

## THE EQUATIONS OF ELLIPSOIDAL STAR VARIABILITY APPLIED TO HR 8427

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### ABSTRACT

The light curves of ellipsoidal variables lack some of the complexities of eclipsing binary light curves, and the equation that describes their theoretical variations is given as a discrete Fourier series. Explicit equations are also found for variations due to the reflection effect.

These results are then used to determine the orbital elements of the ellipsoidal variable HR 8427 (= V365 Lac). It is found that this is a hot reverse Algol system with an unevolved B4 V secondary star, and not a black hole as had been previously suggested. The system has a mass ratio of 0.63 and an orbital inclination of 52°.

*Subject headings:* binaries: spectroscopic — stars: individual (HR 8427)

### 1. ELLIPSOIDAL VARIABILITY

Ellipsoidal variables are close binaries whose components are distorted by their mutual gravitation but whose orbital inclinations as seen from Earth are too small to create eclipses. The Morris method is a procedure that can be used to determine the orbital elements of such systems, as outlined in Morris (1985). This paper also contains an annotated bibliography of known ellipsoidal variables. The present paper will examine the equations governing ellipsoidal variability and the reflection effect in more detail and use these results to analyze HR 8427.

The light curves of ellipsoidal variables typically have amplitudes of only a few percent, as their light variations are due only to the changing projected areas and surface brightnesses of the distorted stars. Such stars are not exactly ellipsoidal in shape but, rather, fill Roche equipotential surfaces. It is worth examining the discrete Fourier series that would be theoretically expected for such stars. Equation (IV.2-37) of Kopal (1959) may be rewritten and expanded to give

$$\begin{aligned}
 L_1(\phi) = L_0 \{ & [1 + (15 + u_1)(1 + \tau_1)(R_1/A)^3(2 + 5q)[2 - 3 \sin^2 i]/60(3 - u_1) \\
 & + 9(1 - u_1)(3 + \tau_1)(R_1/A)^5q[8 - 40 \sin^2 i + 35 \sin^4 i]/256(3 - u_1)] \\
 & + \{15u_1(2 + \tau_1)(R_1/A)^4q[4 \sin i - 5 \sin^3 i]/32(3 - u_1)\} \cos \phi \\
 & + \{-3(15 + u_1)(1 + \tau_1)(R_1/A)^3q \sin^2 i/20(3 - u_1) - 15(1 - u_1)(3 + \tau_1)(R_1/A)^5q \\
 & \times [6 \sin^2 i - 7 \sin^4 i]/64(3 - u_1)\} \cos 2\phi + [-25u_1(2 + \tau_1)(R_1/A)^4q \sin^3 i/32(3 - u_1)] \cos 3\phi \\
 & + [105(1 - u_1)(3 + \tau_1)(R_1/A)^5q \sin^4 i/256(3 - u_1)] \cos 4\phi + \text{small higher order terms} \} .
 \end{aligned} \quad (1)$$

In this equation,  $q = M_2/M_1$  is the mass ratio between the two stars, where star 1 is the distorted star that is varying in light. The phase is represented by  $\phi$ , with  $\phi = 0^\circ$  when star 1 (the primary star) is farthest from the observer. Usually the primary star is brighter and more massive than the secondary, but not always.  $L_1(\phi)$  is the brightness of star 1 as a function of phase, and  $L_0$  is the brightness the star would have if it were not distorted by the secondary star. The orbital inclination is represented by  $i$ , the limb-darkening and gravity-darkening coefficients of the primary star are represented by  $u_1$  and  $\tau_1$  respectively,  $R_1$  is the mean radius of the primary star, and  $A$  is the semimajor axis of the system. This equation applies to tidally locked ellipsoidal variables in circular orbits, and the models of stellar evolution in Zahn (1977) have shown that this condition is quickly reached compared to the lifetimes of the stars. Equation (1) can be used for the variability of the secondary star if the subscripts 1 and 2 are interchanged, the phase is increased by  $180^\circ$  and  $q$  is replaced by  $1/q$ .

