

# The $\delta$ Scuti star FG Virginis

## I. Multiple pulsation frequencies determined with a combined DSN/WET campaign

M. Breger<sup>1</sup>, G. Handler<sup>1</sup>, R. E. Nather<sup>2</sup>, D. E. Winget<sup>2</sup>, S. J. Kleinman<sup>2</sup>, D. J. Sullivan<sup>3</sup>, Li Zhi-ping<sup>4</sup>, J. E. Solheim<sup>5</sup>, Jiang Shi-yang<sup>4</sup>, Liu Zong-li<sup>4</sup>, M. A. Wood<sup>6</sup>, T. K. Watson<sup>2</sup>, W. A. Dziembowski<sup>7</sup>, E. Serkowitsch<sup>1</sup>, H. Mendelson<sup>8</sup>, J. C. Clemens<sup>9</sup>, J. Krzesinski<sup>10</sup>, and G. Pajdosz<sup>10</sup>

<sup>1</sup> Astronomisches Institut der Universität Wien, Türkenschanzstr. 17, A-1180 Wien, Austria

INTERNET: breger@astro.ast.univie.ac.at

<sup>2</sup> Astronomy Department, University of Texas at Austin, Austin, TX 78712, USA

<sup>3</sup> Physics Department, Victoria University of Wellington, P. O. Box 600 Wellington, New Zealand

<sup>4</sup> Beijing Observatory, Academy of Sciences, Beijing, China

<sup>5</sup> Institute of Mathematical and Physical Sciences, University of Tromsø, N-9037 Tromsø, Norway

<sup>6</sup> Dept. of Physics and Space Sciences, Florida Institute of Technology, 150 W. University Blvd., Melbourne, FL 32901-6988, USA

<sup>7</sup> Copernicus Astronomical Center, Bartycka 18, PL-00-716 Warsaw, Poland

<sup>8</sup> Wise Observatory, Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

<sup>9</sup> Dept. of Physics and Astronomy, Iowa State University, Ames, IA 50211, USA

<sup>10</sup> Mt. Suhora Observatory, Cracow Pedagogical University, PL-30-083 Cracow, Poland

Received 14 September 1994 / Accepted 16 October 1994

**Abstract.** A coordinated photometric campaign of FG Vir at nine observatories covering 170 hours was undertaken by DSN (Delta Scuti Network) and WET (Whole Earth Telescope). Two different observing techniques were adopted for the two telescope networks in order to optimize different frequency ranges.

Ten pulsation frequencies between 9.19 and 34.12 c/d (112 and 395  $\mu$ Hz) were detected with amplitudes ranging from 0.8 to 22 mmag. Pulsational instability is observed only in specific frequency regions. Additional frequencies of pulsation within these regions probably exist, but do not reach the significance criterion of amplitude signal/noise adopted by us. Comparisons with previously obtained data show that the amplitudes of the main frequencies are stable over a year or longer.

A preliminary identification of the ten dominant frequencies is proposed in a stellar model with  $1.8M_{\odot}$  in advanced main-sequence phase of evolution. The frequencies correspond to low order p and g modes with  $\ell \leq 2$  and radial order 1 to 6. According to the linear nonadiabatic calculations, the identified modes are driven by the opacity mechanism along with many other modes. For asteroseismology of  $\delta$  Scuti stars, FG Vir is an extremely important candidate, especially because of the probable presence of g modes.

**Key words:**  $\delta$  Scu – stars: oscillations – stars: individual: HD 106384=FG Vir – techniques: photometric

Send offprint requests to: M. Breger

### 1. Introduction

The  $\delta$  Scuti stars are pulsating variables situated inside the classical Instability Strip on and above the main sequence. A number of extensive observing campaigns covering individual  $\delta$  Scuti stars have shown that the majority pulsates with a large number of simultaneously excited modes. While the variables with small rotational velocities tend to be radial pulsators with large amplitudes, the vast majority of the  $\delta$  Scuti stars pulsate nonradially with a multitude of small-amplitude p modes. Photometrically, low-degree ( $\ell \leq 3$ ) and low-order ( $n = 0$  to 4) modes are commonly seen. A good example is the star  $\theta^2$  Tau (Breger et al. 1989). On the other hand, studies of line-profile variations favor the detection of high-degree sectorial modes with  $\ell = |m|$ . For  $\kappa^2$  Bootis, the available spectroscopic data have been matched by a low-degree mode ( $\ell = 0$  to 2) and a high-degree  $\ell = |m| \sim 12$  mode (Kennelly et al. 1991).

The observed short-period limit of the  $\delta$  Scuti stars is consistent with the low-order p mode identification. For the star V624 Tau (Breger 1972; Seeds & Stephens 1977) a period around 34 minutes has been found. Even this period is much longer than those found for the roAp stars (Kurtz 1990), for which periods between about 6 and 15 minutes are detected. As a group, the roAp stars have similar temperatures and luminosities as the  $\delta$  Scuti stars. Because of the different range of periods, for each of the two groups of variable stars two different observing techniques (viz. the three-star and high-speed techniques,

see below), have been adopted, each designed for maximum precision in the respective period domain. It is not surprising that outside these domains the precision of each technique is lower, so that small-amplitude pulsations with unfavorable frequencies could have been missed in previous investigations. It is, therefore, important to examine the question whether  $\delta$  Scuti stars have photometrically detectable high-order pulsation modes with periods shorter than 30 minutes. A promising approach appears to be to concentrate on a single  $\delta$  Scuti star for the presence of both long and short periods of pulsation by multisite campaigns using both techniques.

With the three-star technique adopted by the Delta Scuti Network, the required high photometric accuracy is achieved by alternating measurements of the variable star with those of two carefully chosen comparison stars. The same photometric channel is used for all three measurements. The procedure can produce the required long-term stability of 2 mmag or better (also within different observatories), but yields a variable-star measurement only every five minutes. The technique is working well for periods between 30 minutes and several days and has been described by Breger (1993).

