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NEUTRON STAR DYNAMOS AND THE ORIGINS OF PULSAR MAGNETISM

CHRISTOPHER THOMPSON[1,2]
Theoretical Astrophysics, 130–33 Caltech, Pasadena, CA 91125

AND

ROBERT C. DUNCAN
Department of Astronomy and McDonald Observatory, University of Texas, Austin, TX 78712
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ABSTRACT

Young neutron stars are convective, and amplification of their magnetic fields seems almost inevitable. Fields as strong as 3×10^{15} G are generated during a period of entropy-driven convection, if the ratio of mean magnetic pressure to turbulent pressure equals that observed in the upper convection zone of the Sun. The ~ 1 ms convective overturn time is probably much shorter than the initial rotation period of a typical pulsar. The energy for field amplification comes mostly from convection, not differential rotation, if the initial rotation period exceeds ~ 30 ms, and probably also for significantly shorter periods. Thus rotation plays a smaller role than in the solar dynamo, and the dominant mode of field amplification will have a scale much smaller than the stellar radius. Large-scale $\alpha - \Omega$ dynamo action is nonetheless possible in protopulsars with rotation periods $P_{\rm rot} \lesssim 30$ ms during the last stages of neutrino diffusion, and perhaps earlier if there exists a postbounce phase of slow mixing before the onset of entropy-driven convection.

Neutron star convection is a transient phenomenon and has an extremely high magnetic Reynolds number, $\Re_m \sim 10^{17}$. In this sense, a neutron star dynamo is the quintessential fast dynamo. The convective motions are only mildly turbulent on scales larger than the $\sim 10^2$ cm neutrino mean free path, but the turbulence is well developed on smaller scales. Study of a neutron star dynamo raises several fundamental issues in the theory of fast dynamos, in particular the possibility of dynamo action in mirror-symmetric turbulence. Other issues include the role played by turbulent diffusion, the relevance of the Bondi-Gold theorem, and the degree of intermittency of the field generated. We argue that in any high-$\Re_m$ dynamo, most of the magnetic energy becomes concentrated in thin flux ropes when the field pressure exceeds the turbulent pressure at the smallest scale of turbulence.

Most of the magnetic energy of a young pulsar probably resides on scales smaller than the ~ 1 km dimension of an individual convective cell in the newborn neutron star. The surface field strength significantly exceeds that expected in a simple dipole model if even a tiny fraction of the field generated during the convective epoch is retained. The crustal field of a young pulsar should have many discontinuities, at which reconnection is inhibited by the stable stratification of the star. Diffusive processes in the crust eventually allow the field to reconnect, which may result in persistent, detectable shifts in the spin-down rate. The field should provide sufficient free energy to power the largest observed pulsar glitches.

We also examine the possibilities for dynamo action during the various (precollapse) stages of convective motion that occur in the evolution of a massive star, and contrast the properties of white dwarf and neutron star progenitors. In general, the Rossby number of the convective motions, and the energy density in these motions, both increase as evolution progresses. Thus, the earliest stages of convection provide the most suitable site for an $\alpha - \Omega$ dynamo, while the latest stages of convection generate the most intense fields. On energetic grounds, main-sequence convective episodes are capable of accounting for a neutron star dipole field stronger than 10^{12}–3×10^{13} G. If the dipole field is generated after collapse, then this field strength also arises naturally as the random superposition of many small dipoles of strength 10^{14}–10^{15} G and size ~ 1 km. The postcollapse convection should destroy any preexisting correlation between the magnetic and rotation axes. Magnetically induced fluctuations in the brightness of the neutrinosphere will also impart a recoil to the star, which plausibly is of magnitude ~ 100 km s^{-1}.

We consider the effect of the strong magnetic fields generated in a nascent neutron star on a supernova explosion, including deposition of energy outside the star via the reaction $\nu \rightarrow \nu + e^+ + e^-$, via neutron scattering off electron pairs trapped in magnetic flux ropes, and also via more conventional mechanisms such as magnetic reconnection and MHD waves. Some of these energy sources may suffice to revive a stalled supernova shock, but only under optimistic assumptions concerning the strength of the dynamo-generated field. We also briefly discuss the possible role of extremely strong magnetic fields in producing gamma-ray bursts.

Subject headings: convection — gamma rays: bursts — MHD — pulsars: general — stars: interiors — stars: magnetic fields — stars: neutron — supernovae: general

[1] Richard Chace Tolman Fellow.
[2] Present Address: CITA, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A1.

1. INTRODUCTION

Pulsars are neutron stars endowed with strong magnetic fields, as deduced from their observed period derivatives. In a simple magnetic dipole braking model (Pacini 1967; Gunn & Ostriker 1969), the polar field strength of young pulsars lies in the range $B_{\rm dipole} \sim 10^{12}$–$3 \times 10^{13}$ G (e.g., Lyne, Manchester & Taylor 1985; Narayan & Ostriker 1990). The origin of this characteristic field strength has been the subject of much controversy and speculation.

One possibility is that the flux through the star is simply trapped primordial flux. Although the characteristic dipole field strength is not readily explained, this scenario does at least have some plausibility, in that magnetic flux is conserved during the core collapse events which produce neutron stars, and the ohmic decay time of a large-scale magnetic field in quiescent regions of a massive star greatly exceeds the star's lifetime (Cowling 1945).

There are at least two problems with this "fossil field" hypothesis, however:

1. Massive SN II progenitor stars have convective cores, and nascent neutron stars are themselves convective. Thus the dipole field is promordial only if the fluid motions tangle up the core field without significantly changing its low-order moments. We will argue that this is unlikely: dynamo action can occur in such convective episodes.
2. Only a few percent of white dwarfs have magnetic fields in excess of 10^6 G (Angel, Borra, & Landstreet 1981; Schmidt 1989), which would correspond to fields stronger than a few 10^{11} G if the star were compressed to nuclear density. It does not appear possible to explain the absence of strong fields by field decay (e.g., Wendell, Van Horn, & Sargent 1987). Note that white dwarfs form from progenitor stars with initial masses less than $\sim 8\ M_\odot$, whereas neutron stars form from more massive progenitors, and so the efficiency of the precollapse dynamo is likely to be different in the two cases. Nonetheless, we consider this strong evidence against the primordial field hypothesis.

In this paper we consider turbulent dynamo amplification as a mechanism for generating neutron star magnetic fields. We consider dynamos operating both in convective regions of the progenitor star and in the young neutron star itself. We have not been able to exclude the possibility that the dipole field originates in either phase. However, if the postcollapse dynamo does amplify $B_{\rm dipole}$, then the characteristic dipole field strength arises naturally. It is clear, we will argue, that small-scale magnetic fields *much* stronger than $B_{\rm dipole}$ are generated in young neutron stars, and it appears very likely that enough of this field energy is retained to induce multipolar structure in the surface fields of young pulsars. This amplification would occur in neutron stars formed via accretion-induced collapse of white dwarfs as well as in the collapsed cores of massive stars.

The possibility of a dynamo in a young neutron star was first noted by Flowers & Ruderman (1977) but at that time little was understood about the internal motions of the star. An alternative dynamo mechanism is that of Blandford, Applegate, & Hernquist (1988), who showed that magnetic fields can be generated in cooling neutron star crusts by thermoelectric currents driven by the temperature gradient. This can plausibly produce $\sim 10^{12}$ G fields, but the time scale for the field generation is $\sim 10^5$ yr, which cannot account for the fields of some young pulsars like Crab and Vela.

We begin our study in §§ 2–4 by elucidating the physical conditions in nascent neutron stars, laying the groundwork for understanding dynamo action on them. In § 2 we explain why convection driven by entropy gradients is inevitable in any newly formed neutron star on account of the rapid neutrino cooling, and we estimate properties of this convection using mixing-length theory. In § 3 we consider the viscosity of hot nuclear matter, and show that neutron star turbulence has a "double cascade" structure: neutrino viscosity can partially damp the fluid motions on large scales, where the turbulence is mild, but this does not avert the onset of fully developed turbulence on scales smaller than the neutrino mean-free path. In § 4 we find charge transport coefficients in hot nuclear matter and show that the convection-driven turbulence is strongly in the MHD limit. We also estimate the magnetic Prandtl number, Rayleigh number, and other parameters for the flow.

In § 5 we gather the results from the earlier sections and summarize the main physical ingredients that may engender magnetic field amplification in nascent neutron stars. To put this in perspective, we compare the physical properties of a convective neutron star with those of the convective region of the Sun.

The two sections which follow deal with more general aspects of dynamo theory important for neutron stars. In § 6, we discuss dynamos operating at extremely high magnetic Reynolds number. We conjecture that the magnetic field generated by such a dynamo develops a flux rope structure when the Maxwell stress exceeds the turbulent stress on the smallest turbulent scale allowed by viscosity. In § 7 we discuss the observational and theoretical evidence for and against nonhelical dynamo action in slowly rotating stars.

In § 8 we consider the possibilities for magnetic amplification in massive stars *before* the core collapse events that form neutron stars. We show that precollapse dynamos, especially α-Ω dynamos in convective H-burning cores during the main-sequence phase, could play an important role in determining pulsar dipole fields. At very least, the precollapse dynamos generate very strong seed fields for the neutron star dynamo. Also in § 8, we compare the properties of different convective epochs, both before and after the formation of the neutron star, especially as regards the strength and scale of the dynamo-generated field.

In § 9 we elucidate the physical mechanisms involved in the generation of white dwarf magnetic fields.

In § 10 we apply the results of §§ 6, 7, and 8 to the question of dynamo action in young neutron stars. We argue that small-scale magnetic fields of strength $\sim 3 \times 10^{15}$ G are probably generated in the convection zones of protopulsars, although $B_{\rm dipole}$ is always significantly weaker when the rotation period is much greater than ~ 1 ms.

In § 11 we briefly discuss the effects of these intense fields on supernova explosions. We discuss various neutrino reactions which occur rapidly in supercritical magnetic fields, $B > B_{\rm crit} = m_e^2/e = 4.4 \times 10^{13}$ G.[3] We also consider the more promising possibility of heating of the exterior of the star by magnetic reconnection and MHD waves.

In § 12, we present our conclusions on the origin of pulsar dipole fields. In § 13, we suggest that observed pulsar peculiar velocities result from magnetically induced anisotropic neutrino emission during the first ~ 10 s after collapse. In § 14, we examine how much magnetic energy is entrained in a neutron star, and on what spatial scale, as convection ceases and the

[3] Throughout this paper we employ units in which $\hbar = c = k_B = 1$.

star evolves into a pulsar. Despite the absence of material stresses in the quiescent liquid core of a neutron star, the buoyant motion of flux ropes out of the core is prevented by the stable stratification of the stellar material (Goldreich & Reisenegger 1992). Although a considerable amount of field rearrangement (constrained by the stable stratification) may occur at early times, the surface field remains quite complex. In particular, crustal stresses do not limit the strength of the surface field and play a less central role in fixing the dipole field of a neutron star than is often supposed. We consider the implications of strong high harmonics for the evolution of pulsar fields, and for phenomena such as glitches.

In § 15 we briefly consider the role that extremely strong magnetic fields might play in models for gamma-ray bursts. In § 16 we give conclusions and mention some unresolved questions.

Readers with interests in astronomy rather than dynamo theory may want to skip §§ 6 and 7 on a first reading, and rely on the summary of these sections in § 10. Readers with interests in dynamo theory may wish to focus on §§ 6 and 7, following the basic groundwork of §§ 2–5.

2. CONVECTION IN A NASCENT NEUTRON STAR

2.1. *Why Convection Occurs*

The outer half of a neutron star is born hot, shock-heated after the core bounce to an initial entropy per baryon $S \simeq 5$–10. Convection may, in principle, be driven by a radial gradient in the lepton number per baryon Y_l (Epstein 1979), as well as by an entropy gradient. Convective instability sets in when the quantity

$$\frac{dS}{dr} + \left(\frac{\partial S}{\partial Y_l}\right)_{P,\rho} \frac{dY_l}{dr} \tag{1}$$

is negative.

Numerical models of the collapse indicate that the nascent neutron star and the surrounding mantle can be unstable to convection, for two independent reasons. First, the outgoing shock weakens as it dissociates heavy nuclei, creating a negative radial gradient dS/dr (Mayle & Wilson 1988; Bethe, Brown, & Cooperstein 1987). Second, the outermost layers of the star lose entropy and lepton number faster than the interior, and negative gradients in these quantities are established (Epstein 1979; Burrows 1987; Burrows & Lattimer 1988, hereafter BL88).

The first phase of convection lasts at most ~ 100 ms. It is sensitive to the mass and composition of the precollapse core; in particular, it could be suppressed if dY_l/dr were initially large and positive. Recent two-dimensional simulations (Burrows & Fryxell 1992) indicate that this phase can drive near-sonic flows in the bloated ($r \sim 100$ km) outer layers of the star. However, this phase will probably not persist for a very large number of overturn (mixing) times.

The second phase of convection should occur in any young neutron star that has undergone a hydrodynamic bounce at a density near nuclear density,[4] *irrespective of whether it forms via the collapse of the degenerate-electron core of a massive star, or via accretion-induced collapse of a white dwarf.* This phase exists for quite general reasons and should persist until the star has radiated almost all the heat generated after the collapse. In the layers of the star just below the neutrinosphere, the radiative temperature profile can be related to the pressure by $T(r) \propto P^{1/2}(r)$ once the energy and lepton number transport have settled to a quasi-steady state. (To derive this relation, divide the equation of neutrino radiative transfer by the equation of hydrostatic equilibrium. Just below the neutrinosphere, both nucleons and neutrinos are nondegenerate and the neutrino opacity scales as T^2.) The adiabatic temperature profile is less steep, $\sim T(r) \propto P^{1/3}(r)$ when $Y_l \lesssim 0.1$ and $S \sim 3$ (e.g., Lattimer, Burrows, & Yahil 1985), thus $dS/dr < 0$ in the outermost layers of the star. In this situation, the sign of dY_l/dr is relatively unimportant since the magnitude of the thermodynamic derivative $(\partial \ln S/\partial \ln Y_l)_{P,\rho}$ is small when $Y_l \lesssim 0.1$ (Lattimer & Mazurek 1981), as opposed to $Y_l \sim 0.35$ in the precollapse core. Convection ensues, driving the gradients toward adiabaticity. More detailed cooling models of an isolated neutron star (Burrows & Lattimer 1986, hereafter BL86) indicate that the convectively unstable region encompasses the outer third of the mass of the star 1 s after its formation, and that this region grows as heat diffuses into the unshocked inner core.

We should caution that this type of convective instability is found to be suppressed in the collapse simulations of Mayle & Wilson (1991) until 1 s after bounce, the entropy exhibiting a sharp positive gradient near the neutrinosphere due to neutrino pair annihilation and residual accretion onto the star. A complete model of convective instability in young neutron stars must account for these effects self-consistently. In the absence of rapid ($\gtrsim 0.1\ M_\odot\ \mathrm{s}^{-1}$) fallback to the star or efficient transport of entropy by other hydrodynamical instabilities such as "neutron fingers" (Wilson & Mayle 1988), the conclusion that convective instability occurs before most of the thermal energy is radiated away could be reversed only by a substantial modification of the calculated equation of state of hot nuclear matter. Such a modification is not likely at densities below nuclear matter density.

