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NEUTRON STAR MOMENTS OF INERTIA

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ABSTRACT

An approximation for the moment of inertia of a neutron star in terms only of its mass and radius is presented, and insight into it is obtained by examining the behavior of the relativistic structural equations. The approximation is accurate to $\sim 10\%$ for a variety of nuclear equations of state, for all except very low mass stars. It is combined with information about the neutron-star crust to obtain a simple expression (again in terms only of mass and radius) for the fractional moment of inertia of the crust.

Subject headings: equation of state — stars: interiors — stars: neutron — stars: rotation

In a number of models of glitch behavior the fractional moment of inertia of matter in the crust of a neutron star plays an important role (Alpar et al. 1993). Recently, in collaboration with C. P. Lorenz (Lorenz, Ravenhall, & Pethick 1993, 1994), we have derived simple expressions for some properties of the crust alone, in terms of the mass and radius of the star. In order to obtain simple expressions for the *fractional* moment of inertia of the crust one needs comparable simple results for the moment of inertia of the core of the star. That quantity depends on the equation of state at high densities, about which there is considerable uncertainty, and thus no simple exact analytical result for it can be derived. What we demonstrate in this paper is that for a wide variety of neutron-star models there does exist a simple approximate expression for the moment of inertia in terms of the mass M and radius R of the star. It may be combined with other approximate results (Lorenz et al. 1993, 1994) to obtain a simple expression for the crust fraction of the moment of inertia $\Delta I_c/I$ in terms of M, R, and the matter properties at the crust boundary. For the FPS equation of state[1] the quantitative behavior of the various ingredients in the exact calculation of a moment of inertia is examined, to provide a partial justification of the result. Comparison is made with numerical calculations for many neutron-star models made by Arnett & Bowers (1977) to provide evidence for this as a quite widely applicable estimate, independent of the equation of state, and useful for all except the lightest neutron stars.

We use the general relativistic equations for a slowly rotating star as described by Hartle (1967). For the nonrotating star that provides the radial dependence, the metric used is

$$ds^2 = -e^{\nu(r)}\,dt^2 + e^{\lambda(r)}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\,. \quad (1)$$

[1] The interaction used, hereafter called FPS, is based on the nuclear and neutron matter studies of Friedman & Pandharipande (1981) and is described in Pandharipande & Ravenhall (1989). The version used is that given on p. 116 of the latter reference, with the modification $p_{10}/\rho \rightarrow p_{10}/\rho_0$ discussed on p. 117.

It involves the radial functions $\nu(r)$ and $\lambda(r)$. The Oppenheimer-Volkoff equations for the pressure $P(r)$ and mass function $m(r)$,

$$\frac{dP}{dr} = -\frac{(\rho + P/c^2)G(m + 4\pi r^3 P/c^2)\Lambda(r)}{r^2}\,, \quad \frac{dm}{dr} = 4\pi r^2 \rho\,, \quad (2)$$

where $\Lambda(r) = e^{\lambda(r)} = [1 - 2Gm(r)/rc^2]^{-1}$ and ρ is the mass-energy density, must be supplemented by the equation for $\nu(r)$,

$$\frac{d\nu}{dr} = \frac{2G(m + 4\pi r^3 P/c^2)\Lambda(r)}{r^2}\,, \quad (3)$$

with the boundary condition $e^{\nu(R)} = 1/\Lambda(R)$, and the equation for the rotational drag, ϖ,

$$\frac{d}{dr}\left(r^4 j\,\frac{d\varpi}{dr}\right) = -4r^3\,\frac{dj}{dr}\,\varpi\,. \quad (4)$$

Here $j(r)$ is the quantity $e^{-[\nu(r)+\lambda(r)]/2}$, which has the boundary value $j(R) = 1$. In the limit of slow rotation, such that the rotation angular velocity $\Omega \ll GM/R^2c$, $\varpi(r)$ has the boundary condition $\varpi(R)/\Omega = 1 - 2GI/R^3c^2$. I is the total moment of inertia, given by either of the integrals

$$I = -\frac{2c^2}{3G}\int_0^R r^3\,\frac{dj(r)}{dr}\,\frac{\varpi(r)}{\Omega}\,dr$$
$$= \frac{8\pi}{3}\int_0^R r^4\left(\rho + \frac{P}{c^2}\right)\Lambda(r)j(r)\,\frac{\varpi(r)}{\Omega}\,\mathrm{dr}\,. \quad (5)$$