The high-speed measurements adopted by WET (Whole Earth Telescope) are obtained with two-channel and three-channel photometers. Measurements of the intensities of the target and a nearby comparison star are made simultaneously, and in the case of three-channel photometers, the sky brightness is also measured continuously. The technique works well for periods under about 30 minutes. At longer time-scales, instrumental drifts (and residual transparency variations) rapidly dominate. The operation of WET has been described by Nather et al. (1990).

The two multisite networks and their techniques complement each other in allowing the investigation of periods between a few minutes up to several days. The  $\delta$  Scuti star FG Vir was selected for the present study for two reasons: the presence of a large number of pulsation modes with photometrically visible amplitudes is suspected, and the position of the star in the Hertzsprung-Russell Diagram is similar to those of the known roAp pulsators (Kurtz & Martinez 1993).

The variability of FG Vir = HD 106384 was discovered by Eggen (1971), who deduced a period of 0.07 d and a semi-amplitude of 0.025 mag from one night of observation. In 1982, Lopez de Coca et al. (1984) observed the star for three nights through specially chosen narrowband filters, while one night of  $V$  data was collected in 1986 (Gonzalez-Bedolla & Rodriguez 1990). More extensive photometric data covering 26 nights were obtained by Dawson (1990) during 1985 and 1986. These data, however, are of somewhat lower photometric precision. They show a dominant variation with a period of about 0.08 d and will be the subject of a later paper in this series. Colomba et al. (1991) reported a presently unpublished observational program of FG Vir. During 1992, Mantegazza et al. (1994, hereafter referred to as MPB) measured FG Vir photometrically for 8 nights and spectroscopically for one night. MPB were confident about the correct identification of six frequencies, while a seventh mode of pulsation was also suggested.

The present paper reports a multisite campaign of FG Vir using both the three-star and high-speed techniques and the pulsational properties of the star at the frequencies typical for a  $\delta$  Scuti star. An investigation at higher frequencies will be reported in Paper II.

## 2. New photoelectric measurements

In order to eliminate the serious aliasing caused by regular observing gaps, a multisite campaign was organized utilizing the WET (Whole Earth Telescope) Network for high-speed measurements and the Delta Scuti Network for low frequencies. During 1993 March and April, 170.4 h of usable data were obtained at nine different observatories. The  $V$  filter was used. On six nights, additional measurements with the  $B$  filter were obtained in order to estimate the phase shifts and amplitude ratios between the different wavelength regions for the primary pulsation mode. These nights are indicated with "BV" under the "Technique" column of Table 1, which presents a journal of all observations. Since two different techniques were used, we will discuss the acquisition and reductions of the two data groups separately.

### 2.1. Measurements made with the three-star technique

During 17 nights between 1993 March 12 through April 5, photoelectric measurements of FG Vir were obtained at two observatories with the three-star technique.

For the measurements at McDonald Observatory the following two comparison stars were used: HD 106952 (F8V) and HD 105912 (F5V). No variability of the comparison stars could be detected and the two comparison stars indicated a precision of between  $\pm 3$  and 4 mmag per single measurement. This can also be taken as an estimate of the accuracy of the FG Vir measurements. No serious problems were experienced during either the observations or data reductions. The resulting measurements of the variability of FG Vir is denoted in this paper as data set A.

During four nights of observations at McDonald Observatory, the standard three-star technique was modified in order to examine the accuracy of a hybrid technique in which the telescope was moved to the two comparison stars only once every hour. In principle, this hybrid technique could combine the advantage of the photometric stability of the three-star technique with the high duty cycle of the high-speed technique. The resulting measurements were indeed of very high quality and the comparison of the different techniques will be discussed elsewhere. We only note here that the application of the hybrid technique requires very high atmospheric and instrumental stability and cannot be recommended for general use. The 30.1 hours of data (data set B) obtained with the hybrid technique can be used for both high and low-frequency analyses.

Additional data (data set C) with the three-star technique were also obtained at the Xing-Long observatory located in China. These data are important since the longitude of these observatories complements the longitude of the major other observing site, McDonald Observatory. These data contain sev-

**Table 1.** Journal of the observations of FG Vir

Observatory	Observer(s)	Date (UT)	Length (hrs)	Technique	Data set
Xing Long 0.6m	Jiang shi-yang	12 Mar 93	4.2	Three star	C
Xing Long 0.6m	Jiang shi-yang	14 Mar 93	4.9	Three star	C
McDonald 2.1m	T.K. Watson & R.E. Nather	16 Mar 93	3.9	High speed	D
Siding Spring 0.6m	S.J. Kleinman	16 Mar 93	1.8	High speed	D
SAAO 0.75m	J.E. Solheim	16 Mar 93	3.9	High speed	D
McDonald 0.8m	E. Serkowsch & G. Handler	17 Mar 93	7.5	Three star, BV	A
McDonald 2.1m	T.K. Watson & R.E. Nather	17 Mar 93	6.8	High speed	D
Siding Spring 0.6m	S.J. Kleinman	17 Mar 93	3.2	High speed	D
Xing Long 0.6m	Liu zong-li	17 Mar 93	3.7	Three star	C
SAAO 0.75m	J.E. Solheim	17 Mar 93	6.8	High speed	D
Siding Spring 0.6m	S.J. Kleinman	18 Mar 93	9.8	High speed	D
Xing Long 0.6m	Liu zong-li	18 Mar 93	4.1	Three star	C
Siding Spring 0.6m	S.J. Kleinman	19 Mar 93	2.2	High speed	D
Mauna Kea 0.6m	M.A. Wood	20 Mar 93	2.9	High speed	D
Mt. John 1.0m	D.J. Sullivan	22 Mar 93	6.9	High speed	D
Wise 1.0m	H. Mendelson	25 Mar 93	3.7	High speed	D
McDonald 0.9m	J.C. Clemens	26 Mar 93	3.6	High speed	D
Mauna Kea 0.6m	M.A. Wood	26 Mar 93	4.1	High speed	D
McDonald 0.8m	G. Handler	27 Mar 93	6.1	Three star, BV	A
Xing Long 0.6m	Li zhi-ping	27 Mar 93	5.1	Three star	C
Xing Long 0.6m	Li zhi-ping	28 Mar 93	5.0	Three star	C
McDonald 0.8m	G. Handler	29 Mar 93	4.2	Three star, BV	A
McDonald 0.8m	G. Handler	30 Mar 93	6.6	Three star, BV	A
McDonald 0.8m	G. Handler	31 Mar 93	7.5	Hybrid	B
Xing Long 0.6m	Li zhi-ping	31 Mar 93	3.7	Three star	C
Mt. Suhora 0.6m	J. Krzesinski & G. Pajdosz	1 Apr 93	1.2	High speed	D
McDonald 0.8m	G. Handler	1 Apr 93	7.8	Hybrid	B
Mt. John 0.6m	D.J. Sullivan	1 Apr 93	7.0	High speed	D
Mt. Suhora 0.6m	J. Krzesinski & G. Pajdosz	2 Apr 93	4.0	High speed	D
McDonald 0.8m	G. Handler	2 Apr 93	7.2	Hybrid	B
McDonald 0.8m	G. Handler	3 Apr 93	7.6	Hybrid	B
McDonald 0.8m	G. Handler	4 Apr 93	6.0	Three star, BV	A
McDonald 0.8m	G. Handler	5 Apr 93	7.4	Three star, BV	A

