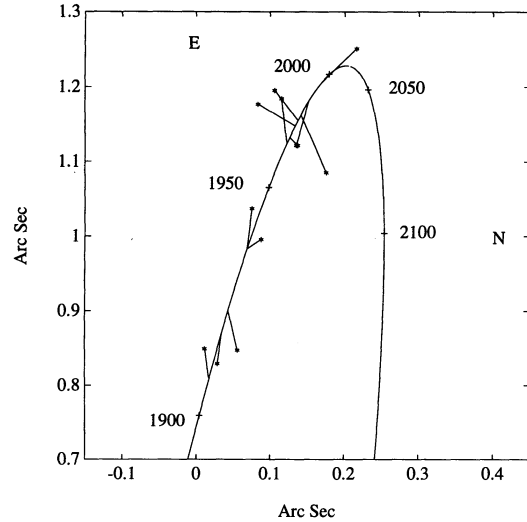


FIG. 3.—Best-fit orbit for ADS 9731 AB. Observed positions are connected to their predicted positions on the orbit.

able scatter, particularly for B, we develop Table 3 from an analysis of data in the WDS catalog.

We prefer the Δmag of 3.5 for AB because the median is more robust than the mean and most of the large magnitude differences were contributed by early investigators when B was closer to A. There is a linear trend of the magnitude of B, or of the Δmag value between A and B, with time. The apparent secular increase in brightness of B of 0.3 mag per decade is most likely due to the increasing separation between A and B. However, there is the possibility that B is variable, since as shown in Figure 6, there is a dramatic drop and rise in Δmag (but without error bars) over 20 yr in modern times.

Simultaneous with the observations at $0.85 \mu\text{m}$, a NICMOS II camera was used by Arcetri Observatory from Florence, Italy (Lisa et al. 1995) to record the closed-loop images of ADS 9731 in the J and H bands. Together with the observations at $0.85 \mu\text{m}$, the component Δmag values at 1.25 and $1.65 \mu\text{m}$, as well as the Δmag from Table 3 at $0.55 \mu\text{m}$, lead to T_{eff} for the

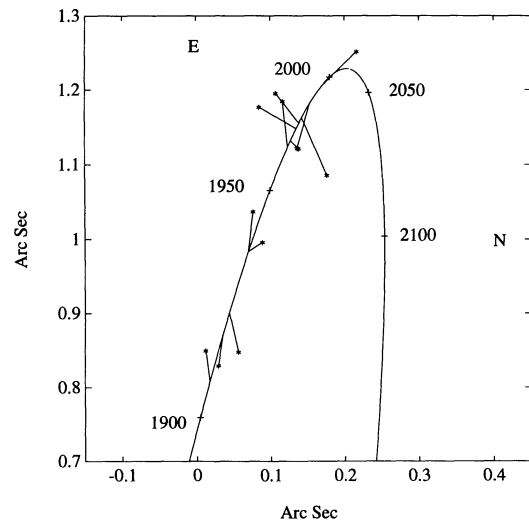


FIG. 4.—Best-fit orbit for ADS 9731 CD. Observed positions are connected to their predicted positions on the orbit.

TABLE 3
A. ADS 9731 APPARENT MAGNITUDES (v) FROM THE WDS CATALOGUE

Parameter	A	AB	B	C	CD	D
ΔV	...	3.79 ± 1.13	...	...	1.08 ± 0.60	...
Median ΔV	...	3.5	...	...	1.3	...
v (mean $\pm$ s.d.)	7.08 ± 0.31	...	11.18 ± 1.33	7.65 ± 0.44	...	8.64 ± 0.43

B. v FOR ADOPTED ΔV

Parameter	$\Delta V = 3.5$	$\Delta V = 1.1$
Adopted v	$7.1 \pm 0.3, 10.6 \pm 1.4$	$7.6 \pm 0.4, 8.7 \pm 0.4$

components based on the HD F5 spectral class of ADS 9731. We assume that the spectral type applies only to component A and that its T_{eff} may range from 6076 to 6540 K appropriate to luminosity classes III to V, and we use the relative instrumental magnitudes of the three remaining components to fit a blackbody curve at the four wavelength points. An initial run revealed that the temperature derived for B was too cool for its absolute magnitude. By not including the uncertain ΔV , we were able to obtain a more reasonable temperature estimate for B.

Table 4 summarizes the data, Table 5 summarizes the results, and Figure 7 shows the derived blackbody curves. Once the apparent magnitude of the component at $0.55 \mu\text{m}$ is determined from the blackbody curve fit, its error in Table 5 depends only on the error in the apparent magnitude of the A component and the standard error of fit at visible wavelengths. The error in T_{eff} from the blackbody fit for each of the components is larger than the range possible if the luminosity class is off by one, so they are adopted as the uncertainty and give rise to the dotted lines around each blackbody curve in Figure 7. For component A, the T_{eff} is chosen for class IV, and the error used covers classes V–III.

The apparent v magnitudes derived from the blackbody fits are very close to the values from Table 3 for all components except B, which is a full magnitude brighter than determined from the literature, indicating a ΔV between A and B of 2.5 mag. With these adopted apparent magnitudes, we can use the

distance modulus to find absolute V magnitudes for trial distances d ,

$$d \text{ (pc)} = 10^{(v-V+5)/5}, \quad (1)$$

which, in turn, gives solar masses (M) from the mass/luminosity relation. We use the following simple third-degree polynomial fit to mass as a function of V for main-sequence and luminosity class III giants according to data in Allen (1983) and/or Zombeck (1990):

$$\log M = a_1 + a_2 V + a_3 V^2 + a_4 V^3, \quad (2)$$

where

$$a_1 = 0.60627, \quad a_2 = -0.14316,$$

$$a_3 = 0.0057159, \quad a_4 = -0.00019308,$$

and the standard error of fit is 0.05.

The sum of the masses for each pair can be compared to the sum of the masses derived from Kepler's third law with the orbital parameters of Table 2:

$$\Sigma M = \frac{[a_r(\text{arcsec})d(\text{pc})]^3}{P(\text{yr})^2}. \quad (3)$$

Figure 8 shows that the intersection of the sum of masses from the two methods occurs at 106.5 pc for AB and at 109.7 for CD. We therefore adopt the distance to ADS 9731 as 108.1 ± 2.3 pc and the sum of the masses (in solar units) from Kepler's law as 3.53 ± 1.53 for AB and 3.31 ± 1.46 for CD. The uncertainties are calculated from propagating the errors in a_r ,

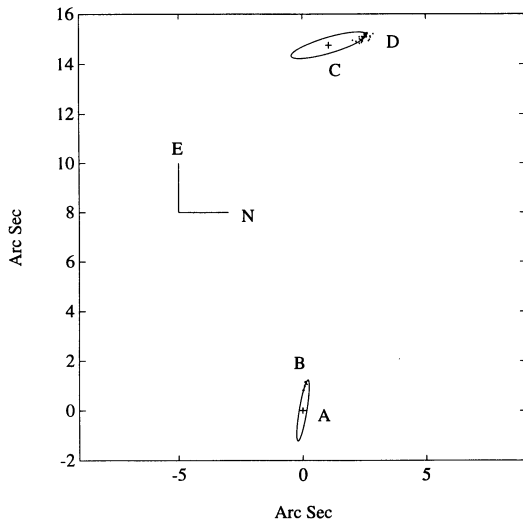


FIG. 5.—Both ADS 9731 orbits are plotted to scale. A and C are separated by 14".8.