2.2. *Convective Velocity*

During the first ~ 3 s of its life, the star radiates $\sim 10^{53}$ ergs in the form of neutrinos. Such a large heat flux drives very vigorous convection when $dS/dr < 0$. Given a convective luminosity $L_{\rm con}$ at radius R and density ρ, the convective velocity is

$$\begin{aligned} V_{\rm con} &= \left(\frac{\Gamma - 1}{2\Gamma}\,\frac{L_{\rm con}}{4\pi R^2 \rho}\right)^{1/3} \\ &= 1.3 \times 10^8 \left(\frac{L_{\rm con}}{3 \times 10^{52}\ \mathrm{erg\ s^{-1}}}\right)^{1/3} \\ &\quad \times \left(\frac{\rho}{10^{14}\ \mathrm{g\ cm^{-3}}}\right)^{-1/3} \left(\frac{R}{15\ \mathrm{km}}\right)^{-2/3}\ \mathrm{cm\ s^{-1}}\,. \end{aligned} \tag{2}$$

Here we have adapted the mixing-length prescription of Böhm-Vitense (1958) to a fluid of semidegenerate fermions with $\Gamma \equiv \partial \ln P/\partial \ln \rho$, and taken $\Gamma = 5/3$. Setting the mixing length equal to the local pressure scale height $l_p = P/\rho g \sim$ 0.3–1 km (here g is the gravitational acceleration), the overturn time of a convective cell is $\tau_{\rm con} \sim l_p/V_{\rm con} \sim 10^{-3}$ s at the base of the convection zone and $\tau_{\rm con} \sim 10^{-4}$ s near the neutrinosphere. Note that the initial rotation period of the star (see Narayan & Ostriker 1990) is almost certainly longer, and probably much longer, than $\tau_{\rm con}$.

[4] A possible exception is a neutron star progenitor that carries enough angular momentum to undergo a rotation-supported bounce at a density well below nuclear density. Such a star may develop rotation-driven meridional circulations and other instabilities which tend to homogenize the entropy in its shocked layers (e.g., Mönchmeyer & Müller 1989). We say a little more about such "fizzlers" in § 15.

3. TURBULENCE IN HOT NUCLEAR MATTER: THE DOUBLE CASCADE

In a hot nuclear fluid, neutrinos induce viscous stresses in the flow on scales that are large compared to the neutrino mean-free path. We assume three flavors of nondegenerate neutrinos and antineutrinos in degenerate nuclear matter, because BL86 find that the electron neutrinos are only mildly degenerate in the part of the star which is unstable to convection. The neutrino viscosity is then

$$\nu[\nu] = \frac{0.4 f(Y_p) T}{G_F^2 m_n^{2/3} \rho^{4/3}}\,, \tag{3}$$

where Y_p is the proton fraction, $f(Y_p) \equiv [Y_p^{1/3} + (1 - Y_p)^{1/3}]^{-1}$, and we have used the scattering cross sections of Iwamoto & Pethick (1982) appropriate to the limit $E_\nu > T$. With the definitions $\rho_{14} \equiv (\rho/10^{14}\ \mathrm{g\ cm^{-3}})$, $T_{30} \equiv (T/30\ \mathrm{MeV})$, this implies

$$\nu[\nu] = 2 \times 10^9 \frac{f(Y_p) T_{30}}{\rho_{14}^{4/3}}\ \mathrm{cm^2\ s^{-1}}\,. \tag{4}$$

Note that $f(Y_p)$ is close to unity: $0.63 < f(Y_p) < 1$. The Reynolds number based on neutrino viscosity is

$$\Re[\nu] = \frac{V_{\rm con} l_p}{\nu[\nu]} \simeq 6 \times 10^3 \frac{V_{\rm con\,8}\, l_{p5}\, \rho_{14}^{4/3}}{T_{30} f(Y_p)}\,, \tag{5}$$

where we use the shorthand $V_{\rm con\,8} \equiv (V_{\rm con}/10^8\ \mathrm{cm\ s^{-1}})$ and $l_{p5} \equiv (l_p/10^5\ \mathrm{cm}) = (l_p/1\ \mathrm{km})$. Thus the flow is turbulent, but only moderately so, at the base of the convection zone when $T_{30} \sim 1$ (BL88); but the Reynolds number increases as the temperature drops.

One may also demonstrate that the convection is turbulent from a more fundamental perspective (i.e., without invoking semiphenomenological mixing-length theory) by calculating the Rayleigh number across one pressure scale height,

$$\mathrm{Ra} = \frac{g\alpha \Delta T l_p^3}{\kappa \nu}\,. \tag{6}$$

Here $\Delta T \simeq |l_p dT/dr|$ is the temperature drop, and $\alpha = (1 - \Gamma^{-1}) C_p/\rho g l_p$ is the coefficient of thermal expansion of a quasi-degenerate Fermi fluid with specific heat C_p per unit volume. Since both momentum and heat are transported primarily by neutrinos, the thermal diffusivity κ is related to the kinematic viscosity by

$$\kappa \sim \frac{\rho}{C_p T}\, \nu = 1.5 \times 10^{10} T_{30}^{-1} \rho_{14}^{-2/3}\ \mathrm{cm^2\ s^{-1}}\,, \tag{7}$$

where $C_p = (\pi/3)^{2/3} [f(Y_p)]^{-1} \rho^{1/3} m_n^{2/3} T$ in a noninteracting gas of semidegenerate nucleons. The Prandtl number may be expressed in terms of the momentum $p_{\rm F} = (3\pi^2 \rho/m_n)^{1/3}$ as

$$\mathrm{Pr} \equiv \frac{\nu}{\kappa} \simeq \frac{1}{f(Y_p)} \left(\frac{\pi T}{p_{\rm F}}\right)^2 = \frac{0.15}{f(Y_p)}\, T_{30}^2 \rho_{14}^{-2/3}\,. \tag{8}$$

In general, low Prandtl number fluids are not able to support steady, laminar convection (e.g., Busse 1985, Fig. 5.5). The convective cells develop oscillatory modes and break up as Ra is raised above the critical value for convective instability.

Substituting expression (7) into (6), the Rayleigh number is

$$\mathrm{Ra} \sim \left(\frac{\Gamma - 1}{\Gamma}\right)\left(\frac{l_p}{\kappa}\right)^2 \frac{\Delta T}{T} \sim 1.6 \times 10^{10} l_{p,5}^2\, T_{30}^2 \rho_{14}^{4/3} \frac{\Delta T}{T} \tag{9}$$

for $\Gamma = 5/3$. What does this result imply? For Pr = 7, the convective motions become well mixed when the Rayleigh number is larger than $\sim 10^7$ (Busse 1985, Fig. 5.7). Since the degree of turbulence of the convection depends on the quantity Ra/Pr (e.g., Landau & Lifshitz 1959), we conclude that the critical Rayleigh number for turbulent, mixing-length convection in a young neutron star is $\sim 10^5$, well below our estimate (9).

Assuming a Kolmogorov spectrum, the turbulent cascade would terminate via neutrino viscous damping at a scale

$$l_{\rm vis}[\nu] = \Re[\nu]^{-3/4} l_p \simeq 200 \frac{l_{p5}^{1/4}\, T_{30}^{3/4} [f(Y_p)]^{3/4}}{\rho_{14} V_{\rm con\,8}^{3/4}}\ \mathrm{cm}\,. \tag{10}$$

However, neutrino viscosity is only effective in terminating the cascade when $\lambda_\nu \ll l_{\rm vis}[\nu]$, where

$$\lambda_\nu \simeq \frac{2.3 f(Y_p)}{G_{\rm F}^2 m_n^{2/3} \rho^{1/3} T^3} \simeq 200 \rho_{14}^{-1/3} T_{30}^{-3} f(Y_p)\ \mathrm{cm}\,. \tag{11}$$

is the neutrino mean free path weighted by the neutrino energy density in a spectral average. This value of λ_ν is small compared to the size $\sim l_p$ of the convective cells, but not compared to $l_{\rm vis}[\nu]$. Thus the turbulent cascade actually continues to much smaller scales. It finally is damped by the viscosity of electrons, which is given by

$$\nu[e] = \frac{\pi n_e}{20 \ln \Lambda\, \alpha_{\rm em}^2 \rho} \simeq 0.4 \frac{Y_{0.2}}{\ln \Lambda}\ \mathrm{cm^2\ s^{-1}} \tag{12}$$

(Nandkumar & Pethick 1984). In this equation, $\alpha_{\rm em}$ is the fine structure constant, the Coulomb logarithm $\ln \Lambda$ is of order unity, and $Y_{0.2} \equiv Y_e/0.2$. The Reynolds number $\Re[e]$ on a scale $l < \lambda_\nu$ is, when $\lambda_\nu > l_{\rm vis}[\nu]$,

$$\Re[e](l) \simeq 5 \times 10^9 \frac{V_{\rm con\,8}}{l_{p5}^{1/3} Y_{0.2}} \left(\frac{l}{100\ \mathrm{cm}}\right)^{4/3}\,. \tag{13}$$

On scales as small as this the turbulence is well developed.

If λ_ν is less than $l_{\rm vis}[\nu]$, significant dissipation occurs on the scale $l_{\rm vis}[\nu]$. The flow velocity difference $\Delta V(l)$ drops by a factor of approximately $\xi \equiv \lambda_\nu / l_{\rm vis}[\nu] \simeq V_{\rm con\,8}^{3/4} l_{p5}^{-1/4} \rho_{14}^{2/3} T_{30}^{-15/4}$ between the scale where neutrino damping sets in ($l \simeq l_{\rm vis}[\nu]$) and the scale where small-scale turbulence begins ($l \simeq \lambda_\nu$). Thus if $\xi < 1$, the small-scale Reynolds number (eq. [13]) should be multiplied by $\xi^{2/3}$; however, usually $\xi \gtrsim 1$.

This situation, in which the viscosity on large scales differs from that on small scales, has only one parallel that we are aware of, namely, the universe at recombination, where the viscosity of the cosmic background radiation is enhanced by the large increase in the photon mean free path at redshift $z \simeq 10^3$. However, it is not known whether turbulence is excited at such an early time. In this respect, neutron star turbulence may be unique.

4. MAGNETOHYDRODYNAMICS OF HOT NUCLEAR MATTER

Electric charge is transported via degenerate relativistic electrons in a hot neutron star. The nuclear fluid is an extremely good conductor. Taking into account electron-proton scattering, the conductivity is $\sigma \approx \mu_e (4\pi\alpha_{\rm em} \Lambda)^{-1}$ (Lee 1950), where μ_e is the electron fermi energy, and we assume that the protons are nondegenerate. Electron-electron scattering (Urpin & Yakovlev 1980) reduces σ by at most a factor ~ 2.

The magnetic Reynolds number of the convective motions is $\mathfrak{R}_m = l_p V_{\rm con}/v_m$ [where $v_m = (4\pi\sigma)^{-1}$ is the magnetic diffusivity]. Equivalently,

$$\mathfrak{R}_m = 3 \times 10^{17} V_{\rm con\,8}\, l_{p5}\, Y_{0.2}^{1/3} \rho_{14}^{1/3} . \tag{14}$$

Thus neutron star convection is strongly in the magnetohydrodynamic limit, $\mathfrak{R}_m \gg 1$.

The magnetic Prandtl number on scales where the neutrino viscosity operates is

$$\frac{\mathfrak{R}_m}{\mathfrak{R}[v]} = \frac{1.3 Y_p^{1/3} f(Y_p) T}{\alpha_{\rm em} G_{\rm F}^2 \rho m_n} = 6 \times 10^{13} \frac{Y_p^{1/3} f(Y_p) T_{30}}{\rho_{14}} . \tag{15}$$

That is, the viscous diffusion rate of vortex lines through the flow greatly exceeds the ohmic diffusion rate of magnetic field lines on scales larger than λ_v. On smaller scales,

$$\frac{\mathfrak{R}_m}{\mathfrak{R}[e]} \simeq \frac{\pi}{20\Lambda^2} \frac{Y_e}{\alpha_{\rm em}^3} \frac{\mu_e}{m_n} = 1.2 \times 10^4 Y_{0.2}^{4/3} \rho_{14}^{1/3} . \tag{16}$$

This magnetic Prandtl number is still large compared to that of the solar convective zone, where $\mathfrak{R}_m/\mathfrak{R} \approx 0.03$, but not nearly as large as equation (15).

Finally, we note that the scale $l_{\rm diff} \sim \mathfrak{R}_m^{-1/2} l_p$ over which the magnetic field lines can diffuse during one overturn of the energy bearing eddies,

$$l_{\rm diff} \simeq 2 \times 10^{-4} \frac{l_{p5}^{1/2}}{V_{\rm cons\,8}^{1/2} Y_{0.2}^{1/6} \rho_{14}^{1/6}}\ {\rm cm} , \tag{17}$$

is slightly larger than the ultimate viscous damping scale

$$l_{\rm vis}[e] = \mathfrak{R}[e]^{-3/4} l_p \simeq 8 \times 10^{-6} \frac{l_{p5}^{1/4} Y_{0.2}^{3/4}}{V_{\rm con\,8}^{3/4}}\ {\rm cm} . \tag{18}$$

5. DYNAMOS IN NEWBORN STARS: AN OVERVIEW OF THE PHYSICAL CONDITIONS

A nascent neutron star convects vigorously and rotates differentially, thus is a possible site for dynamo action. The efficiency of the dynamo will depend on the initial spin rate of the star. Many of the physical properties of a young neutron star are distinctly different from those of the Sun and other magnetically active main-sequence stars (Table 1). We now briefly summarize the similarities and differences.

1. A newly formed neutron star rotates differentially for two reasons: first, neutron stars are less centrally condensed than the degenerate electron pressure-supported cores out of which they form, owing to the hardness of the equation of state above nuclear density; and second, the angular momenta of various mass shells are conserved (to a first approximation) in the collapse. Thus, even if the core rotates as a solid body prior to collapse, the young neutron star will rotate differentially. The initial gradient $d\Omega/dr$ is positive in the unshocked inner core, and negative in the hot, extended outer layers of the star.

2. Once the neutrinosphere has shrunk to a radius of 10–20 km, and heat and lepton number transport have reached a quasi-steady state in the outermost layers of the star, the treatment of entropy-driven convection given in § 2 is applicable. The ~ 1 ms overturn time of the convective cells at the base of the convection zone is comparable to rotation period $P_{\rm rot}$ only if the nascent neutron star is rotating near break-up. The actual rotation period in most protopulsars is probably longer (e.g., Ruderman 1972), yielding a Rossby number ${\rm Ro} \equiv P_{\rm rot}/\tau_{\rm con} = 10\ (P_{\rm rot}/10\ {\rm ms})$, as compared with ${\rm Ro} \simeq 2$ in the Sun. At this stage, differential rotation plays a less important role than it does in the solar dynamo. However, neutron stars which are born rotating fast, $P_{\rm rot} \sim 1$ ms, *can* support an efficient $\alpha - \Omega$ dynamo and plausibly develop dipole fields as strong as $\sim 10^{15}$ G (Duncan & Thompson 1992 hereafter DT).