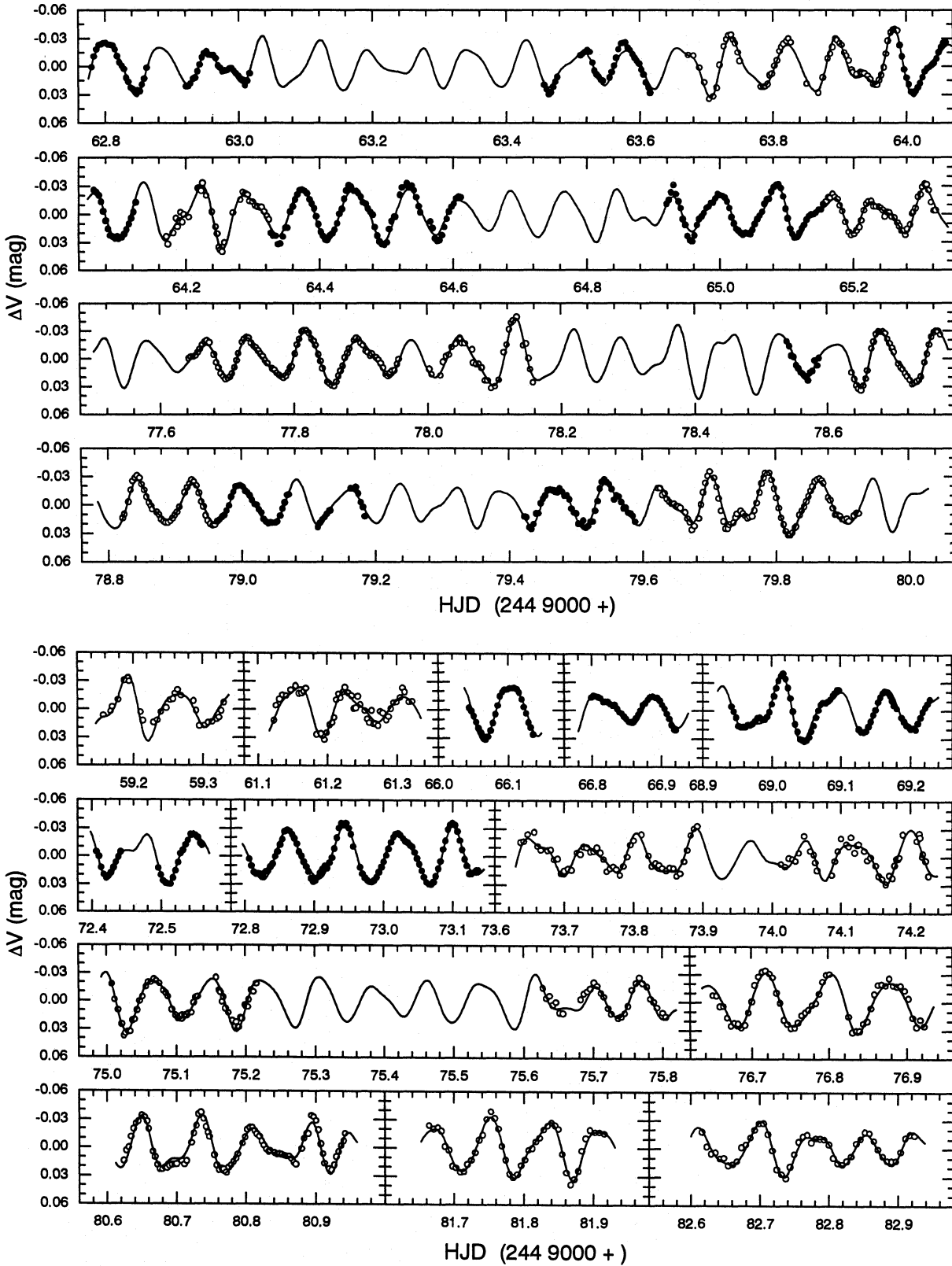
eral individual data points which deviate by large amounts from both their neighboring data points as well as the overall solution. Since the problem is present in this data set only, it should be interpreted as observational errors. Consequently, we have applied a conservative  $3.5\sigma$  criterion and eliminated those data points which deviated from the overall solution by more than 3.5 standard deviations.