It is of interest to note that the expression in the first pair of braces is the change in the average luminosity of the ellipsoidal star, independent of phase. The term containing the  $(1 - u_1)$  factor is much smaller than the term containing the  $(15 + u_1)$  factor for all reasonable values of  $u_1$ ,  $\tau_1$ ,  $q$ , and  $R_1/A$ , and it equals zero at inclinations of  $30.6^\circ$  and  $70.1^\circ$ . When this term is discarded, the average luminosity of the star is given by

$$L_{\text{ave}} = L_0 \{ 1 + (15 + u_1)(1 + \tau_1)(R_1/A)^3(2 + 5q)[2 - 3 \sin^2 i]/60(3 - u_1) \} . \quad (2)$$

It is a remarkable fact that the second term on the right-hand side of this equation will equal zero when  $i = 54.7^\circ$ , independent of mass ratio or stellar radius. It is clear that a low-inclination binary member will appear brighter than if it were a single star with the same limb- and gravity-darkening.

Most ellipsoidal variables change in luminosity by only a few percent, and it is not productive to try a Fourier analysis of a light curve to match the coefficients in equation (1). The most useful information that can be obtained from a light curve is the value of

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$\Delta M$ , the difference between the magnitude of maximum and the mean magnitude of the two minima, which may be found directly from the light curve. The value of  $\Delta M_1$  (the value  $\Delta M$  would have if the secondary star were nonluminous) can then be calculated from equation (4) of Morris (1985). Solutions should be found assuming the primary star is the variable, and a second set of solutions should be found assuming the secondary is the variable. If this is done for light curves taken at different wavelengths, the correct assumption will be the one that gives a consistent and realistic set of solutions. If both stars do contribute to the variability, then a more complex technique such as Wilson-Devinney computer modeling may be used. Even in this worst-case scenario, the Morris method provides a useful set of initial values.

Equation (1) can be used to give

$$\begin{aligned} q(R_1/A)^3 \sin^2 i = & [\Delta M_1 / \log_{10} e] \{ 960(3 - u_1) + 16(15 + u_1)(1 + \tau_1)(R_1/A)^3(2 + 5q)[2 - 3 \sin^2 i] \\ & + 45(1 - u_1)(3 + \tau_1)(R_1/A)^5 q[6 - 30 \sin^2 i + 35 \sin^4 i] \} / \{ 720(15 + u_1)(1 + \tau_1) \\ & + 1125(1 - u_1)(3 + \tau_1)(R_1/A)^2[6 - 7 \sin^2 i] \}. \end{aligned} \quad (3)$$

The term  $\log_{10} e$  is the logarithm to the base 10 of the natural logarithm  $e$ . Equation (6) of Morris (1985) was derived with the simplification that the average luminosity of a star was independent of its inclination. This simplification is now dropped, and it is suggested that in future the more accurate equation (3) of the present paper be used in place of equation (6) of the earlier paper in deriving the system parameters of ellipsoidal stars.

Some ellipsoidal variables have sufficiently large amplitudes that one more quantity may be determined from their light curves; the magnitude difference of the primary star between the minimum at  $\phi = 0^\circ$  and the minimum at  $\phi = 180^\circ$ , which can be written as  $M_1(0^\circ) - M_1(180^\circ)$ . Equation (1) can be used to give

$$\begin{aligned} M_1(0^\circ) - M_1(180^\circ) = & [1500 \log_{10} e] \{ u_1(2 + \tau_1)(R_1/A)^4 q[-3 \sin i + 5 \sin^3 i] / \{ 480(3 - u_1) \\ & + 16(15 + u_1)(1 + \tau_1)(R_1/A)^3[2 + 5q - 3 \sin^2 i - 12q \sin^2 i] \\ & + 45(1 - u_1)(3 + \tau_1)(R_1/A)^5 q[3 - 30 \sin^2 i + 35 \sin^4 i] \} \}. \end{aligned} \quad (4)$$

The term containing the  $(1 - u_1)$  factor in the denominator is much smaller than the other terms for all reasonable values of  $u_1$ ,  $\tau_1$ ,  $q$ , and  $R_1/A$ , and it equals zero for  $i = 19^\circ.9$  and  $59^\circ.4$ . The term containing the  $(15 + u_1)$  factor is larger but still not too significant and will equal zero at some inclination between  $40^\circ.2$  and  $54^\circ.7$ , depending on the mass ratio.