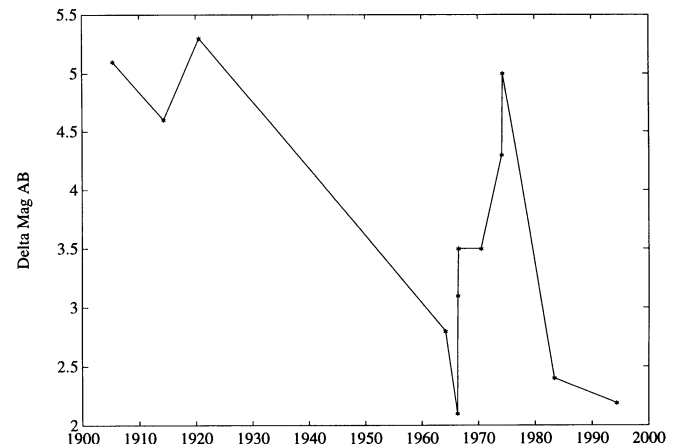


FIG. 6.—Visual magnitude difference between ADS 9731 A and B as reported in the WDS catalog. The latest point is ours, made at a slightly longer wavelength of $0.85 \mu\text{m}$.

TABLE 4
ADS 9731 Δ mag VALUES

λ (μ m)	AB	AC	CD
0.55.....	3.5 ^a $\pm$ 1.4	0.5 $\pm$ 0.5	1.1 $\pm$ 0.6
0.85.....	2.19 $\pm$ 0.09	0.40 $\pm$ 0.01	0.90 $\pm$ 0.02
1.25.....	2.05 $\pm$ 0.08	0.58 $\pm$ 0.01	1.05 $\pm$ 0.05
1.65.....	1.93 $\pm$ 0.05	0.58 $\pm$ 0.08	0.83 $\pm$ 0.04

^a Not used in fit for blackbody temperature.

TABLE 5
ADS 9731 T_{eff}

Component	T_{eff}	Predicted v
A ^a	6324 $\pm$ 248	7.1 $\pm$ 0.30
B	5258 $\pm$ 124	9.59 $\pm$ 0.30
C	6276 $\pm$ 300	7.58 $\pm$ 0.31
D	5824 $\pm$ 510	8.66 $\pm$ 0.33

^a Assumed T_{eff} appropriate for F5 IV with uncertainties covering V to III.

and P in Table 2, as well as their covariance σ_{arP}^2 (25.86 for AB and -1.51 for CD), and the 2.3 pc uncertainty in d , through equation (3):

$$\sigma_{\Sigma M}^2 = (\Sigma M)^2 \left[\left(\frac{3\sigma_{ar}}{a_r} \right)^2 + \left(\frac{3\sigma_d}{d} \right)^2 + \left(\frac{2\sigma_P}{P} \right)^2 - 2 \frac{6\sigma_{arP}^2}{a_r P} \right]. \quad (4)$$

Individual masses of 2.24 ± 0.12 , 1.17 ± 0.05 , 1.96 ± 0.10 , and $1.47 \pm 0.08 M_{\odot}$, as determined from the mass/luminosity relation for A, B, C, and D, respectively (where the errors are determined from contributions by the distance uncertainty and the individual magnitude uncertainties of Table 5), are scaled up to match the sum found from Kepler's law and are given in Table 6. The final mass uncertainties are found by propagating the errors from the individual masses through the scale up (σ')

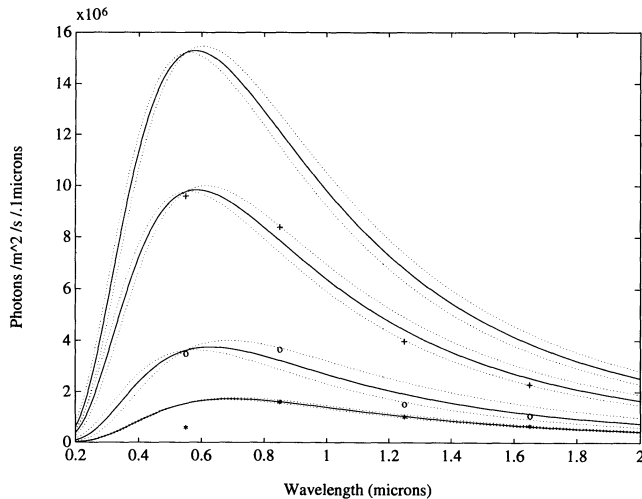


FIG. 7.—Blackbody curves for ADS 9731 components: (top to bottom) A ($T_{\text{eff}} = 6324$ K), C (6276 K), D (5824 K), and B (5258 K). Dotted error lines are from the uncertainty of T_{eff} derived from each fit at the four wavelength points of 0.55, 0.85, 1.25, and 1.65 μ m, except that the 0.55 point was not used for B. The T_{eff} of A, however, is for an assumed F5 IV object, while its error lines cover luminosity classes III to V.

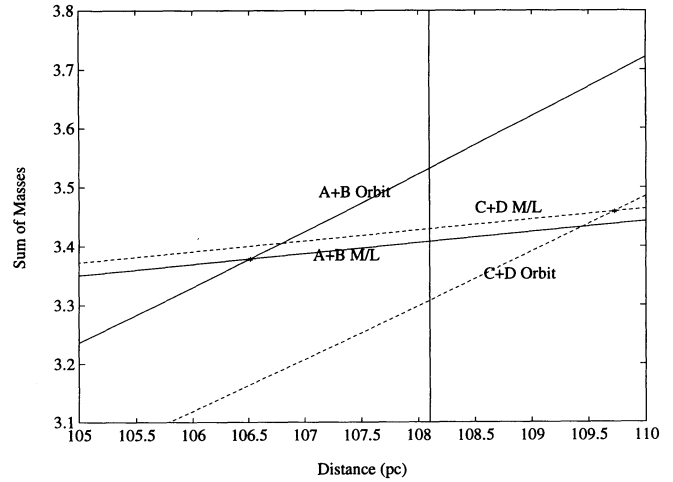


FIG. 8.—Trial distances to ADS 9731 from the sum of the masses using the mass/luminosity relation (M/L) and from Kepler's law using the orbital elements (Orbit) in Table 2. Distances are determined separately for AB (solid lines) and CD (dashed line). The AB pair indicates a distance of 106.5 pc while CD indicates 109.7; we adopt the mean of 108.1 pc.

and then scaling the resulting errors to the error of the sum from Kepler's law. Denoting masses from the mass/luminosity law by small letters a and b , the sum of masses from Kepler's law by ΣM , and final masses by capital letters A and B , the final masses and their uncertainties are given by

$$M_A = M_a \frac{\Sigma M}{(M_a + M_b)}; \quad M_B = M_b \frac{\Sigma M}{(M_a + M_b)}, \quad (5)$$

$$\sigma_{M_A}^2 = M_A^2 \left[\left(\frac{\sigma_{M_a}}{M_a} \right)^2 + \left(\frac{\sigma_{\Sigma M}}{\Sigma M} \right)^2 + \left(\frac{\sigma_{(M_a + M_b)}}{M_a + M_b} \right)^2 \right], \quad (6)$$

$$\sigma_{M_A}^2 = \sigma_{M_A}^2 \frac{\sigma_{\Sigma M}^2}{\sigma_{M_A}^2 + \sigma_{M_B}^2},$$

and similarly for σ_{M_B} .