## 2.2. Measurements made with the high-speed technique

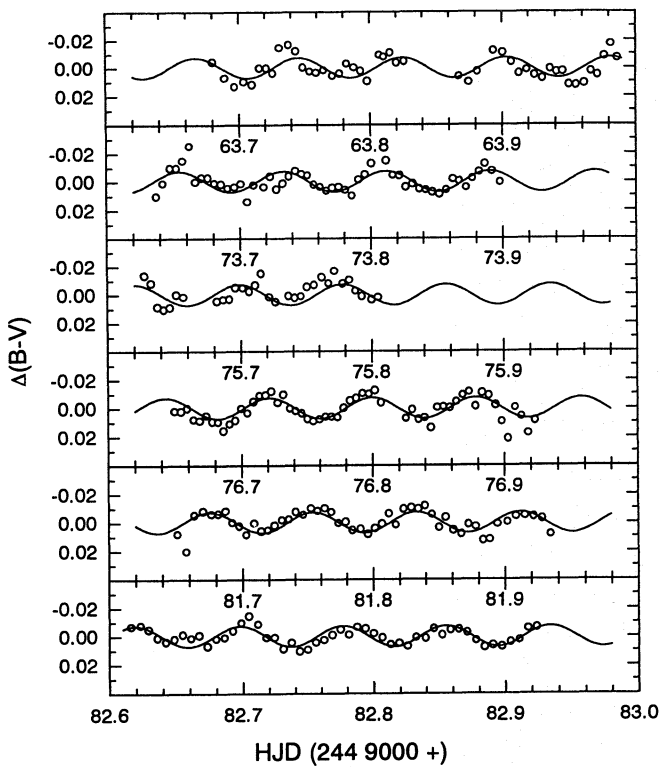
In order to adapt the WET data for searching at low frequencies, the standard procedure to reduce WET photometry needed to be extended. For data obtained with multichannel photometers, sky subtraction was performed on a point-by-point basis after an adjustment of the sensitivity ratios of the different photometer channels. For two-channel photometers the background measurements were interpolated. After the background corrections were applied, the data were edited and incorrect data points elim-

inated. With the high-speed technique such editing is necessary, since the measurements are made continuously and include even those times when the telescope is moving away from the star.

Since FG Vir varies with a relatively high amplitude and the possibility of tube sensitivity drift could not always be ruled out, extinction corrections are not straightforward. A synthetic light curve predicted from the three-star data was subtracted from the variable star data substantially decreasing the variance of the data. Bouguer diagrams of these residuals and the comparison star data were examined for variable extinction or tube drift, and an appropriate extinction coefficient was determined for each night. In cases where trends in the residuals due to atmospheric or equipment problems remained, an additional polynomial fit was subtracted from the data. The two data sets from Poland showed substantial transparency variations and scintillation noise (caused by the high air masses at which the measurements were obtained), so that the polynomial fit was replaced by a spline fit determined from the channel 2 data. We



**Fig. 1a and b.** Multisite photometry of FG Vir obtained during the 1993 campaign.  $\Delta V$  is defined to be the magnitude difference (variable - comparison stars) normalized to zero. The open circles refer to measurements obtained with the three-star technique, while the filled circles are averages of continuous (high-speed) measurements. The fit of the ten-frequency solution derived in this paper is shown as a solid curve. Note the excellent agreement between the measurements and the fit



**Fig. 2.**  $B - V$  variations of FG Vir obtained during six nights. The solid curve represents the fit obtained with the dominant frequency of 12.72 c/d

found that the Polish data reduced in this manner could be used for low-frequency analyses (but not for high frequencies).

The final data were converted to Heliocentric Julian date (HJD) and summed into 5 minute bins for the low-frequency analysis.

The observed variability of FG Vir obtained during the 1993 campaign with both the three-star (open circles) and high-speed techniques (filled circles) is shown in Fig. 1 together with the predicted ten-frequency fit derived below. The color variations are presented in Fig. 2.

### 3. Pulsation frequencies up to 30 c/d

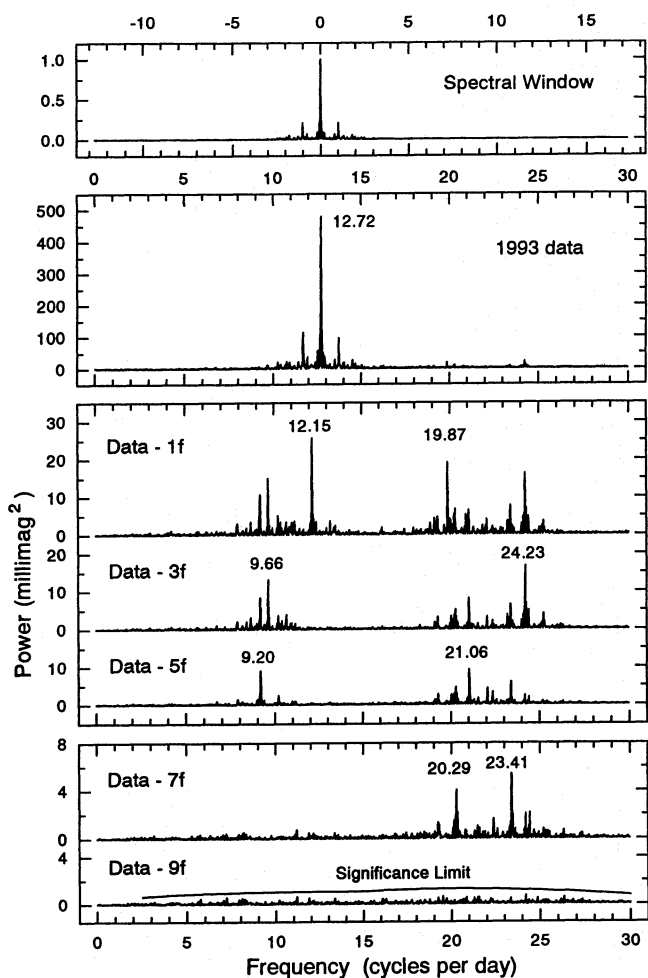
The pulsation frequency analyses were performed with a package of computer programs with single-frequency and multiple-frequency techniques (program PERIOD, Breger 1990a), which utilize Fourier as well as multiple-least-squares algorithms. The latter technique fits a number of simultaneous sinusoidal variations in the magnitude domain and does not rely on prewhitening. For the purposes of presentation, however, prewhitening is required if the low-amplitude modes are to be seen. Therefore, the various power spectra are shown as a series of panels, each with one or two additional frequencies removed relative to the panel above. The analyses were performed using the traditional units of magnitude. We note that for the small amplitudes present in FG Vir any differences between using intensity and magnitude variations are negligible.



