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OBSERVATIONS OF PULSATING OB STARS

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Abstract. Using examples from the literature, I try to show how astrophysical parameters of pulsating stars are derived and how certain (or, occasionally, uncertain) they are. The examples are arranged in the order of increasing input of *a priori* knowledge. Thus, I start with pulsation periods, because in most cases they can be obtained directly from observations. Then I consider the two stars for which pulsation constants have been determined without recourse to photometric calibrations. Next I show how multicolour photometry can help identify pulsation modes. Finally, I discuss the photometric $T_{\rm eff}$, $\log g$, and pulsation constants. This leads to conclusions which contain some recommendations for the observers, including a list of promising targets for asteroseismology.

1. Introduction

Observations of pulsating stars can be used to derive information about their (1) equilibrium state, (2) eigenvalues, and (3) eigenfunctions. The equilibrium state is characterized by such parameters as mass, age, chemical composition, effective temperature and surface gravity, the latter two averaged over the pulsation cycle. The eigenvalues are the period (or frequency) and the pulsation constant (or the dimensionless frequency). The eigenfunction determines whether the mode of pulsation is radial or nonradial, fundamental or overtone, pressure or gravity, etc.

2. Pulsation Periods

They can be derived by induction or by deduction. In the present context, induction consists in deriving a period from observations, and then proving that it can only be accounted for by pulsations. Deduction, on the other hand, starts with assuming that pulsation in a specific mode is the cause of the observed variations. A phase in the pulsation cycle can then be found for each observation by a comparison with the assumed model. Finally, the phases yield the period.

2.1. AN EXAMPLE OF DERIVING PULSATION PERIODS BY INDUCTION: 16 (EN) LAC

This B2 IV star is a single-lined spectroscopic binary (Lee 1910, Struve & Bobrovnikoff 1925) and an eclipsing variable with orbital period $P_{orb} = 12\overset{d}{.}09684$ (Jerzykiewicz *et al.* 1978, Jerzykiewicz 1980). The primary eclipse is a partial transit; it lasts only $0\overset{d}{.}37$ and has a depth of about $0\overset{m}{.}04$ in B. The secondary eclipse has not been detected. The primary component of the system is the well-known β Cep-type star EN Lac. As most

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other β Cep variables, EN Lac is multiperiodic: periodogram analyses of its light and radial velocity observations have revealed three sinusoidal terms with periods $P_1 = 0\overset{d}{.}16917$, $P_2 = 0\overset{d}{.}17079$, and $P_3 = 0\overset{d}{.}18171$. These terms are always present, but their amplitudes vary on a time scale of years (Fitch 1969, Jerzykiewicz *et al.* 1984).

The three terms are believed to represent normal pulsation modes because (1) this accounts for the multiperiodicity, and (2) the periods are too short to be explained otherwise.

Six fainter terms were recently found in the light variation of 16 Lac by Jerzykiewicz (1993b, 1993c). The data consisted of *UBV* observations obtained at the Lowell Observatory in the summer and autumn of 1965. Observations falling between the first and the fourth contact were, of course, omitted. The method was Lomb's (1976) least squares frequency analysis. The results can be summarized as follows.

While the 1965 *V* amplitudes (half-ranges) of the P_1, P_2, and P_3 terms amount to 18.0 ± 0.14, 9.6 ± 0.14 and 10.5 ± 0.14 mmag, the amplitudes of the six fainter ones range from 2.1 ± 0.14 to 0.7 ± 0.14 mmag. The strongest has a period equal to half the orbital period, a value to be expected if an "ellipticity effect" were present. However, neither the amplitude nor the phase of the observed variation agree with such an explanation. Two obvious possibilities of accounting for this term are: rotational modulation by a pair of spots placed on the opposite sides of the stellar surface or pulsation in a *g*-mode.

The second of the six faint terms has a period equal to $0\overset{d}{.}139$. It may be an overtone of any of the three strongest ones. The next term is the lowest order harmonic of the P_1 term, and the last three are the first order combination terms with frequencies $1/P_2 + 1/P_3$, $1/P_1 + 1/P_3$ and $1/P_1 + 1/P_2$.