This rather intricate set of equations is integrated from $r = 0$ to the value $r = R$ where the presure becomes negligible, with a given equation of state $P = P(\rho)$, and a central density $\rho(0)$ chosen to give the desired neutron-star mass. One then has also the radius and, after satisfying the boundary conditions, the moment of inertia.

Numerically there are no problems with this procedure, but conceptually it is difficult to intuit answers except in simple but remote cases such as the Newtonian limit and/or the incompressible fluid. From what is known observationally, however, it is clear that some neutron stars show strong effects of general

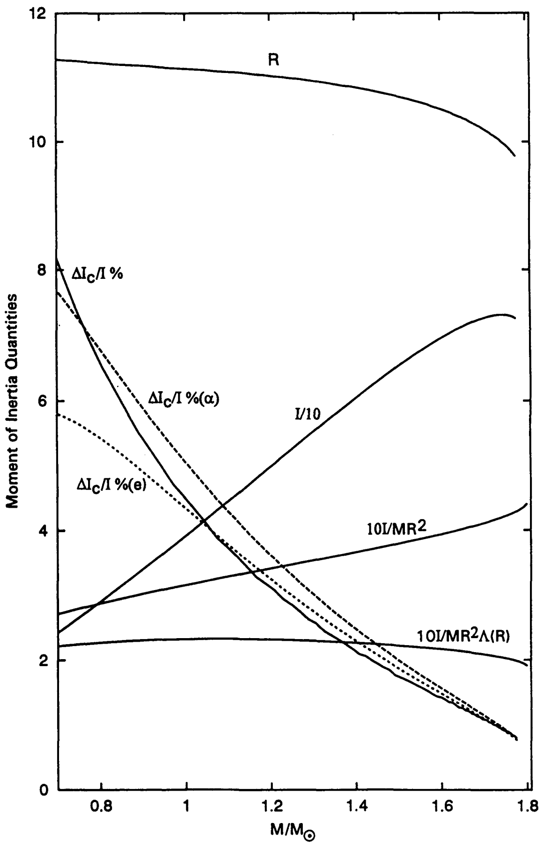

FIG. 1.—For the FPS equation of state (see footnote 1), neutron star properties as a function of the mass M in $M_\odot$. The radius R is in km; the moment of inertia I is in $M_\odot$ km^2; the ratios I/MR^2 and $I/MR^2\Lambda(R)$, and the crust fractions $\Delta I_c/I$ obtained numerically and by our approximation (see text), are dimensionless.

relativity, that is, $\Lambda(R)$ is considerably bigger than one. Given this fact, and the variability arising from the equation of state, it seems too much to hope that, as with a Newtonian star of constant density, there should be some relationship between the whole-star quantities M, R, and I. In Figure 1 we show the dependence of I and R on M for one equation of state, FPS (see footnote 1). One of the accompanying curves on that figure represents a successful attempt to guess such a relationship: we find that over a wide range of M, I can be approximated by

$$I \simeq 0.21 MR^2\Lambda(R) = 0.21\,\frac{MR^2}{1 - 2GM/Rc^2}\,. \tag{6}$$

(We recall that for an incompressible fluid in the Newtonian limit $I/MR^2 = \frac{2}{5} = 0.4$. The general relativistic result has been explored by Chandrasekhar & Miller [1974], and its representation in terms of our variables is given below in Fig. 4.) We now try to see under what range of conditions equation (6) may be expected to hold.