**Fig. 3.** Power spectrum of FG Vir in the 0 to 30 c/d range using the new multisite measurements obtained with the three-star technique (data sets ABC). This technique alternates measurements of FG Vir with those of two comparison stars. The spectra are shown before and after applying multiple frequency solutions

One of the most important questions in the examination of multiperiodicity concerns the decision of which of the detected peaks in the power spectrum can be regarded as variability intrinsic to the star. Due to the presence of nonrandom errors in photometric observations and because of observing gaps the predictions of standard statistical false-alarm tests give answers which are considered by us to be overly optimistic. In a previous paper (Breger et al. 1993) we have argued that a ratio of amplitude signal/noise = 4.0 provides a useful criterion for judging the reality of a peak. Subsequent comparisons have confirmed that this restrictive limit of 4.0 cannot be lowered significantly for typical photometric data. This means that peaks below signal/noise values of 3.5 should be regarded with suspicion, although some of them may be intrinsic to the star.

In the present study the noise was calculated by averaging the amplitudes (oversampled by a factor of 20) over 10 c/d regions centered around the frequency under consideration. The rather large range of 10 c/d was chosen in order to deemphasize the effects of a single additional pulsation mode on the computed noise level. The curves shown in the diagrams of the



**Fig. 4.** Power spectrum of FG Vir in the 0 to 30 c/d range using all the new multisite measurements (data sets ABCD). The spectra are shown before and after applying multiple frequency solutions

power spectra entitled “significance limits” are smoothed fits of the power values corresponding to amplitude signal/noise ratios of 4.

As a first step we should restrict the analysis to the data obtained with the three-star technique because of the relative accuracy in the low-frequency domain given by the regular observing of comparison stars. This corresponds to data sets ABC. Of course, the spectral window of the partial data is not as clean as that for the three-star and high-speed data together. Nevertheless, the uncertainties caused by 1 c/d aliasing are quite small for the main frequencies of pulsation. This is demonstrated at the top of Fig. 3, which shows the spectral window pattern based on the times of available measurements. The next panels show the power spectra of the data before and after subtraction of one, three, five and seven frequencies. We note that the star pulsates with a dominant frequency at 12.72 c/d (147  $\mu$ Hz) and at least eight additional frequencies. Seven frequencies can already be regarded as certain.

However, an additional important result of the frequency analyses of the data obtained with the three-star technique is

the absence of variability below 9 c/d. This means that the high-speed data, for which measurements of comparison stars had not been obtained with the same photometer channel, can now be included. Addition of the high-speed data lowers the noise level in the power spectrum and also improves the spectral window.

Figure 4 shows the power spectra of all the 1993 campaign data before and after subtracting the best one, three, five, seven and nine-frequency solutions. These nine frequencies can be regarded as well-established and should be free of 1 c/d aliasing. The best multi-frequency solution obtained with PERIOD is listed in Table 2. We have also repeated the analysis with the inclusion of the 53 hours of MPB data obtained during 1992 (called data set E). The resulting power spectrum looks essentially identical to the power spectrum of the 1993 data alone shown in Fig. 4 and is consequently not presented as a separate diagram. Nevertheless, the existence of a data set obtained a year earlier significantly improved the frequency resolution. The last two decimal places listed in Table 2 were determined from comparing the 1992 and 1993 data sets. In spite of the improved resolution, the large gap between the observations causes annual aliasing ( $\Delta f = 1/365$  c/d = 0.0027 c/d = 0.032  $\mu$ Hz). While we have selected the values which gave the lowest residuals between the measurements and the prediction, the values of the frequencies with smaller amplitudes may be affected by annual aliasing.

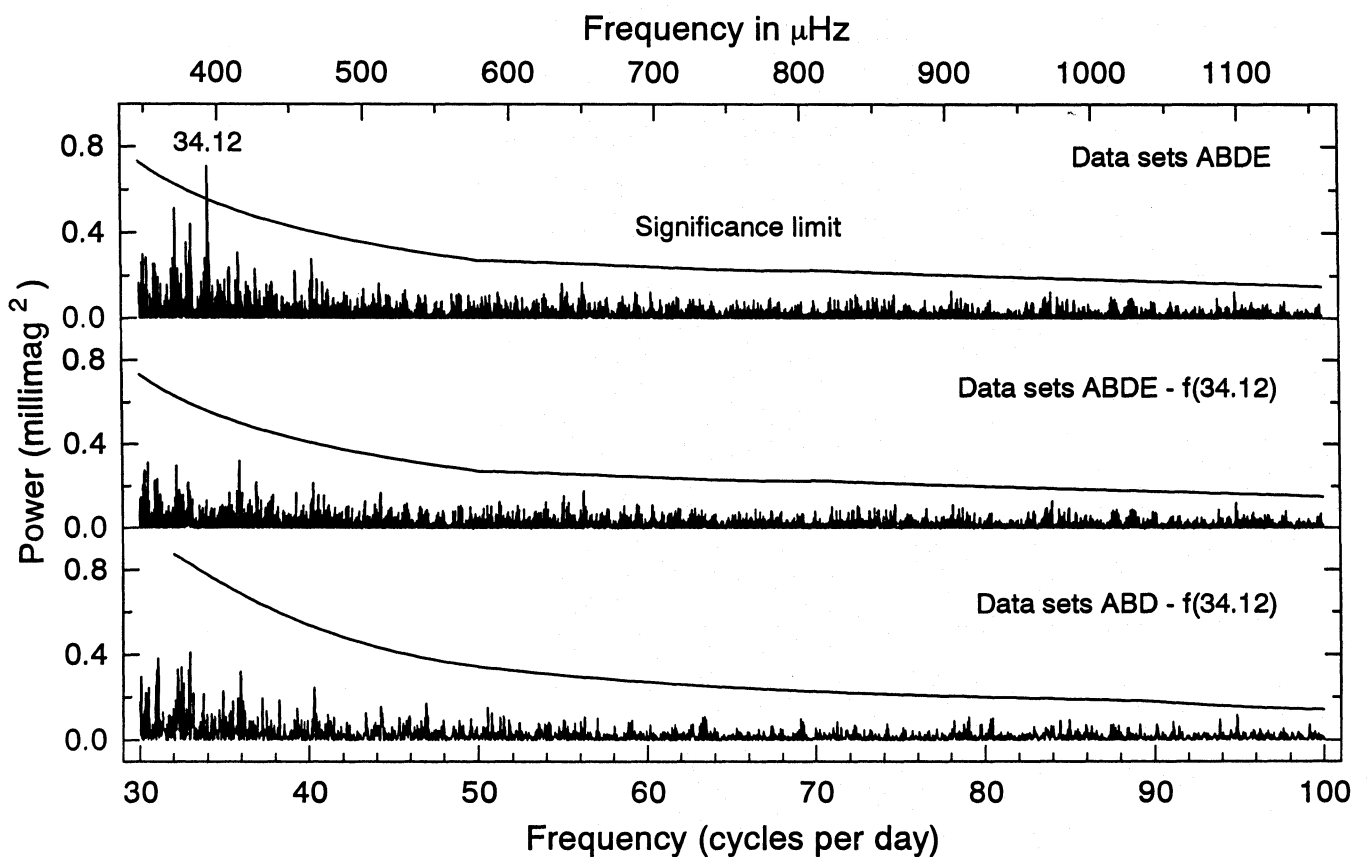
#### 4. Pulsation frequencies larger than 30 c/d