The main contribution to the error in the individual masses comes from the uncertainty in the sum of the masses, which is large in turn because of the relatively poorly known orbital periods. The uncertainty in the period for pair AB is approximately $\frac{1}{3}$ of the period, and for CD it is $\frac{1}{5}$. Only time will improve the estimates of the masses.

A plot of absolute magnitude (from the distance and the apparent magnitudes derived from the blackbody curves) versus $\log T_{\text{eff}}$ is shown in Figure 9, a version of the H-R diagram with data taken or derived from Allen (1983) and/or Zombeck (1990). Each component is plotted along with error bars from Table 5. Both A and C lie near the evolved F5 line, agreeing with the classification apparently given to both bright components. Components B and D look like G8 and G0, respectively, and also appear to be evolved. Figure 10 is a plot

TABLE 6
ADS 9731 MASSES

Parameter	Value	Parameter	Value
M_A	2.32 $\pm$ 1.36	M_C	1.89 $\pm$ 1.17
M_B	1.21 $\pm$ 0.71	M_D	1.42 $\pm$ 0.88
ΣM	3.53 $\pm$ 1.53	ΣM	3.31 $\pm$ 1.46

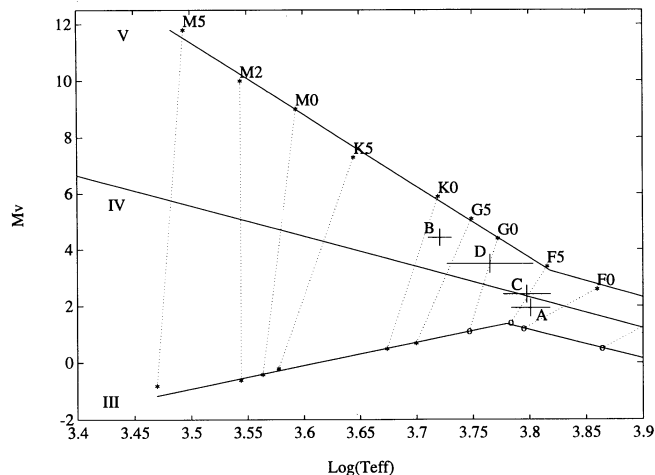


FIG. 9.—Absolute V magnitude as a function of T_{eff} for luminosity classes III, IV, and V. ADS 9731 components plot as having evolved away from the main-sequence (V) line.

of log mass versus log T_{eff} and also supports the notion that all four components are evolved.

However, when comparing these results to the stellar models of Vandenberg (1985), assuming solar abundances, it appears that the four components of ADS 9731 do not fall on a single isochrone. In the absolute bolometric magnitude–log T_{eff} plane, either B, C, and D are brighter than the main sequence by 1.5 mag, all components are overmassive by $\sim 33\%$ for their positions, or all are underluminous by 1 mag for their orbital masses—all inconsistent with isochrones. If the distance to ADS 9731 is halved to 54 pc, then B, C, and D fall along the main sequence, A appears to have left it, and all fall along the 3×10^9 yr isochrone. But at this distance, the orbital mass sums for AB and CD would be 0.45 and 0.42 $M_{\odot}$, respectively, which is obviously unrealistic. In order to preserve the apparent evolutionary sense implied from Vandenberg's models, the orbital periods would have to decrease to 380 and 536 yr, respectively, or the semimajor axes would have to increase to $2''.11$ and $2''.75$, both well outside the range of uncertainty of

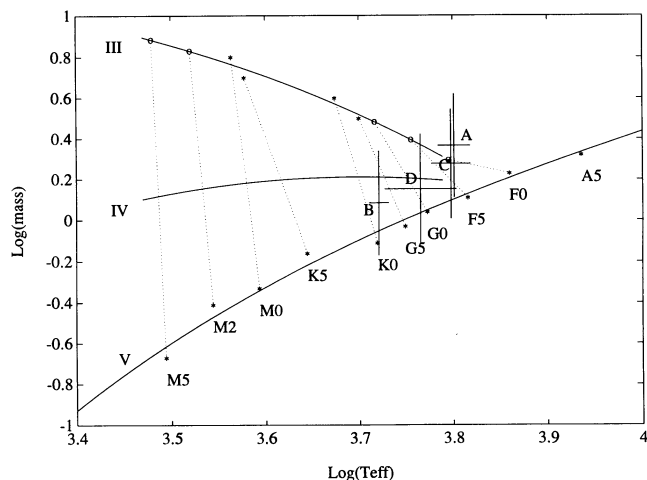


FIG. 10.—Solar masses as a function of T_{eff} . Again, components of ADS 9731 appear to be more massive than main-sequence stars of similar spectral types.

Table 2. Perhaps further study may reveal if this system is peculiar (greater than solar metal content?), or more time may improve the orbits. Handicapped by the short arcs, we were only able to find orbits by assuming that they were circular. If indeed they are not, and if they are currently near apastron, then we should soon start to see the companions accelerate about their primaries, perhaps leading to periods short enough to accommodate modern evolutionary models. At any rate, ADS 9731 is one of the brightest multicomponent evolving systems whose members can be studied separately at this time.

Although the full resolution power of AO is not required to split the components of ADS 9731, this system does help illustrate the other benefit of adaptive optics—more light in separate buckets. The contrast enhancement of close, normally overlapping images allows more efficient studies of objects separated cleanly with adaptive optics.

4. μ CASSIOPEIAE

4.1. Observations

At the suggestion of D McCarthy (private communication; McCarthy et al. 1993) and J. Angel (private communication), we made AO observations of the nearby Population II binary μ Cas, but without the laser, since the A component was bright enough to close the higher order loop, which always provides better performance. We easily detected the faint component at $\lambda = 0.85 \mu\text{m}$ on 1994 August 28 on several closed-loop images of 5–240 s integration times taken around 5:52 and 11:16 UT, but the best unsaturated images were those of less than 15 s. Further processing of these then allows accurate photometry to be performed on the two components as well as better estimates of separations and position angles. When a faint component falls on the halo around its much brighter companion, there is a tendency to underestimate the separation using centroiding or the brightest pixel. We observed μ Cas again on 1994 October 22 at $\lambda = 0.85 \mu\text{m}$ and determined that component B had moved along its orbit.

