Astron. Astrophys. 292, 183–190 (1994)

ASTRONOMY
AND
ASTROPHYSICS

Stability of the multimode pulsation in the β Cephei-type variable 12 (DD) Lacertae

A. Pigulski

Wroclaw University Observatory, ul. Kopernika 11, PL-51-622 Wroclaw, Poland
E-mail: pigulski@ii.uni.wroc.pl (Internet)

Received 29 December 1993 / Accepted 4 August 1994

Abstract. Using all available photometric and spectrographic data of 12 (DD) Lac we studied secular stability of periods and amplitudes of all pulsation components of this star. By means of the LS periodogram analysis we confirmed the reality of all 6 previously known components. From the O-C diagrams we found all periods to be variable. Three of them (P_1, P_4, and their combination, P_5) change in the same manner. Moreover, P_2 changes inversely to P_1. Such a phenomenon was previously found in only one β Cephei-type star, i.e. β CMa. Observed period changes cannot be explained by a constant-rate evolutionary effect or by a light-time effect. However, a degree of regularity seen in the O–C diagrams suggests that the mechanism(s) causing period changes probably distinguishes modes with different l. The photometric amplitudes of the pulsation components do not vary, except for that of the f_2 component, which shows an increase by about 30% during the last 20 years.

Key words: β Cephei stars – individual stars: 12 Lac – oscillations of stars – rotation of stars

1. Introduction

12 (DD) Lacertae (HR 8640, $V = 5\overset{\mathrm{m}}{.}6$, B1.5 III) is a multiperiodic β Cephei-type variable. The variability of its radial velocity and light was discovered by Adams (1912) and Stebbins (1917), respectively. Following the detection of the primary period of variations in the radial velocities (Young 1915), the star was extensively monitored spectroscopically and photometrically in the hope that additional periodicities will be found. The second and third period were indeed detected a few decades later by de Jager (1953) and Barning (1963), respectively. Jerzykiewicz (1978, hereafter J78) confirmed the reality of these three periods and found three new ones. The semi-amplitudes of the pulsation components, as found by J78, ranged from 3 to 41 mmag (millimagnitudes) in light, and from 1 to 19 km s^{-1} in radial velocity.

Both light and radial-velocity variations of 12 Lac are dominated by the primary pulsation component with the period equal to about $0\overset{\mathrm{d}}{.}1931$. This is why almost all studies of the secular period changes this star were confined to the investigation of this period. All investigators (Pozigun 1966; Sato 1973; Heard et al. 1976; Chapellier 1985) found the rate of change of the primary period to be negative and ranging from -0.2 to -0.6 s cen^{-1} (second per century). Some authors (Heard et al. 1976; Chapellier 1985) pointed out, however, that the O–C diagram for this period can be equally well explained by an abrupt change. In the most recent study, Ciurla (1987) found a periodic variation in the O–C diagram and proposed the light-time effect in a binary system as a possible explanation. Sato (1973; 1977) investigated also the secondary period ($0\overset{\mathrm{d}}{.}1974$), and found it to be very nearly constant.

The main goal of this paper is the investigation of the secular period changes of all six pulsation components present in the light and radial-velocity variations of 12 Lac. The existing observations cover about 80 years, so that this type of analysis can be already performed. We describe the results of this analysis in Sect. 3.2. Before the O–C diagrams for pulsational components are constructed (Sect. 3.1), we check whether all pulsation components can be detected by means of periodogram analysis in the largest sets of observations (Sect. 2). Finally, the changes of the amplitudes are investigated in Sect. 4, and possible sources of the observed period and amplitude variations are briefly discussed in Sect. 5.

2. Pulsational frequency spectrum of 12 Lac

As we mentioned in the Introduction, the latest search for the periodicities in the light variations of 12 Lac was performed by J78. This author analyzed yellow magnitude observations made during the 1956 international campaign (de Jager 1963). The analysis yielded six pulsation components. Their average periods, frequencies, and semi-amplitudes can be found in Table 3 of J78. In the present paper we follow the designation of components introduced by J78. J78 pointed out that components f_1, f_4, and f_3 form an almost equidistant frequency triplet, whereas the frequency f_5 is equal to the sum of two others: $f_5 = f_1 + f_4$.

J78 made his periodogram analysis for only one set of photometric observations, i.e., for the 1956 campaign yellow filter

Table 1. Data sets selected for periodogram analysis. N stands for the number of individual observations. The references are given in the footnote to Table 4

Set	Range of Julian days	N	References
PHOTOMETRY			
A	2428750–2429927	860	(1), (2)
B	2435683–2435806	1331	(3), yellow filter
C	2435700–2435822	1175	(3), blue filter
D	2436817–2437979	680	(4)–(6)
E	2441231–2441965	807	(7)–(10)
F	2443400–2444174	1247	(11)–(13)
RADIAL VELOCITIES			
G	2419265–2421886	273	(14)–(16)
H	2423381–2424777	277	(17), (18)
I	2433478–2435726	520	(3), (19)–(22)
J	2447771–2448228	614	(23)

photometry. All six components were also found by means of a similar analysis in the 1977 photometry by Jarzębowski et al. (1979) and in the combined 1977 and 1979 photometry by Jerzykiewicz et al. (1984). In the present analysis we would like to check, however, whether all periodicities (especially those with the smallest amplitudes) could be detected in the most numerous data sets. For this purpose, we selected 6 photometric data sets which include at least 500 individual observations spread over at most 4 consecutive observational seasons. Unfortunately, these two conditions are fulfilled by only one radial velocity data set. Therefore, we decided to perform periodogram analysis for three additional radial velocity data sets, containing at least 250 observations and covering up to 8 consecutive seasons. We would like to stress, however, that due to the limited number of observations, periodograms for the radial velocity data should be interpreted with caution. All data sets used in periodogram analysis are listed in Table 1.