The standard deviation of the nine-term least squares fit to the *V* observations amounts to 3.0 mmag, a number close to the mean error of a photoelectric observation of good quality. Thus, the light variation of 16 Lac outside eclipse is satisfactorily accounted for by the nine sinusoidal terms.

2.2. ANOTHER EXAMPLE OF DERIVING PULSATION PERIODS BY INDUCTION: HD 74560 = HY VEL

This is one of the seven stars used by Waelkens & Rufener (1985) to define the class of "mid-B variables." These objects have spectral types from B3 to B8, MK luminosity classes from V to III, and low values of the projected rotation velocity. They vary in light with amplitudes of a few hundredths of a magnitude and periods between one and three days.

Waelkens & Rufener (1985) suggested that variability of mid-B stars is caused by pulsations in *g*-modes of high radial order. An observational proof of this suggestion was provided by Waelkens (1991), who found all seven above-mentioned stars to be multiperiodic. Hence, Waelkens (1991) proposed to call them "slowly pulsating B stars."

Waelkens' (1991) data consisted of observations in the seven filter Geneva system. In the case of HD 74560, there were 408 observations obtained between 1976 and 1989. However, three or less observations per year were taken before 1981 and none in 1982, 1984, 1987 and 1988. The average density of observations was about 0.2 per night in 1986, 0.5 per night in 1981, 1983 and 1985, and 1.3 per night in 1989. The analysis was carried out by means of the phase dispersion minimization (PDM) method of Stellingwerf (1978). Three sinusoidal terms were derived, with periods equal to $1\overset{d}{.}5511$, $1\overset{d}{.}6455$ and $2\overset{d}{.}3571$. Their amplitudes amount to 13.5, 6.5 and 4.0 mmag in *V*, and 22.5, 9.8 and 6.5 mmag in *U*. The standard deviations of the three-term least squares fits range from 7.2 mmag in *V* to 11.8 mmag in *U*. These numbers significantly exceed the mean error of a photoelectric observation of good quality. Thus, the three terms do not account for all of the light variation of HD 74560.

In the remaining six cases the situation is similar. In fact, there is a rough correlation between the number of periods and the standard deviation of the least squares fits: HD 143309 and HD 160124, for which Waelkens (1991) found the largest numbers of periods, also have the greatest residual standard deviations, while HD 177863, with only two periods, has the smallest.

The question of what the unaccounted component of the light variation of the mid-B variables is due to is difficult to answer. Unfortunately, the data are not available for an independent analysis. Perhaps it is erratic in character, with a time scale of the order of a day or shorter. For HD 74560 this possibility is suggested by the fact that in the yearly PDM periodograms (Waelkens' Fig. 17), the depths of the minima corresponding to the dominant period are inversely proportional to the average density of observations: the 1986 minimum is the deepest, while the 1989 one is the shallowest.

2.3. DERIVING PULSATION PERIODS BY DEDUCTION: THE 53 PER VARIABLES

The term "53 Per variables" has been introduced by Smith (1980b) to denote the sharp-lined late O to mid B stars that show low-order line profile variations with a time scale of the order of a day. Smith (1977, 1980a) attributed the variations to g-modes of low l. Assuming a specific model, usually an $l = 2$ sectorial mode, he was then able to affix phases in the pulsation cycle to the observed line profiles. According to Smith (1980a), a few profiles suffice to determine the period, provided that only one mode is visible at a time.

Applying this deductive procedure to half a dozen stars, Smith (1980b) derived periods that ranged from about 5 to 45 hours. He concluded that the periods are unstable, the dominant one switching approximately every month to another value, with period ratios 2:1 seen most frequently.