4.2. Image Analysis and Postprocessing

The quality of the compensated images can be quantified by two parameters, the Strehl ratio and the full width at half-maximum (FWHM). In order to compute the former, we compare the measured point-spread function (PSF) to that of a “perfect” unaberrated PSF through the same pupil at the same wavelength—the Strehl ratio being the ratio of the peak power such that the perfect PSF is at 100%. The FWHM is computed by model-fitting the PSF with two elliptical Gaussians which find, among other parameters, the maximum and its location. The FWHM is then found by linear interpolation. For the observations of 1994 August 28 Figure 11 shows the azimuthally averaged radial profiles for seven PSFs along with their models, Strehl ratios, and FWHMs. The mean FWHM was computed to be $0''.231 \pm 0.010$. Note the trend showing the FWHM to increase as the Strehl ratio decreases with increasing integration time. The correlation between FWHM and Strehl ratio is as expected, and the decrease in image quality with integration time is due to accumulation of tracking errors. Note that at this wavelength the diffraction limit (λ/D) is $0''.117$, which is ~ 2 times smaller than the measured FWHMs.

The difference between a measured and perfect PSF can be better understood by looking at its Fourier transform, i.e., the modulation transfer function (MTF). Radial profiles of the MTFs for the PSFs shown in Figure 11 are presented in Figure

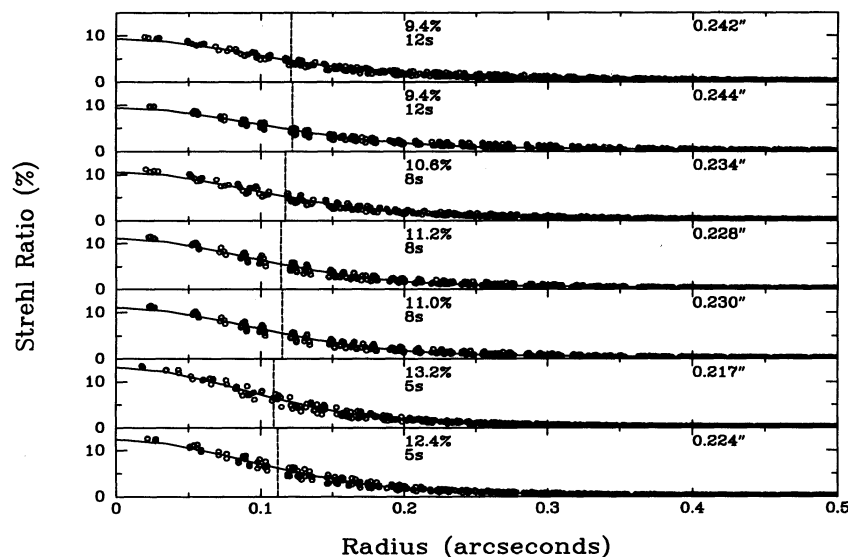


FIG. 11.—Azimuthally averaged radial profiles of seven μ Cas PSFs from 1994 August 28. The solid lines are the profiles for the model fits. The half-width at half-maximum is indicated by the vertical line for each profile, and the value of the FWHM is given on the right-hand side. Between the two, the Strehl ratio (percent) and intergration times (seconds) are displayed.

12. Note that the PSFs contain high spatial frequency power but reduced by ~ 1 – 2 times compared to the true diffraction-limited MTF. Also note that the highest spatial frequency sampled is $\sim 84\%$ of the cutoff frequency (D/λ). The differences between the measured MTFs correspond to the different image FWHMs. The high spatial frequency plateau at spatial frequencies $> 7.2 \text{ arcsec}^{-1}$ is due to the detector noise.

The images at top left (1994 August 28) and bottom left (1994 October 22) in Figure 13 clearly show the secondary, but superposed against a low-level background extending out to large radii, and the low-level fixed waffle pattern produced by the deformable mirror. Both of these make it difficult to extract the relative photometry of the two components without further processing. AO images are characterized by a variability of the PSF, as shown by the radial profiles in Figure 11. Furthermore, closing the loop on a nearby star to use as a reference PSF is not successful because the PSF is also quite object and time (note the radial profiles in Fig. 11) dependent. Therefore, in order to deconvolve the AO images, we applied an algorithm developed by Jefferies & Christou (1993) known as IBD,

which assumes that both the object and PSF are unknown. The algorithm makes use of positivity and finite support constraints as well as band-limit constraints to solve for both unknowns. The use of multiple frames is also a major constraint as the algorithm solves for a single object and n PSFs simultaneously. The greater the difference in the PSFs, the better the object solution. A full description of the algorithm and its applications can be found in Jefferies & Christou (1993) and Christou et al. (1995).

Seven images for the August observations, and 10 for October, were reduced by this algorithm, using their average as the initial object estimate and a Gaussian PSF fit for initial PSF estimates. The object estimate and final IBD result are shown in Figure 13. Note the removal of the waffle and low-level structure isolating the two components. The “ringing” is due to the application of the bandpass limit constraint which solves for the PSF out to the highest spatial frequency sampled by the pupil, corresponding to $\sim 7 \text{ arcsec}^{-1}$ compared to the pupil’s cutoff frequency of 8.6 arcsec^{-1} . Further processing with the Lucy-Richardson algorithm (L-R) removes the ringing.

Magnitude difference estimates were obtained by aperture photometry on the IBD and IBD/L-R images shown in Figure 13. These Δmag values for August 28 of 4.9 and 5.1 yield a mean of 5.0 ± 0.1 at $\lambda = 0.85 \mu\text{m}$, consistent with previous estimates from speckle interferometry. We note that the companion is ~ 5 times brighter than residual noise in the reconstructions. On 1994 October 22, we obtained Δmag values at $0.85 \mu\text{m}$ of 4.7 and 4.9, for a mean of 4.8 ± 0.1 .

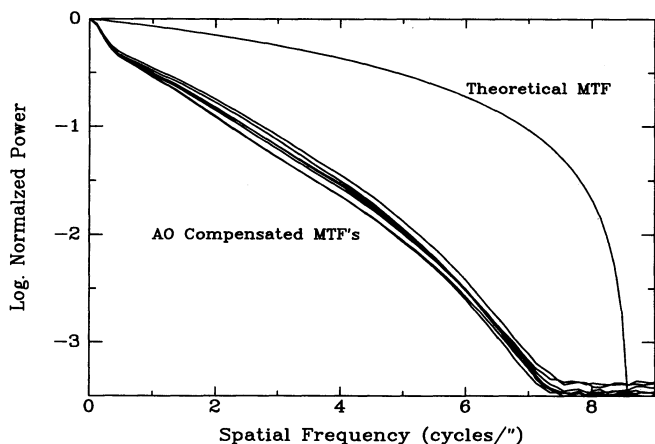


FIG. 12.—Azimuthally averaged radial profiles of the seven MTFs of August compared to the theoretical MTF.