In Figure 2 are shown some of the radial functions that are ingredients in a moment of inertia calculation. They are for the FPS equation of state (see footnote 1) and a star mass of $M = 1.445\ M_\odot$, for which $R = 10.8$ km and $\Lambda(R) = 1.65$, a quite relativistic object. The metric-related functions $\Lambda(r)$, $j(r)$, and $\varpi(r)/\Omega$ displayed in Figure 2a are seen to be not constant, nor close to one (their Newtonian limit). In view of that fact, it is perhaps surprising that, as is shown in Figure 2a, the products $\Lambda(r)j(r)$ and $j(r)\varpi(r)/\Omega$, and thus the ratio $[\varpi(r)/\Omega]/\Lambda(r)$, are remarkably constant in the interior of the star. As to the reason for this, it is straightforward to show that

$$\frac{d}{dr}\,\Lambda(r)j(r) = \Lambda(r)j(r)\,\frac{G\Lambda(r)}{r^2c^2}\left\{4\pi r^3\left[\rho(r) - \frac{P(r)}{c^2}\right] - 2m(r)\right\}. \tag{7}$$

The behavior of the various quantities on the right of this equation are plotted in Figure 2b. If they are evaluated at $r \to 0$, the equation becomes

$$\frac{d}{dr}\,\Lambda(r)j(r) = 2\Lambda(0)j(0)\,\frac{G\Lambda(0)}{c^2}\,4\pi r\rho(0)\left[\frac{1}{3} - \frac{P(0)}{\rho(0)c^2}\right], \quad (r \to 0)\,. \tag{8}$$

For the case illustrated, the second ratio in the square bracket has the value -0.184. For a star mass of $M = 1.70\ M_\odot$, it has