3. An earlier phase of convection, triggered by the weakening of the shock, may also occur (e.g., Bethe et al. 1987; Burrows & Fryxell 1992). The details of this phase are not yet

TABLE 1

PROPERTIES OF THE CONVECTION ZONES OF A NEUTRON STAR AND THE SUN, AND THE MAIN-SEQUENCE CONVECTIVE CORE OF A 9 $M_\odot$ STAR

Quantity	Neutron Star[a]	Sun[a]	Convective Core	Description
R_* (km)	15	7×10^5	6×10^5	Outer radius
T (MeV)	$\lesssim 30$	1.7×10^{-4}	3×10^{-3}	Temperature
l_p (km)	1	5×10^4	3×10^5	Pressure scale height
$\ln (P_{\rm bottom}/P_{\rm top})$	6	20	~ 1	Zone depth, in P-scale heights
$V_{\rm con}$ (km s^{-1})	$\sim 1000 F_{39}^{1/3}$	0.03	0.2	Convective cell velocity[b]
$\tau_{\rm con} \equiv l_p/V_{\rm con}$ (s)	$10^{-3} F_{39}^{-1/3}$	2×10^6	1.7×10^6	Convective turnover time
$R_* V_{\rm con}^2/GM$	$10^{-4} F_{39}^{2/3}$	5×10^{-9}	4×10^{-8}	Convective/gravitational energy
v (cm^2 s^{-1})	$3 \times 10^8 T_{30}$	20	330	Viscosity[c]
$\mathfrak{R} = V_{\rm con} l_p/v$	$10^4 T_{30}^{-1} F_{39}^{1/3}$	8×10^{11}	2×10^{12}	Reynolds number
v_m (cm^2 s^{-1})	1×10^{-4}	8×10^2	12	Magnetic diffusivity
$\mathfrak{R}_m = V_{\rm con} l_p/v_m$	$1 \times 10^{17} F_{39}^{1/3}$	2×10^{10}	5×10^{13}	Magnetic Reynolds number
$l_{\rm vis} = \mathfrak{R}^{-3/4} l_p$ (cm)	$100 T_{30}^{3/4} F_{39}^{-1/4}$	10	20	Viscous dissipation scale[c]
λ_v (cm)	$100 T_{30}^{-3}$	...	...	Neutrino mean-free path
$\mathfrak{R}[e](l \sim \lambda_v)$	$6 \times 10^9 T_{30}^{-4}$	...	...	Reynolds number at scale λ_v[d]
${\rm Ro} \equiv P_{\rm rot}/\tau_{\rm con}$	$10 P_{10} F_{39}^{1/3}$	2	$0.05(P_{\rm rot}/1\ {\rm day})$	Rossby number
$B_{\rm sat}$ (G)	$9 \times 10^{15} F_{39}^{1/3}$	5×10^3	2×10^5	Convective saturation field

[a] Except for lines 1 and 4, all quantities refer to the base of the convection zone.

[b] We use the shorthand temperature $T = 30 T_{30}$ MeV; heat flux $F = 10^{39} F_{39}$ ergs cm^{-2} s^{-1}; rotation period $P = 10 P_{10}$ msec.

[c] For the Sun, ion viscosity; for the neutron star, neutrino viscosity (eq. [3]).

[d] $\mathfrak{R}[e]$ calculated using electron viscosity (eq. [12]).

fully understood, but since it lasts a shorter period of time, it is less likely to amplify a seed magnetic field to the dynamical saturation strength. If such a convective phase is absent, then the effects of differential rotation still have to be considered. A star born with $P_{\rm rot} < 30$ ms may, in principle, generate a stronger field from differential rotation than it can from the subsequent convective episode (§ 7.4). Eventually, the outermost layers of the star cool to the point that entropy-driven convection develops. When $P_{\rm rot} \gg 1$ ms, this convection has high Rossby number; whatever differential rotation remains then has a small influence on the field compared to the convective motions (§ 7.4).

4. Thus, dynamo action in a young neutron star is a transient phenomenon. It operates for $\sim 3 \times 10^4$ convective overturn times and $\sim 3 \times 10^3\,(P_{\rm rot}/10\ {\rm ms})^{-1}$ rotation periods. The angular momentum distribution in the star changes, with some of the free energy of differential rotation released by growing turbulent and magnetic stresses. However, a zone of significant $d\Omega/dr$ may remain at the base of the convection zone, as in the Sun.

5. The seed field is *not* infinitesimal: a dynamo almost certainly operated during one or more of the previous phases of stellar evolution. In general, the convective motions which occur after the collapse are capable of generating much stronger magnetic fields than any phase of convection before the collapse (§ 8.4).

6. The magnetic Reynolds number in a young neutron star far exceeds that in any other astrophysical dynamo: $\mathfrak{R}_m \sim 10^{17}$ as compared to $\mathfrak{R}_m \sim 10^{10}$ in the Sun (Stix 1976). This means that the ohmic damping of magnetic fields requires smaller scale gradients in the field in a neutron star than in the Sun. In particular, the stretching of *macroscopic* flux tubes is limited by Lorentz forces before ohmic losses become significant. However, the required small-scale gradients can be provided by turbulent motions, if the field is not strong enough to suppress these motions. Note that $\mathfrak{R}_m \gg \mathfrak{R}$ in a young neutron star, whereas $\mathfrak{R}_m \ll \mathfrak{R}$ in the Sun. The seed field is almost certainly strong enough that the small-scale fluid motions required for efficient ohmic dissipation of magnetic energy are suppressed by magnetic tension (§ 6).

7. The density drop across the convection zone is much smaller in a neutron star (a factor $\sim$30–100) than it is in the Sun (a factor $\sim 10^6$). Equivalently, the neutron star convection zone extends over $\sim$6 pressure scale heights (BL88), whereas the Solar convection zone extends over $\sim$20 pressure scale heights (e.g., Stix 1989). This implies that the ratio l_p/R_* at the stellar surface is much larger in a neutron star than it is in the Sun. That is, the size of the magnetic structures formed at the surface is, proportionately, *much* larger in the neutron star.

8. The convective motions cease when the star becomes transparent to neutrinos, which occurs first in the outermost layers of the star. During the last stages of convection, the convection zone is buried under a stably stratified layer whose mass is a reasonable fraction of the mass of the star. In such a layer, radial displacements of magnetized fluid tend to be suppressed by the very long time required to relax to β equilibrium (Reisenegger & Goldreich 1991). Understanding how the field evolves in this situation is a delicate matter, since the dynamo is deformed *adiabatically*. That is, the cooling time of the young neutron star is longer by three orders of magnitude than the overturn time of the convective cells (§ 14).

6. DYNAMO ACTION AT VERY HIGH $\mathfrak{R}_m$

A dynamo in which the growth rate of the magnetic field approaches a finite positive value as $\mathfrak{R}_m \to \infty$ is termed a *fast* dynamo (e.g., Vainshtein & Zel'dovich 1972). In a young neutron star $\mathfrak{R}_m \sim 10^{17}$ is very high, and so we need an *operational* definition of a fast dynamo. This task is complicated by our uncertainty as to how the dynamo operates, in particular by questions concerning the role played by turbulent diffusion. Other efforts to understand this question are presented in Kulsrud & Anderson (1992) and Vainshtein & Rosner (1991).

The conventional formulation of the turbulent dynamo problem starts with an infinitesimal magnetic field which is convected passively by the fluid. Small-scale gradients in the field are smoothed by ohmic losses, and the field is the product of both stretching and microscopic diffusion (e.g., Moffatt 1978; Zel'dovich, Ruzmaikin, & Sokoloff 1983). The success of Kolmogorov's theory of turbulence has inspired an analogous approach to the dynamo problem, in which it is assumed that the dynamo, if it exists, operates as an inverse cascade of magnetic energy upward from the ohmic scale $l_{\rm diff} \sim \mathfrak{R}_m^{-1/2} l_p$ (e.g., Kraichnan & Nagarajan 1967). This approach rests on two assumptions: first, that the weak seed field is stretched fastest on the smallest scales; and, second, that microscopic diffusion is needed to amplify the magnetic moment of the star, as is implied by the Bondi-Gold (1950) theorem. This theorem states that the magnetic moment μ_* of a star composed of perfectly conducting material is bounded above by

$$\mu_* \leq \frac{3}{4\pi} R_* \Phi_{\rm in} , \tag{19}$$

where $\Phi_{\rm in}$ in the flux entering (equal to minus the flux leaving) the star. We will argue that, in fact, this theorem is made irrelevant by the effects of magnetic buoyancy: the key assumption made in deriving the bound (19) is that the radial component of the fluid velocity vanishes at the stellar surface, so that $\Phi_{\rm in}$ does not change.

6.1. *Flux Rope Dynamo*

Given that such a turbulent dynamo does indeed exist, let us consider how the dynamo mechanism is modified when the field becomes strong enough to react back on the fluid. This modification may be substantial.[5] Once $B^2/4\pi\rho \gtrsim V^2(l)$ on a scale l, the turbulent diffusion of magnetic flux is suppressed on that scale, but not on larger scales. Further stretching of magnetic field lines strengthens the field and further suppresses diffusion transverse to the field lines. However, individual flux ropes still can move through the surrounding fluid.

The essential point here is that, although both the magnetic field and the vorticity obey the same time evolution equation in the dissipationless limit (Batchelor 1950), the nonlinear properties of magnetic flux ropes and vortex lines differ substantially. A flux rope is a stable entity compared to a vortex line, since it has no long-range attractive self-interaction when immersed in a conducting fluid. It is not subject to those instabilities, such as the Crow (1970) instability, which cause a

[5] This issue is confused by the fact that the Sun (as well as most other stars in our Galaxy) formed when the Galactic field was presumably strong enough to provide a seed field even larger than its present field. The existence of the solar activity cycle provides strong evidence that the solar field is continuously regenerated by dynamo action, but does not necessarily imply that the conventional turbulent dynamo is responsible.

vortex line to cancel itself. One therefore expects the distribution of magnetic flux in a turbulent conducting fluid to be more highly intermittent than the distribution of vorticity.

Both observations of the solar magnetic field and high-$\mathfrak{R}_m$ numerical simulations (e.g., Meneguzzi & Pouquet 1989) provide some evidence for the greater intermittency of magnetic structure. The extreme intermittency of the solar surface field (e.g., Stenflo 1989) has been traditionally viewed as a surface phenomenon, but this intermittency appears to *increase* with depth at least as far as ~ 6 pressure scale heights below the photosphere (Goldreich et al. 1991, hereafter GMWK). Indeed, this number is comparable to the total number of scale heights in the neutron star convection zone (§ 5).

This leads us to the following conjecture, which is not yet supported by detailed dynamical calculations:

A high magnetic Reynolds number dynamo is transformed into a flux rope dynamo when $B^2/4\pi \gtrsim \rho V^2$ on a scale $l \sim l_{\rm visc}$.

In an idealized flux rope dynamo, the magnetic energy is localized at a discrete set of lines, and interactions between these lines occur only at a discrete set of points. A roughly analogous physical system is a light superconducting cosmic string network embedded in the Sun (Chudnovsky & Vilenkin 1988). The formation of flux ropes is also enhanced by the tendency of convective cells to expel the magnetic field which permeates them (Weiss 1966). [This process occurs only on a rather long time scale $\sim \mathfrak{R}_m^{1/3}(l_p/V_{\rm con})$ if the flow is laminar (Moffatt & Kamkar 1983), but on the eddy overturn time $l_p/V_{\rm con}$ if the flow is turbulent.] We plan to examine the statistical properties of magnetic flux ropes immersed in homogeneous turbulence.

In what sense can one speak of an inverse cascade of magnetic energy from smaller to larger scales in the flux rope limit, where the magnetic tension suppresses turbulent diffusion within the rope? As a flux rope is stretched in the flow, its radius a decreases in inverse proportion to the square root of its length when $B^2/8\pi \ll P$ inside the tube. However, the corresponding increase in the rope's tension leads to an increase in its radius of curvature R_c. To see this, balance the ram pressure force $C_D 2a\rho(\Delta V)^2$ per unit length of the tube with the curvature force $\pi a^2(B^2/4\pi)R_c^{-1}$. [Here $C_D = O(1)$ is the coefficient of drag at high $\mathfrak{R}$, and ΔV is the velocity of the rope with respect to the ambient fluid.] Suppose that the field has an initial strength B_0, the corresponding Alfvén velocity being comparable to the turbulent velocity on scale l_0, which is small compared to the scale l_p of the energy bearing eddies if $B_0^2/4\pi \ll \rho V_{\rm con}^2$. Since the flux $\pi a^2 B$ through the rope is conserved, and $\Delta V \lesssim V(R_c) = V(l_0)(R_c/l_0)^{1/3}$, the radius of curvature of the flux rope increases as it is stretched:

$$\frac{R_c}{l_0} \gtrsim \left(\frac{a}{l_0}\right)^{-9/5} . \tag{20}$$

The medium outside the flux ropes is assumed to be weakly magnetized on scales $l \gtrsim l_0$, so we have chosen a Kolmogorov scaling of turbulent velocity. Kolmogorov scaling is violated if energy is exchanged between the medium and the flux ropes on scales $R_c \gtrsim l \gtrsim l_0$. Smaller scale ripples on the flux rope represent traveling kink waves, which are damped by acoustic radiation.

When the radius of curvature R_c becomes comparable to the size of the energy-bearing turbulent eddies, the flux rope can be stretched no further. *The saturation field strength in a flux rope dynamo is determined by a dynamical balance between the drag of the fluid on the rope and the tension of the rope.* The rms field will approach the saturation strength $B_{\rm rms}^2/4\pi \sim \rho V_{\rm con}^2$ in the case where the rope thickness is not much smaller than the size of the energy-bearing eddies.

The physical scale of a field must be carefully defined when the field is confined to thin, strong ropes. A Fourier decomposition is not particularly useful when the field has a flux rope structure. Each flux rope requires two scales for its description: its radius of curvature and its thickness (or equivalently the flux and the thickness). The stretching of strong flux ropes in a turbulent fluid causes simultaneously *both* an increase and a decrease in the scale of the field.

6.2. *Ohmic Dissipation in a Flux Rope Dynamo*

The simplest initial condition for the dynamo is a weak, uniform seed field B_0, where weak means $B_0/(4\pi\rho)^{1/2} \ll V_{\rm con}$. Microscopic ohmic diffusion may be neglected only if the smallest structure in the field imparted by the fluid motions is larger than $l_{\rm diff}$, or equivalently if

$$\frac{\mathfrak{R}_m}{\mathfrak{R}} \gtrsim \left(\mathfrak{R}\,\frac{4\pi P}{B_0^2}\right)^{1/2} . \tag{21}$$

This relation may be derived as follows. The turbulence wrinkles the field lines at scales $\gtrsim l_{\rm visc}$. Wrinkles in the directions transverse to the field lines are subsequently *reduced* in scale as the lines are stretched in the flow. Thus, stretched flux ropes can have diameters smaller than $l_{\rm visc}$. In the case of a young neutron star, the turbulent cascade continues to scales much smaller than the neutrino mean-free-path, and so condition (21) can never be satisfied.