Of more interest is the surviving numerator of equation (4), which shows that both minima will be equally deep if  $i = 50^\circ.8$ . This reveals an important point that has not been recognized before in work on ellipsoidal variables: the minimum at  $\phi = 0^\circ$  will be the deeper one *only* if the inclination is greater than  $50^\circ.8$ , and this is true regardless of the mass ratio and the radius of the star. If  $i < 50^\circ.8$ ,  $M_1(0^\circ) - M_1(180^\circ)$  will be a negative number. Of course this assumes that the ephemeris is based on the spectroscopic orbit, with  $\phi = 90^\circ$  when the radial velocity of the primary star is at its most negative value.

This creates an important diagnostic when trying to determine if a system is a well-behaved ellipsoidal variable. Once the parameters of the system have been calculated, equation (4) can be used to check if the observed values of  $M_1(0^\circ) - M_1(180^\circ)$  are reasonable. One should also check that  $R_1 + R_2 < A \cos i$  if an eclipse is to be avoided, and that the value of  $\sin i$  found from solving the equations is less than unity. If the solution shows that  $i < 34^\circ$ , then no eclipses are possible even if this is a contact system (Tables 3–4 of Kopal 1959). There are some situations in which a star may seem to be an ellipsoidal variable, but the above analysis gives inconsistent results. There may be some reflection effect, which will have to be subtracted before the system is resolved (see next section). The binary may suffer grazing eclipses which deepen the minima, but this may be detectable if the minima are carefully reexamined. If the two systems are of comparable brightness, it may be the fainter star that is distorted. It is a good idea to get light curves in several wavelengths to help determine if any of these possibilities are likely. If independent solutions have very different results, then the system may contain a pulsating star or have its light curve distorted by the presence of streaming gas.

## 2. THE REFLECTION EFFECT

In a close binary system, some of the light from one star will be absorbed and thermalized by its companion and then reemitted. Each star will be brighter and hotter on the side facing its companion, and this will affect the light curve. The result is rather misleadingly called the reflection effect. While it is the bolometric luminosity that is responsible for this heating, the observer measures the result in some wave band centered on a wavelength  $\lambda$  (in angstroms). The luminous-efficiency factor  $f_\lambda$  (not to be confused with the mass function  $f$ ) can be calculated for the binary system as:

$$f_\lambda = \left( \frac{T_2}{T_1} \right)^4 \frac{\exp(1.43879 \times 10^8 / \lambda T_2) - 1}{\exp(1.43879 \times 10^8 / \lambda T_1) - 1}, \quad (5)$$

where  $T_1$  and  $T_2$  are the temperatures of the two stars in degrees Kelvin.

With this information in hand, equations (IV.6-87) and (IV.6-100) of Kopal (1959) can be used to give the difference between the maximum and the mean of the minima due to the reflection effect alone, to third-order accuracy in the radii:

$$\begin{aligned} \Delta M = & -(5/144\pi) \log_{10} e \sin^2 i \{ [24(R_2/A)^2 + 27\pi(R_2/A)^3 + 2(R_2/A)^2 \sin^2 i] / f_\lambda \\ & + 10^{-0.4\Delta m} [24(R_1/A)^2 + 27\pi(R_1/A)^3 + 2(R_1/A)^2 \sin^2 i] f_\lambda \} / (1 + 10^{-0.4\Delta m}). \end{aligned} \quad (6)$$

If the secondary star is fainter than the primary star,  $\Delta m$  (the magnitude difference between the two components) is a positive number.