In this frequency range the long-term stability of the photometric equipment and the atmospheric conditions over several hours become less important, so that the regular observations of two comparison stars with the same photometer channel are not essential. Consequently, the high duty cycle of the high-speed observations with their essentially continuous coverage of FG Vir, makes these observations very important. On the other hand, those three-star measurements with relatively lower accuracy (data set C) can be omitted, since they do not improve the signal/noise ratio in the high frequency range. The analyses of our new data were then repeated while including data set E.

Figure 5 shows the power spectrum in the 30 to 100 c/d range. In order to avoid power leakage from the lower frequencies found in the previous section, we have prewhitened the best low-frequency solution for each data set. The power spectrum shows significant power only in the 30 - 40 c/d region. Of these peaks, (only) the frequency at 34.12 c/d is significant according to our adopted signal/noise criteria. The amplitude of  $f_{10}$  derived from the ABD data alone is 0.80 mmag, in agreement with the 0.84 mmag derived from the larger ABDE set. The associated amplitude signal/noise ratio of 4.7 makes the identification as a frequency intrinsic to the star quite easy. In order to exclude the possibility that the peak is caused by systematic errors in a particular data set, solutions with locked phase were made for each individual data set A to E. The restriction of fixed phase reduced the number of free parameters for these solutions to only one, viz. the amplitude. We find that the frequency is dominant in four of the five data sets (A, B, D and E), and absent in the

**Table 2.** Multiple-frequency V-filter solution for FG Vir

	Frequency		Q value days	Amplitude (1993) <sup>1</sup>		Epoch (HJD) 244 9000+	Amplitude <sup>2</sup>	
	c/d	$\mu$ Hz		mmag	S/N		mmag	S/N
$f_1$ ,	12.7162	147.2	0.027	22.4	85.6	72.2369	22.0	91.6
$f_2$ ,	19.8679	230.0	0.017	4.4	15.2	72.2882	4.4	16.4
$f_3$ ,	12.1542	140.7	0.028	4.4	16.9	72.2808	4.1	17.0
$f_4$ ,	24.2312	280.5	0.014	4.0	14.2	72.2752	4.2	15.4
$f_5$ ,	9.6562	111.8	0.036	3.7	14.4	72.2301	3.8	15.6
$f_6$ ,	9.1962	106.4	0.037	3.0	11.6	72.2604	2.8	11.4
$f_7$ ,	21.0576	243.7	0.016	3.0	10.2	72.2980	2.7	9.7
$f_8$ ,	23.4063	270.9	0.015	2.6	9.0	72.2700	2.4	8.5
$f_9$ ,	20.2878	234.8	0.017	2.3	7.9	72.2617	2.0	7.4
$f_{10}$ ,	34.1159	394.9	0.010	(0.6)	(3.2)	72.3036	0.8	4.7
Residuals				$\pm 3.7$		$\pm 3.8$		

<sup>1</sup> Data sets ABCD<sup>2</sup> Data sets ABDE (see text)

**Fig. 5.** Power spectrum of FG Vir in the 30 to 100 c/d range, after prewhitening the nine frequencies in the low-frequency 9 to 24 c/d region. The power spectra before and after prewhitening the frequency at 34.12 c/d are shown. See the text for the definition of the different combined data sets. The lower panel corresponds to data restricted to our 1993 multisite campaign and confirms that no further significant peaks are present

relatively noisy data set C. This also affects the value derived for the combined data set ABCD and we have bracketed the value of the amplitude (0.64 mmag) listed in Table 2.

No other peaks in the 0 to 100 c/d (0 to 1160  $\mu$ Hz) range exceed the significance limit adopted for our studies. Some of the peaks evident in the data below of the significance level are probably real and intrinsic to the star, but we cannot make definite decisions concerning individual peaks. Table 3 lists the most promising of the additional peaks. Most of the additional peaks are in 19 - 27 c/d (220 - 313  $\mu$ Hz) range. We note that the frequency of the peak at 11.19 c/d corresponds to  $(f_1 + f_5)/2$ , but the astrophysical interpretation of this numerical agreement should wait until the existence of this peak has been confirmed with a higher level of significance. Additional photometric studies of sufficient length to lower the noise level should be able to examine the richness of the pulsation modes of this  $\delta$  Scuti star.

The large number of additional peaks in the 19 - 27 c/d region increases the computed noise figure for this frequency region. If some of these peaks are real, the noise will have been overestimated for this region. Since it is uncertain which of the peaks are intrinsic to the star, the computed noise figure cannot be lowered. This explains in part why in this frequency range peaks with amplitudes of 0.9 mmag are not statistically significant, while at higher frequencies pulsation modes with smaller amplitudes can already be detected reliably.