4.3. μ Cassiopeiae Astrometric and Relative Orbits

Table 7 adds our two position angles, separations, and Δmag values for 1994 August 28 (8:34 UT) and October 22 (4:50 UT) to the six other detections of component B accepted and used by McCarthy et al. (1993) plus the four detections recently reported by Haywood, Hegyi, & Gudehus (1992) from their speckle interferometry. Binaries γ Ari ($0^\circ; 7''.8$) and ξ Cep ($275^\circ; 8''.1$) were used to set the orientation and scale ($0''.0348 \text{ pixel}^{-1} \pm 0.0001$) of our images on August 28. For the obser-

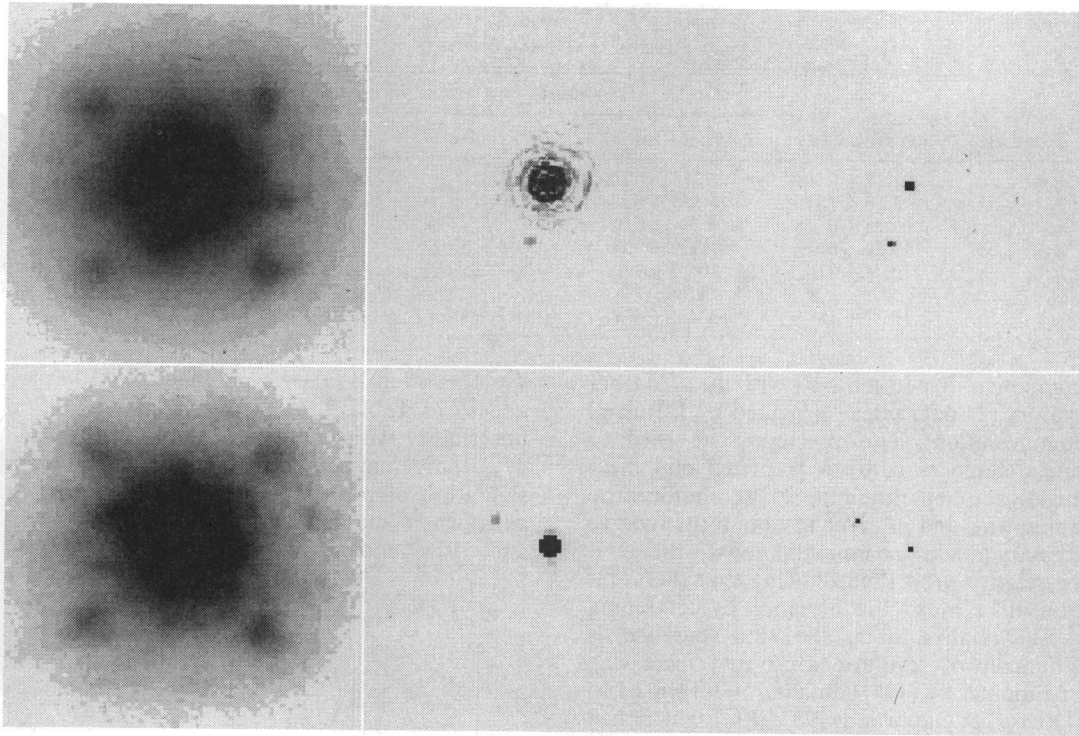


FIG. 13.—Adaptive optics images of μ Cas for 1994 August 28 (*top*) and 1994 October 22 (*bottom*). Component B is located $162^{\circ}.4$ counterclockwise (east) from up, centered on A, in the top panel, and north is at $133^{\circ}.9$. On October 22, B is $63^{\circ}.9$ counterclockwise from up, and north is at $36^{\circ}.9$. The orientation of north and east changes between the two dates because of the altazimuth mount. From left to right, the frames are the average of the several AO closed loop images used as the initial object estimate (in addition to the secondary, note the presence of the waffle and other low-level extended structure), the IBD result (which removes the extended structure), and a Lucy-Richardson deconvolution of the IBD result (which reduces the ringing).

uations on October 22, the scale was determined from the diffraction pattern produced by the deformable mirror at three wavelengths and the orientation from 8 Mon (27° ; $13''.4$), 59 And (35° ; $16''.6$), and 2 Pup (339° ; $16''.8$).

Photocentric orbits for μ Cas (component A about the center of mass) have been derived from data from Sproul Observatory (Lippincott 1981) and from Allegheny Observatory (Russell & Gatewood 1984). Since component B was unseen, it was known to be faint and small. McCarthy et al. (1993) derived a relative orbit of the faint component B around A after detecting B in 1983 (McCarthy 1984), 1990, and 1991

with speckle interferometry in the infrared and combining their data with three other detections (see Table 7). We compute a new orbit based on the now 12 detections of B and list the elements in Table 8; the standard error of fit is $0''.062$.

However, rather than independently determine the ratio of the relative semimajor axis to the photocentric semimajor axis at the time of each of the 12 detections of the secondary (which gives the ratio of the sum of the masses to the smaller mass), we prefer to make a simultaneous fit for both the relative and photocentric orbits from the data of Sproul Observatory (29 individual positions given by Lippincott), combined with the

TABLE 7
DETECTIONS OF μ CASSIOPEIAE B

Date	P.A.	Separation	Δmag	λ (μm)	Reference
1973.789	$24^{\circ} \pm 3^{\circ}$	$0''.35 \pm 0''.04$	5.5 ± 0.7	0.55	1
1974.650	333.9 ± 2	0.23 ± 0.01	4.5 ± 1.0	0.75	2
1983.2	55.8 ± 0.7	0.98 ± 0.024	3.2 ± 0.1	3.4	3
1983.2	...	...	3.5 ± 0.1	2.2	3
1983.7072	61.9 ± 3.0	1.074 ± 0.042	5.4 ± 0.2	0.810	4
1984.13	63 ± 2	1.118 ± 0.023	4.9 ± 0.2	0.85	5
1984.9132	59.3 ± 1.3	1.251 ± 0.030	5.1 ± 0.2	0.810	4
1985.0842	60.6 ± 1.1	1.320 ± 0.027	5.1 ± 0.2	0.810	4
1985.8448	59.47 ± 0.51	1.425 ± 0.016	4.8 ± 0.1	0.858	4
1990.69	48.2 ± 3.1	1.36 ± 0.076	4.3 ± 0.25	1.65	6
1991.73	48.6 ± 6.8	1.41 ± 0.051	4.2 ± 0.25	1.25	6
1994.66	28.5 ± 0.7	0.73 ± 0.02	5.0 ± 0.1	0.846	7
1994.81	27.0 ± 0.8	0.66 ± 0.02	4.8 ± 0.1	0.846	7

REFERENCES.—(1) Wickes & Dicke 1994; (2) Wickes 1975; (3) McCarthy 1984; (4) Haywood et al. 1992; (5) Karovska et al. 1986; (6) McCarthy et al. 1993; (7) This work.