In order to exclude the influence of bad nights on the final periodogram, the quality of the input observations was carefully checked. First, all nights shorter than $0\overset{d}{.}1$ were rejected from photometric observations. Second, some nights with large scatter, indicating poor weather were also excluded. Because in some photometric data sets observations from different observatories, obtained with different equipment and even with different comparison stars were combined, they were first reduced to a common zero point.

In order to detect periodicities in the data, we used the least-squares (LS) periodogram (Lomb 1976). The resulting periodograms were calculated in the range from 0 to 12 c d^{-1}, with the step in frequency equal to $(10T)^{-1}$, where T is the time interval spanned by observations. In the first periodograms, the strongest peak in the spectrum was selected (in all cases its frequency was f_1). Then, the data were prewhitened with this frequency using the non-linear least-squares iterative fitting, with amplitude, phase, period, and average magnitude (or magnitudes, in the case when several subsets were combined in one data set) as unknowns. Having prewhitened the data with the f_1 component, we recalculated the LS periodogram and again selected the strongest peak, the frequency of which turned out to be in all but one case equal to f_2. Then, the original data were prewhitened again, but this time simultaneously with 2 components (f_1 and f_2). This procedure of calculating the LS spectrum and prewhitening was repeated until 4 strongest components were detected and prewhitened. All these components, namely f_1, f_2, f_3, and f_4, have been detected in all data sets. Thus, these frequencies undoubtedly represent the real periodicities in the variations of the light and radial velocity of 12 Lac.

In the search for low-amplitude components, the LS periodograms for each data set were calculated for the fifth time. The peak at frequency $f_5 = 10.514$ c d^{-1} was easily detected in the periodograms of sets B, D, F, I, and J. It was, however, hardly distinguishable from the noise in the periodogram of set C, and was not visible at all in the periodograms of sets A, E, G, and H. The latter two sets consisted, however, the oldest radial velocity data and were much less numerous than other sets.

The situation was much worse as far as the f_6 component is concerned. The peak at frequency $f_6 = 4.241$ c d^{-1} appeared distinctly in only two periodograms (of sets B and F), where it was originally detected by J78 and Jerzykiewicz et al. (1984), respectively. No significant peak at frequency f_6 appeared in any other periodogram, except perhaps those of sets A and C, where it was barely visible. This picture of the visibility of peaks at frequency f_6 has not changed after prewhitening with the f_5 component (we could not verify this for set G, because iteration with 5 components did not converge). The f_6 component has not been detected in any of the periodograms of the radial velocity data sets either.

Except for the six previously known components, no significant peaks were found in the periodograms. This conclusion is drawn from the LS periodograms after prewhitening with all 6 components. In fact, the final LS periodograms were not calculated for two data sets (G and E), because the iterative least-squares fitting with 6 components did not converge for these two sets. In these two cases the calculations were terminated after prewhitening with 4 and 5 components for set G and E, respectively.

As we mentioned earlier, both low-amplitude components (f_5 and f_6) were detected in the periodograms of the data of best quality (set B and F). This is because the quality of the data is the main factor which determines the detection threshold of the periodogram. Some other factors like the number of individual observations and their distribution in time also affect the visibility of peaks in a periodogram. The f_6 component was undoubtedly detected in two periodograms of two independent data sets separated by about 20 years. As it is rather unlikely to obtain two high random peaks at the same frequency in two independent periodograms, we conclude that f_6 represent a real periodicity in the light variations of 12 Lac. The same conclusion is valid for the f_5 component, which was detected in 5 out of 10 analysed data sets. The reality of these two low-amplitude components will be supported by another argument in Sect. 3.1.

In some periodograms there was a considerable signal at frequencies about 1, 2, and even 3 c d^{-1}. This can be the effect of night-to-night variations of 12 Lac or the comparison star, 10 Lac. Such variations were, in fact, suspected to be present by J78. The mean magnitude of 12 Lac was also studied by Sato (1979), who found it to be almost constant, with only small irregular variations. Some extinction effects which were not properly accounted for may, however, produce similar features in the periodogram (see Poretti & Zerbi 1993).

In the course of the iterative prewhitening we obtained also the periods of all components for each data set (with two exceptions, as mentioned above). As we will need later good approximations of the average periods, we calculated these averages as the weighted means (with weight inversely proportional to the error of individual period) of the values obtained for each data set. There were however two exceptions. In the calculation of the average periods we did not use the periods obtained for sets B and C, because their errors were large due to the short time intervals covered by observations. Moreover, P_6 for sets G and E, and P_5 for set G were not used in averaging, because they were not obtained. The average periods are equal to: $\overline{P_1}$ = 0$^{\rm d}$.1930880(07), $\overline{P_2}$ = 0$^{\rm d}$.1973802(06), $\overline{P_3}$ = 0$^{\rm d}$.1821544(24), $\overline{P_4}$ = 0$^{\rm d}$.1874653(04), $\overline{P_5}$ = 0$^{\rm d}$.0951177(04), $\overline{P_6}$ = 0$^{\rm d}$.2358029(33), where the numbers in parentheses denote the error of last two digits.

3. Period changes

3.1. The method

In the investigation of secular period changes in variable stars, the O–C diagrams are commonly in use. This method of the detection of period changes is favoured because even very subtle period changes accumulate after many thousands of cycles, leading to detectable changes in the O–C diagram.