## 5. Steps towards identifying the pulsation modes

The identification of the observed frequencies of pulsation with particular pulsation modes requires the knowledge of the basic parameters of the star. The values of narrowband photometry adopted here take into account the misprint of the definition of  $[c_1]$  in Eggen (1971), see the notes below Table I in Breger (1979). The new  $c_1$  average, therefore, differs slightly from those listed in the *uvby $\beta$*  catalogs, e. g. Hauck & Mermilliod (1990). The measurements by Eggen (1971), Olsen (1983) and Olsen & Perry (1984) give the following average values for FG Vir:  $b - y = 0.160$ ,  $m_1 = 0.180$ ,  $c_1 = 0.840$  and  $\beta = 2.766$ .

The calibrations given by Crawford (1979) indicate no interstellar reddening and  $M_V = 1.71$ . The *uvby $\beta$*  photometry can also be used to derive the  $T_{\text{eff}}$  and  $\log g$  values. We adopt the values derived by MPB with a slight adjustment to account for the increased  $c_1$  value. The relative shift was calculated by using the Kurucz (1991) models. We find  $T_{\text{eff}} = 7500 \pm 150\text{K}$ ,  $\log g = 3.89 \pm 0.15$ .

The values of the pulsation constants  $Q$  can be estimated from the following equation:

$$\log Q_i = -6.456 + \log P_i + 0.5 \log g + 0.1 M_{\text{bol}} + \log T_{\text{eff}}.$$

The observed  $Q$  values, listed in Table 2, range from 0.010 ( $\pm 0.002$ ) to 0.037 ( $\pm 0.007$ ) d (for details on the error estimate for this method see Breger 1990b). This generally unavoidable uncertainty in the  $Q$  values is caused by the uncertainty of the photometry and its calibrations. It is this uncertainty which makes an identification of the observed frequencies with specific pulsation modes based on the values of the frequencies

**Table 3.** Additional promising peaks for FG Vir

c/d	Frequency $\mu$ Hz	Amplitude (1993) <sup>1</sup>		Amplitude <sup>2</sup>	
		mmag	S/N	mmag	S/N
11.19	129.5	0.8	3.1	0.8	3.3
19.23	222.5	0.8	3.0	0.9	3.4
21.48	248.6	0.7	2.5	0.8	3.5
24.19	280.0	0.8	3.0	0.9	3.4
26.33	304.8	0.7	2.9	0.9	3.6
40.29	466.3	0.5	3.0	0.5	3.0

<sup>1</sup> Data sets ABCD

<sup>2</sup> Data sets ABDE (see text)

alone impossible. Additional information for this identification is provided by (i) an examination of amplitude ratios and phase differences between measurements at different wavelengths (see Garrido et al. 1990; Watson 1988; Balona & Stobie 1980), (ii) spectroscopic analyses, (iii) recognition of measured frequency patterns with those from computed models. Here we can apply all three techniques.

The color variations were measured during six nights of this campaign. These data are sufficient to derive reliable phase shift and amplitude ratio values for (only) the primary frequency. The uncertainty can be estimated by calculating the effect on the phase and amplitude of using only the primary frequency (instead of all ten frequencies) and by calculating the effects of using only six nights of  $V$  data as opposed to the whole  $V$  data set. For the primary frequency,  $f_1$ , we find

$$\text{Color phase shift, } \phi_{B-V} - \phi_V = -10.4 \pm 2^\circ.$$

Amplitude ratio,  $A_{B-V}/A_V = 0.33 \pm 0.03$ , where the listed errors are estimates of internal errors.

A comparison of the observed negative color phase shift with the models of Watson (1988) and Garrido et al. (1990), scaled to  $B - V$ , excludes the possibility of radial pulsation ( $\ell = 0$ ) for  $f_1$  (12.7 c/d). Both  $\ell = 1$  and 2 are possible. However, from spectroscopy, MPB identified  $f_1$  with radial pulsation. These contradictory results cannot be resolved at this stage. MPB also suggested that two other observed modes (9.7, 19.9 c/d) are non-axisymmetric and a fourth mode (24.2 c/d) is axisymmetric. They determined  $v \sin i = 21 \text{ km s}^{-1}$  and estimated  $i \sim 30^\circ$  from their spectroscopic data. This leads to a rotational period of 2.8 d, or  $\Omega \sim 0.36$ , which can be included for calculations of rotationally split m modes.

## 6. Pulsation models

A powerful tool to assist with mode identifications is given by the calculation of stellar models, especially when the uncertainty in the  $Q$ ,  $\ell$  and  $m$  values are considered. Such models should include the effects of convective overshooting and allow for a nonuniform rotation rate. We have calculated a series of equilibrium stellar models and their oscillation frequencies. The range of models takes into account the uncertainties of the observationally determined input parameters in  $\log T_{\text{eff}}$ ,  $\log g$  as well as the chemical composition parameters,  $X$  and  $Z$ , and



**Table 4.** Preliminary mode identifications (Model A)

Observed frequency c/d	$\mu\text{Hz}$	Mode type	n	$\ell$	m	Predicted frequency c/d	$\mu\text{Hz}$
9.20	106	g	5	2	1	9.27	107
9.66	112	g	4	2	-2	9.84	114
12.15	141	g	3	2	1	12.15	141
12.72	147	p	1	0	0	12.67	146
19.87	230	p	2	2	-1	19.90	230
20.29	235	p	3	0	0	20.33	235
21.06	244	p	3	1	1	21.19	245
23.41	271	p	3	2	0	23.54	272
24.23	281	p	4	0	0	24.26	281
34.12	395	p	6	1	1	34.24	396

the rotation rate,  $\Omega$ . Some additional information on the physics of the models can be found in Dziembowski & Goode (1992) and Dziembowski & Pamyatnykh (1991).