However, the neutron star magnetic field will already have undergone some dynamo amplification during a previous stage of stellar evolution and may already be concentrated in ropes. A rope of initial radius a_0 can be compressed no further when the magnetic pressure equals the ambient pressure. Ohmic diffusion across the final width of the rope can be neglected if

$$\frac{a_0}{l_p} \gtrsim \mathfrak{R}_m^{-1/2}\left(\frac{4\pi P}{B_0^2}\right)^{1/4} . \tag{22}$$

In a young neutron star, this bound becomes $a_0 \gtrsim 10^{-3}$ $(B_0/10^{16}\ {\rm G})^{-1/2}$ cm. We emphasize that, in this case, the transverse size of the flux ropes is determined dynamically and is *not* determined by ohmic diffusion (which would yield a rope thickness $\sim \mathfrak{R}_m^{-1/2} l_p$).

If the magnetic field is concentrated in strong flux ropes, then the dynamo does not involve microscopic ohmic diffusion, except where the ropes meet and reconnect. The behavior of a flux rope dynamo should be approximately independent of $\mathfrak{R}_m$ when $\mathfrak{R}_m$ is very large, a necessary characteristic of a fast dynamo. Thus, if both the solar dynamo and a neutron star dynamo were flux rope dynamos, then their main distinguishing features would be the depth of the convection zone and the value of Ro, and not the value of $\mathfrak{R}_m$ (or $\mathfrak{R}_m/\mathfrak{R}$).

6.3. *Stellar Dynamo Action with Vanishing Resistivity*

Although the ohmic dissipation of magnetic energy is suppressed inside a strong flux rope, the rope can still move through the surrounding fluid, and turbulent diffusion of the *mean* field—which we define as the spatial average over scales large compared to the mean separation between flux ropes—need *not* be suppressed. The usual interpretation of the turbulent diffusivity as a renormalized (enhanced) ohmic diffusivity confuses this point: turbulent diffusion of the mean field is possible even in a *perfectly* conducting fluid. The idea of a renormalized ohmic diffusion coefficient has meaning only when $B^2/8\pi \ll \rho V^2(l)$ on *all* scales above the diffusion length $l_{\rm diff}$ (cf. Kulsrud & Anderson 1992).

More generally, a fast dynamo can operate in a perfectly conducting star which is surrounded by a medium of finite conductivity. In such a star, the magnetic field lines are stretched by convection, and the Parker (1975) buoyancy force then drives them across the surface of the star, where they diffuse into the surrounding medium. This change in the topology of the field may be achieved with a vanishingly small cost in energy dissipated at the surface. In this manner the rms surface field, as well as the dipole field of the star, may be significantly enhanced; the physical transport of magnetic field lines across the stellar surface contravenes one of the assumptions of the Bondi-Gold (1950) bound on the magnetic moment (eq. [19]). In principle, a fast *stellar* dynamo need not generate a field whose scale is of order $\sim \Re_m^{-1/2} l_p$ almost everywhere in the flow, as deduced by Moffatt & Proctor (1985) for a fast dynamo in an unbounded region of high conductivity.

Consider a magnetic field configuration in which buoyant flux loops densely fill the stellar surface, with density $l_{\rm loop}^{-2}$. We imagine that each loop consists of a rope of flux Φ which leaves and reenters the star. In the vacuum approximation, the magnetic moment of each loop is $\mu_{\rm loop} \simeq 3\Phi l_{\rm loop}/8\pi$. The number of loops is $N_{\rm loop} = 4\pi(R_*/l_{\rm loop})^2$. If the loops are oriented randomly, then the net dipole moment of the star is

$$\mu_* \simeq \sqrt{N_{\rm loop}}\,\mu_{\rm loop} = \frac{3\Phi R_*}{4\pi^{1/2}} . \tag{23}$$

The star is initially threaded by flux Φ_0, corresponding to an initial dipole moment $\mu_0 \simeq \Phi_0 R_*/2\pi$. If the field is randomly stretched inside the star but not subject to true dynamo amplification, then $\Phi < \Phi_0$, since each individual flux system cannot carry more flux than initially threads the star. *We conclude that the dipole moment of the star can be significantly increased by the buoyant loss of flux ropes only if the ropes are superimposed coherently by a dynamo operating within the star*, or *if they tend to be aligned in a certain direction.* (The only preferred direction is the axis of rotation, but spontaneous symmetry breaking could select a random direction.)

7. DYNAMO ACTION IN A SLOWLY ROTATING STAR

Let us now examine how the dynamo mechanism depends on the stellar rotation rate. There are two ways to measure the effectiveness of a dynamo. The first is to calculate the ratio of magnetic stresses to turbulent stresses, given approximately by the dimensionless ratio $B_{\rm rms}^2/4\pi\rho V^2$, where V is the total fluid velocity associated with convection and differential rotation. The second is to calculate the mean field generated on a relevant physical scale, such as the radius of the star, $\langle |B| \rangle / B_{\rm rms} \leq 1$.

Differential rotation, acting alone on a seed magnetic field, generates a toroidal field which wraps around the star. However, when differential rotation is small, in the sense that $l_p^2 |\partial\Omega/\partial r| \ll V_{\rm con}$, the field is stretched mainly on the scale of an individual convective cell or by smaller scale turbulence. According to the mixing-length theory of convection, each convective cell extends over roughly one pressure scale height l_p. Thus, any dynamo which operates at high Rossby number $\mathrm{Ro} \equiv P_{\rm rot}/\tau_{\rm con}$ generates a field whose coherence length is small compared to the stellar radius, since in general $l_p \ll R_*$. Moreover, there is no separation in scales between the field and the turbulence. Thus the mean field approach to the dynamo problem is not appropriate at $\mathrm{Ro} \gg 1$.

We base our considerations on the Rossby number Ro because this quantity is easy to calculate and because stellar magnetic activity is observed to correlate with Ro to a remarkable extent (e.g., Simon 1990). The dynamo number, on the other hand, presupposes a specific *theoretical* approach to dynamo action—the mean-field theory—and its function of other derived quantities (the helicity α and turbulent diffusivity ν_T) which cannot be calculated from first principles except to within an order of magnitude. If we make conventional assumptions about how α and ν_T are related to convection and rotation ($\nu_T \propto l V_{\rm con}$ and $\alpha \propto \Omega l$ when $\mathrm{Ro} > 1$), then the $\alpha - \Omega$ dynamo number on scale R_* is

$$D_{\alpha\Omega} \equiv \frac{\alpha \Delta\Omega R_*^3}{\nu_T^2} \propto \mathrm{Ro}^{-2}\left(\frac{R_*}{l}\right)^3 . \tag{24}$$

A threshold minimum value of $D_{\alpha\Omega}$ is then equivalent to a *maximum* threshold Ro below which dynamo action occurs, a result which can be obtained using more direct arguments.

Note that the critical Rossby number for dynamo action scales as $[\mathrm{Ro}]_{\rm crit} \propto (R_*/l_p)^{3/2} (\Delta\Omega/\Omega)^{1/2}$ when $[\mathrm{Ro}]_{\rm crit}$ exceeds unity. Thus expression (24) predicts $[\mathrm{Ro}]_{\rm crit} \gg 1$ when $R_* \gg l_p$ under the simplest assumptions about α and ν_T, whereas observations of magnetically active main-sequence stars suggest $[\mathrm{Ro}]_{\rm crit} \sim 1$. Expression (24) implies that dynamo action becomes easier when the active region extends over a larger number of scale heights, and when differential rotation is stronger. Given the considerable uncertainty in the normalization of $[\mathrm{Ro}]_{\rm crit}$, we will not try to predict whether $[\mathrm{Ro}]_{\rm crit}$ is greater or less than unity in a young neutron star.

7.1. *A High Rossby Number Dynamo: Observational Evidence*

Does such a high Rossby number dynamo exist? Because large-$\Re_m$ turbulent flows cannot be studied in the laboratory, our main observational clues about such systems come from the Sun and other magnetically active stars.

At first glance, it would appear that slowly rotating stars do not support efficient dynamos. The degree of chromospheric activity provides a measure of the field strength at the surface, and in the upper convection zone, of a star. This activity is observed to be correlated with rotation rate, and in particular with Ro (e.g., Simon 1990). We are interested in the behavior of this correlation well above the solar value $\mathrm{Ro} \sim 2$. For late-type dwarfs there is little data at such high Ro (Simon 1990), but it is clear that chromospheric emission declines rapidly with Ro above $\mathrm{Ro} \sim 1$. (On the Sun, the emission through chromospheric lines is proportional to the square root of the surface magnetic energy density [Schrijver et al. 1989], and so the implied drop of magnetic energy with Ro is even steeper.)

More recent evidence suggests that the turbulent motions in the upper convection zone of the Sun can amplify a preexisting magnetic field to a value close to the saturation strength $B_{\rm sat} = (4\pi\rho V_{\rm con}^2)^{1/2}$. Observed increases in solar p-mode frequencies between 1986 and 1989 (Libbrecht & Woodard 1990) have been attributed to the presence of a strong field in the Sun's upper convection zone, whose mean pressure scales in constant proportion to the turbulent pressure of convection, $B_{\rm rms}^2/8\pi \propto \rho V_{\rm con}^2 \propto \rho^{1/3}$, with $f \equiv (B_{\rm rms}/B_{\rm sat})^2 \simeq 0.1$ (GMWK). This perturbing field is concentrated at active latitudes (Libbrecht & Woodard 1990). That is, magnetic flux generated at the base of the convection zone (the conventional picture for solar activity, Parker 1979) is amplified in the upper convection zone. This secondary amplification mechanism does not depend on rotation, since the local Rossby number at the top of the Solar convection zone is $\mathrm{Ro} \sim 5 \times 10^3$.

It is important to understand whether or not this secondary amplification mechanism acts as a *true* dynamo, that is, whether it can also cause the *spontaneous* growth of a magnetic field in the upper convection zone. The only thing that is clear (see Murray 1991) is that a strong, tangled field can be *maintained* in the upper solar convection zone for a significant fraction of the Solar cycle (equivalently, $\sim 10^3$ supergranule lifetimes).

7.2. *A High Rossby Number Dynamo: Theoretical Motivation*

Now let us place these remarks in a broader context. Convection and rotation together induce helicity in a flow, that is, an expectation value for the quantity $\boldsymbol{v} \cdot (\nabla \times \boldsymbol{v})$. In the mean-field dynamo theory (Steenbeck et al. 1966; Krause & Radler 1980), the growth term in the induction equation is $\nabla \times (\alpha\langle \boldsymbol{B}\rangle) + \boldsymbol{r} \times (\langle \boldsymbol{B}\rangle \cdot \nabla)\boldsymbol{\Omega}$, where in cylindrical coordinates $\Omega(r, z)$ is the angular velocity of rotation about the z-axis and α is proportional to the mean helicity. When $\tau_{\mathrm{con}} \ll P_{\mathrm{rot}}$, the coefficient $\alpha \sim \Omega l_p$ is approximately independent of the convective velocity. The α and Ω effects are of comparable importance in generating toroidal field from poloidal field if there is significant differential rotation, $|\partial \ln \Omega/\partial \ln r| \sim 1$.

The esssential difficulty with a large Ro *mean-field* dynamo, is that both the α and Ω effects amplify the field only on a time scale $\sim P_{\mathrm{rot}}$, whereas turbulent diffusion weakens the field on the much shorter timescale $\sim \tau_{\mathrm{con}} = \mathrm{Ro}^{-1}\, P_{\mathrm{rot}}$. This suggests that at high Ro both $\alpha - \Omega$ and α^2 dynamos are ineffective, and that there is no growth of a mean field on scales larger than the largest turbulent eddies. Even when $\mathrm{Ro} \lesssim 1$, the viability of an α^2 dynamo is in doubt. Hoyng (1987) has noted that, in the mean-field approximation, the ensemble average $\langle B_i B_j\rangle$ grows without bound in an α^2 dynamo, if the parameters are chosen so that $\langle B_i\rangle$ is constant. Hoyng interprets this result to mean that an α^2 dynamo can *only* build a small-scale field, with the dipole field appearing as a random sum of small-scale fields. The direction of this effective magnetic moment wanders around on a time scale $\sim R_*^2/v_T$, where $v_T \sim V_{\mathrm{con}}\, l_p$ is the turbulent diffusivity.

Further understanding of high Ro stellar dynamos is provided by a study of the dynamo properties of isotropic, mirror-symmetric turbulence, first pursued by Batchelor (1950). This simplified model lacks a spatial gradient in the velocity of the energy bearing eddies, as is present in any stellar convection zone, but has the important property that the flow has vanishing mean helicity. There is some numerical evidence (e.g., Meneguzzi & Pouquet 1989) that above a critical magnetic Reynolds number $\mathfrak{R}_m \sim 100$, a magnetic field grows spontaneously in mirror-symmetric turbulence. A similar result is obtained in the approximation that the turbulence has a very short correlation time (Ruzmaikin & Sokoloff 1981). In this respect, it is interesting to note that during the minimum of the solar cycle, the mean absolute field is $\langle |B| \rangle \simeq 4$ G over most of the solar surface (Murray 1991), corresponding to $B_{\mathrm{rms}} = 90$ G and $B_{\mathrm{rms}}^2/4\pi\rho V_{\mathrm{con}}^2 \simeq 0.05$. At the same point in the cycle, the mean surface field is very close to zero, except near the poles. It is not yet clear whether this minimal field is merely generated at the base of the convection zone in an $\alpha - \Omega$ dynamo and subsequently amplified by the turbulence, or whether an independent, high Rossby number dynamo can operate in the upper convection zone. Observations such as these provide an important test of the role played by helicity in a stellar dynamo.

7.3. *Dynamo Action in Transient High-*Ro *Convection*

Let us suppose that high-Ro convection does not amplify the mean field on the scale of a convective cell. What then does happen? In the absence of such dynamo action, the field must eventually dissipate. However, the energy in the field can be significantly increased for a short period of time, as the convective motions stretch the field and reduce its scale. If the convective motions themselves *turn off* after a certain period of time, then it is essential to compare the decay time of the amplified field, with the lifetime of the convection.

This point is illustrated by a two-dimensional example. As is well known, there is no dynamo in two spatial dimensions, but considerable amplification of the magnetic field energy is possible for a short period of time (Zel'dovich 1957). To see this, note that the MHD equation can be reduced to an equation for the vector potential $A(B_i = \epsilon_{ij}\, \partial A/\partial x_j)$ in two dimensions,

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \boldsymbol{v} \cdot \nabla A = \nu_m \nabla^2 A \ . \tag{25}$$

In the absence of dissipation, each fluid particle carries around a fixed value of A. In a fluid volume of size L^2 with initial field B_0, no two particles have values of A which differ by more than $\Delta A \sim B_0 L$. No matter how the particles are rearranged within this volume, the mean field on a scale $l \leq L$ cannot exceed $(l/L)^{-1} B_0$. Thus, the turbulent motion of the fluid cannot increase the mean field on a scale L, but *can* significantly tangle the field on smaller scales. Indeed, from equation (25), one obtains by integrating over the entire volume (Zel'dovich 1957),

$$\frac{\partial}{\partial t} \int_{L^2} A^2 d^2x = -2\nu_m \int_{L^2} B^2 d^2x \ , \tag{26}$$

assuming that no flux enters or leaves the volume. The energy in the field must decay asymptotically, but only after undergoing considerable amplification. The turbulence mixes fluid particles with values of A differing by $\Delta A \sim B_0 L$ down to the ohmic diffusion scale $l_{\mathrm{diff}} = \mathfrak{R}_m^{-1/2} L$ when $l_{\mathrm{diff}} \gg l_{\mathrm{visc}}$. Thus, the field energy increases by the factor $(B/B_0)^2 = \mathfrak{R}_m$, and A decays on the overturn time L/V of the largest eddies.