In the first step, we calculated the amplitudes and phases of all 6 pulsation components for 10 sets selected in Sect. 2. In order to do this, we simply fitted a sum of six sine-wave components with assumed periods to each data set. The values of assumed periods were taken from the final iterative prewhitening, i.e., were different for different data sets. The only exceptions were P_6 for set G and E, and P_5 for set G, where the average values $\overline{P_6}$ and $\overline{P_5}$ were used, respectively. This was because the iterations with these components, for the mentioned two sets, did not converge during the prewhitening procedure. The remaining parameters, i.e., the amplitudes and phases were left as unknowns and were calculated in the fit. Then, the phases were converted to the times of maximum. The time of maximum nearest to the average time of the observations in a given set was selected as representative for the set. In this way, we obtained 6 photometric and 4 radial velocity times of maximum for each component. They are listed in Table 2.

The mathematical model for light and radial-velocity variations of 12 Lac, adopted in the above calculations, is a sum of six sine-wave components. This assumption is based on the fact that six components were detected by means of periodogram analysis. As in β Cephei-type stars strange changes of pulsational spectrum are not observed, and the changes of amplitudes in 12 Lac are small (see Sect. 4), we can be almost sure that 12 Lac pulsates all the time with all 6 components. At this point, however, an important question arises: are we able to obtain reliable amplitudes and phases for low-amplitude components (f_5 and f_6) from the data in which these components were not detected by means of the periodogram analysis?

In order to answer this question we performed some numerical tests. First, we calculated light changes with amplitudes and periods taken from J78, and arbitrarily chosen phases. These "artificial observations" were distributed in time in the same manner as the real observations in set E. Then, the Gaussian noise with different standard deviation (SD) was added and such "data" were analyzed. This noise standard deviation (NSD) was the only parameter which was different for different sets we created. Next, the analysis of all these artificial data with noise was performed in the same way as for the real data. If we adopt that the phase is still reliable if it differs less than $0.1P$ from the assumed one, the conclusion is the following. Reliable phases for f_5 and f_6 components can be obtained even from the data with NSD roughly 3 times larger than NSD at which peaks at f_5 and f_6 disappear among the noise peaks in the periodogram of these data. Thus, we can answer the above question in the affirmative: we are able to obtain reliable phases (and the amplitudes) for the low-amplitude components from the data in which these components were not detected by means of the periodogram analysis. Needless to say, the data can be so poor that the phases and amplitudes may become unreliable. However, in such a case the phase errors become comparable to the whole period.

For the above given reason we used also less numerous data sets in order to get additional times of maximum of light or radial velocity. Therefore, as the second step, we selected the sets which fulfilled two conditions: (i) consisted of at least 200 individual observations (in fact, the observations of one set, namely P7, consisted of only 199 observations), and (ii) covered one observing season, or exceptionally, two consecutive seasons. In this step we again fitted all sets with a sum of six sine-curves with amplitudes and phases as unknowns. This time, however, the assumed periods were for all sets equal to the $\overline{P_i}$ values derived in Sect. 2. Because of condition (ii), the time interval covered by observations was for these sets always shorter than 500 days. Thus, the small difference between the average and real periods did not affect the times of maximum light and radial velocity or the amplitudes calculated in this step. We would like to note also that some of the sets used in this step were one-year subsets of larger data sets analyzed in the first step. Table 2 presents the times of maximum for 17 photometric (P1 to P17) and 3 radial velocity (RV1 to RV3) data sets analyzed in the second step.

3.2. Results

Having calculated the times of maximum, we could construct the O–C diagrams for all six pulsation components. The O–C

Table 2. Heliocentric times of maximum light and radial velocity for all six components of the variation of 12 Lac. The time of maximum for the ith component can be obtained by adding T_i to T_0, where T_0 is Julian day after HJD 2400000. The numbers in parentheses denote the errors of adjacent times of maximum. All sets are referenced in Table 4