The problem is that neither the values of the periods nor the period switching could be confirmed photometrically. An attempt by Smith *et al.* (1984) to reconcile the line profile and photometric observations of 53 Per itself was not entirely successful. The two photometric periods found by these authors, equal to about $1^{\mathrm{d}}\!.7$ and $2^{\mathrm{d}}\!.1$, were different from the ones determined earlier by Smith (1980b). In addition, the photometric fits were far from satisfactory, leaving a component of the light variation unaccounted for. An investigation by Balona & Engelbrecht (1985) of three 53 Per stars accessible from the southern hemisphere resulted in detecting light variability, but with periods different from those found by Smith (1980b) and, in addition, with amplitudes much smaller than in 53 Per. A similar result was obtained for the northern 53 Per variable ι Her by Chapellier *et al.* (1987). One 53 Per variable, ζ Cas, was found constant in light to within ± 1 mmag by Jerzykiewicz (1993a), and to within ± 2 mmag by Sadsaoud *et al.* (these Proceedings).

These discrepancies show the line-profile periods to be spurious. Thus, the assumption of nonradial pulsations, on which Smith's deductive method is based, may be questioned. The issue is somewhat confused by the fact that 53 Per itself is similar in its photometric behaviour to the mid-B variables. However, in this respect the prototype clearly differs from all other members of the group it is supposed to represent.

3. Empirical Pulsation Constants and Pulsation Modes

For two B stars, the pulsation constants can be derived directly from the definition $Q = P\sqrt{<\rho>}$, where P is the period and $<\rho>$ is the (equilibrium) mean density. These stars are the primary component of Spica (α Vir A), the β Cep variable that has apparently stopped pulsating (Lomb 1978, Balona 1985, Sterken *et al.* 1986), and EN Lac. In the first case, the mean density can be computed from the mass and radius obtained by combining interferometric and spectroscopic orbits (Herbison-Evans *et al.* 1971, Code *et al.* 1976). In the second case, the photometric and spectroscopic orbital elements lead to a mass-radius relation such that $<\rho>$, and hence Q, are virtually independent of the assumed mass (Pigulski & Jerzykiewicz 1988). For the three strongest terms, mentioned in Subsection 2.1, the pulsation constants are $Q_1 = 0.0335 \pm 0.0023$, $Q_2 = 0.0339 \pm 0.0023$, and $Q_3 = 0.0360 \pm 0.0024$.

A comparison with the results of linear nonadiabatic calculations of Dziembowski & Pamyatnykh (1993) shows that these three numbers are all very close to the theoretical Q values for the lowest radial order acoustic modes of low spherical harmonic order l. Thus, one of the three strongest terms in the light variation of 16 Lac may represent pulsation in the radial fundamental mode. However, additional information is required to find out which of them, if any, it is. One possibility is to use multicolour observations.

Such data contain information about l because the wavelength dependence of the light amplitude and phase is determined by the surface amplitudes and phases of the eigenfunctions.

The first attempt to take advantage of this possibility was made by Stamford & Watson (1977). An improved and extended version of this work has been recently published by Watson (1988). Using an analytic formula, originally derived by Dziembowski (1977), Watson (1988) expressed the light variation, $\Delta m(l,t)$, as a sum of two cosine terms, $A_1 \cos 2\pi t/P$ and $A_2 \cos(2\pi t/P + \Psi_T)$, where P is the pulsation period, and Ψ_T is the phase shift between local effective temperature variation and local radius variation. A_1 and A_2 are functions of l, the ratio of local fractional effective temperature amplitude to local fractional radius amplitude, henceforth denoted $\mathcal{B}$, and parameters obtainable from model stellar atmospheres. Adopting stationary model atmospheres of Kurucz (1979), Watson (1988) was able to use the expression for $\Delta m(l,t)$ to obtain the colour amplitude to visual amplitude ratio, A_{U-V}/A_V, and the colour phase minus visual phase difference, $\Phi_{U-V} - \Phi_V$, as a function of l, Ψ_T, and $\mathcal{B}$. Since the nonadiabatic eigenfunctions for models of β Cep stars were not available at the time, all Watson (1988) could do was to allow Ψ_T and $\mathcal{B}$ to vary within suitable limits in order to trace outlines of "areas of interest" in the $\Phi_{U-V} - \Phi_V$, A_{U-V}/A_V plane, that is, areas corresponding to a given l that can be occupied by β Cep stars.