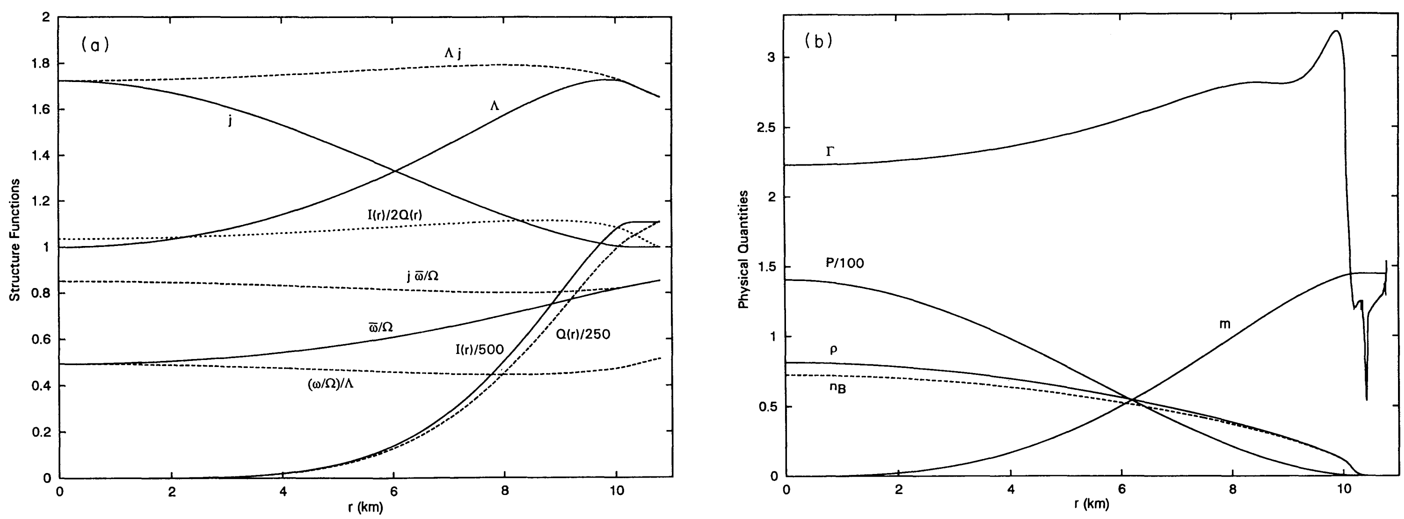

FIG. 2.—For the FPS equation of state (see footnote 1), radial functions occurring in the moment of inertia calculation as a function of the radial coordinate r, in km. The neutron star mass is $M = 1.445\ M_\odot$. (a) Functions $\Lambda(r)$, $j(r)$, and $\varpi(r)/\Omega$ occurring in the metric of the space; the moment of inertia integral eq. (5) and that of the approximation $Q(r)$, eq. (6) (in $M_\odot$ km^2); also ratios or products of these quantities, as functions of the radial coordinate r in km. (b) The baryon density n_B (in fm^{-3}); the density ρ (in m_n fm^{-3}); the pressure P (in MeV fm^{-3}); the mass function $m(r)$ (in $M_\odot$); and the adiabatic index Γ.

the value -0.34. Thus the square bracket, and the second derivative of $\Lambda(r)j(r)$ at $r = 0$ (the equation has a factor r also) is small, and may be very small. But generally it cannot be identically zero. On the other hand, the curve of $\Lambda j(r)$ shown in Figure 2*a* is quite flat at small r.

The equation of state, $P = P(\rho)$, is one of the constraints in the equations solved to obtain the above result. As an alternative stellar model, one can require of equation (7) that the quantity $\Lambda(r)j(r)$ be exactly constant, i.e., that

$$4\pi r^3\left[\rho(r) - \frac{P(r)}{c^2}\right] - 2m(r) = 0 \,, \quad (0 < r < R) \qquad (9)$$

in place of the equation of state. (All of the other equations are as before.) An alternative way of writing this equation is

$$P(r) = [\rho(r) - \tfrac{2}{3}\bar{\rho}(r)]c^2 \,, \qquad (10)$$

where $\bar{\rho}(r) = m(r)/(4\pi r^3/3)$ may be regarded as some average density over the region interior to r. We thus see that equation (9) is equivalent to a nonlocal equation of state, since the pressure at any point r depends on the density at all points interior to r. A family of homologous stellar shapes results, whose vital quantities are plotted in Figure 3. For a given central density $\rho(0)$ they are characterized by a length scale a defined in terms of the central density $\rho(0)$ by $a = [4\pi G\rho(0)/c^2]^{-1/2}$. They have a mass $M = 1.96[\rho(0)/m_n 1 \text{ fm}^{-3}]^{1/2} M_\odot$ and a radius $R = 10.0[\rho(0)/m_n 1 \text{ fm}^{-3}]^{-1/2}$ km, and all have $\Lambda(R) = 2.37$. As a function of r/a they all have the adiabatic index $\Gamma = d\ln P/d\ln\rho$ shown in Figure 3: Γ has the value 9/5 at small radii, and it increases monotonically with r/a. This is quite similar to the Γ of FPS shown in Figure 2*b*, although that function is somewhat dependent on M. This shows that the approximation we suggested, $\Lambda(r)j(r) \simeq$ a constant, does not lead to grossly unphysical behavior of stellar models. It is clear, however, that for a given equation of state such as FPS there is no analytic identity that can result in the precise constancy of the quantity $\Lambda(r)j(r)$ and the other quantities, or the relationship (6) that we have guessed, but only quantitative coincidences. Ultimately we must proceed numerically, and test the approximation against actual models with different equations of state. It is

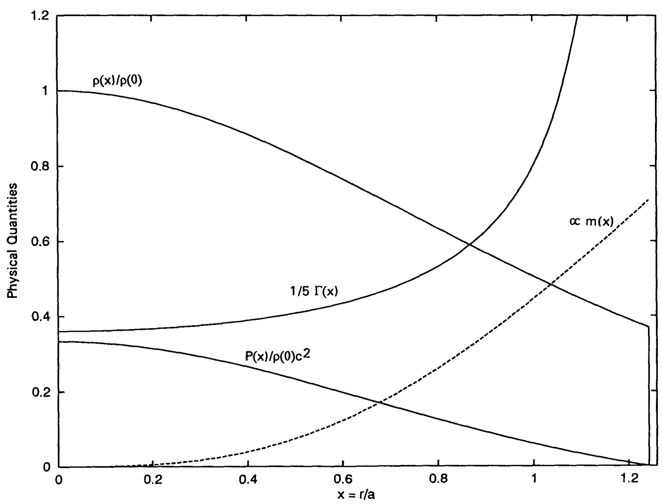

FIG. 3.—Quantities proportional to the density, the pressure and the mass vs. the dimensionless radial distance $x = r/a$, for the homologous model described in the text. Also shown is its adiabatic index Γ.

nonetheless amusing to use the approximations to bend the formalism into a slightly more familiar shape.