Set	T_0	Heliocentric times of maximum light and radial-velocity					
		T_1	T_2	T_3	T_4	T_5	T_6
				PHOTOMETRY			
A	29477	.2996(.0005)	.2795(.0015)	.2636(.0016)	.3292(.0036)	.2965(.0047)	.3760(.0059)
B	35729	.4918(.0003)	.5318(.0010)	.5051(.0011)	.5253(.0019)	.4869(.0019)	.5204(.0043)
C	35734	.3198(.0004)	.2680(.0011)	.2369(.0013)	.2150(.0026)	.2384(.0030)	.2326(.0051)
D	37133	.4371(.0004)	.5087(.0010)	.4969(.0013)	.4673(.0018)	.4286(.0014)	.4779(.0059)
E	41446	.6361(.0004)	.4893(.0010)	.5767(.0015)	.5126(.0026)	.5490(.0030)	.5117(.0063)
F	43775	.6818(.0003)	.5791(.0006)	.5644(.0009)	.5945(.0013)	.6179(.0010)	.7554(.0035)
P1	28770	.7828(.0014)	.6528(.0038)	.6816(.0045)	.7742(.0075)	.7372(.0138)	.6684(.0079)
P2	29483	.2841(.0010)	.4006(.0027)	.4561(.0032)	.3340(.0147)	.3853(.0090)	.2967(.0125)
P3	29902	.2917(.0008)	.2369(.0022)	.2336(.0025)	.3122(.0046)	.2013(.0053)	.2842(.0094)
P4	33893	.8110(.0011)	.8814(.0040)	.8405(.0028)	.8555(.0042)	.8096(.0046)	.8364(.0138)
P5	33894	.0042(.0010)	.0782(.0030)	.0211(.0031)	.0380(.0033)	.0993(.0047)	.0792(.0331)
P6	33911	.1884(.0012)	.2518(.0033)	.3291(.0035)	.2820(.0068)	.2176(.0055)	.3146(.0071)
P7	36498	.7564(.0012)	.9271(.0028)	.8936(.0038)	.8834(.0051)	.8019(.0034)	.8995(.0088)
P8	36835	.3115(.0004)	.2660(.0011)	.3242(.0014)	.2119(.0019)	.2332(.0021)	.1880(.0060)
P9	37717	.5265(.0010)	.5592(.0027)	.4659(.0030)	.6048(.0047)	.5418(.0021)	.5287(.0128)
P10	41254	.8976(.0007)	.8301(.0013)	.9463(.0023)	.9298(.0074)	.8900(.0040)	.8007(.0090)
P11	41255	.8605(.0013)	.8181(.0014)	.8491(.0074)	.6972(.0099)	.8333(.0046)	.7513(.0110)
P12	41339	.4714(.0006)	.5060(.0012)	.4675(.0018)	.4672(.0049)	.4449(.0039)	.4398(.0069)
P13	41392	.3785(.0009)	.4025(.0022)	.4711(.0029)	.3528(.0070)	.4301(.0076)	.5148(.0128)
P14	41552	.4525(.0020)	.4830(.0035)	.4138(.0102)	.4140(.0103)	.4975(.0070)	.5809(.0321)
P15	43433	.7248(.0004)	.7197(.0010)	.8547(.0016)	.8450(.0019)	.7669(.0015)	.8396(.0059)
P16	44142	.7400(.0003)	.7029(.0008)	.5924(.0011)	.6517(.0017)	.6747(.0013)	.6664(.0041)
P17	45581	.8392(.0008)	.8173(.0015)	.9238(.0027)	.8522(.0034)	.9205(.0034)	.7877(.0144)
				RADIAL VELOCITIES			
G	20661	.7634(.0010)	.8489(.0025)	.8063(.0034)	.8071(.0034)	.7971(.0111)	.8431(.0254)
H	24231	.7978(.0008)	.6635(.0021)	.7123(.0023)	.7359(.0031)	.6860(.0030)	.7883(.0137)
I	34597	.3705(.0012)	.2973(.0023)	.3169(.0032)	.3715(.0028)	.2849(.0042)	.2359(.0112)
J	47967	.5875(.0006)	.5027(.0020)	.5948(.0024)	.5163(.0016)	.4679(.0014)	.5360(.0055)
RV1	33522	.2611(.0007)	.1587(.0018)	.2126(.0025)	.2514(.0020)	.1759(.0021)	.2175(.0121)
RV2	35721	1.1038(.0008)	.9808(.0024)	.9881(.0047)	.9779(.0025)	1.0483(.0027)	1.1441(.0127)
RV3	48177	.2818(.0006)	.1411(.0020)	.2446(.0018)	.1047(.0011)	.2029(.0009)	.1576(.0139)

values for the light and radial velocity times of maximum were calculated according to the equation of the form:

$$T_{\rm max}(E) = T^0_{\rm max} + P_0 E, \qquad (1)$$

where $T_{\rm max}(E)$ is the calculated time of maximum after E cycles, $T^0_{\rm max}$ — an initial time of maximum, and P_0 — a reference period. As the first values of reference periods, the $\overline{P_i}$ values were used. They were subsequently corrected by trial-and-error in order to get as small scatter in the O–C diagram as possible.

Moreover, in order to use both light and radial velocity times of maximum in a single O–C diagram, we had to find the value of the phase lag between the light and radial-velocity curves. This had to be done for each component separately. Fortunately, there are simultaneous photometric and spectrographic observations of 12 Lac, which can be used to get the phase lags for all pulsation components. These observations come from the 1956 campaign. The photometry consists of two numerous sets (B and C, see Sect. 2), while set RV2 includes the 1956 radial velocity observations. The phase lags for all components were calculated as the difference between the O–C values for the 1956 radial velocity observations and the mean O–C for sets B and C.

Table 3. Initial times of maximum, $T^0_{\rm max}$, reference periods, P_0, and phase lags, $\Delta\phi$, for all pulsational components of 12 Lac. The first two parameters were used in Eq. (1), from which the O–C values for the components were calculated

Component i	$T^0_{\rm max}$ HJD 2420661.+	P_0 [d]	$\Delta\phi$ [d]
1	.7634	0.1930886	.051 ± .001
2	.8489	0.1973810	.050 ± .004
3	.8063	0.1821544	.047 ± .006
4	.8071	0.1874670	.050 ± .005
5	.7971	0.0951181	.017 ± .005
6	.8431	0.2357982	.065 ± .015

These phase lags, $\Delta\phi$, are listed in the last column of Table 3, together with initial times of maximum and the final reference periods used in Eq. (1).

