DYNAMICS OF TWO INTERACTING OBJECTS ORBITING THE SUN

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The investigations of orbits of two gravitationally interacting objects moving around the Sun in one plane were made basing on the results of numerical integration of the equations of motion of the planar three-body problem. Regions of initial data corresponding to different types of evolution were considered.

Several types of evolution of orbital elements of two interacting close planets are known [1-6]. Below these types are denoted by letters *N*, *M*, *A*, *C*, and *P*. For *N*- and *M*-types, graphs of the time dependence of the semimajor axes are "*N*" and "*M*"-shaped, respectively. If one of the axes of synodic coordinates coincides with the direction from the Sun to the first object, then for these types we have tadpole and horseshoe synodic orbits of the second object. One triangular point of libration is inside the synodic orbit for *N*-type, and two triangular points are inside this orbit for *M*-type. Close encounters of objects and their collisions can take place only for *A*-type. Semimajor axes of objects cross each other during the evolution. Orbits of objects often become resonant for a rather long time if the mass of the larger object exceeds that of the Earth [1]. Variations in orbital elements are chaotic for *A*- and *C*-types, but for *C*-type there are no close encounters and semimajor axes are not crossing. *P*-type corresponds to such periodic variations in orbital elements when the Sun is inside the synodic orbit.

The aim of our investigations was to determine regions of $\varepsilon=|a_2^o-a_1^o|/a_1^o$ and φ_o corresponding to various types, where a_1^o and a_2^o are initial values of semimajor axes of objects and φ_o is the initial value of the angle φ with the apice in the Sun between directions to the objects. For one series of computer runs we have $\mu_1=\mu_2$, and $\mu_2=0.01\mu_1$ for another series of runs, where μ_1 and μ_2 are the mass ratios of the two objects to the Sun ($10^{-9}\le\mu_1\le$ $\le10^{-3}$). The considered time span N_c exceeds 25,000 revolutions of objects around the Sun. Let α, β, γ, δ be the values of ε corresponding to boundaries between *N*-, *M*-, *A*-, *C*-, and *P*-regions, respectively. The case of initially circular orbits is considered below. Earlier it was obtained that $\delta=2.1\mu^{1/3}$ at $\varphi_o=0$ [2], $\delta=2.4\mu^{1/3}$ at $\varphi_o=180°$ [3,4], and $\gamma=1.49\mu^{2/7}$ [5], where $\mu=\mu_1+\mu_2$. The values of $\alpha^*=\alpha/\mu^{1/2}$, $\beta^*=\beta/\mu^{1/3}$, $\gamma^*=\gamma/\mu^{1/3}$, and $\delta^*=\delta/\mu^{2/7}$ obtained by us are presented in Figs. *a-d*. The values of δ at $\varphi_o=60°$ are, as a rule, greater than those at $\varphi_o=0$ and $\varphi_o=180°$ (Fig. *d*). For most of runs at $\varphi_o=60°$ and $\mu\le2\cdot10^{-4}$, we obtained that $\alpha^*\approx1.63$-1.64, $\beta^*\approx0.77$-0.81, $\gamma^*\approx2.1$-2.45, and $\delta^*\approx1.45$-1.64. For the considered time spans, a small *P*-region usually was obtained inside the *C*-region and several small *C*-regions might be located inside the *A*-region. The value of φ_o corresponding to the boundary between *M*- and *A*-regions at $a_2^o=a_1^o$ is denoted by φ_M. The values of $\varphi_M^*=\varphi_M/\mu^{1/3}\approx3.9$-$4.4$ are presented in Fig. *e*. If $\mu_1\gg\mu_2$

than $\varepsilon^2 \approx \varepsilon_*{}^2 - \frac{8}{3}\mu\{(1-\cos\varphi)+[2(1-\cos\varphi)]^{-1/2}-1.5\}$, where ε_* is the value of ε at $\varphi=60°$ [1]. By analyzing this equation we obtained that α^* is maximum at $\varphi=60°$ and this maximum value is equal to $\sqrt{8/3}\approx 1.633$. This value is presented in Fig. *a* by a solid line. The dashed and solid lines in Fig. *e* correspond to the values of φ_M^* obtained from the above equation for $\varepsilon_*=0.8\mu^{1/3}$ and for the values of $\varepsilon_*=\beta$ presented in Fig. *b*, respectively. If $\mu_1 \le 10^{-5}$, then $e_M = e_{max}/\mu_1^{1/3}$ usually does not exceed 7–8 at $\mu_1 \gg \mu_2$ and 4–6 at $\mu_1=\mu_2$ for *A*–type, and 4–6 at $\mu_1 \gg \mu_2$ and 4 at $\mu_1=\mu_2$ for *C*–type, where e_{max} is the maximum eccentricity during the evolution. For *A*– and *C*–types and $a_2^o > a_1^o$, the values of $\Delta r_\delta = (\max\{r\} - a_1^o)/a_1^o\delta$ are about 3, 6.5, 2, and 3.5 at $\mu_1=10^{-9} \gg \mu_2$, $\mu_1=10^{-5} \gg \mu_2$, $\mu_1=\mu_2=10^{-9}$, and $\mu_1=\mu_2=10^{-5}$, respectively, where $\max\{r\}$ is the maximum value of the distance r from the Sun to the second object. For *C*–type and $\mu_1 \gg \mu_2$, it is obtained that $\min\{r\} - a_1^o \approx 0.4\gamma a_1^o$. For three objects orbiting the Sun e_{max} can be several tens times larger than that for two objects [7]. For initially eccentric orbits, the values of α and β are smaller and the values of δ and γ may be considerably larger than those obtained for initially circular orbits. The values of e_{max} are greater for greater initial eccentricities.

This work was supported in part by National Science Foundation under Grant No. PHY89–04035 in 1992 and by Russian Fund of Fundamental Investigations under Grant 93–02–17035 in 1993.

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φ_o	60°	60°	0	180°
N_c	2500	25000	2500	2500
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