As a consequence of the approximate constancy of $j(r)\varpi(r) \simeq \varpi(R)$, it is straightforward to simplify the second version of the exact definition (5) for I into the form

$$I \simeq \frac{J}{1 + 2GJ/R^3c^2} \,, \quad J = \frac{8\pi}{3}\int_0^R r^4\left[\rho(r) + \frac{P(r)}{c^2}\right]\Lambda(r)dr \,. \qquad (11)$$

This is even closer to the familiar Newtonian integral, with the extra factor $\Lambda(r)$ and other, minor changes. (Because of our reliance on the numerical guidance provided in the figures, we do not attempt to transform the integrals by modifying the metric.) It is still an integral, however. By arguments similar to those that produced equation (8) one can show that the total derivative $q(r)$,

$$q(r) = \frac{d}{dr}[m(r)r^2\Lambda(r)j(r)] \,, \qquad (12)$$

has a very similar radial dependence to the integrand of equation (5), and, for the FPS equation of state, we show in Figure 2*a* the indefinite integrals of I itself, equation (5), and the integral $Q(r)$ of $q(r)$,

$$Q(r) \propto \int_0^r q(r)dr \,, \quad Q(R) = MR^2\Lambda(R) \,. \qquad (13)$$

[We recall that $j(R) = 1$.] Thus $Q(R)$ is, apart from a constant, the approximation (6) we described earlier. The almost constant value of the ratio $I(r)/Q(r)$ demonstrates that there is a quantity, $q(r)$, that mimics the moment of inertia integrand for all r, and whose integral has the value given by our guess (6). Because of this fact, the approximation is more likely to extend to other models, for which the functional dependence $P(\rho)$ is different.

As an example of exact calculations with other equations of state against which we can test equation (6) we turn to a compilation reported some time ago by Arnett & Bowers (1977). They give results for the 11 equations of state listed in abbreviated form in Table 1. We use those results, without necessarily endorsing the equations of state that they represent, because they provide a reasonable variety of cases against which to make a comparison. In Figure 4 we have plotted the ratio $I/MR^2\Lambda(R)$ as a function of M/R for the values in the Tables of Arnett & Bowers (1977), to permit a visual evaluation. Indicated on each curve, when the range of values given in Arnett & Bowers (1977) allows it, are the points closest to the maximum mass and to $M = 1\, M_\odot$. Also marked are the points where the central density is equal to nuclear saturation density $\rho_s/m_n \simeq 0.16 \text{ fm}^{-3}$, and where it is equal to the density $\rho_B/m_n \simeq 0.97 \text{ fm}^{-3}$ of the liquid-solid phase boundary (Lorenz et al. 1993, 1994). The latter stars consist entirely of crust. Figure 4 includes also the functional relationship obtained using FPS.

Curves for the moment of inertia of an incompressible fluid (invariant to the density on this particular plot) and for a gas of noninteracting neutrons, which has a polytropic equation of state $P \propto \rho^{5/3}$, are also included in Figure 4. They clearly represent limits outside which the relativistic models (larger values of M/R) cannot lie. Their contrasting behavior, with respect to the realistic models, in the Newtonian limit points up the effect of the low-Γ region the realistic models possess in their less-dense parts, which becomes relatively more important as M/R

TABLE 1

KEY TO THE EQUATIONS OF STATE REFERRED TO BY ARNETT AND BOWERS 1977[a]