After subtracting the phase lags from all photometric O–C values, the single O–C diagrams, including both photometric

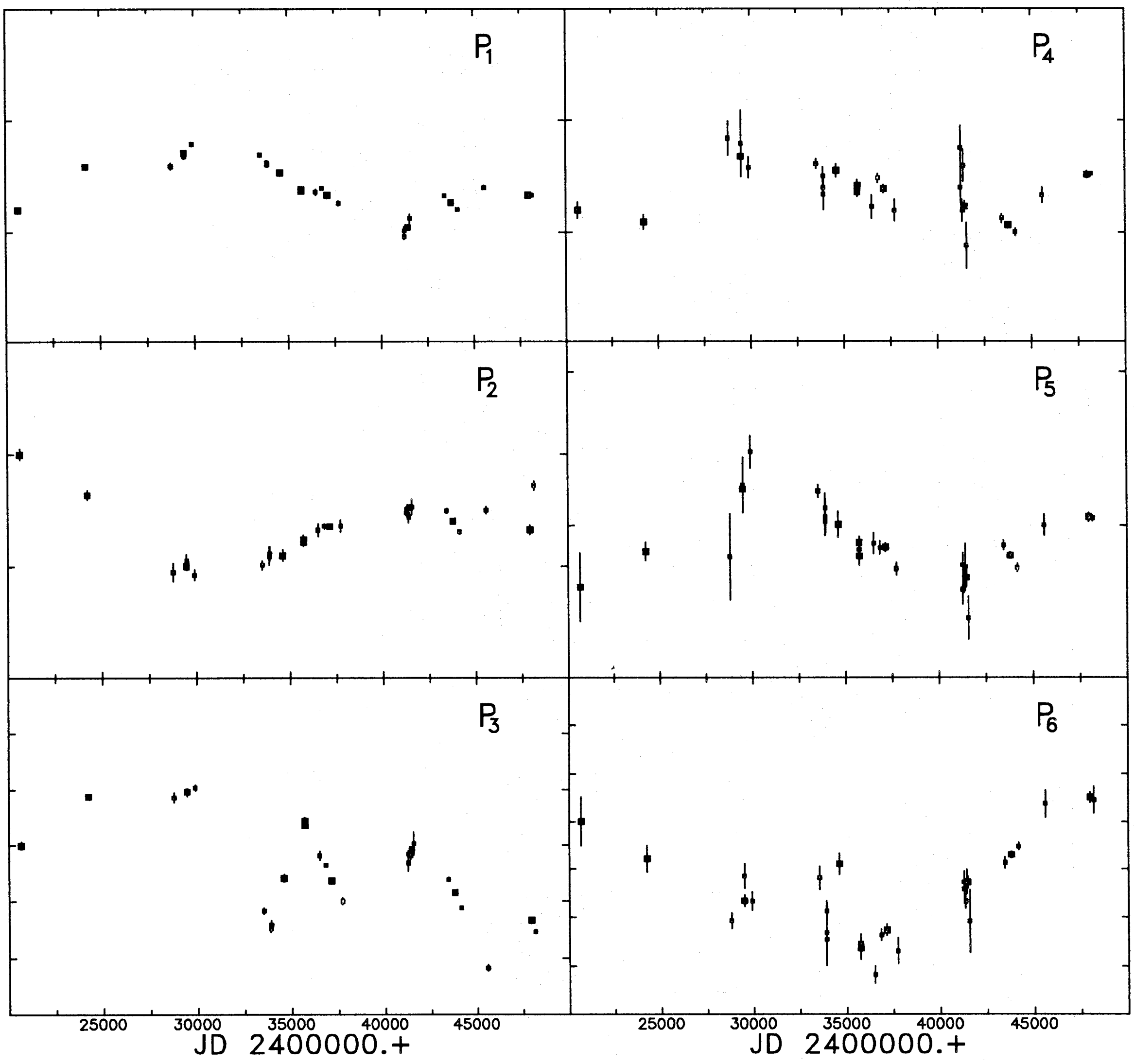

Fig. 1. The O–C diagrams for the 6 pulsation components of the light and radial-velocity variations of 12 Lac. The large symbols refer to sets A to J, while the small symbols, to the remaining ones. The ordinate is O–C (in days); the tick marks are spaced every $0^{d}.05$. Note that the ordinate scale is different for different components. The O–C values were calculated according to Eq. (1) with parameters taken from Table 3

and radial velocity O–C values, were constructed. These O–C diagrams for all components are shown in Fig. 1.

It can be clearly seen from Fig. 1 that not only P_1, but *all* periods of 12 Lac show appreciable changes.

The fact that despite the large errors we could obtain a reliable O–C diagram for the f_5 and f_6 components is a serious argument in favour of the reality of both these components, as well as of the reality of the calculated phases (see also Sect. 3.1). If these periods or phases were not reliable, we would obtain the O-C values spread over the range of the whole period. This is not the case, as the scatter of points in the O-C diagrams for both low-amplitude components is of the order of at most $0.2P$.

The O–C diagram for the strongest component (f_1) has a wave-like shape with a quasi-period of the order of 50 years. This confirms the result obtained by Ciurla (1987). However, it is possible that P_1 changes also on a shorter time scale (15–20 years), although the data presented in the O-C diagram are insufficient to show this clearly.

Comparing the O–C diagrams for P_1 and P_2 (Fig. 1) one can see that P_2 changes inversely to P_1. This resembles the effect which was found by Shobbrook (1973) for the two strongest pulsational components in β CMa.

On the other hand, changes of P_4 are very similar to those of P_1 in both the time scale and amplitude. Moreover, as found by J78, $f_5 = f_1 + f_4$, i.e., f_5 represents a combination component. What follows, P_5 should behave similarly to P_1 and P_4, with approximately the same amplitude. This is exactly what is observed. This fact confirms that our analysis is reliable even for this small-amplitude component.

The largest period changes can be seen for the f_3 component (note also large error of $\overline{P_3}$). The time scale of these changes is slightly shorter than 20 years, but it is not clear whether these changes are strictly periodic or not. P_3 is also the only period for which an ambiguity in the cycle count may be present.