TABLE 8
μ CASSIOPEIAE ORBITAL ELEMENTS

ELEMENTS	RELATIVE ONLY	LIPPINCOTT		RUSSELL & GATEWOOD	
		Photocentric	Relative	Photocentric	Relative
<i>P</i> (yr).....	21.97 ± 0.30	21.715 ± 0.100	...	21.855 ± 0.100	...
<i>T</i> ₀	1977.20 ± 1.12	1975.855 ± 0.229	...	1975.938 ± 0.241	...
<i>e</i>	0.425 ± 0.048	0.544 ± 0.027	...	0.525 ± 0.025	...
<i>a</i>	1".084 ± 0".056	0".1979 ± 0".0074	1".018 ± 0".022	0".1879 ± 0".0067	1".024 ± 0".021
<i>i</i>	103°.4 ± 2°.1	106°.2 ± 1°.1	...	106°.0 ± 1°.1	...
<i>Ω</i>	52°.1 ± 2°.8	47°.8 ± 1°.4	...	48°.4 ± 1°.3	...
<i>ω</i>	176°.7 ± 18°.7	334°.3 ± 4°.6	154°.3 ± 4°.6	336°.3 ± 4°.7	156°.3 ± 4°.7

12 detections of component B, and similarly with the data from Allegheny Observatory (34 data points provided by J. Russell in a private communication). This precludes the need to separately evaluate differences between predicted and measured position angles and separation ratios. A photocentric orbit has seven unknowns, and in order to obtain the relative orbit it is only necessary to add one more unknown—the semimajor axis for the relative orbit (but allowing that the arguments of periastron differ by π). Furthermore, by combining the astrometric and relative data, the time baseline is lengthened—the number of revolutions covered increasing from 1.9 to 2.6 for Lippincott's (1981) data (L) and from 2.1 to 2.8 for the data of Russell & Gatewood (1984) (RG), which then gives a better handle on the period and epoch of periastron passage.

We give equal weight to each astrometric measurement and to each measurement of B. In effect, this makes the astrometric measurements more important than the detections of B—by 29:12 (L) and 34:12 (RG)—but in our opinion it would be difficult to justify any other treatment. The results for these preferred nonlinear least-squares fits are given in Table 8; the standard error of fit for the 12 relative points combined with the 29 data points of Lippincott (1981) is 0".035, and for the 12 points and the 34 Russell & Gatewood (1984) points it is 0".036.

Finally, we combine all 75 points into one least-squares fit for a master orbit. The standard error of fit for the orbit given in Table 9 is 0".030. Figure 14 shows both the photocentric and relative orbits from this simultaneous fit and the data connected to their predicted positions.

4.4. μ Cassiopeiae Masses

To derive masses from the semimajor axes and period in Table 9, we find the weighted mean of the parallaxes (π) from Sproul and Allegheny Observatories (0.132 ± 0.002 and 0.1368 ± 0.0033 , respectively), $\pi = 0.1333 \pm 0.0017$, where the uncertainty is the error of the mean and is derived from the individual observatory's uncertainties. The distance, then, is

TABLE 9
μ CASSIOPEIAE MASTER ORBIT

Elements	Photocentric	Relative
<i>P</i> (yr).....	21.753 ± 0.069	...
<i>T</i> ₀	1975.738 ± 0.145	...
<i>e</i>	0.561 ± 0.017	...
<i>a</i>	0".1908 ± 0".0043	1".009 ± 0".016
<i>i</i>	106°.8 ± 0°.9	...
<i>Ω</i>	47°.3 ± 1°.0	...
<i>ω</i>	332°.7 ± 3°.1	152°.7 ± 3°.1
π	0".1333 ± 0".0017	...

$d = 1/\pi = 7.502(\pm 0.096)$ pc which, together with the period and a_r of Table 9, leads to the sum of the masses and its uncertainty with equations (3) and (4): $\Sigma M = 0.916 \pm 0.058 M_\odot$. Individual component masses can be found from a consideration of both the relative and photocentric semimajor axes (where the covariances are $\sigma_{a_r P}^2 = -2.4 \times 10^{-5}$, $\sigma_{a_r a_p}^2 = 2.6 \times 10^{-5}$, and $\sigma_{a_p P}^2 = -1.6 \times 10^{-5}$):

$$M_B = d^3 \left(\frac{a_r}{P} \right)^2 a_p; \quad M_A = \Sigma M - M_B, \quad (7)$$

$$\left\{ \begin{aligned} \sigma_{M_B}^2 &= M_B^2 \left[\left(\frac{3\sigma_d}{d} \right)^2 + \left(\frac{2\sigma_P}{P} \right)^2 + \left(\frac{2\sigma_{a_r}}{a_r} \right)^2 + \left(\frac{\sigma_{a_p}}{a_p} \right)^2 \right] \\ &\quad + 4 \frac{\sigma_{a_r a_p}^2}{a_r a_p} - 4 \frac{\sigma_{a_p P}^2}{a_p P} - 8 \frac{\sigma_{a_r P}^2}{a_r P} \end{aligned} \right\}, \quad (8)$$

$$\sigma_{M_A}^2 = \sigma_{\Sigma M}^2 + \sigma_{M_B}^2$$

Table 10 gives the final individual component masses from the fits with the data from the individual observatories (Table 8) and for the master orbit (Table 9). The largest contribution to the uncertainty in the sum of the masses for the master orbit comes from the uncertainty in the relative semimajor axis, whereas the largest contribution to the error in determining the mass of component B is the uncertainty in the distance.

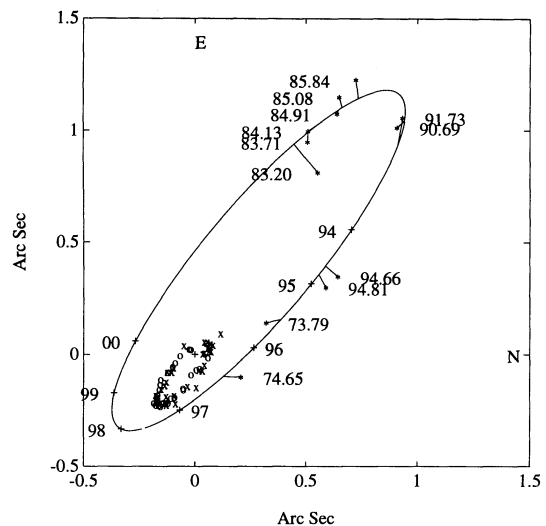


FIG. 14.—μ Cas photocentric and relative orbits from a simultaneous fit of 12 detections of B (asterisks), 29 Sproul Observatory points (circles), and 34 Allegheny Observatory points (crosses). B positions are connected to their predicted points on the master orbit. Predicted B positions with respect to A (plus sign at 0, 0) for 1994–2000 are shown on the orbit with plus signs.