However, these conclusions hold only if the seed field is sufficiently small, since there is not enough energy in the fluid to amplify the field beyond $B^2/4\pi \sim \rho V^2$ (this point has also been noted by Cattaneo & Vainstein 1991). If $B_0^2/4\pi \gtrsim \mathfrak{R}_m^{-1} \rho V^2$, then the magnetic energy decays on a longer time scale

$$\frac{t_{\mathrm{diff}}}{L/V} \sim \mathfrak{R}_m \left(\frac{B_0^2}{4\pi\rho V^2}\right) \tag{27}$$

Most of the field energy resides in thin flux ropes, and the tension of the ropes prevents them from being stretched to the point that their mean separation becomes comparable to l_{diff}. If the turbulence extends only to a maximum scale $l_{\mathrm{max}} < L$, with V now the eddy velocity on the scale l_{max}, the right side of equation (27) is increased by a factor L/l_{max}.

Now let us return to three dimensions. The growth of the mean field is now a possibility, since for each fluid particle A is not a conserved quantity in the dissipationless limit. Magnetic flux ropes can be stretched and rotated in three directions, and ropes of like sign can be superimposed. In isotropic MHD turbulence, growth of the magnetic energy seems likely, at least temporarily, but the spatial distribution of the field is far from clear. The dynamo is a result of a competition between the self-smoothing property of flux ropes under stretching (see eq. [20]), and the formation of small-scale structure on flux ropes via reconnection. The basic uncertainty concerns the relative orientation of the flux ropes, and the magnitude of the mean field generated.

7.4. *Relative Energy Density in Convection and Differential Rotation*

So far in this section we have avoided discussing the role played by differential rotation in neutron star dynamos. In this final subsection we quantify its importance, using energetic arguments. The most rapidly-rotating nascent neutron stars probably do support efficient α-Ω dynamos. A more detailed study of these stars, which we argue have properties distinct from those of ordinary radio pulsars, is presented in DT.

Both convection and differential rotation provide energy to a dynamo. The relative amounts of energy available in these two forms depends on the strength of the differential rotation, and on Ro. Since the initial distribution of angular momentum in the star differs significantly from that which can be supported in a convective zone, most of the free energy associated with differential rotation is used only once. In contrast, the energy in convection is constantly regenerated by a superadiabatic temperature gradient.[6] Angular momentum may be redistributed throughout the star by viscous and magnetic stresses on a time scale shorter than the cooling time, in which case differential rotation plays a more important role in the dynamo at the beginning.

A body of fixed mass M and angular momentum J has the least rotational energy E_Ω when it rotates as a solid body (Lynden-Bell & Pringle 1974), and so the energy[7] in differential rotation may be defined to be $E_{d\Omega} = E_\Omega - E_{\Omega,\min}(J, M) \equiv \epsilon_{d\Omega} E_{\Omega,\min}(J, M)$. The differential rotation is strong when $\epsilon_{d\Omega} = O(1)$.

The energy in differential rotation, relative to the energy in convection, is

$$\frac{E_{d\Omega}}{E_{\rm con}} \simeq \epsilon_{d\Omega} \frac{I(2\pi/P_{\rm rot})^2}{MV_{\rm con}^2} \simeq \frac{4\pi^2}{3} \epsilon_{d\Omega} \left(\frac{R_*}{l_p}\right)^2 {\rm Ro}^{-2} \simeq 10^3 \epsilon_{d\Omega} {\rm Ro}^{-2} . \tag{28}$$

Here, we have made use of the relation $I \simeq \frac{1}{3} MR_*^2$ for the moment of inertia of a realistic model neutron star, and assumed that the star is almost entirely convective. The numerical coefficient in equation (28) is large because $E_{d\Omega}$ is distributed nonlocally throughout the star. (When Ro ~ 1 and $\epsilon_{d\Omega} \sim 1$, the differential angular velocity across one pressure scale height is $\sim 2\pi V_{\rm con}/R_*$, but the differential angular velocity over a distance $\sim R_*$ is larger by R_*/l_p.)

[6] In this discussion, we avoid the possibility that the dynamo exhibits a cyclic behavior, during which energy can be transferred between convection and differential rotation.

[7] We ignore all complications associated with defining E_Ω in the presence of convection.

However, the magnetic field can tap only a fraction of the energy in differential rotation, unless the characteristic radial scale ΔR_B of the field is comparable to the stellar radius. Suppose that the differential rotation is smoothed out within cylindrical shells of radial thickness $\Delta\varpi = \Delta R_B$, but the differential velocity between shells is maintained. The energy made available by this smoothing process is then

$$\frac{\Delta E_{d\Omega}}{E_{d\Omega}} = \frac{1}{12} \left(\frac{d \ln \Omega}{d \ln \varpi}\right)^2 \left(\frac{\Delta\varpi}{\varpi}\right)^2 . \tag{29}$$

Since the Rossby number is Ro $\simeq (P_{\rm rot}/1 \text{ ms})$, we see that the convective kinetic energy is larger than the total energy in differential rotation when the initial rotation period is longer than $30\epsilon_{d\Omega}^{1/2}$ ms. If the radial scale of the field is comparable to one pressure scale height, then substituting $\Delta\varpi/\varpi \sim l_p/R_* \sim 0.1$ and $|d \ln \Omega/d \ln \varpi| \sim 1$ in equation (29), we find that only a fraction $\sim 10^{-3}$ of $E_{d\Omega}$ is available for conversion into a magnetic field at a given time. In this case, the available energy in differential rotation $\Delta E_{d\Omega} \sim {\rm Ro}^{-2} E_{\rm con}$ approaches $E_{\rm con}$ only when $P_{\rm rot}$ is as small as ~ 1 ms.

8. DYNAMO ACTION IN THE PROGENITORS OF TYPE IIa SUPERNOVAE

The origin of pulsar magnetic fields is sometimes ascribed to dynamo action in the progenitor star (e.g., Ruderman & Sutherland 1973). At the very least, the progenitor should bequeath a substantial seed field to a dynamo operating in a newborn neutron star. Convection driven by nuclear burning certainly plays a key role in generating white dwarf magnetic fields, since these stars do not form in sudden gravitational collapse events.

A massive star undergoes several episodes of convection, both in its core, in exterior thin shells, and in an extended envelope on successive giant branches. We begin this section by analyzing core convection in the two extreme precollapse phases.

8.1. *Convection in a Hydrogen-burning Core*

In Table 1 we display various physical characteristics of the H-burning convective core of a moderately massive star, $M = 9\ M_\odot$. We use the evolutionary models of Castellani, Chieffi & Straniero (1990, hereafter CCS) at the zero-age main sequence, when the convective core is largest. For comparison, we display the analogous properties of the solar convection zone and of the convection zone in a newly formed neutron star. The convective velocity in the core at the zero-age main sequence is $V_{\rm con} \simeq (C_p)^{-1/3}(L_{\rm cc}/4\pi\rho_c R_{\rm cc}^2)^{1/3} \simeq 2.5 \times 10^4$ cm s^{-1}, where the core has radius $R_{\rm cc} = 6 \times 10^{10}$ cm, central density $\rho_c = 10$ g cm^{-3}, and total luminosity $L_{\rm cc} = 1.5 \times 10^{37}$ ergs s^{-1}. The convective overturn time is therefore $\tau_{\rm con} \sim R_{\rm cc}/V_{\rm con} \simeq 30$ days (comparable to $\tau_{\rm con}$ in the bottom scale height of the solar convection zone).

We may summarize as follows. In the convective core of a 9 $M_\odot$ main-sequence star:

1. The Rossby number is small, Ro $\simeq 0.03(P_{\rm rot}/1$ day), as compared to Ro $\simeq 2$ in the bottom scale height of the solar convection zone.

2. The Reynolds number is comparable to that in the Sun, but the magnetic Reynolds number is larger by some three orders of magnitude (mainly because the temperature and convective velocities are higher). As a result, $\mathfrak{R} < \mathfrak{R}_m$ in the convective core, whereas $\mathfrak{R} > \mathfrak{R}_m$ in the Sun.

3. The ratio $V_{\rm con}^2 R/GM \simeq 4 \times 10^{-8}$ is somewhat larger than it is in the Sun. That is, the limiting ratio of magnetic energy to gravitational potential energy is somewhat larger.

8.2. *Convection during Silicon Burning*

We now examine convection in the final phase of nuclear burning, silicon burning, adopting parameters from the detailed calculations of Arnett (1977) and Thielemann & Arnett (1985). Given a central energy generation rate of $\epsilon_{\rm nuc,c} \simeq 10^{13}$ ergs g^{-1} cm^{-3} during the final stages of silicon burning (Thielemann & Arnett 1985), and an average core burning rate $\langle \epsilon_{\rm nuc} \rangle = 0.009 \epsilon_{\rm nuc,c}$ (Arnett 1977), one finds that the total core luminosity is $L_{\rm nuc} \simeq 2.7 \times 10^{44}$ ergs s^{-1}. Since the neutrino losses are smaller by an order of magnitude in the silicon-burning core (Thielemann & Arnett 1985), the convective heat flux is given by

$$8\rho(R) V_{\rm con}^3 \simeq \frac{L_{\rm nuc}}{4\pi R^2} . \tag{30}$$

Here, we make use of equation (2), and note that $\Gamma \simeq 4/3$. The radius enclosing 0.5 $M_\odot$ is $R_{0.5} \simeq 1.6 \times 10^8$ cm, the density at this radius is $\rho(R_{0.5}) \simeq 1.3 \times 10^8$ g cm^{-3}, and so the convective velocity is $V_{\rm con} \simeq 9 \times 10^5$ cm s^{-1}. Equivalently, $V_{\rm con}^2 R/GM \simeq 2 \times 10^{-6}$ at $R = R_{0.5}$. This ratio is smaller by a factor of ~ 100 during silicon burning than during the early stage of entropy-driven convection in a newborn neutron star.

We should note that an individual convective eddy undergoing a burst of nuclear burning will reach higher velocities (Arnett 1977), but such bursts will be sporadic. Use of equation (30) is more appropriate for estimating the rms magnetic field which may be generated in the convection zone.

8.3. *Precollapse Dynamos: An Overview of the Physical Conditions*

We now outline some basic features of a precollapse dynamo, and how these compare to those of the solar dynamo. (See cols. [3] and [4] of Table 1).

1. Intermediate- and high-mass stars rotate rapidly while on the main sequence, with mean rotation periods $P_{\rm rot} \sim 0.5$–2 days in spectral types F to B, rising to ~ 5 days in spectral type O5 (Tassoul 1978). If the angular velocity is approximately independent of radius, then the overturn time of the convective core is much longer than $P_{\rm rot}$; that is, the Rossby number $\text{Ro} = P_{\rm rot}/\tau_{\rm con}$ is smaller than unity (§ 8.1).

2. The core tends to lose angular momentum as the star evolves. Core contraction together with envelope expansion generates a negative $d\Omega/dr$. The core then experiences braking torques mediated by viscosity and magnetic fields. General arguments (§ 9 below) indicate that the smallest Rossby number and hence the most favorable conditions for *core* $\alpha - \Omega$ dynamo action occurs on the main sequence, although later stages of convection may generate stronger rms magnetic fields. This is certainly the case if the core and envelope are tightly coupled, and the star rotates approximately as a rigid body. We should emphasize that the core rotation rate is very sensitive to the size of the angular velocity gradient in a convective envelope when the envelope spans a large range in radius. The theorem (Lynden-Bell & Pringle 1974) which states that solid body rotation minimizes the (rotational) kinetic energy does not strictly apply in the presence of convection, and so *rapid* convection (with $\text{Ro} \gg 1$) may be able to sustain negative $d\Omega/dr$. Solid body rotation is plausibly attained in radiative regions of the star, due to the magnetic shearing instability (Balbus & Hawley 1991 or, even in the absence of this instability, by linear winding of a magnetic field.

3. Both a convective core and a convective nuclear-burning shell extend over roughly one pressure scale-height. In contrast, an outer convective envelope extends over many pressure scale heights (because the photospheric temperature of the star is much smaller than the central temperature). This is the basic reason that the surface field of the Sun has a coherence length small compared to the solar radius (e.g., Murray 1991). For the same reason, if a dynamo operates in a convective core, it should generate a magnetic field whose coherence length is comparable to the radius of the core.

4. A dynamo probably operates in a convective core at low Rossby number, but the nature of the dynamo mechanism is unclear, since the convection is substantially modified by the rotation. There is an obvious but crucial difference in topology between the convective core of a massive star, and the outer convective envelope of a low-mass star like the Sun. Most of the energy in the solar magnetic field apparently is generated in a thin shear layer at the interface of the convection zone and the radiative core, since the radial angular velocity gradient in the solar convection zone is small (Dziembowski, Goode, & Libbrecht 1989; Brown et al. 1989; Libbrecht & Woodard 1990). If the radial angular velocity gradient is likewise suppressed in the convective core of a massive star, then the shear should also be localized in a layer between the core and the radiative exterior. But an essential ingredient in the solar dynamo, the buoyant rise of magnetic flux ropes into the convection zone, is not present in this case, since the convective zone lies *beneath* the shear layer rather than the other way around. This indicates that the ratio B_p/B_ϕ will be smaller in the convective core of a massive star than it is in the Sun; here B_p is the strength of the poloidal field which threads the convection zone, and B_ϕ is the toroidal field generated in the shear layer. This effect, acting alone, would tend to favor a dynamo operating in convective, nuclear-burning shells, or in the outer convective envelope which forms while the star is on the asymptotic giant branch or (sometimes) the red giant branch, over a core dynamo. At $\text{Ro} \sim 1$, a core dynamo is qualitatively similar to an α^2 mean-field dynamo (although mean-field theory does not strictly apply because there is little separation in scale between the core and the convective motions; see § 7). The behavior at $\text{Ro} \gg 1$ is very uncertain. It is possible that a preexisting field is expelled from the core by the diamagnetic property of MHD turbulence (see Vainshtein & Zel'dovich 1972). The value of Ro during successive stages of evolution depends on the efficiency with which angular momentum is transported throughout the star (§ 8.1.2).