For 16 Lac, the observed A_{U-V}/A_V and $\Phi_{U-V} - \Phi_V$ values, obtained from the *UBV* data mentioned in Subsection 2.1, place the P_1 term in the $l = 0$ area, and the P_2 and P_3 terms in the $l = 2$ or 3 area. Thus, the P_1 pulsation is the radial fundamental mode. Moreover, the close P_1, P_2 doublet is not caused by rotational splitting of two m states belonging to the same l, but results from a coincidence of two modes of different l.

Given the nonadiabatic eigenfunctions (Dziembowski & Pamyatnykh 1993), the "areas of interest" in the $\Phi_{U-V} - \Phi_V$, A_{U-V}/A_V plane reduce to points. Apart from l, the coordinates of the points are determined by the equilibrium parameters. In this case, comparison with the observations may yield not only l, but also $T_{\rm eff}$, $\log g$, *etc.* This approach, especially promising for multiperiodic variables, has been explored by Cugier *et al.* (these Proceedings).

4. $T_{\rm eff}$, $\log g$ and Q from Photometric Indices

In the following discussion, I shall use the effective temperatures and surface gravities derived from the Strömgren photometric indices, c_0 and β. From the several temperature and gravity calibrations of these indices available in the literature, I chose the recent one of Napiwotzki *et al.* (1993; henceforth NSW). I obtained the NSW $T_{\rm eff}$ and $\log g$ values by means of a FORTRAN

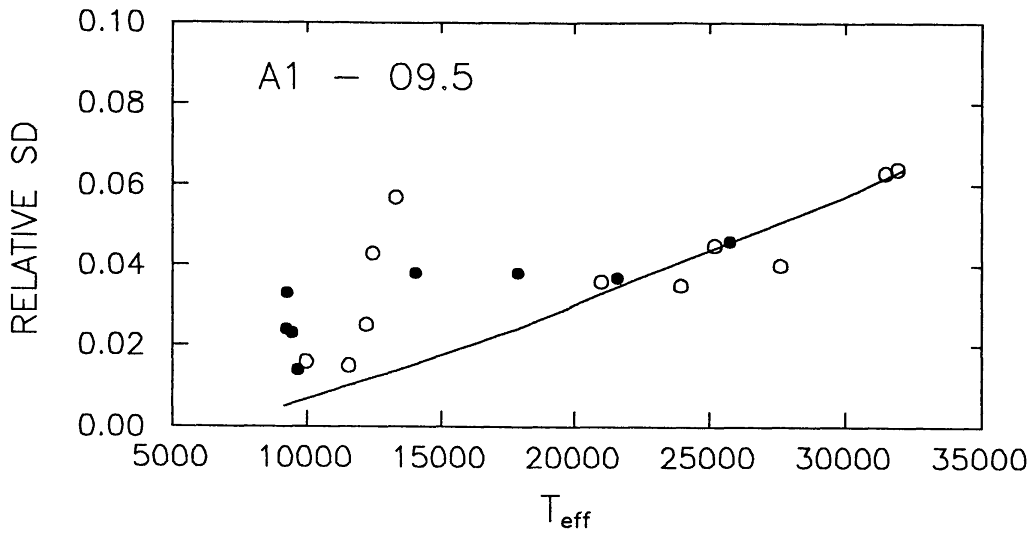

Fig. 1. Relative standard deviations of the empirical effective temperatures of Code *et al.* (1976) and propagation of photometric errors. Filled circles represent stars that were used as calibration standards by NSW. Solid line shows the relative standard deviation in the NSW effective temperature, produced by standard deviations of 20 mmag in c_0 and 5 mmag in β.