Their Table Number	Equation of State
2	Pandharipande 1971a (neutron), BBP[b] and BPS[c]
3	Pandharipande 1971b (hyperon), BBP[b] and BPS[c]
4	Bethe & Johnson 1974, *I*; BBP[b] and BPS[c]
5	Bethe & Johnson 1974, V and BPS[c]
6	Moszkowski 1974 and BPS[c]
7	Pandharipande 1971b, Arponen 1972, and BPS[c]
8	Canuto & Chitre 1974, BBP[b] and BPS[c]
9	Pandharipande & Smith 1975 (mean field model)
10	Pandharipande & Smith 1975 (tensor interaction)
11	Walecka 1974 (neutrons)
12	Bowers, Gleeson, & Pedigo 1975

[a] Properties of the equations of state are plotted in Fig. 4.
[b] Baym, Bethe, & Pethick.
[c] Baym, Pethick, & Sutherland.

decreases. (For example, when with the FPS model $M/R = 0.013$, the radius is $R \simeq 15$ km, and matter outside $r = 7$ km is all in the solid phase, with the considerably reduced adiabatic index shown at large r in Fig. 2*b*.)

Those results are not surprising. What is unexpected and noteworthy about Figure 4 is the remarkable unanimity of all except two of the equations of state (Walecka 1974; Bowers, Gleeson, & Pedigo 1975). (We comment about these exceptions later.) This unanimity is apparently not a function of the mass, but of the quantity M/R, which determines $\Lambda(R)$. Consequently the grouping together of the curves seen in Figure 4 would also occur if $I/MR^2\Lambda(R)$ were plotted against other functions that differed only by factors involving M/R. For our particular choice, however, the ratio $I/MR^2\Lambda(R)$ has the further property that it is approximately constant, to within an accuracy of $\sim 10\%$, for a range of masses that includes those relevant to present observations.[2] The numerical factor 0.21 in equation (6) is negotiable. It was deduced by inspection of Figure 1 and was not refitted to the results of Figure 4.

The two sets of high values of $I/MR^2\Lambda(R)$ in Figure 4 come from Arnett and Bowers' Tables 11 and 12. In each case (Walecka 1974; Bowers et al. 1975) the $P(\rho)$ curve for that particular equation of state has a plateau below $n = n_s$, corresponding to a phase transition for *neutron* matter. Our approximation, based on a total-derivative argument, will clearly fail for such discontinuous profiles. (There is no experimental or