5. It should be recalled, however that the rotation rate of the star slows appreciably when it enters the giant phase, and the motions in the convective envelope probaby have a high Rossby number. Take, for example, the 1.75 $M_\odot$ red-giant model shown in Table 7 of Sweigart & Gross (1978). From mixing-length theory, we deduce that the convective overturn time at the base of the convective envelope is $\tau_{\rm con} \simeq 9 \times 10^5$ s at 30 Myr before helium-core flash. At this point in its evolution, the star has spectral type K6 Adopting the maximum observed projected rotation velocity for K3 giants (Gray & Pallavicini 1989; there are no measurements for later spectral types) implies a lower bound to the surface rotation period, $P_{\rm rot} \gtrsim 3 \times 10^7$ s. If $d\Omega/dr$ vanishes in the convective envelope, then the Rossby number at the base of the convective envelope is $\text{Ro} \sim 30$, and probably much larger. The fate of the magnetic

field in the convective envelope is tied to the existence or absence of a high-Ro dynamo. If such a dynamo does *not* exist (or is much less efficient than a low-Ro dynamo, as observations of late-type main-sequence stars suggest), then the convective envelope substantially reduces any preexisting field generated during a previous convective phase. This would also eliminate the possibility of a dynamo in nuclear burning shells, assuming that these shells are torqued down to the same angular velocity as the convective envelope. The only caveat we would raise here is the possibility of negative radial $d\Omega/dr$ in the convective envelope, which would allow an envelope that is slowly rotating (Ro $\gg 1$) in its outermost parts but rapidly rotating at its base. (It has been suggested that the rotational velocities of class III giants decline sharply at spectral type GO, and that this decline is due to the onset of dynamo activity in the convective envelopes of these stars [Gray 1989], as is commonly believed to occur in dwarfs and subgiants. The evidence for such a sharp break in giants is controversial, however, and the decline in rotational velocities on the giant branch can be ascribed to simple conservation of angular momentum; see Rutten & Pylyser 1988. Moreover, Gray's definition of the Rossby number differs from ours by a factor $[2\pi]^{-1}$. If the onset of dynamo action in the convective envelope of a giant occurs at the same Rossby number, Ro ~ 1, as is observed in the convection zones of late-type main-sequence stars [Simon 1990], then Gray's calculation of the Rossby number does not suggest efficient dynamo action. Finally, it should be noted that even if dynamo action did occur in the envelope during the early stages of giant evolution, our calculation of Ro suggests that an antidynamo could operate further up the giant branch, when the stellar rotation had slowed even more.)

8.4. *Relative Efficiency of Different Dynamo Epochs*

Two useful parameters which characterize a stellar dynamo are, first, the energy density in the fluid motions in convection and differential rotation and, second, the Rossby number Ro $\equiv P_{\rm rot}/\tau_{\rm con}$. We now compare the values of these parameters associated with different evolutionary stages of a massive star.

8.4.1. *Energy in Convection at Different Phases in Stellar Evolution*

When convection is the main energy source for the dynamo, the relative importance of precollapse and postcollapse dynamos depends on the ratio $\epsilon_v \equiv V_{\rm con}^2 R_{\rm cc}/GM_{\rm cc}$, since the ratio of magnetic to gravitational binding energy is not changed by compression or expansion of a star. (Here $R_{\rm cc}$ and $M_{\rm cc}$ are the radius and mass of the convective core.) This quantity scales with core luminosity $L_{\rm cc}$ and central density ρ_c as $\epsilon_v \propto L_{\rm cc}^{2/3}\rho_c^{-4/9}M_{\rm cc}^{-8/9}$. During the first few stages of nuclear burning, ρ_c increases faster than $L_{\rm cc}$ (in part due to the onset of neutrino cooling from the core) and so ϵ_v declines slightly. We found $\epsilon_v \simeq 4 \times 10^{-8}$ for the zero-age main-sequence convective core of a 9 $M_\odot$ star (§ 8.1). A similar calculation gives $\epsilon_v \simeq 1 \times 10^{-8}$ during central carbon burning (using the 8.8 $M_\odot$ model calculated by Nomoto 1987). During the last stages of nuclear burning $L_{\rm cc}$ grows much more rapidly, and ϵ_v shoots up to 2×10^{-6} during silicon burning in the core of a somewhat more massive star (§ 8.2). Convection is much more vigorous in the postcollapse phase, because the neutrinos drive a large heat flux through the nascent neutron star. During this phase, one finds $\epsilon_v \sim 2 \times 10^{-4}$ (§ 2). Thus, a precollapse dynamo can generate only a relatively weak field as compared to the postcollapse dynamo, although this field may still exceed the characteristic dipole field of young pulsars (when the effects of compression during core contraction are taken into account).

8.4.2. *Rossby Number at Different Phases of Stellar Evolution*

The time dependence of Ro during stellar evolution is more difficult to determine because the rotation rate of the stellar core is not directly measurable. If the star rotates as a solid body while on the main sequence, then it is clear that the core must lose a large fraction of its angular momentum as later stages of nuclear burning occur and the core contracts. For example, if the core of a 9 $M_\odot$ main-sequence star were to contract from a central density 10 g cm^{-3} to a central density $\simeq 10^{15}$ g cm^{-3} typical of a neutron star, while conserving angular momentum, its angular velocity would increase by a factor 2×10^9. With these assumptions, the neutron star could form only if the initial rotation period were greater than 25 days, much longer than the mean surface rotation period of ~ 1 day (Tassoul 1978). Thus, the formation of pulsars with initial rotation periods $P_{\rm rot}$ significantly in excess of 1 ms (e.g., Narayan & Ostriker 1990) requires spin-down of the stellar core by a factor $800(P_{\rm rot}/30\ {\rm ms})$.

We found that Ro ~ 0.03 is likely in the convective core of a 9 $M_\odot$ main-sequence star, whereas Ro $\simeq 10(P_{\rm rot}/10\ {\rm ms})$ is the Rossby number in a newborn, convective neutron star. Given that angular momentum is conserved during the rapid core collapse which forms the neutron star, the Rossby number of the pre-collapse core must also significantly exceed unity, since Ro (core)/Ro (ns) $\sim [\epsilon_v({\rm core})/\epsilon_v({\rm ns})]^{1/2}(R_{\rm core}/R_{\rm ns})^{1/2} \sim 2$. This implies that the Rossby number *increases* substantially[8] during successive stages of core nuclear burning. Whereas the latest stages of convection generate the strongest magnetic fields, it is the earliest stages of convection which support the most vigorous $\alpha - \Omega$ dynamo, since Ro < 1 is required for such a dynamo. Assuming that the rotation of the core and envelope remains well coupled as the star evolves, one finds that Ro increases by a factor $\sim 10^3$ from hydrogen to helium core burning (for the 9 $M_\odot$ model of CCS), which implies that Ro > 1 during core helium burning. Thus, *if low Rossby number is necessary for amplification of the dipole field in a stellar dynamo, then the phase of convection in the progenitor star which supports the most efficient dynamo is main sequence core convection.* From Table 1, the dynamical saturation field strength in the convective core of a 9 $M_\odot$ main-sequence star is $\simeq 2 \times 10^5$ G. Upon compression to a central density of 10^{15} g cm^{-3}, this field would increase to 4×10^{14} G, in excess of the strongest pulsar dipole fields. Note, however, that the spin period of the convective core could be substantially reduced while the envelope is *also* convective (as it often is during the later stages of nuclear burning), if $d\Omega/dr < 0$ in the slowly rotating convective envelope. Indeed, a moderately efficient $\alpha - \Omega$ dynamo would operate during core Si burning if the resulting neutron star were formed rotating near breakup, $P_{\rm rot} \sim 1$ ms. The Rossby number during Si burning would be Ro(Si) $\sim 2(P_{\rm rot}/1\ {\rm ms})$ (using the convective velocity calculated in § 8.2). Finally, a binary companion will sometimes impart angular momentum to the core at the end of a common envelope phase.

[8] The convective envelope of a red giant almost certainly has a large Rossby number (§ 8.3).

8.5. *The End of Convection and the Burial of Magnetic Fields*

Although the later stages of nuclear burning proceed rapidly from an evolutionary point of view, they last for many time scales P_{rot} and τ_{con}. That is, as stellar evolution progresses, the dynamo mechanism is deformed *adiabatically*. A key question is the following. As a convective zone shrinks in size, convective material becomes stably stratified. How strong is the field left behind in this stably stratified material, compared to the field which the active dynamo was able to generate?

There is one obvious constraint on this remnant field, namely, that it is not strong enough to overcome the stable stratification and rise to the surface of the star. The magnetized fluid remains stably stratified if the quantity

$$D \equiv \frac{\kappa}{\nu_m} \frac{V_A^2}{\omega_{BV}^2 l_p^2} \tag{31}$$

is less than or of order unity (Acheson 1979). Here κ is the thermal diffusivity, V_A is the Alfvén velocity, ω_{BV} is the Brunt-Vaisälä frequency, and $P_{rot} < l_p/V_A$ is assumed. This expression has a simple physical motivation. Thermal conduction helps overcome stable stratification: when $\kappa \neq 0$, the thermal diffusion time is smaller than the inverse of ω_{BV} on a sufficiently small scale. This effect is counterbalanced by ohmic diffusion, which tends to smooth out small-scale wrinkles in the field. Introducing parameters appropriate to the $\simeq 1.3\ M_\odot$ helium core of a 9 $M_\odot$ star at the end of central hydrogen burning (CCS), one finds that $D < 1$ when $B < B_{max} \simeq 3 \times 10^7$ G. Compressing the core from a central density $\rho_i = 16$ g cm^{-3} to a central density $\rho_f \simeq 10^{15}$ g cm^{-3} typical of a neutron star, the upper bound on the field strength becomes $(\rho_f/\rho_i)^{2/3} B_{max} \simeq 4 \times 10^{16}$ G, much higher than typical pulsar dipole fields.

The field generated in a convective core will remain buried in a degenerate remnant that forms later, unless the convective core extends *beyond* the mass shell which forms the outer boundary of the degenerate remnant. If, on the other hand, the field is generated in a convective envelope, then the degenerate remnant traps some of this field only if the base of the convection zone lies *inside* the mass shell which forms the surface of the degenerate remnant.

9. ORIGINS OF WHITE DWARF MAGNETISM

One objective of stellar dynamo theory is to explain why only a fraction $\sim 3\%$ of white dwarfs have detectable magnetic fields stronger than 10^6 G (Angel, Borra, & Landstreet 1981; Schmidt 1989), when it does not appear possible to explain the absence of these fields by field decay (e.g., Wendell et al. 1987). This explanation must be consistent with scenarios for the origin of pulsar magnetism. This is a nontrivial constraint: if a white dwarf of mass 0.6 $M_\odot$ (Hamada & Salpeter 1961) and field strength 10^6 G were compressed to central density 10^{15} g cm^{-3}, it would have a field of 5×10^{11} G, roughly one order of magnitude smaller than the median field strength of young pulsars.

We have seen that a precollapse dynamo may occur in a convective core or, what is less likely, in a convective envelope. Note that the progenitors of neutron stars are distinguished from the progegnitors of white dwarfs, in that the convective envelope *does not* extend into the material which later forms the neutron star. *A white dwarf is more or less likely to be endowed with a strong magnetic field depending on whether or not the convective envelope can sustain a magnetic field.*

A dynamo operating in a convective core creates a detectable field at the surface of the white dwarf only if the convective core extends outside the mass shell corresponding to the surface of the dwarf. In this respect, it is amusing to note that the maximum extent of the convective core (e.g., CCS) is smaller than the white dwarf remnant (e.g., Weidemann 1987) if $M < M_{crit} \simeq 3\ M_\odot$. From the initial mass function of Scalo (1986), we deduce that only $\sim 5\%$ of white dwarfs form from progenitors this massive, comparable to the fraction of magnetic white dwarfs.

The inner radius of the convective envelope depends very sensitively on the metallicity of the star: the lower the metallicity, the larger is this radius (e.g., CCS). For example, a 3 $M_\odot$ mass star is believed to form a $\sim 0.65\ M_\odot$ white dwarf (Weidemann 1987). The minimum mass enclosed by the convective envelope is $\simeq 0.44\ M_\odot$ when $Y = 0.27$, $Z = 0.02$, and $\simeq 0.74\ M_\odot$ when $Y = 0.27$, $Z = 0.002$ (CCS). Alternatively, fixing Y and Z, and varying the mass, one finds that above a certain mass $M_{crit,2}$, the convective envelope never encloses a mass smaller than the white dwarf mass. For $Y = 0.30$ and $Z = 0.01$, one finds $M_{crit,2} \simeq 3.5\ M_\odot$ (Sweigart, Greggio, & Renzini 1990). Note that this cutoff lies in the right direction if a dynamo does *not* operate in the convective envelope: only the most massive ($M > M_{crit,2}$) and least numerous stars should form magnetic white dwarfs. Since the critical mass $M_{crit,2}$ depends sensitively on the metallicity, a strong correlation between degree of magnetic activity and white dwarf mass is not predicted.

The relative absence of magnetic white dwarfs is a significant constraint on models in which pulsar dipole fields are generated before the collapse. The existence of magnetic white dwarfs might indicate that a dynamo could operate in a neutron star progenitor, but we should caution that the role of the convective envelope in the progenitor star is quite different in the two cases.

10. STRONG MAGNETIC FIELDS IN YOUNG NEUTRON STARS

We now reexamine the question of magnetic field amplification in a protoneutron star, making use of results derived in previous sections.

10.1. *Application of Dynamo Theory to Convective Neutron Stars*

First, we summarize the discussion given in §§ 6 and 7 of high Rossby number, high magnetic Reynolds number dynamos.

1. We conjectured that the magnetic field becomes localized in a discrete set of stable flux tubes, within which turbulent motons are suppressed, when the Maxwell stress exceeds the Reynolds stress on the smallest scale of turbulent motion. The dynamo is then the result of a competition between two effects: the tendency of neighboring flux ropes to reconnect and to create small-scale structure in the field, and the increase in the radius of curvature of an isolated flux rope associated with stretching. After the field becomes highly intermittent, ohmic diffusion is suppressed (except at reconnection sites). However, flux ropes still move through the fluid with their entrained matter, and these motions engender turbulent diffusion of the *mean* field without any significant ohmic dissipation.

2. Magnetic buoyancy tends to drive flux ropes across the surface of the star, where they diffuse into the surrounding medium. This can change the topology of the field and poten-

tially can lead to amplification of the dipole moment and the rms surface field of the star with negligible *internal* diffusion. Thus, in a neutron star dynamo, Parker buoyancy can play a role crudely analogous to that played by ohmic diffusion in conventional fast-dynamo theory.

3. Mean-field dynamo theory treats the coherent alignment and superposition of field lines on scales much larger than that of the largest turbulent eddies. When the Rossby number exceeds unity (as in a protopulsar undergoing entropy-driven convection) this theory is *not* applicable because flux-stretching on the convective eddy scale dominates all tendencies for large-scale alignment. Recent analyses suggest that in the upper convection zone of the Sun, the rms field does approach the dynamical saturation strength. It is not clear whether the mean field on the scale of the eddies is increased by dynamo action. Analytical and numerical studies of mirror-symmetric, large-$\mathfrak{R}_m$ flows suggest this to be the case, but the evidence from observations of solar and stellar magnetic fields is mixed.

4. Even if high-Ro convection does not amplify the mean field on the scale of a convective cell, the energy in the field will be increased by stretching. Were the convective motions to continue indefinitely, the field would eventually be dissipated. However, the seed field is almost certainly strong enough that the field tension suppresses fluid motions on the scale $\sim \mathfrak{R}_m^{-1/2} l_p$. In the simple two-dimensional model discussed in § 7.3, the time scale for ohmic dissipation is longer than $l_p/V_{\rm con}$ when the seed field is stronger than $B_0^2/4\pi \gtrsim \mathfrak{R}_m^{-1} \rho V_{\rm con}^2$. Substituting parameters appropriate to a young neutron star ($L \sim 10^6$ cm, $l_{\rm max} \sim 10^5$ cm, $\mathfrak{R}_{\rm m} \sim 10^{17}$, $\rho \sim 3 \times 10^{14}$ g cm^{-3}) in equation (27), we have $t_{\rm diff} \sim 10^2\,(B_0/10^9\ {\rm G})^2$ s, longer than the neutrino cooling time if $B_0 \gtrsim 2 \times 10^8$ G. This suggests that ohmic dissipation will also be suppressed in the presence of full three-dimensional turbulent motions, if $B_0 \gg 10^9$ G.