TABLE 10
 μ CASSIOPEIAE MASSES

Masses ($M_{\odot}$)	L	RG	Combined
M_A	0.783 ± 0.080	0.718 ± 0.086	0.742 ± 0.059
M_B	0.189 ± 0.015	0.161 ± 0.015	0.173 ± 0.011
ΣM	0.972 ± 0.079	0.879 ± 0.084	0.916 ± 0.058

Using the Δmag values of Table 7, we fitted (weighted) for a blackbody temperature for component B, using an apparent magnitude at $0.55 \mu\text{m}$ of 5.14 ± 0.02 and a T_{eff} of 5260 K for A as determined by Carney & Aaronson (1979). The data from Haywood et al. (1992) are combined to give $\Delta\text{mag} = 5.10 \pm 0.16$ at $0.81 \mu\text{m}$ and $\Delta\text{mag} = 4.88 \pm 0.11$ at $0.858 \mu\text{m}$. Our Δmag value at $0.846 \mu\text{m}$ is 4.9 ± 0.1 . The blackbody fit is shown in Figure 15 for a T_{eff} of 3025 K (± 78), which is in excellent agreement with the $3034 \pm 160 \text{ K}$ found by McCarthy et al. (1993) and the $2977 \pm 42 \text{ K}$ by Haywood et al. (1992). Notice that the curve is well below the point at $0.55 \mu\text{m}$ measured by Wickes & Dicke (1974) and indicates that at V the actual apparent magnitude of the B component is 11.45 ± 0.35 , a Δmag of 6.31 from A, and agrees with the findings of Karovska, Nisenson, & Stachnik (1986) ($\Delta V = 7$), McCarthy et al. ($\Delta V = 6.5$), and Haywood et al. ($\Delta V = 6.5$). We use this magnitude and plot, as we did for ADS 9731, the components of μ Cas on a modified H-R diagram (Fig. 16) and on a plot of mass versus T_{eff} (Fig. 17). Both figures show that they are on or slightly below the main sequence for Population I stars. Notice in Figure 16 that component A, a G5 VI spectral type, has an absolute visual magnitude and temperature of a Population I K0 V star, while component B is cooler than an M5 V star but would be classified by Henry, Kirkpatrick, & Simons (1994) as an M3.5 dwarf in comparison to other nearby M stars. (The error bars for the temperature of the A component arise from the 125 K difference between our independent blackbody fit of the data from Carney & Aaronson 1979 and their temperature.)

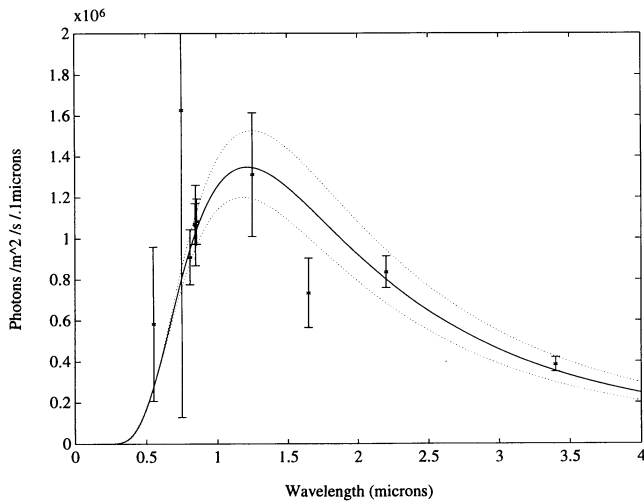


FIG. 15.—Same as Fig. 7, but blackbody curve for μ Cas B. The T_{eff} of $3025 \pm 78 \text{ K}$ is derived from the Δmag values of Table 7 and the 5260 K temperature for A from Carney & Aaronson (1979).

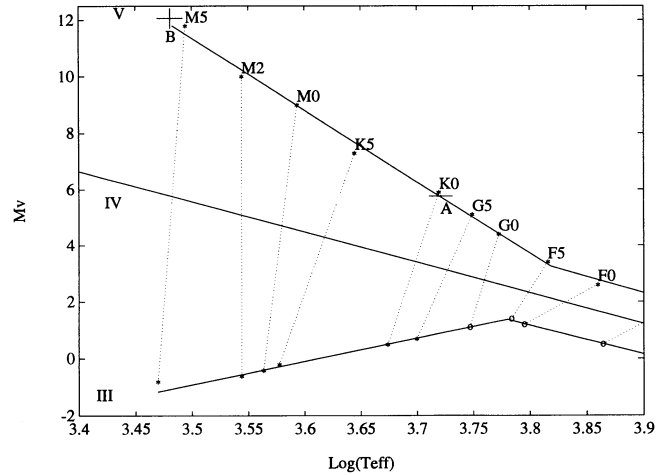


FIG. 16.—Same as Fig. 9, but for the components of μ Cas. Both components lie on the Population I main sequence.

5. LUMINOSITIES AND RADII

Except for the use of the rather tight mass/luminosity relation on ADS 9731, all of the parameters derived thus far have been from directly observable variables and so represent fundamental quantities. In order to derive other quantities it is necessary to resort to relations that are less certain than the mass/luminosity law. There are several ways to make bolometric corrections, but from Allen (1983) we use

$$BC = -42.54 + 10 \log T_{\text{eff}} + \frac{29000}{T_{\text{eff}}} \quad (9)$$

to correct the absolute V magnitude of each star to its absolute bolometric magnitude, and we then find its luminosity and radius in solar units from the Stefan-Boltzmann relation,

$$R^2 = \left(\frac{5780}{T_{\text{eff}}} \right)^4 10^{-0.4(V - BC - 4.7)}, \quad (10)$$

where the second term is the luminosity of the star. For the A component of μ Cas, however, we use the bolometric correction of 0.22 found by Carney & Aaronson (1979).

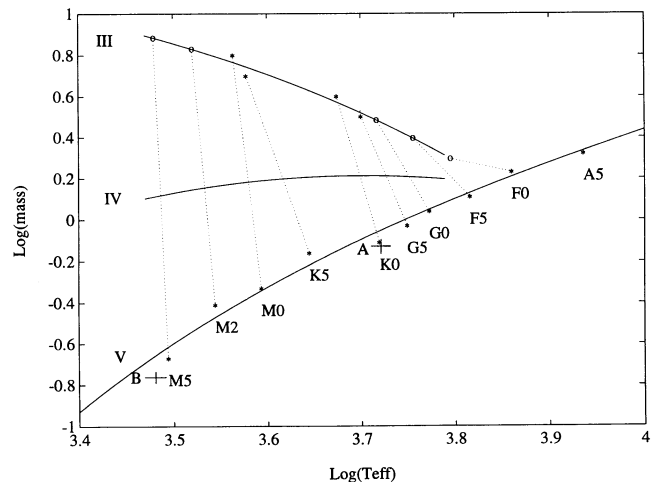


FIG. 17.—Same as Fig. 10, but for the components of μ Cas. Both components lie very slightly below the main sequence.