TABLE 1  
WAVELENGTH-DEPENDENT PARAMETERS FOR LIGHT-CURVE ANALYSIS

Parameter	<i>U</i>	<i>B</i>	<i>V</i>	<i>R</i>
Observational Data				
$u_1$ .....	0.337	0.360	0.294	0.227
$\tau_1$ .....	0.5505	0.4865	0.4322	0.3883
$\sigma$ of data points .....	0.0089	0.0095	0.0074	0.0081
$\Delta M$ .....	0.0738	0.0684	0.0684	0.0649
$M(0^\circ) - M(180^\circ)$ .....	-0.0018	-0.0007	-0.0004	-0.0009
System Parameters Assuming a B4 V Secondary Star				
<i>i</i> .....	53°	55°	50°	49°
$A(R_\odot)$ .....	17.7	17.7	17.9	17.9
$R_1(R_\odot)$ .....	7.5	7.4	7.9	8.0
<i>q</i> .....	0.61	0.60	0.65	0.67

The reflection effect also influences the magnitude difference between the minima:

$$M(0^\circ) - M(180^\circ) = (5/12) \log_{10} e \sin i \{ [4(R_2/A)^2 + 3(R_2/A)^3]/f_\lambda - 10^{-0.4\Delta m} [4(R_1/A)^2 + 3(R_1/A)^3]/f_\lambda \} / (1 + 10^{-0.4\Delta m}). \quad (7)$$

Although this effect has been incorporated into computer models of binary systems, it is useful to have explicit equations. For example, it is interesting to see how sensitive these variations are to the ratio of the temperatures (approximately to the fourth power). The values of  $\Delta M$  from equations (3) and (6) may be added together (as may eqs. [4] and [7]) to obtain the light-curve behavior due to both ellipsoidal variability and the reflection effect.

These equations do not take re-reflection or the increase in average luminosity into account, as these are fourth-order terms in the radii. Also,  $T_1$  and  $T_2$  should, strictly speaking, be the mean effective temperatures of the illuminated hemispheres only. These equations should be especially useful in studying double-lined spectroscopic binary systems, in which  $T_2/T_1$  and  $\Delta m$  can be more reliably determined.

### 3. OBSERVATIONAL HISTORY OF HR 8427

The star HR 8427 (=V365 Lac) is a bright ( $V = 6.27$ ) single-lined spectroscopic binary that is well placed for Northern Hemisphere observers. This B2 V star is a member of the Lac OB1 Association (formerly called the I Lacerta Association) and is of some interest, as Trimble & Thorne (1969) have pointed out that the mass function of this system implies that the unseen secondary star has a minimum mass of  $5.2 M_\odot$  and may therefore be a collapsed object such as a black hole. If this were the case, it would be a remarkable discovery, especially in such a young association.

Three sets of radial velocity curves were obtained in 1919, 1954, and 1959, which were analyzed separately by van Albada & Klomp (1969). They found that the period was stable at  $P = 2.172710$  days, and the eccentricities were so small and uncertain that the orbit was probably circular. Although they did not publish mass functions, their orbital elements can be used to calculate them as  $f = 0.476 \pm 0.031 M_\odot$ ,  $0.406 \pm 0.026 M_\odot$ , and  $0.48 \pm 0.18 M_\odot$ , respectively. The third set of observations contains only nine data points, and its mass function can be disregarded as too uncertain. The first two values are somewhat discordant, but as van Albada & Klomp point out (1969, p. 209), "The observed velocities show systematic deviations from the velocity curve, in the same sense as would be expected from blending of the lines of the primary and secondary component." For the present paper, a value of  $f = 0.441 \pm 0.035 M_\odot$  will be used. It is rather surprising that the authors did not attempt to combine the data to create one set of orbital elements or solve for a circular orbit. A new set of radial velocities and a comprehensive orbit are needed. It would be particularly valuable if spectra in the red region of the spectrum were taken, where the lines of the fainter secondary star could be observed and measured.