program kindly provided by Dr. Napiwotzki, using as data the c_0 and β indices from Balona (1984), Clausen & Giménez (1991), Davis & Shobbrook (1977), Giménez *et al.* (1990), and Shobbrook (1978, 1985). A comparison of the NSW effective temperatures with the empirical ones of Code *et al.* (1976) shows a good systematic agreement. In most cases the NSW effective temperatures agree with the empirical ones to within one standard deviation (SD) of the latter. However, the SD's of the empirical effective temperatures are quite large. This can be seen from Fig. 1, where the relative SD's are plotted as a function of $T_{\rm eff}$. Taking into account the fact that effective temperatures derived from photometric indices (on the Strömgren or any other system) must be at least as uncertain as the empirical $T_{\rm eff}$ values, I conclude from this figure that the minimum relative SD of a photometric $T_{\rm eff}$ amounts to about 3 percent between 9000 and 15000 K, 4 percent between 15000 and 25000 K, and 5 to 6 percent between 25000 and 32000 K. It can also be seen from Fig. 1 that the SD's of c_0 and β would have to be as large as 20 and 5 mmag, respectively, to produce a 4 percent relative SD in the photometric $T_{\rm eff}$ at 25000 K. Since the SD's of c_0 and β (or similar indices in other systems) are normally smaller than the above-mentioned values, improving the effective temperature scale of B stars would require better empirical $T_{\rm eff}$ values than those available at present.

A comparison of NSW surface gravities with the empirical ones (that is, the ones derived directly) is presented in Fig. 2. The objects shown are: the components of the detached double-lined eclipsing binary CW Cep (Clausen

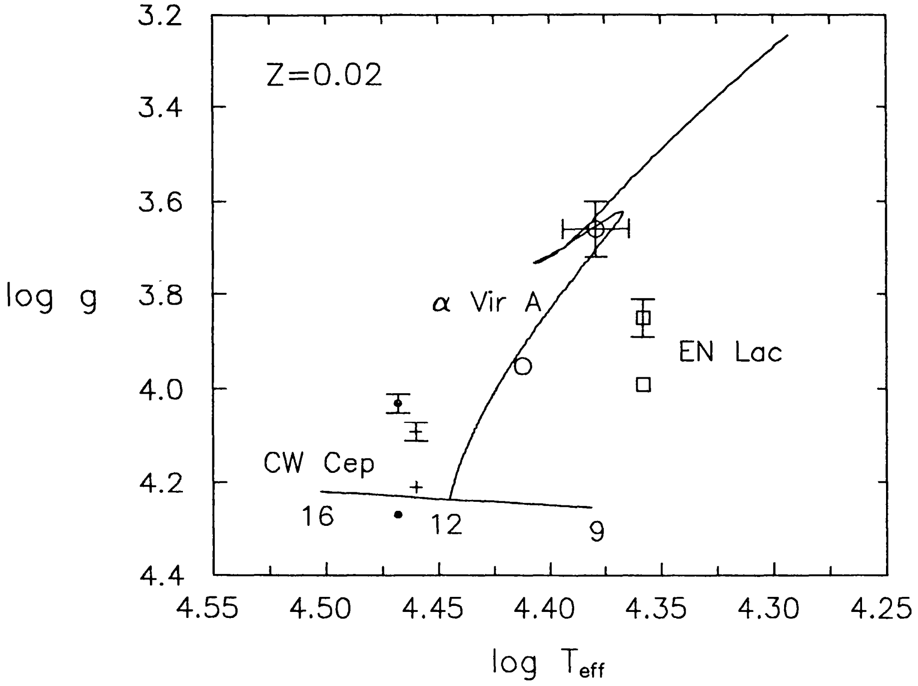

Fig. 2. Comparison of the photometric and empirical surface gravities in the log $T_{\rm eff}$ –log g plane. Symbols with error bars were plotted using the empirical values of log g. For α Vir A, two values of $T_{\rm eff}$ were used: the empirical (open circle with error bars) and the NSW one (open circle). For the other stars, that is, the A and B components of CW Cep (points and plus signs, respectively) and EN Lac (squares), the NSW temperatures were used. Theoretical ZAMS in the range from 9 to 16 $M_\odot$ and a 12 $M_\odot$ evolutionary track for X = 0.7 and Z = 0.02, computed by means of the Warsaw-New Jersey stellar evolution code (see Dziembowski & Pamyatnykh 1993), are shown with solid lines.

& Giménez 1991), α Vir A and EN Lac. I selected these systems because they illustrate the situations in which photometric indices of the components can be reliably derived from the combined light photometry: CW Cep has components of nearly equal brightness, Spica can be resolved, and 16 Lac has components that differ in brightness so much that the secondary's contribution is negligible. The empirical values of log g for the components of CW Cep and for α Vir A could be computed directly from the masses and radii. In the case of EN Lac, I used the mass–radius relation mentioned in Section 3. The value shown in Fig. 2 corresponds to the mass of 11 $M_\odot$, while the error bar covers the range from 8.5 to 14.5 $M_\odot$.