10.2. *How Strong?*

The implications of the previous sections for neutron star dynamos is clear: a very strong magnetic field can be generated in a nascent neutron star undergoing entropy-driven convection. The saturation field strength is

$$B_{\rm sat} \simeq \sqrt{4\pi\rho V_{\rm con}^2} = 6 \times 10^{15} V_{\rm con\,8} (\rho_{14}/3)^{1/2}\ {\rm G}\ . \qquad (32)$$

If the ratio $f \equiv (B_{\rm rms}/B_{\rm sat})^2$ is anywhere near its value in the upper convection zone of the Sun, $f \simeq 0.1$ (GMWK), then the field is very strong indeed. Equivalently, the value $B_{\rm rms} \sim 10^{12}$–3×10^{13} G conventionally associated with pulsar dipole fields corresponds to a very inefficient dynamo, $f \sim 10^{-8}$–10^{-5}. Note that the magnetic energy density is only a fraction $\sim 10^{-4} f$ of the thermal energy density generated after the collapse.

In principle, fields stronger than $B_{\rm sat}$ can be generated by differential rotation in regions of the star which are stable to Ledoux convection (eq. [1]), as long as the stellar rotation period is shorter than ~ 30 ms (the total free energy in differential rotation is larger than the convective kinetic energy by a factor $\sim 10^3\ {\rm Ro}^{-2}$; § 7.4). Angular momentum is redistributed through the star on a time scale $\tau_J \sim \Omega R_* l_p / V_{{\rm A},p} V_{A,\phi}$, where $V_{\rm A} = \boldsymbol{B}/(4\pi\rho)^{1/2}$ and the magnetic field is separated into poloidal and toroidal components, $\boldsymbol{B} = (B_p, B_\phi)$. For example, τ_J is comparable to the neutrino cooling time $\tau_{\rm cool} \sim 3$ s when the rms field is $B_\phi \sim B_p \sim 3 \times 10^{14} (P_{\rm rot}/10\ {\rm ms})^{-1/2} (\tau_{\rm cool}/3\ {\rm s})^{-1/2}$ G; stronger fields redistribute angular momentum even more rapidly.

We may summarize as follows. In convective regions of the star, the magnetic field must compete with the fluid turbulence as a sink for this energy. When high Ro convection is combined with differential rotation, the amplified field is not likely to have a radial scale larger than $\sim l_p$, and B should not exceed the limiting strength $B_{\rm sat} \simeq (4\pi\rho)^{1/2} V_{\rm con}$ determined by the convective motions (§ 7.4). Stronger fields may be generated by the shear motion in regions of the star where convective instability is delayed, but only if $P_{\rm rot} \lesssim 30$ ms. Once convective instability sets in, regions of the fluid containing fields stronger than $B_{\rm sat}$ will rise buoyantly to the surface, where some, and perhaps most of the field energy will be dissipated. In this way, the star only retains a partial memory of its initial rotational profile. When the initial rotation period is very short, $P_{\rm rot} \sim 1$ ms, the field generated by differential rotation may be strong enough to suppress convection, but only in regions where convective instability is delayed for a substantial fraction of the cooling time. Except in this case, the small-scale pulsar field is determined essentially by the properties of the high Rossby number convective dynamo.

Whatever mechanism is responsible for amplifying the magnetic field, it need not be very efficient. To raise the field to $B_{\rm sat}$ from an initial strength B_i in the time $\tau_{\rm cool} \sim 3$ s required for the star to radiate half its thermal energy, the amplification factor Υ per convective overturn must exceed

$$\Upsilon - 1 > \left(\frac{\tau_{\rm con}}{\tau_{\rm cool}}\right) \ln\left(\frac{B_{\rm sat}}{B_i}\right) \simeq 3 \times 10^{-4} \ln\left(\frac{B_{\rm sat}}{B_i}\right) . \qquad (33)$$

For any reasonable value of B_i, a fractional increase of less than 1% per convective overturn is sufficient to generate $B_{\rm sat} \sim 10^{16}$ G.

As we argued in § 8 the strength of the seed field may vary greatly depending on the nature of the progenitor. It is possible that most neutron stars are formed with a initial dipole field in excess of 10^{12} G; it is also possible that the precollapse dynamo does not leave behind a very strong seed field.

11. IMPLICATIONS FOR SUPERNOVAE

This work was originally motivated by a question which we address in more detail in a companion paper (Thomson & Duncan 1992, hereafter TD2). Namely does a dynamo-generated magnetic field help provide energy to the shock in a type II supernova explosion?

The answer to this question depends on a detailed understanding of neutrino transport in a strong magnetic field, as well as energy transport by the field across the stellar neutrinosphere.

The magnetic field mediates neutrino scattering in two ways. First, the reaction[9]

$$\nu \to \nu + e^+ + e^- \qquad (34)$$

is not kinematically forbidden in the presence of a magnetic field (which can absorb momentum from electrically charged particles). The pair energy deposited via this reaction increases linearly with time in the presence of a constant magnetic field. Second, a closed magnetic flux loop can trap electron pairs close to the neutrinosphere, where their energy density increases *exponentially* with time via neutrino-electron scattering, until this source of heating is balanced by cooling via pair annihilation, $e^+ + e^- \to \nu + \bar{\nu}$.

[9] The rate for the reaction $\nu \to \nu + \gamma$ is smaller by a factor $\sim \alpha_{\rm em}$.

11.1. *Neutrino Magnetic Pair Production*

Feynman diagrams for the reaction (34) are shown in Figure 1. The rate of this process in the regime appropriate for supernovae is (Borizov, Zhukovskii, & Lysov 1983; TD2):

$$\Gamma = \frac{g_V^2 + g_A^2}{(3\pi)^3}\,\alpha_{\rm em}\,G_F^2 B^2 E_\nu \ln\left(\sin\theta\,\frac{E_\nu eB}{m_e^3}\right). \qquad (35)$$

In this equation, $\alpha_{\rm em} = e^2$ is the fine-structure constant, G_F is Fermi's constant, E_ν is the initial neutrino energy, m_e is the electron mass, and g_V, g_A are vector and axial coupling constants which depend on the neutrino flavor. Note that this rate is proportional to the magnetic energy density, and depends only weakly on the orientation angle θ of the neutrino momentum with respect to the magnetic field. The mean energy of the pairs emitted is $(7/16)E_\nu$. Thus the pair energy deposited via reaction (34) in a supernova is proportional to the field energy outside the neutrinosphere $E_B = \frac{1}{2}\int_{R_\nu}^{\infty} dr\, r^2 \langle B^2(r)\rangle$, where the brackets denote an angular and temporal average. One finds, assuming that the field is concentrated in a thin layer at the surface of the star (TD2),

$$\frac{E_{\rm pair}}{E_B} = 1.6\left(\frac{R_\nu}{15\ {\rm km}}\right)^{-2}\left(\frac{\Sigma}{10^{60}\ {\rm MeV}^2}\right). \qquad (36)$$

In this equation, Σ is

$$\Sigma = \sum_i \mathcal{N}_i \langle E_{\nu i}^2\rangle (g_V^2 + g_A^2)_i\,, \qquad (37)$$

where the sum runs over neutrino types $i = 1$–6 (three flavors of neutrinos and antineutrinos) and $\mathcal{N}_i$ is the total number of each type emitted. For a uniform magnetic layer of depth ΔR,

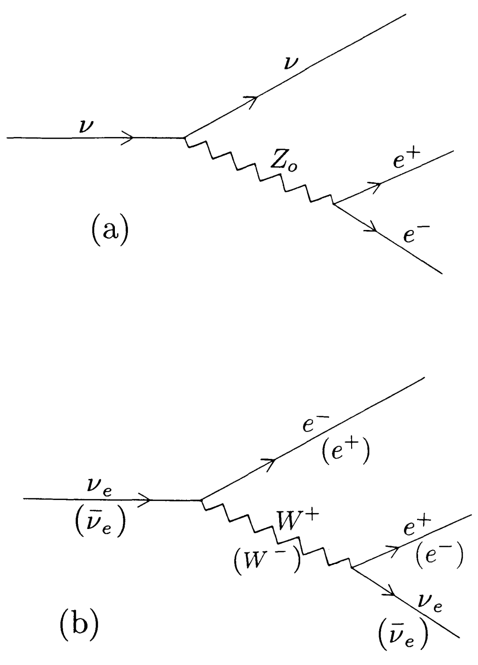

FIG. 1.—Feynman diagrams for neutrino magnetic pair production. (*a*) Neutral current process, which occurs for all neutrino flavors. (*b*) Charged current process, which occurs only for electron neutrinos. Analogous processes for μ and τ neutrinos and leptons do not occur in supernovae since $E_\nu < 2m_\mu < 2m_\tau$.

the total pair energy deposited outside the neutrinosphere is

$$E_{\rm pair} \simeq 10^{50}\left(\frac{B}{10^{16}\ {\rm G}}\right)^2\left(\frac{\Delta R_\nu}{3\ {\rm km}}\right)\left(\frac{\Sigma}{10^{60}\ {\rm MeV}^2}\right)\ {\rm ergs}\,. \qquad (38)$$

which is unlikely to significantly energize a supernova even if the magnetic field exterior to the star is close to the saturation strength in the deep convection zone.

11.2. *Neutrino-Electron Scattering in Flux Ropes*

The energy density of a static pair fluid trapped in a magnetic flux rope just above the neutrinosphere *e*-folds on a time scale (TD2) $\tau_{\rm pair} \sim 100$ ms. Let us suppose that pair energy equivalent to the energy in the magnetic field is released in a time $3\tau_{\rm pair}$. Then to release enough energy in electron pairs (at least 10^{51} ergs) to prevent the supernova shock from stalling in a time $\tau_{\rm cool} \sim 3$ s, requires a rms surface field $\langle B^2(R_\nu)\rangle^{1/2} \gtrsim 3 \times 10^{16}$ G. This field is at least one order of magnitude larger than the likely interior saturation field (eq. [32]). We conclude that the presence of closed magnetic field lines above the neutrinosphere is not likely to enhance the neutrino-electron scattering rate sufficiently to have a strong influence on the shock.

11.3. *Neutron Star Analogues of Chromospheric and Coronal Heating*

11.3.1. *Reconnection Heating*

Above the neutrinosphere, the magnetic field is strong enough, and the matter density is small enough, that the Alfvén speed should approach the speed of light. The *e*-folding time $\tau_{\rm pair}$ is accordingly much longer than the Alfvén travel time across the star. Since $\tau_{\rm pair}$ is also much longer than $\tau_{\rm con}$, one would expect that the energy in neighboring magnetic structures is relased via reconnection in a time short compared to $\tau_{\rm pair}$. A release of 10^{51} ergs in this way corresponds to a small fraction $\gtrsim 3 \times 10^{-3}$ of the energy carried to the surface by convection. However, the corresponding fraction of the solar constant radiated by the solar corona (which is probably heated by some form of reconnection, e.g., Parker 1991) is only $\sim 5 \times 10^{-6}$ (e.g., Stix 1989), and Ro is likely to be much larger in the neutron star. One caveat here is that the entire neutron star is rotating differentially at first, so that the interior of the star contains many shear layers equivalent to that at the base of the solar convection zone.

11.3.2. *MHD Wave Heating*

One final possibility is that the magnetic field can transport energy across the neutrinosphere in the form of MHD waves, in analogy to the role played by magnetic fields in heating the solar chromosphere (e.g., Narain & Ulmschneider 1990). The solar chromosphere radiates more energy than the corona, but still only a fraction 10^{-4} of the solar bolometric luminosity (e.g., Stix 1989), while the corresponding fraction for stars with the highest Rossby numbers is closer to $\sim 10^{-5}$ (Simon 1990). In this case as well as the case of the corona, the solar heating mechanism is not yet well enough understood to construct analogous heating models for a neutron star.

11.4. *Conclusion: Magnetic Supernovae?*

Our tentative conclusions is that a magnetic field generated by a convective dynamo in the first few seconds of the life of a neutron star can only play an important role in energizing the supernova shock under optimistic assumptions about the rates of magnetic reconnection or MHD wave heating outside the neutrinosphere. These physical phenomena are complex, however, and this possibility should be investigated further. In particular, it is important to understand the role of differential rotation at short rotation periods (see DT). For example, the total free energy in differential rotation is $E_{d\Omega} \simeq 10^{52}(P_{\rm rot}/1\ {\rm ms})^{-2}$ ergs. Even if all this energy is converted into a magnetic field and then dissipated outside the neutron star, the effect on the shock is significant only if $P_{\rm rot}$ is as small as ~ 3 ms.

12. ORIGIN OF PULSAR DIPOLE FIELDS

Building on the results of previous sections, we discuss the relative merits of different scenarios for the evolution of magnetic fields in the progenitor of a neutron star, and in the star itself up to the end of the convective epoch at ~ 30 s after core collapse. The key question is whether the dipole field is generated before or after the formation of the neutron star. The key theoretical uncertainty is the effectiveness of a dynamo at high Rossby number.

12.1. *High-Ro Convection Does Support a Dynamo*

A dynamo operating during a period of high-Ro convection would be analogous to a dynamo operating in mirror-symmetric turbulence, as evidenced by numerical (Meneguzzi & Pouquet 1989) and analytical (Ruzmaiken & Sokoloff 1981) studies (§§ 6.1 and 7.2). If such a dynamo does exist, then the *mean* field should reach a value close to $B_{\rm sat}$ on the scale of a convective cell. As we have already discussed (§ 7.1), observations of the surface field of the Sun reveal the presence of a strong, small-scale field at solar minimum (Murray 1991), which *may* be due to a high-Ro number dynamo operating independently at the top of the solar convection zone.

A high-Ro dynamo becomes more effective during the last stages of stellar evolution, in the sense that the limiting ratio of magnetic energy to gravitational binding energy increases (the ratio $V_{\rm con}^2 R/GM$ increases). Entropy-driven convection in a young neutron star generates the strongest fields, $B_{\rm sat} \sim 10^{16}$ G. However, the onset of entropy-driven convection may be delayed for a substantial fraction of the neutrino cooling time (J. R. Wilson 1992, private communication).

12.2. *High-Ro Convection Does Not Support a Dynamo*

The most conservative possibility is that high-Ro convection significantly stretches the field but does not amplify the mean field on the scale of a convective cell (§ 7.2). This significantly limits possibilities for both precollapse and postcollapse dynamos:

1. The dynamical saturation field $B_{\rm sat}$ in the convective core of a massive star on the main sequence corresponds to a $\sim 10^{14}$ G field in the neutron star after amplification by collapse (§§ 8.4 and 10). Since the Rossby number of the convective core is small (assuming that the star rotates roughly as a solid body), a precollapse origin of the dipole field is not implausible. The Rossby number exceeds unity during core helium burning, and later stages of core burning, if the core and envelope remain well coupled as the star evolves. Dynamo action may be possible during these later burning stages, if the core rotates faster than the envelope, or if the core is spun up by interaction with a binary companion (§ 8.4.2).