4. Amplitude changes

As we mentioned in Sect. 3.1, the light and radial-velocity amplitudes were derived together with the times of maximum. These amplitudes are listed in Table 4. Photometric amplitudes for observations made in the blue and yellow spectral regions are put together in Table 4, since there is no significant difference between them for any of the components.

No changes of the photometric amplitudes exceeding the amplitude errors were found. The only exception is the f_2 component, for which the amplitude increase of the order of 30% during the last 20 years may have taken place (Fig. 2). On the other hand, there are too few radial-velocity amplitudes available to draw any reliable conclusion on their changes. The more so that systematic effects in the reductions and different quality of the spectra may considerably influence observed radial-velocity amplitudes.

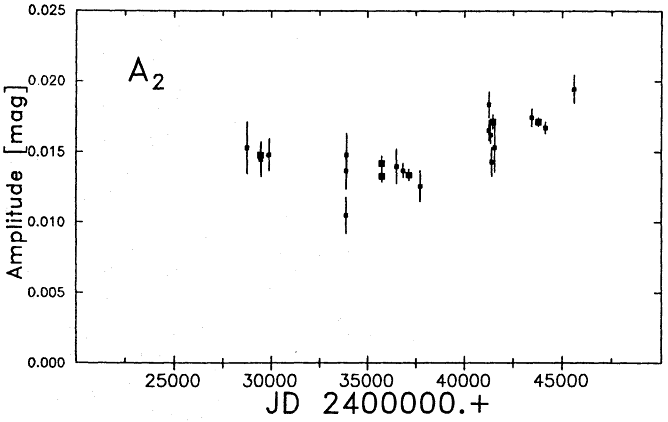

Fig. 2. Photometric semi-amplitudes for the f_2 component of 12 Lac. The small and large symbols refer to the same data as in Fig. 1

5. Discussion and conclusions

In order to explain period changes in β Cephei stars, two effects were usually invoked. The first depends on the evolutionary changes of the radius of the star, which force the changes of pulsation periods. The changes due to this effect take place on the evolutionary time scale and therefore, if detectable, should give a constant-rate of period change. Such period changes are indeed observed for some of the β Cephei stars (see Sterken & Jerzykiewicz 1993). The second effect is the light-time effect, and although there are β Cephei-type stars, in which it is observed (Pigulski & Boratyn 1992; Pigulski 1992), this effect appears only in some unique double systems. The light-time effect should produce, of course, the same (and periodic) changes for all pulsation components of a given multiperiodic star. As we mentioned earlier, Ciurla (1987) proposed this effect to be responsible for the observed O–C diagram for the changes of P_1 in 12 Lac. However, in the view of the changes of other periods in this star (see Fig. 3), one can see that this explanation cannot be retained. In particular, this effect alone is not able to explain why P_2 changes inversely to P_1. A close examination of Fig. 1 leads also to the conclusion that the changes of all periods cannot be explained even by a combination of evolutionary and light-time effects.

Apart from the above-mentioned two effects, there is another one, which should be considered as a possible source of period changes. As was shown by Moskalik (1985), resonant mode coupling of an unstable mode to two stable modes of lower frequency may lead, under some conditions, to periodic modulation of the amplitude and period of the excited mode. The period changes are in this case associated with periodic amplitude changes, which are not, in fact, observed for 12 Lac. Yet, the theory of this effect is not elaborated for the more complex case of a multiperiodic star. It is therefore possible that in a multiperiodic star the changes of periods and amplitudes are different from the case described by Moskalik (1985).

We cannot explain the changes of periods seen in Fig. 1. However, we may suggest that—regarding the regularities seen in period changes (P_4 changes like P_1, P_2 inversely to P_1)—the unknown effect causing these changes probably distinguishes modes with different l and/or m. Whether it is the effect considered by Moskalik (1985) or a different one, remains an open question. A proper mode identification will certainly help understand this effect and vice versa: understanding this effect will probably lead to the correct l and/or m for the observed modes. Securing observational data of good quality during the next decades will be necessary to solve the problem.

As we mentioned in Sect. 2, three of the pulsating components (f_1, f_3, and f_4) form an almost equidistant frequency triplet. This triplet was explained in terms of a rotationally split nonradial mode by J78 and Smith (1980). However, as was originally pointed out by Jerzykiewicz et al. (1984), the frequencies of the triplet components do not agree with the second-order theory of rotational splitting. According to this theory (Saio 1981; Dziembowski & Goode 1992), the difference $f_4 - f_1$ should be larger than $f_3 - f_4$, while the opposite inequality is observed. It can be concluded that the triplet is not rotationally split one, although it is still posssible that a pair of triplet frequencies have the same l. The most probable possibility is that f_1 and f_4 are rotationally split, because their periods change in a similar way.