A good tactic for stars such as FG Vir is to look first for possible identifications of the observed periodicities with radial modes. For these modes, effects of rotation and convective overshooting are relatively unimportant and may be ignored initially. Furthermore, the period ratios of radial modes are relatively insensitive to variations of the parameters adopted for the models. The preliminary model of FG Vir was selected from a family of stellar main-sequence models with  $T_{\text{eff}} = 7500\text{K}$ ,  $X = 0.7$ ,  $Z = 0.02$ , without convective overshooting. The stellar mass was considered as an adjustable parameter within the observed uncertainty range of  $\log g = 3.89 \pm 0.15$ . In the model selected, three radial modes have frequencies within  $\pm 0.05$  c/d of the observed frequencies. The model (Model A) has the following mean parameters:  $M = 1.80M_{\odot}$ ,  $\log L = 1.13$ ,  $\log g = 4.01$ .

In Table 4, the identifications of all ten observed periodicities are given. The rotational splitting was calculated assuming  $v_{\text{rot}} = 46.3$  km/s and a uniform rotation rate. All nonradial modes, except for  $f_{10}$ , are of a dual nature. Their identification as a p or g mode follows the rule adopted by Dziembowski & Pamyatnykh (1991), which is based on the nature of the modes on the ZAMS, where the two types of modes are well separated in frequency. With such a definition, p modes are modes that are partially trapped the acoustic propagation zone i.e. in the outer envelope. Their frequencies decrease during the main-sequence evolution approximately  $\propto R^{-1.5}$ . On the other hand, the g modes penetrate the deep interior and their frequencies increase during the evolution. In the present model, which is in an advanced phase of core hydrogen burning ( $X_c = 0.22$ ), the  $g_1$  mode occurs between  $p_3$  and  $p_4$  at  $\ell = 1$  and between  $p_6$  and  $p_7$  at  $\ell = 2$ .

All identified modes were found to be unstable in the model. Their frequencies span almost the whole frequency range in which the model shows instability. However, the observed modes represent only 10 % of all the unstable modes with  $\ell \leq 2$ . There are two wide frequency ranges (13 to 19, 25 to 33 c/d) in which no mode is observed to be excited to a detectable level.

This selection of which modes are excited by the star presents a challenge to the nonlinear theory of stellar oscillations.

The mode identification based on model A is in agreement with the constraints provided by the spectroscopic determinations (radial overtone, one axisymmetric and two nonradial non-axisymmetric modes) given by MPB. In spite of this consistency it must be pointed out that both the model adopted for FG Vir as well as the mode identification given in Table 4 must still be regarded as preliminary and that alternative identifications of the peaks should be considered as well. In the previous section, the color phase shift indicated that  $f_1$  could be a dipole or quadrupole (rather than a radial) mode. This could be modelled with  $M = 1.98M_{\odot}$ ,  $\log g = 3.86$ . In that model,  $f_1$  would be identified with  $p_2$ ,  $\ell = 1$  and  $f_4$  with  $p_6$ ,  $\ell = 0$ . The different model and mode identification would not substantially change the conclusions of the present paper, but demonstrate the need towards observationally identifying additional pulsation modes in order to make the model selection more unique.

Efforts towards achieving an accurate fit of the calculated and measured frequencies should yield constraints on the internal structure and rotation of the star. FG Vir is one of the most promising of the known  $\delta$  Scuti stars for seismic probing of the stellar interior due to the relatively slow rotation and the large number of detectable modes. Of particular interest are data on g modes, which probe the deep interior.

*Acknowledgements.* Part of the investigation has been supported by the Austrian Fonds zur Förderung der wissenschaftlichen Forschung, project number P8543-GEO. Observations at the Wise Observatory are partially supported by The Basic Research Foundation of the Israeli Academy of Sciences, while observations at Mt. Suhora were supported in part by the Polish National Committee for Scientific Research Grant 2-2109-91-02.

## References

- Balona, L. A., Stobie, R. S., 1980, MN 190, 931
- Breger, M., 1972, ApJ 176, 367
- Breger, M., 1979, PASP 91, 5
- Breger, M., 1990a, Comm. Asteroseismology (Vienna) 20, 1
- Breger, M., 1990b, Delta Scuti Star Newsletter (Vienna) 2, 13

- Breger, M., 1993, Proc. IAU Coll. 136, Cambridge University Press, New York, p. 106
- Breger, M., Garrido, R., Huang, L., et al., 1989, A&A 214, 209
- Breger, M., Stich, J., Garrido, R., et al., 1993, A&A 271, 482
- Colomba, A., De Benedetto, G., Ielo, A., 1991, IBVS 3597
- Crawford, D. L., 1979, AJ 84, 1858
- Dawson, D. W., 1990, BAAS 22, 831
- Dziembowski, W. A., Goode, P. R., 1992, ApJ 394, 670
- Dziembowski, W. A., Pamyatnykh, A. A., 1991, A&A 248, L11
- Eggen, O. J., 1971, PASP 83, 762
- Garrido, R., Garcia-Lobo, E., Rodriguez, E., 1990, A&A 234, 262
- Gonzalez-Bedolla, S. F., Rodriguez, E., 1990, IBVS 3426
- Hauck, B., Mermilliod, M., 1990, A&AS 86, 107
- Kennelly, E. J., Walker, G. A. H., Hubeny, I., 1991, PASP 103, 1250
- Kurucz, R. L., 1991, Van Vleck Obs. Contr. 11, 27
- Kurtz, D. W., 1990, ARA&A 28, 607
- Kurtz, D. W., Martinez, P., 1993, ASP. Conf. Ser. 44, 561
- Lopez de Coca, P., Garrido, R., Costa, V., Rolland, A., 1984, IBVS 2465
- Mantegazza, L., Poretti, E., Bossi, M., 1994, A&A 287, 95
- Nather, R. E., Winget, D. E., Clemens, J. C., Hansen, C. J., Hine, B. P., 1990, ApJ 361, 309
- Olsen, E. H., 1983, A&AS 54, 55
- Olsen, E. H., Perry, C. L., 1984, A&AS 56, 229
- Seeds, M. A., Stephens, C. J., 1977, IBVS 1273
- Watson, R. D., 1988, Ap&SS 140, 255