TABLE 11

LUMINOSITIES, BOLOMETRIC MAGNITUDES, AND RADII

Component	$L_{\odot}$	M_b	$R_{\odot}$
ADS 9731 A	13.49 ± 3.77	1.88 ± 0.30	3.07 ± 0.49
ADS 9731 B	1.53 ± 0.43	4.24 ± 0.30	1.50 ± 0.22
ADS 9731 C	8.68 ± 2.43	2.35 ± 0.30	2.50 ± 0.42
ADS 9731 D	3.31 ± 0.94	3.40 ± 0.31	1.79 ± 0.40
μ Cas A	0.46 ± 0.02	5.54 ± 0.04	0.82 ± 0.04
μ Cas B	0.0062 ± 0.0008	10.22 ± 0.14	0.29 ± 0.02

Errors are obtained from propagation of the uncertainties in T_{eff} and the absolute bolometric magnitudes through equations (9) and (10). Once the apparent magnitude of the component at $0.55 \mu\text{m}$ is determined from the blackbody curve fit, its absolute bolometric magnitude error depends only on the error in the apparent magnitude of the A component, the uncertainty in the temperature derived from the blackbody fit, and the magnitude uncertainty due to the distance uncertainty.

Table 11 contains the luminosities and radii for ADS 9731 and μ Cas. Overall, the results for μ Cas are not significantly different from Karovska et al. (1986), McCarthy et al. (1993), or Haywood et al. (1992), but by including all the astrometry and including new observations, we have produced a more uniform solution. Using the mass/luminosity law as expressed by equation (2), the A and B components of μ Cas are either overluminous (by ~ 1 and 1.5 mag, respectively, at $\lambda = 0.55 \mu\text{m}$) for their masses or are undermassive (by $\sim 80\%$ and 70%) for their luminosities (or some combination thereof), at least in comparison to Population I stars.

6. HELIUM CONTENT AND AGE OF μ CASSIOPEIAE

Closely following the arguments of Haywood et al. (1992), we attempt to find the age of μ Cas from the presumably primordial helium content (Y), mass, luminosity, and metal content of its components, as Dennis (1965) first suggested. The metal content (Z) is assumed to be the same for B as measured for component A: $Z = 0.0041 \pm 0.0008$ from Catchpole, Pagel, & Powell (1967), Cohen (1968), and Hearnshaw (1974), or $Z = 0.0021 \pm 0.0005$ from later determinations by Perrin et al. (1977) and Tomkin & Lambert (1980). For its mass and M_b , however, A appears to require a higher metallicity, close to solar values, from the plots of Vandenberg, than measured, or else, for the measured Z values μ Cas A is too cool by about 500 K. Interpolating the low-mass stellar model tables of Rappaport & Joss (1984) over mass, metallicity (Z), and log luminosity, but not T_{eff} which runs 500–1000 K hotter than μ Cas B for the range of masses considered, we find a helium mass fraction for component B of $Y = 0.11 \pm 0.16$ for $Z = 0.0041$ or $Y = 0.09 \pm 0.16$ for $Z = 0.0021$. Applying these Y values to the stellar models of Vandenberg (1985) for component A, with the interpolation formula provided by Haywood et al. (1992), we then find from component A that the age of the system is $4.1 \pm 5.2 \times 10^{10}$ or $4.6 \pm 5.8 \times 10^{10}$ yr for $Z = 0.0021$ and 0.0041 , respectively.

Obviously, the age calculations are virtually meaningless due primarily to the large uncertainty in the determinations of Y . If the uncertainties in the mass, luminosity, and Z of component B were each reduced by a factor of 12 then a reduction to a fractional error in Y of $\sim 10\%$ would result. The prospects for such improvement appear poor. Inverting the problem, however, the helium content of the system can be deduced

TABLE 12

HELIUM CONTENT OF μ CASSIOPEIAE A

Age ($\times 10^9$ yr)	$Z = 0.0041$	$Z = 0.0021$
12	0.262 ± 0.061	0.231 ± 0.067
10	0.274 ± 0.060	0.244 ± 0.065
8	0.287 ± 0.058	0.258 ± 0.064

rather accurately for a given age, mass, luminosity, and Z . Table 12 suggest a slightly lower primordial helium content than derived by Haywood et al. (1992) due primarily to the larger system mass that we find.

7. SUMMARY AND CONCLUSION

The results from AO imaging of the quadruple system ADS 9731 and the nearby Population II astrometric binary μ Cas have been presented. Postprocessing of the AO images was necessary in order to minimize the PSF contamination of the close components so that photometry could be performed. AO imaging is not characterized by a stable PSF; unfortunately, its variability depends on atmospheric parameters, e.g., atmospheric coherence length and coherence time, and on the object itself. In other words, PSFs show temporal, spatial, and object dependence.

Application of a blind deconvolution algorithm, which solves simultaneously for both the object and PSF, followed by further processing with a L-R algorithm, separates the overlapping PSFs into distinct components, demonstrating that AO is a viable technique for the measurement of very close stellar systems. The binaries studied here are of two very different types. The binary μ Cas is a case of extreme magnitude difference at a separation much smaller than the seeing disk. With AO we have obtained the first direct real time images of the secondary at $\lambda < 1 \mu\text{m}$ (previous detections have been through interferometry or at $\lambda > 1 \mu\text{m}$). Based on this example, we estimate that we should be able to clearly detect a companion of $\Delta\text{mag} \sim 6\text{--}7$ at I -band wavelengths at these separations of $0''.7$. Observations of ADS 9731, with smaller Δmag values and separations greater than the seeing disk, illustrate that a significant gain in signal-to-noise ratio can be accomplished with adaptive optics and postprocessing. For example, ADS 9731 A showed an increase in Strehl ratio of ~ 10 times, or 2.5 mag.

By combining our position angles and separations of ADS 9731 with others, we have derived preliminary orbits and masses for the two pairs. The distance to this system derived from dynamical parallax is twice the distance derived from main-sequence fitting using the stellar models of Vandenberg (1985). This implies that the orbital periods must be shorter by 2 to 3 times the uncertainty derived here, which will be manifested in the next 50–100 yr by increasing orbital motion. Thus, this system bears further study and will be useful in testing many aspects of stellar evolution. Should the orbital period and dynamical distance turn out to be correct, then the system of four stars appears to be overluminous by 1–1.5 mag.

We have demonstrated that although it is not possible to derive a reliable estimate for the age of the Population II stars comprising μ Cas, it is possible to obtain a good estimate of the presumably primordial helium content of the binary from its brighter component. Assuming an age of 10^{10} yr and a metal content of $Z = 0.0021$, we find a helium content for the A component of $Y = 0.244 \pm 0.065$ contrasted with the more

uncertain $Y = 0.09 \pm 0.16$ for the B component found from the models of Rappaport & Joss (1984). By performing a simultaneous fit to the astrometric data from two different observatories and from the 12 detections of the faint component we have made a good determination of the individual component masses of 0.742 ± 0.059 and $0.173 \pm 0.011 M_{\odot}$, respectively. These measurements should prove helpful in refining relationships among various stellar parameters, with questions remaining about the apparent ~ 1 mag overluminosity of μ Cas.

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