As can be seen from Fig. 2, empirical surface gravities are in all cases larger than the photometric ones. However, the sample is small. In addition, this figure shows that the popular method of deriving stellar masses and radii by comparing their positions in the log $T_{\rm eff}$ – log g plane with the theoretical evolutionary tracks may yield unreliable results. Masses obtained in this way for the two components of CW Cep and for α Vir A from the photometric log g values would be about 10 percent too large, while the radii would be too small by 10, 20 and 30 percent for CW Cep B, CW Cep A and α Vir A, respectively. As far as the masses are concerned, the situation would be even worse if the photometric $\log g$ values agreed with the empirical ones: the discrepancy would then increase to 20 percent.

Masses and radii obtained from the evolutionary tracks in the $\log T_{\rm eff} - \log g$ plane and the photometric $T_{\rm eff}$ and $\log g$ values can be used to derive pulsation constants. Clearly, such photometric pulsation constants must be treated with caution.

A mass obtained from an evolutionary track in the $\log T_{\rm eff} - \log g$ plane is sometimes combined with $T_{\rm eff}$ and $\log g$ in order to compute the luminosity, which is then used as the ordinate in the H-R diagram. Apart from its dubious accuracy, the procedure is based on a circular argument. A recent example of an application of this incorrect procedure can be found in a PhD thesis from a well known European university.

The vicious circle just mentioned can be broken by adding information, for example, the absolute magnitude. However, the two components of Spica are the only B stars for which the distance has been derived directly (Herbison-Evans *et al.* 1971), so that their absolute magnitudes can be obtained from the observed magnitudes and the text-book definition. For other early-type stars, methods of varying degrees of indirectness are used, ranging from main sequence fitting for galactic clusters and associations to MK classification for field stars. In practice, the absolute magnitudes for plotting early-type stars in the H-R diagram are often obtained from photometric indices calibrated by means of the main sequence fitting procedure. Of course, the resulting plot is not really an H-R diagram at all, but a $\log T_{\rm eff} - \log g$ diagram in disguise.

Since the empirical $T_{\rm eff}$ value of the fainter component of Spica is not known, α Vir A is the only B star that can be plotted in the H-R diagram without recourse to indirect methods. Whether HIPPARCOS will help to change this situation remains to be seen.

5. Conclusions

(1) Since the pulsation mechanism for the β Cep and mid-B variables is now known (see Dziembowski, these Proceedings), nonadiabatic eigenfunctions can be computed so that these objects will become important targets for asteroseismology. Obviously, asteroseismology should be attempted in the first place for stars which have (some of) their equilibrium parameters known directly. For example:

- α Vir A (if it recovers) and EN Lac,
- HD 92024, which is an eclipsing binary similar to 16 Lac and, in addition, a member of NGC 3293 (Engelbrecht & Balona 1986, Jerzykiewicz & Sterken 1992, Balona 1994),

- other β Cep and mid-B variables in open clusters with well known distances,
- the three β Cep stars with the empirical effective temperatures determined by Code *et al.* (1976), namely, β CMa, β Cru A and ϵ Cen.

(2) More and better empirical $T_{\rm eff}$ values of OB stars are needed.

(3) In order to better calibrate the photometric $\log g$ scale, good line profile and photometric data are needed for OB components of spectroscopic eclipsing binaries with known orbits.

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Discussion

Smith: I have two comments:

(1) I think you have been a little hard on the "deductive" (line profile fitting) method of inferring NRP models. The principal problem you describe derived not so much from the methodology as from my own over-optimism in the early days in translating spectrophotometric errors into pulsation phase errors from too few observations, particularly when multiple modes are present. As for 53 Per itself, both line profile and photometric data in the mid-1980's and also recent photometric data are in very good agreement with the original two-period result determined by Buta and Smith in 1979. The two methodologies (photometric and line profiles) should be seen as complementary and indeed they seem to be in agreement whenever the amplitudes are large.