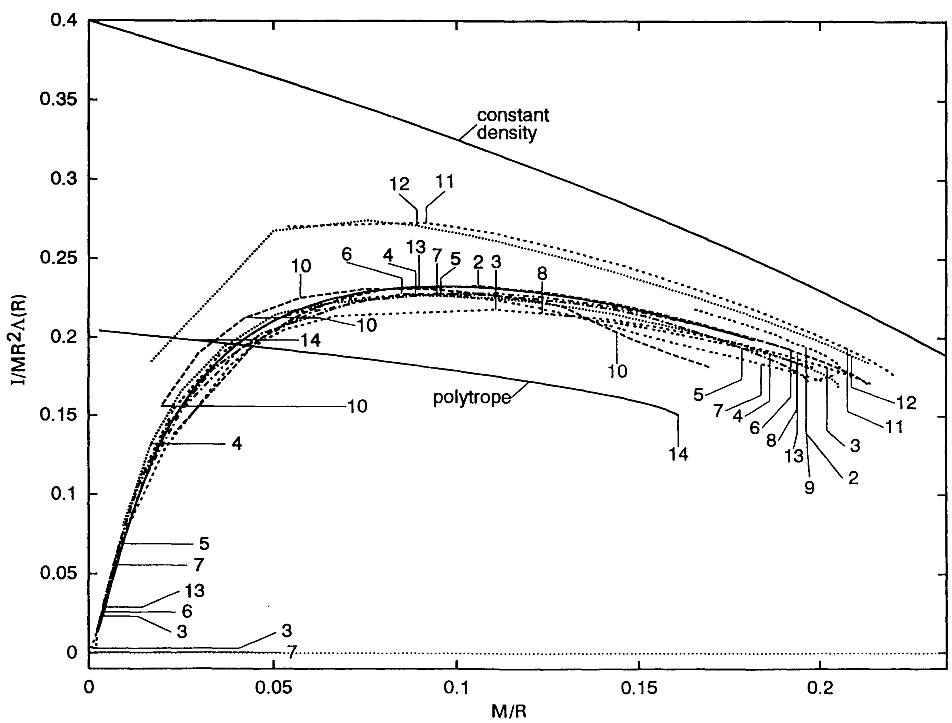

FIG. 4.—The quantity $I/MR^2\Lambda(R)$ as a function of M/R (M in $M_\odot$, R in km), for the values contained in the Tables of Arnett & Bowers (1977). Indicated on each curve is the point closest to the maximum mass (downward vertical line) and to $M = 1\ M_\odot$ (upward vertical line); also, where available, the point at which the central baryon density is close to $n_s = 0.16\ \mathrm{fm}^{-3}$, and that where it is close to the density of the liquid-solid phase transition (short and long horizontal lines, respectively). The number attached to the points refers to the Tables in Arnett and Bowers cited in our Table 1. Curves are also included for the FPS equation of state (see footnote 1), numbered 13, the general-relativistic incompressible-fluid model (Chandrasekhar & Miller 1974), and the polytrope $\nu = 5/3$, respresenting a gas of free neutrons, numbered 14.

other theoretical evidence for such a phase transition.) For most of the cases, however, equation (6) provides a useful approximation to the moment of inertia, and if necessary the FPS curve suggests ways to make a better functional fit (see footnote 2).

The FPS equation of state (see footnote 1) represents, we believe, the best fit currently available to nuclear and neutron matter properties at densities up to nuclear saturation density n_s, and consequently to the properties of the inner crust of neutron stars—the density range from neutron "drip" to the solid-liquid phase boundary. It is unclear, however, how reliable it is at supernuclear densities, since it includes as hadron components only nonrelativistic nucleons, and the the three- and higher-body interactions between between nucleons are uncertain. Thus the best procedure for calculating neutron-star properties may be to graft the FPS crust onto some other core. The procedure this suggests for calculating the observationally accessible quantity $\Delta I_c/I$ is as follows.

As has been shown earlier (Lorenz et al. 1993, 1994), the OV equations may be approximated in the region of the neutron-star crust to obtain for the crust mass ΔM_c the expression

$$\Delta M_c \simeq 4\pi \bar{R}^2 \int_0^{\rho_B} \rho(r) dr = \frac{4\pi(R - \Delta R)^4}{GM\Lambda} P_B . \quad (14)$$

Here ρ_B and P_B are the density and pressure at the phase boundary separating the solid crust from the fluid core. The quantity ΔR allows for the fact that the effective radius of the crust is smaller than the radius R measured to the surface of the star. A relevant dimension, the thickness of the inner crust $\Delta R_{\rm ic}$, depends on star properties and the neutron chemical potential μ_B at the phase boundary as follows (Lorenz et al. 1994):

$$\Delta R_{\rm ic} = \frac{R^2}{m_n GM\Lambda(R)^{3/2}} \mu_B . \quad (15)$$

That distance, from the onset of neutron drip ($\mu_n = 0$) to the phase boundary, presumably underestimates the ΔR needed since it does not include the outer crust. But however ΔR is chosen, it must scale with M and R in the same way as the distance in equation (15). One may also obtain by a similar procedure to that used to derive equation (14) the expression for the crust moment of inertia:

$$\Delta I_c \simeq \frac{2}{3} \Delta M_c (R - \Delta R')^2 \frac{1 - 2GI/R^3c^2}{1 - 2GM/Rc^2} , \quad (16)$$

where because of the somewhat different integrand the radius adjustment $\Delta R'$ will not be precisely the same as the ΔR of equation (14). These expressions can be combined with the result (6) to give the relationship

$$\frac{\Delta I_c}{I} \simeq \frac{8\pi}{3\alpha} \frac{(R - \Delta R)^6 Q_I P_B}{GM^2R^2\Lambda(R)} , \quad Q_I = \left(1 - \frac{2GI}{R^3c^2}\right) \quad (17)$$

where $\alpha \simeq 0.21$ is the numerical factor in equation (6), and ΔR is some average radius adjustment.