Two careful efforts have classified this star as B2 V without evidence of spectral peculiarities (Levato & Abt 1976). According to Straizys & Kuriliene (1981) a B2 V star has a mass of  $9.8 \pm 1.5 M_\odot$ . Gulati, Malagnini, & Morossi (1989) used three independent temperature calibrations, which all gave the same temperature of  $21500 \pm 2000$  K. This temperature is used to find the limb-darkening coefficients  $u_1$  from the tables of Al-Naimiy (1978) and the gravity-darkening coefficients  $\tau_1$  from equation (10) of Morris (1985). These coefficients are listed in Table 1. The primary star's projected rotational velocity is  $160 \pm 20 \text{ km s}^{-1}$  according to Abt & Hunter (1962). McCrosky & Whitney (1982) observed this star photoelectrically and announced that this system showed variations characteristic of ellipsoidal variability, with a peak-to-peak amplitude of 0.08 in the *V* bandpass. Although they unfortunately do not give their data, they do give the epoch for minimum light as JD 2,444,392.0. McAlister et al. (1987) used speckle interferometry to show that this star does not have any close companions with a separation  $\geq 0''.038$  and  $\Delta m \leq 2$ , limiting the possibility of third light.

### 4. NEW PHOTOMETRIC OBSERVATIONS

Figure 1 shows the four light curves of HR 8427 which were obtained on 10 consecutive nights of photoelectric photometry with the 0.6 m reflector at Table Mountain Observatory. Standard Johnson *UBVR* filters were used, and the data were corrected for sky background and first-order atmospheric extinction.

The comparison star chosen was HD 210353. The check star HD 210697 was also observed 3 or 6 times each night and proved the comparison star to be constant. These two stars were chosen because of their proximity and because their color indices are very