The frequency f_5 (equal to the sum of f_1 and f_4) is the only combination frequency observed in 12 Lac. Such combination frequencies may appear in the data due to mode coupling. Smith (1980) pointed out that coupling between two pulsation modes with (l_1, m_1) and (l_2, m_2) is dictated by the selection rule: $m_1 = -m_2$. It is interesting to note that although the amplitudes of f_2 and f_3 are larger than that of f_4, only the latter interacts with f_1, leading to appearance of the combination component, f_5. It can be an additional argument in favour of the hypothesis that f_1 and f_4 belong to the mode with the same l, but opposite m (most probably $l = 1$, $m = \pm 1$). There is, however, another β Cephei-type star, namely 16 Lac, for which Jerzykiewicz (1993) found all three possible combination fre-

Table 4. The light and radial-velocity semi-amplitudes, A_i, for all six components of the variation of 12 Lac. The photometric amplitudes are given in mmag, while those for radial-velocities, in km s^{-1}. The numbers in parentheses are the mean errors

Set	N	Year(s)	Semi-amplitudes of light and radial-velocity						References
			A_1	A_2	A_3	A_4	A_5	A_6	
			PHOTOMETRY						
A	860	1937–1940	42.1(0.7)	14.8(0.7)	12.8(0.7)	6.2(0.7)	2.3(0.7)	4.6(0.7)	see Table 1
B	1331	1956	41.2(0.4)	13.3(0.4)	11.7(0.4)	6.8(0.4)	3.3(0.4)	3.8(0.4)	see Table 1
C	1175	1956	40.5(0.5)	14.2(0.5)	10.5(0.5)	5.9(0.5)	2.4(0.5)	3.6(0.5)	see Table 1
D	680	1959–1962	41.1(0.5)	13.4(0.4)	10.7(0.5)	8.0(0.5)	4.8(0.4)	3.0(0.5)	see Table 1
E	807	1971–1973	41.0(0.6)	17.1(0.5)	10.4(0.5)	7.0(0.6)	2.3(0.5)	2.9(0.5)	see Table 1
F	1323	1977–1979	39.2(0.3)	17.1(0.3)	10.5(0.3)	7.4(0.3)	4.8(0.3)	3.4(0.3)	see Table 1
P1	213	1937	37.8(1.7)	15.3(1.8)	10.4(1.6)	6.2(1.6)	1.5(1.4)	7.0(1.4)	(1)
P2	297	1939	43.1(1.4)	14.5(1.2)	13.2(1.4)	3.0(1.5)	2.1(1.2)	3.9(1.3)	(1)
P3	350	1940	43.9(1.1)	14.8(1.1)	12.4(1.1)	7.7(1.2)	2.9(1.0)	4.5(1.1)	(1),(2)
P4	275	1951	38.2(1.5)	10.5(1.3)	14.4(1.4)	9.8(1.4)	4.3(1.3)	3.9(1.5)	(24)
P5	277	1951	44.2(1.4)	13.7(1.3)	12.4(1.4)	11.9(1.3)	4.1(1.3)	1.6(1.5)	(24)
P6	405	1951	40.4(1.6)	14.8(1.5)	13.1(1.6)	6.8(1.5)	4.2(1.5)	7.8(1.6)	(24),(25)
P7	199	1958	37.7(1.4)	14.0(1.2)	11.2(1.5)	9.4(1.5)	4.5(1.0)	4.8(1.2)	(26)
P8	421	1959	40.2(0.6)	13.7(0.5)	11.1(0.6)	9.0(0.6)	3.5(0.5)	3.4(0.5)	(4)
P9	209	1961-1962	41.4(1.3)	12.6(1.1)	10.5(1.1)	6.5(1.0)	5.7(0.8)	3.2(1.1)	(5)
P10	524	1971	45.1(1.9)	18.3(0.9)	7.4(2.0)	9.7(3.0)	2.0(0.6)	2.8(0.8)	(7),(9)
P11	344	1971	41.5(0.9)	16.5(0.7)	10.8(0.9)	4.6(1.1)	2.3(0.6)	2.8(0.7)	(7)
P12	732	1971-1972	41.9(1.2)	14.3(1.0)	11.6(1.2)	5.6(1.4)	1.9(0.9)	3.2(1.1)	(7),(8),(9)
P13	348	1971-1972	41.1(0.7)	16.2(0.6)	10.9(0.7)	5.2(0.8)	2.1(0.5)	3.4(0.6)	(9)
P14	208	1972	41.7(2.8)	15.3(1.7)	7.2(2.7)	8.4(2.1)	2.7(1.2)	2.4(2.1)	(8),(9)
P15	685	1977	39.7(0.6)	17.4(0.6)	10.4(0.6)	7.9(0.5)	5.1(0.5)	3.3(0.5)	(11)
P16	638	1979	38.7(0.4)	16.7(0.4)	10.6(0.4)	7.0(0.4)	4.5(0.4)	3.7(0.4)	(13)
P17	323	1983	37.7(1.0)	19.4(1.0)	11.5(1.0)	9.1(1.0)	3.7(0.8)	2.5(1.0)	(27)
		Average:	40.3(0.4)	15.5(0.7)	10.9(0.2)	7.8(0.4)	3.7(0.3)	3.5(0.1)	
			RADIAL VELOCITIES						
G	273	1911–1918	16.7(0.6)	7.0(0.6)	4.7(0.6)	4.8(0.5)	0.7(0.5)	0.8(0.6)	see Table 1
H	277	1922–1926	18.8(0.5)	7.1(0.5)	6.2(0.5)	4.4(0.4)	2.0(0.4)	1.3(0.5)	see Table 1
I	520	1950–1956	18.3(0.3)	6.6(0.3)	3.8(0.3)	5.6(0.3)	2.2(0.3)	0.9(0.3)	see Table 1
J	614	1989–1990	20.6(0.4)	5.5(0.3)	3.8(0.3)	7.1(0.4)	3.4(0.3)	2.8(0.4)	see Table 1
RV1	250	1950	18.7(0.4)	7.2(0.4)	4.6(0.4)	6.0(0.4)	2.8(0.4)	1.3(0.4)	(19)
RV2	286	1956	18.7(0.5)	7.0(0.5)	3.2(0.5)	5.9(0.5)	2.6(0.5)	1.5(0.5)	(3),(21),(22)
RV3	294	1990	18.8(0.4)	4.9(0.3)	4.0(0.2)	8.8(0.3)	3.8(0.2)	1.2(0.4)	(23)
		Average:	18.8(0.4)	6.2(0.3)	4.1(0.2)	6.4(0.9)	3.0(0.3)	1.4(0.2)	