(2) The second point is semantical only: I would like to appeal to the photometric community to avoid using physical mechanisms in defining variable star classes. There are two reasons for this. The first is historical, namely, that 53 Per stars were discovered by northern observers via line profile variations, whereas the "SPB" stars were discovered by southern observers photometrically. However, more fundamentally, it is perhaps ill-considered to define a class of variable stars in terms of an expected physical mechanism (which will be surely wrong in some cases, e.g., for "ellipsoidal variable" binaries). This is why astronomers have fallen back on a prototypical star's name in their nomenclature. In this case, the term "53 Per" still fits as a prototype because the two periods found years ago are still the values found in modern analyses (e.g. Huang *et al.*, these Proceedings).

Jerzykiewicz: I agree that using physical mechanisms in defining variable star classes should be avoided. This is why I refer to the Waelkens & Rufener stars as the mid-B variables, the term they introduced in their discovery paper. However, using 53 Per as a prototype of this class of variable stars would be historically incorrect and, in addition, misleading. The point is that your 53 Per stars, except for 53 Per itself, are photometrically different from the mid-B variables.

Balona: I would like to point out that it is very dangerous to claim multiperiodicity unless you have a well-sampled data set. In the 3 best-studied 53 Per stars the photometry seems to suggest that there is only one period which is coherent over many seasons. The other periods change amplitude and period in the matter of weeks (i.e., they are not coherent). Under these circumstances it will be very difficult to obtain the correct eigenfrequencies for all except the single coherent pulsation.

Jerzykiewicz: I agree. Historically, the first β Cep star in which multiperiodicity (called "the beat phenomenon" at the time) has been discovered was β CMa. The discovery, made by Meyer in 1934, consisted in showing that two short periods, both close to 6 hours, account for the long period modulation of the radial velocity amplitude. In addition, only one of the periods was seen in the line width variation. Thus, there was no doubt that the two periods are physically distinct. Later, these periods were also found in the light variation (see, e.g., Sterken & Jerzykiewicz 1993 for the references and further details). As far as multiperiodicity of the mid-B variables is concerned, photometry is all we have so far.

Harmanec: In a search for multiperiodicity one must be cautious about the way of prewhitening the data for individual periods in cases when variations are significantly nonsinusoidal. Otherwise the multiperiodicity can only mean a Fourier decomposition of finite data set.

Jerzykiewicz: I agree. However, consistent prewhitening with sinusoids is also OK since harmonics, if they are present, will show up in the later runs anyway. Of course, the data should be adequate.

Henrichs: (1) Are the amplitude changes in the different periods that you mentioned for 16 Lac typical or exceptional among these stars?

(2) You showed in your diagram that when the amplitude of one period increased, the amplitudes of the other periods decreased at the same time. Can one say something quantitatively about these amplitude changes, for instance: did the sum of the squares of the amplitudes remain constant?

Jerzykiewicz: (1) They are believed to be an exception rather than a rule. However, apart from the extreme cases like Spica or 16 Lac, the amplitude changes may escape detection if adequate observations over a number of years are not available. Long-term stability of the periods and amplitudes of the well known multiperiodic β Cep star 12 (DD) Lac has been recently studied by Pigulski (these Proceedings).

(2) No, in 1977 all three amplitudes were by a factor of about 2 smaller than in 1965.

Bolton: (1) Be careful in equating 53 Per with SPB stars until we have equivalent data for both types of stars.

(2) I agree with Dr. Jerzykiewicz's comment that the so called 53 Per stars around the β Cep strip are different from the cooler 53 Per stars. The line profile survey carried out by Mike Fieldus and I shows that LPV has a sharp blue edge near $\log T_{\rm eff} = 4.3$ for $v \sin i \leq 100$ km s^{-1}.

Jerzykiewicz: Thank you.

Percy: One problem of identifying 53 Per stars with SPB stars is that most photometry is being done in the southern hemisphere and most spectroscopy in the northern. A good approach is to study one or two stars simultaneously with both techniques: 53 Per and ϵ Per are good candidates.

Jerzykiewicz: I agree.