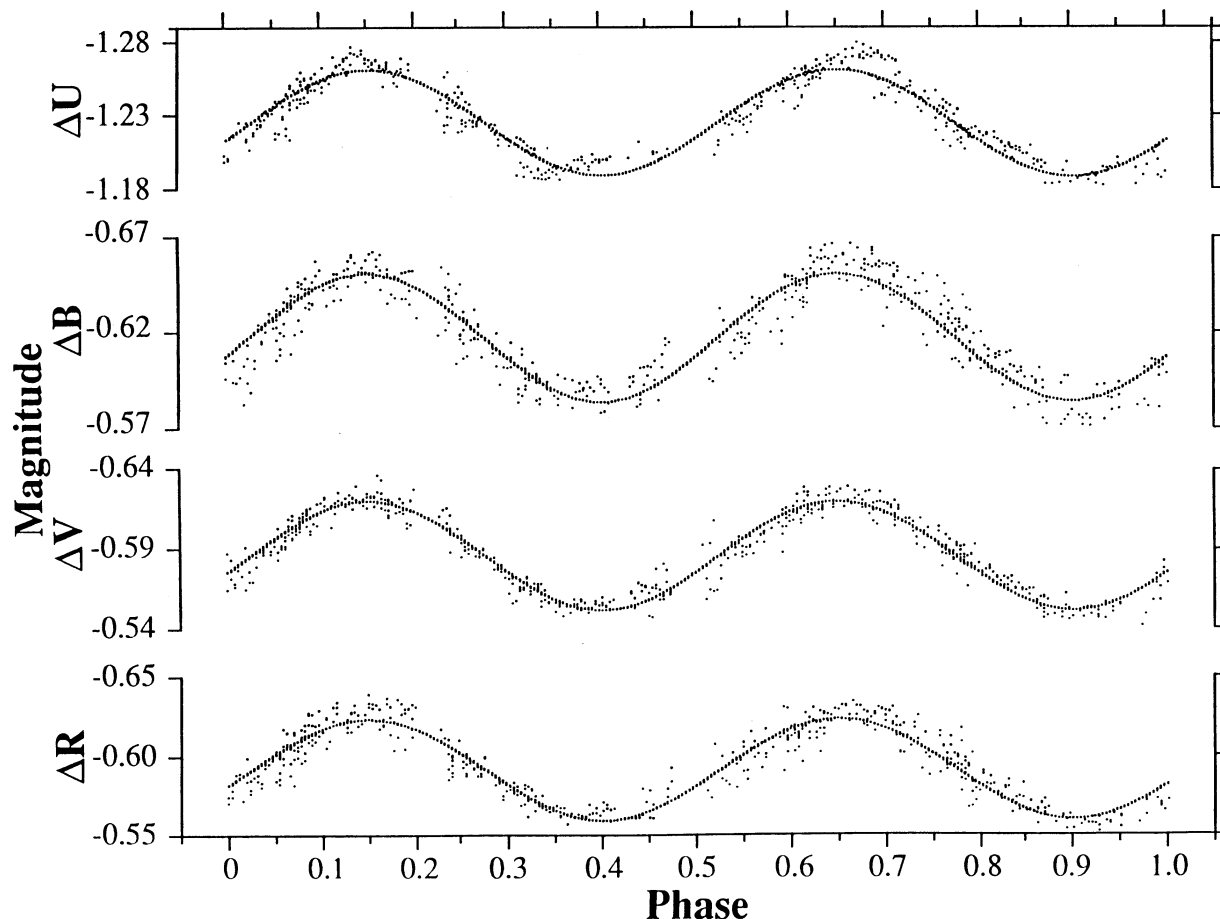


FIG. 1.—The differential light curves of HR 8427, in the Johnson *UBVR* bandpasses, given as HR 8427 minus HD 210353. The phase is calculated with a period of 2.172710 days and an epoch of HJD 2,435,001.92.

close to those of HR 8427. A least-squares fit of a  $\cos \phi$  and a  $\cos 2\phi$  curve was found using the technique of Gill & Murray (1978). Based on this curve, Table 1 gives the standard deviation of the data points  $\sigma$ , the difference between the maxima and the mean of the minima  $\Delta M$ , and the difference between the two minima  $M(0^\circ) - M(180^\circ)$ . The phases are calculated from the ephemeris of van Albada & Klomp (1969) with epoch  $T = \text{HJD } 2,435,001.92$ , and tables of the photometric data have been deposited as file number 239 in the IAU archives of unpublished photoelectric observations, as described by Breger, Jaschek, & Dubois (1990).

It may be seen from the displacement of the light-curve minima from  $\phi = 0^\circ$  that the published period of 2.172710 days is slightly too large. The epoch of primary minimum for these light curves was found to be HJD 2,448,061.886, according to the widely accepted method of Kwee & Van Woerden (1956). The secondary minima were found to occur one-half period later than the primary minima. The technique given by Ghedini (1982) is useful when the two branches of the light-curve minima are asymmetrical, and it produced essentially the same result. A recalculation of the ephemeris is not justified until another set of radial velocities is obtained.

##### 5. ANALYSIS OF HR 8427

Ignoring the light curves of the system for the moment, equations (7), (8), and (9) of Morris (1985) can be applied to this star's parameters as given in § 3. When this is done, it is found that the radius of the primary star is much larger than the critical Roche lobe for all possible values of the mass ratio  $q$ . It was found by varying the parameters that uncertainties in the value of  $v \sin i$  dominated the variations in the radius of the primary. Stothers (1973) has noted that the uncertainty of  $v \sin i$  measurements is typically  $\pm 15\%$ , which matches the large uncertainty given for the single available measurement for this star. Indeed, all values of  $v \sin i \geq 150 \text{ km s}^{-1}$  give solutions with large inclinations that generate eclipses. It was also found that all values of  $v \sin i \leq 130 \text{ km s}^{-1}$  give solutions with primary stars significantly larger than their critical Roche lobes, which is physically unrealistic. Only a very small range of values centered on  $v \sin i = 140 \text{ km s}^{-1}$  are permitted, and this value is used for the rest of the paper. Such an alteration is not unrealistic, as this assumed value is only  $1 \sigma$  smaller than the measured value.

Since the secondary-star visible-band absorption line pattern is unobserved, a grid of solutions was obtained for each color, assuming different values of  $\Delta M_2$  (the amount of variation due to the secondary star) and assuming different values of  $\Delta m$  (the magnitude difference between the two stars). It was found from equation (3) that the only realistic solutions were those that had the secondary star contributing 5% or less of the variability. If the secondary star is large enough ( $> 3 R_\odot$ ) to contribute more



variability than this, then it is large enough to create significant eclipses. If the secondary star is assumed to be small, all solutions for  $\Delta m > 1.5$  in the  $V$  bandpass are realistic. Clearly, the secondary star contributes almost nothing to the variability but may contribute a large constant brightness to the system.