References: (1) – Fath (1947), (2) – Green (1941), (3) – de Jager (1963) and references therein, (4) – Opolski & Ciurla (1961), (5) – Opolski & Ciurla (1962), (6) – Jerzykiewicz (1963), (7) – Ciurla (1973), (8) – Ciurla (1987), (9) – Sato (1973), (10) – Sato (1979), (11) – Jarzębowski et al. (1979), (12) – Jarzębowski et al. (1980), (13) – Jerzykiewicz et al. (1984), (14) – Young (1915), (15) – Young (1918), (16) – Beardsley (1969), (17) – Christie (1925), (18) – Christie (1927), (19) – Struve (1951), (20) – Struve & Zebergs (1955), (21) – Struve et al. (1957), (22) – Hack (1957), (23) – Mathias et al. (1992), (24) – Nekrasova (1952), (25) – de Jager (1953), (26) – Rakosch (1960), (27) – unpublished Białków observations carried out by B. Musielok, S. Ratajczyk, E. Szuszkiewicz, M. Tomczak, and J. Włodarczyk.

quencies, resulting from the coupling of three largest-amplitude modes excited in this star. Because these modes are suspected to belong to different l (Jerzykiewicz 1993), in order to fulfill the above-mentioned selection rule, they all should have $m = 0$. Clearly, more multiperiodic stars with proper mode identification and the combination frequencies detected are needed to clarify the conditions under which the combination components appear.

Acknowledgements. I would like to thank Prof. M. Jerzykiewicz for his comments made upon reading the manuscript and Dr. P. Mathias for sending unpublished radial velocity observations of 12 Lac. This work was supported by research grant No. 2 P304 001 04 from the Polish Scientific Research Commitee (KBN).

References

Adams W.S., 1912, ApJ 35, 179

Barning F.J.M., 1963, Bull. Astron. Neth. 17, 22

Beardsley W.R., 1969, Publ. Allegheny Obs. vol. VIII, No. 7

Chapellier E., 1985, A&A 147, 135

Christie W.H., 1925, Publ. Dominion Obs. Victoria 3, 209

Christie W.H., 1927, Publ. Dominion Obs. Victoria 4, 59

Ciurla T., 1973, Acta Astron. 23, 367

Ciurla T., 1987, Acta Astron. 37, 53

de Jager C., 1953, Bull. Astron. Neth. 12, 81

de Jager C., 1963, Bull. Astron. Neth. 17, 1

Dziembowski W.A., Goode P.R., 1992, ApJ 394, 670

Fath E.A., 1947, Publ. Goodsell Obs. No. 12, 1

Green H.E., 1941, MNRAS 101, 43

Hack M., 1957, Contr. Oss. Astron. Milano-Merate, Nuova Ser. , No. 109, 8
Heard J.F., Hurkens R.J., Percy J.R., Porco M., 1976, J. R. Astron. Soc. Can. 70, 213
Jarzębowski T., Jerzykiewicz M., le Contel J.-M., Musielok B., 1979, Acta Astron. 29, 517
Jarzębowski T., Jerzykiewicz M., Ríos Herrera M., Ríos Berumen M., 1980, Rev. Mex. Astron. Astrofis. 5, 31
Jerzykiewicz M., 1963, Acta Astron. 13, 253
Jerzykiewicz M., 1978, Acta Astron. 28, 465 (J78)
Jerzykiewicz M., 1993, Acta Astron. 43, 13
Jerzykiewicz M., Borkowski K.J., Musielok B., 1984, Acta Astron. 34, 21
Lomb N.R., 1976, Ap&SS 39, 447
Mathias P., Gillet D., Crowe R., 1992, A&A 257, 681
Moskalik P., 1985, Acta Astron. 35, 229
Nekrasova S.V., 1952, Izv. Krymskoj Astrof. Obs. 9, 126
Opolski A., Ciurla T., 1961, Acta Astron. 11, 231
Opolski A., Ciurla T., 1962, Acta Astron. 12, 269
Pigulski A., Boratyn D.A., 1992, A&A 253, 178
Pigulski A., 1992, A&A 261, 203
Poretti E., Zerbi F., 1993, A&A 268,369
Pozigun V.A., 1966, Perem. Zvezdy 16, 555
Rakosch von K., 1960, Astron. Nachr. 285, 211
Saio H., 1981, ApJ 244, 299
Sato N., 1973, Ap&SS 24, 215
Sato N., 1977, Ap&SS 48, 453
Sato N., 1979, Ap&SS 66, 309
Shobbrook R.R., 1973, MNRAS 161, 257
Smith M.A., 1980, ApJ 240, 149
Stebbins J., 1917, Popular Astronomy 25, 657
Sterken C., Jerzykiewicz M., 1993, Space Sci. Rev. 62, 95
Struve O., 1951, ApJ 113, 589
Struve O., Zebergs V., 1955, ApJ 122, 134
Struve O., Sahade J., Ebbighausen E., 1957, AJ 62, 189
Young R.K., 1915, Publ. Dominion Obs. Ottawa 3, 65
Young R.K., 1918, Publ. Dominion Obs. Victoria 1, 105

This article was processed by the author using Springer-Verlag TEX A&A macro package 1992.