[2] The behavior shown by FPS in Fig. 4 can be represented approximately over most of the range of M/R by a curve of the form $I/MR^2\Lambda(R) \simeq 0.23$–$0.07(1-10M/R)^2/(M/R)^{1/2}$, with M in $M_\odot$ and R in kilometres. For very small M/R, but M greater than the neutron-star minimum mass, a linear expansion is sufficient. It leads to $I \simeq 75M^2R$, in the same units. The result expected for a polytrope is $I \propto MR^2$, with a proportionality constant that depends on the polytropic index. Our seemingly paradoxical relationship results from the varying value of the adiabatic (polytropic) index Γ in the crust, shown in Fig. 2*b*.

Assuming the FPS value (Lorenz et al. 1993a) $P_B = 0.374$ MeV fm^{-3} for the pressure at the phase boundary, and a choice of ΔR that we will discuss later, we may obtain the following expression for $\Delta I_c/I$:

$$\frac{\Delta I_c}{I} \simeq 9.06 \times 10^{-4} \frac{(R - \Delta R)^6 Q_I}{M^2R^2\Lambda(R)} \% ,$$
$$\Delta R \simeq 1.74 \times 10^{-2} \frac{R^2}{M\Lambda(R)^{3/2}} \text{ km} ,$$
$$\Lambda(R) = \left(1 - 2.953 \frac{M}{R}\right)^{-1} , \quad Q_I \simeq 1 - 0.62 \frac{M\Lambda(R)}{R} . \quad (18)$$

In this expression R is in km, M in $M_\odot$, and $\Lambda I_c/I$ is a percentage.

We show in Figure 1 the fractional moment of inertia of the crust given by equation (18) for the FPS equation of state (see footnote 1), compared with the numerical results for that model. Since in equation (18) $R - \Delta R$ is raised to the sixth power, the results depend rather sensitively on ΔR. The value of $R - \Delta R$ represents an average radius for each integral approximated, so ΔR should be somewhat less than the total crust thickness. The curve in Figure 4 labeled *e*, and the expression for ΛR in equation (18), correspond to a choice ΔR_e that gives best agreement for ΔM_c, equation (14) at the mass $M = 1.445\ M_\odot$. Alternatively, the crude assumption that the density is falling exponentially near the crust boundary, $\rho(r) \propto \exp(-\alpha r)$, leads to the choice $\Delta R_\alpha = \Delta R_c - 1/\alpha$. The resulting prediction for $\Delta I_c/I$ is labeled α in Figure 1. For the mass $M = 1.445\ M_\odot$, these changes in radius have the values $\Delta R_c = 0.75$ km, $\Delta R_e = 0.66$ km and $\Delta R_\alpha = 0.54$ km; they all scale with M and R in the manner given in equation (18). The resulting versions of $\Delta I_c/I$ shown in Figure 1 are noticeably different, but version *e* differs from the numerical evaluation by less than $\sim 10\%$ for all masses greater than 1 $M_\odot$. At the lowest mass illustrated in Figure 1, the star radius is 11.3 km, and the crust thickness is 2.4 km. Clearly when $\Delta R_c/R \gtrsim 20\%$ the first-order expansion with which we have approximated the crust properties is strained somewhat.

Accepting the uncertainty that the sensitivity on ΔR causes, we present equation (18) as an approximation for the observable quantity $\Delta I_c/I$ that depends only on the neutron star mass and radius and is (within the limits we have described) independent of the equation of state. Used with the M and R from other equations of state, it represents a simple way to graft onto those equations the FPS (see footnote 1) description of the crust.

We conclude by referring again to the results of Fig. 4: for the large majority of the equations of state considered, although they differ considerably in how their masses are related to radii and what their maximum masses are, the moment of inertia as a function of the ratio M/R is surprisingly similar. Exploitation of this result has produced a simple and model-independent expression for the crust fraction of the moment of inertia, and its mode of derivation may lead to insights concerning other general properties of slowly rotating neutron stars.

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