The following three extreme cases could be identified from the grid of solutions.

1. *Black hole solution.*—If the secondary star contributes no light to the system, the Morris method gives  $q = 0.55$ ,  $i = 60^\circ$ ,  $R_1 = 6.9 R_\odot$ , and  $A = 17.5 R_\odot$ . Such a large inclination gives a *positive* value of  $M(0^\circ) - M(180^\circ)$ , contrary to the negative (and almost negligible) values given in Table 1. The black hole solution may be rejected.

2. *Secondary star is constant with respect to phase.*—The smallest scatter among the solutions of the four light curves implies that  $\Delta m = 2.0$  in the  $V$  bandpass, and this solution gives  $q = 0.61$ ,  $i = 51^\circ$ ,  $R_1 = 7.5 R_\odot$ , and  $A = 17.7 R_\odot$ . Smaller values of  $\Delta m$  give a primary star that overflows its Roche lobe. This solution provides no information about the secondary star, except that it must be significantly smaller than its Roche lobe ( $6 R_\odot$ ).

3. *Secondary star varies with respect to phase.*—As the secondary star is allowed to grow, the solutions that match the observed values of  $\Delta m$  and  $M(0^\circ) - M(180^\circ)$  shift to slightly larger values of  $\Delta m$ . If the secondary star is assumed to generate just 1% of the system's ellipsoidal variability, the resulting orbital elements ( $q = 0.87$ ,  $i = 39^\circ$ ,  $R_1 = 9.7 R_\odot$ ,  $A = 18.6 R_\odot$ , and  $R_2 = 3.2 R_\odot$ ) demand that the primary star overflow its critical Roche lobe. The secondary star's contribution to the variability of the system must be less than this.

All of these solutions imply that the secondary star has a mass appropriate for a B4 V star. Using the  $UBVR$  colors of such a star (Allen 1973), the solutions that give the least scatter among the four light curves are given in Table 1. The average of these solutions in Table 1 gives the best solution for this system:  $q = 0.63$ ,  $i = 52^\circ$ ,  $R_1 = 7.7 R_\odot$ , and  $A = 17.8 R_\odot$ . The primary star is either filling its Roche lobe or is close to doing so. The radius of the secondary star's critical Roche lobe is  $5.9 R_\odot$ , and since B4 V stars typically have radii of about  $3.6 R_\odot$  (Straizys & Kuriliene 1981), the secondary star will not contribute a significant ellipsoidal variation or reflection effect to the light curves. If the secondary star is still on the zero-age main sequence, its radius may be only  $2.5 R_\odot$ .

What is the evolutionary status of the system? Blaauw (1964) has found that the association this star is in has an age of 16 Myr based on its color-magnitude diagram and 7 Myr based on kinematic data, so the stars cannot have evolved much. On the other hand, the primary star has a significantly larger radius than an ordinary B2 star has on the main sequence ( $4.8 R_\odot$  according to Straizys & Kuriliene 1981). The system is apparently a hot reverse Algol, as described by Leung (1989). This implies that the more massive primary has begun to evolve off the main sequence, but has not yet exchanged mass with the secondary star. According to Figure 4 of Leung (1989), the primary has reached the terminal-age main sequence.

It would be interesting to know how close this star is to filling its Roche lobe. A modern radial velocity curve should be obtained to determine a definitive mass function for this system and to resolve the differences among previous studies. The measurement of the radial velocities of the secondary star are probably within reach of modern detectors. Once that information is in hand, another analysis (perhaps using the Wilson-Devinney computer code) would be in order.

*Note added in manuscript (1993 July 26).*—The authors have just learned that Aslanov & Khruzina (1990) give a value of the mass ratio determined from a direct measurement of the faint spectral lines of the secondary. They state that  $q$  is between 0.60 and 0.65, in excellent agreement with the value of  $q = 0.62$  determined in the present paper.

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