2. Entropy-driven convection during the early cooling phase of a nascent neutron star has a Rossby number in excess of unity, unless the star rotates close to breakup, $P_{\rm rot} \sim 1$–3 ms. Mean field dynamo models (e.g., Moffatt 1978; Krause & Rädler 1980) indicate that an $\alpha - \Omega$ dynamo is not very efficient when Ro $\gg 1$. Note, however, that the entire neutron star is at first differentially rotating, whereas the radial differential rotation in the Sun is localized at the base of the convection zone. An $\alpha - \Omega$ dynamo should generate stronger fields, for a particular value of Ro, in the presence of such strong differential rotation. More importantly, the one-time release of a significant fraction of the free energy in differential rotation may generate much larger fluxes than is possible later on (after the internal rotation has relaxed to an equilibrium profile in which differential rotation is maintained by convection). The total free energy in differential rotation corresponds to a field $B \sim 10^{16}\ (P_{\rm rot}/30\ {\rm ms})^{-1}$ G. The conversion of this energy into magnetic fields would occur more effectively if the onset of high-Rossby number convection were delayed for a substantial fraction of the cooling time (§ 10.2). In particular, a significant amount of flux would be generated by radial shearing of poloidal magnetic field lines. Once high-Ro convection turns on, it (by hypothesis) stretches the field without amplifying the *mean* field on the scale of a convective cell. The magnetic field energy is greatly increased, albeit at the expense of a reduction in the scale of the field. For example, a flux rope of initial diameter $l_p \sim 1$ km and flux density $B_i \sim 10^{13}$ G has final diameter $\sim (B_i/B_f)^{1/2} l_p \sim 0.1 l_p$, for $B_f \sim 10^{15}$ G.

Efficient $\alpha - \Omega$ dynamo action will also occur for rotation periods much longer than 1 ms, during the last stages of neutrino cooling. The convective velocity declines by a factor ~ 10 from 1 s to 10 s after collapse (§ 14.1), and so the dynamo will operate if $P_{\rm rot}$ is as long as 10–30 ms. Rotators this slow would also support an $\alpha - \Omega$ dynamo at earlier times if a slower form of convection (perhaps due to a double-diffusive instability triggered by lepton-number gradients; Mayle & Wilson 1988) occurred prior to the appearance of a negative entropy gradient. We can set general constraints on $P_{\rm rot}$ and $\tau_{\rm con}$, independent of the precise nature of the convective instability, by applying mean field theory. For a given rotation period, turbulent diffusion becomes *less* of an impediment to mean field amplification as $\tau_{\rm con}$ increases and Ro decreases. The convection cannot, however, be too slow or the dynamo will not be able to iterate sufficiently many times before the convection turns off, and will not be able to generate a strong enough poloidal field. We consider these points in turn.

The free growth rate of the field on a scale R_* due to $\alpha - \Omega$ dynamo action is $\Gamma_{\alpha\Omega} \simeq ({\rm Ro}\ \gamma l/R_*)^{1/2} P_{\rm rot}^{-1}$ (e.g., Moffatt 1978; Durney & Robinson 1982). Here we have assumed that the helicity α is a significant fraction of $V_{\rm con}$, as appropriate for Ro $\ll 1$ (e.g., Tout & Pringle 1992), and parametrized $d\Omega/dr = \gamma(\Omega/R_*)$. The initial differential rotation is expected to be strong, $\gamma \sim 1$ (§§ 5 and 7.4). We emphasize that this expression is uncertain to within a numerical coefficient of order unity, which we ruthlessly ignore. We normalize to $P_{\rm rot} = P_{10} \times 10$ ms, mixing length $l = l_5 \times 10^5$ cm and $\tau_{\rm con} = \tau_{100} \times 100$ ms. Then for Ro $\ll 1$, $\Gamma_{\alpha\Omega} \sim 1(\gamma l_5/P_{10}\tau_{100})^{1/2}\ {\rm s}^{-1}$. The range of $\tau_{\rm con}$ within which the dynamo experiences at least 10 *e*-foldings in the first 10 s is roughly $10 P_{10}$ ms $< \tau_{\rm con} < 100 \gamma l_5 P_{10}^{-1}$ ms.[10]

[10] Here we have made the idealization that growth of the mean field disappears sharply at a critical Rossby number $[{\rm Ro}]_{\rm crit} \simeq 1$, although in some situations Ro moderately larger than 1 might support an $\alpha - \Omega$ dynamo (but not $[{\rm Ro}]_{\rm crit} \gg 1$; cf. § 7; Simon 1990). Insofar as one can extrapolate the Ro $\ll 1$ formula for turbulent diffusivity, $\nu_T \sim l V_{\rm con}\ {\rm Ro}^2$ (Goldreich & Keeley 1977), into the regime where ν_T limits mean-field growth, the cutoff for the growth rate is fairly sharp: $\Gamma_{\alpha\Omega} \simeq \Gamma_{\alpha\Omega}^0 \{1 - ({\rm Ro}/[{\rm Ro}]_{\rm crit})^{5/2}\}$.

Notice that this parameter space vanishes at $P_{rot} > 30$ ms and becomes very large for $P_{rot} \sim 1$ ms. The limiting field strength allowed by the convective motions alone is $B_{sat} = (4\pi\rho)^{1/2} V_{con} = 6 \times 10^{13} (\rho_{14}/3)^{1/2} l_5 \tau_{100}^{-1}$. The toroidal field B_ϕ during the linear growth phase is related to the poloidal field B_p by $B_\phi/B_p \simeq (\gamma R_*/l \text{ Ro})^{1/2} \sim 10(\gamma\tau_{100}/l_5 P_{10})^{1/2}$. The saturation field B_{sat} is stronger than observed pulsar dipole fields for $\tau_{con} \lesssim 300$ ms. Note, however, that the limiting value of B_{dipole} is likely to be somewhat smaller than B_{sat} as evaluated at the base of the convection zone. (Exactly how much smaller depends on the efficiency with which toroidal flux generated by shearing motions inside the star rises buoyantly above the stellar surface.) We conclude that $B_{dipole} \sim 10^{12}$–10^{13} G is plausibly generated during the last stages of entropy-driven convection in a reasonably rapid rotator, $P_{rot} \sim 10$ ms. Moreover, a postcollapse $\alpha - \Omega$ dynamo is a real possibility for rapid rotators, $P_{rot} \lesssim 3$ ms, where it could produce a supercritical dipole field. In general, these stars would have properties quite distinct from classical radiopulsars (DT; § 15).

12.3. *Origin of the Characteristic Dipole Field*

Do any of these dynamo mechanisms offer a simple explanation for the observed range of young pulsar dipole fields, $10^{12} \lesssim B_{dipole} \lesssim 3 \times 10^{13}$ G? The saturation field strength during main-sequence core convection yields a flux which lies comfortably above this characteristic dipole flux (since the core extends over only ~ 1 scale height). This field could also conceivably be generated by $\alpha - \Omega$ dynamo action in a rapidly rotating ($P_{rot} \sim 10$ ms) neutron star, once the convective motions have slowed to $\tau_{con} \sim 10$ ms as the star is becoming transparent to neutrinos. Finally, the observed range of dipole fields does arise naturally from a postcollapse, *high* Rossby number dynamo, as we now describe.

After magntostatic equilibrium is established, one expects that magnetic flux loops densely fill the surface of the star, although their strength may vary considerably depending on the degree of intermittency of the generated field. We assume that, on average, the dipole loops are comparable in size to the convective cells which generated them, $l_{loop} \sim 1$ km. The key question is the value of the mean field within each loop. Assuming that the high Ro dynamo does exist, the mean field B_{loop} within each loop is not much smaller than B_{sat}, and $B_{loop} \sim 10^{15}$ G is plausible (scaling to the field strength in the upper convection zone of the Sun; GMWK). The flux per loop is $\Phi_{loop} \simeq \frac{1}{2} B_{loop} l_{loop}^2$, and the stellar dipole field is (see [eq. 23])

$$B_{dipole} \simeq \frac{3}{2\pi^{1/2}} \frac{\Phi_{loop}}{R_*^2} = 4 \times 10^{12} \left(\frac{B_{loop}}{10^{15}\text{ G}}\right)\left(\frac{l_{loop}}{1\text{ km}}\right)^2 \text{ G}, \quad (39)$$

under the assumption that the dipole loops are randomly oriented. Of course, the rms surface field is much larger than B_{dipole}. This dynamo does not directly generate a mean field on scales larger than l_p; thus the dipole field is derived from smaller scale structures. (The same is *not* true for the solar dynamo, where the rotation rate is high enough to create a coherent toroidal field at the base of the convection zone, and thence to create a coherent poloidal field at the poles.)

13. NEUTRINO STARSPOTS AND PULSAR RECOILS

Long-baseline interferometry (Lyne, Anderson, & Salter 1982; Bailes et al. 1990; Harrison, Lyne, & Anderson 1992) and interstellar scintillation studies (Cordes 1986) have determined transverse velocities V_{trans} for a large number of radio pulsars, revealing a broad distribution with mean $\langle V_{trans} \rangle \sim 150$ km s^{-1}, and a tail extending beyond 300 km s^{-1}. There is evidence that V_{trans} is correlated with $P\dot{P} \propto B_{dipole}^2$, which several of the authors listed above argue is not a consequence of selection effects. This conclusion is supported by the statistical analysis of Stollman & van den Heuvel (1986).

One possible source for pulsar recoils is binary disruption (Gott, Gunn, & Ostriker 1970), but studies of the genesis of the pulsar population indicate that this mechanism alone cannot account for the observed V distribution and the $V - B_{dipole}$ correlation (Dewey & Cordes 1987). Bailes (1989) has proposed a model in which pulsar recoils are mostly intrinsic (i.e., not due to binary disruption), but the recoil mechanism is independent of B_{dipole}. In this model, the $V - B_{dipole}$ correlation arises because some pulsars initially stay bound in massive binaries and have their fields reduced via accretion until a second supernova disrupts the system, releasing both the newly formed, high-field, high-velocity ($V \sim V_{recoil}$) pulsar and the old, slow ($V \sim V_{oribital}$), weak-field one.

In any case, some mechanism for producing intrinsic recoils seems called for. One innovative suggestion is anisotropic magnetic dipole radiation (Harrison & Tadmaru 1975); however, this can only produce velocities as large as

$$V_{rocket} = 4\left(\frac{\xi}{0.16}\right)\left(\frac{P_{rot,i}}{10\text{ ms}}\right)^{-2} \text{ km s}^{-1}, \quad (40)$$

where $P_{rot,i}$ is the initial rotation period. In this equation ξ is the ansiotropy factor, determined by the geometry of an off-center, oblique dipole radiator; $\xi = 0.16$ is the maximum possible value (in vacuum). Thus $V_{rocket} \gtrsim 100$ km s^{-1} only if $P_{rot,i} \lesssim 2$ ms. Furthermore, this scenario predicts directional alignment between the velocity and the rotation axis, for which the observational evidence is contradictory (Anderson & Lyne 1983; Pskovsky & Dorofeev 1989).

Several authors, beginning with Shklovskii (1970), have noted that a small fractional anisotropy in the momentum distribution of a supernova's ejecta, if imparted to the central pulsar, could induce a substantial recoil. However, scenarios based on this idea have not yet produced recoils of the observed magnitude (Fryxell 1979), nor is it known how an intrinsic $V - B_{dipole}$ correlation would arise.

An alternative possibility is that anisotropic *neutrino* radiation imparts the recoil. The total scalar neutrino momentum lost by the cooling star is $p_\nu = E_B/c$, where $E_B \simeq 3 \times 10^{53}$ ergs is the binding energy. This is large enough that a dipole anisotropy $\xi_\nu \gtrsim 0.003$ would produce $V \gtrsim 100$ km s^{-1} in a 1.4 $M_\odot$ neutron star.

"Neutrino starspots" might cause this anisotropy (DT). Convective flows induce temperature inhomogeneities of order the Mach number squared, perhaps creating significant brightness fluctuations at the neutrinosphere in the early postbounce period ($t \lesssim 100$ ms). However, the Mach number is small once the energy and lepton-number transport have settled to quasi-steady state (see eq. [2]). In this later phase, during which most of the star's binding energy is radiated away, magnetic fields could modulate the surface neutrino brightness.

Recall that a significant fraction of the energy in a young neutron star is transported via convection. Magnetic fields which exceed $B_{sat} \sim 10^{16}$ G locally suppress convective energy transport, as in sunspots. Very strong fields, $B \gg B_{crit}$, also affect neutrino cross sections (§ 11; TD2), potentially causing concurrent modulations in the radiative component of flux. If the temperature fluctuations have characteristic size λ_s and coherence time τ_s, then the total number of starspots which form on the star during the neutrino cooling time $t_{\rm cool}$ is $\mathcal{N}_s \simeq (R/\lambda_s)^2(t_{\rm cool}/\tau_s)$, and the resulting stochastic anisotropy is of order

$$\xi_v \sim \frac{\sqrt{\mathcal{N}_s}}{\mathcal{N}_s} \simeq \frac{\lambda_s}{R}\left(\frac{\tau_s}{t_{\rm cool}}\right)^{1/2} . \tag{41}$$

(Reasonable assumptions about the geometry of the spots yield a numerical coefficient in eq. [41] close to unity.) If $\lambda_s \sim l_p \sim 0.1R$, comparable to the pressure scale height inside the convective zone, then $V \gtrsim 100$ km s^{-1} when the spots persist for $\tau_s \gtrsim 10^{-3} t_{\rm cool} \sim 3$ ms. This is plausible, since τ_s is not much longer than $\tau_{\rm con}$. Because $\tau_s \ll P_{\rm rot}$ in most cases, one would not expect a strong directional alignment between the velocity and the rotation axis (see Anderson & Lyne 1983). We conclude that neutrino starspots provide a promising mechanism for generating observed pulsar recoils.

Stars with larger scale and more intense spots would acquire larger recoils. Such stars would also tend to have larger $B_{\rm dipole}$, since λ_s in the neutrino cooling phase should be positively correlated with the dimension $l_{\rm loop}$ of the magnetic surface features after crust-freezing, and since the starspot field strengths should be correlated with $B_{\rm loop}$ (see eq. [39] and § 14 below). In this way, a correlation between V and $B_{\rm dipole}$ could be generated.

Significantly larger recoils, plausibly $V \sim 10^3$ km s^{-1}, could be imparted to neutron stars born with $P_{\rm rot} \sim 1$ ms, through the action of several mechanisms which are ineffective in ordinary pulsars born with $P_{\rm rot} \gg 1$ ms (DT). This leads to the prediction that if some neutron stars are indeed formed with short rotation periods, then the proper motion distribution of neutron stars will show two maxima. There should exist a *second* class of neutron stars, which have higher mean proper motions and stronger mean dipole fields than typical pulsars (DT).

14. EARLY EVOLUTION OF A PULSAR MAGNETIC FIELD

We now consider how the neutron star magnetic field relaxes after the convective motions in the star cease at ~ 30 s after collapse. We have shown that convective motions can generate a field much stronger than $B_{\rm dipole} \sim 10^{12.5}$ G, on scales less than or equal to the size of the convective cells, $l \lesssim 1$ km. In this section we argue that a significant fraction of this field energy may remain trapped, and we outline the implications of this for pulsar phenomenology.

14.1. *Relaxation of the Field to Magnetostatic Equilibrium*

After the convective motions inside the star stop, the magnetic field which remains relaxes to hydrostatic equilibrium. The final disposition of the field is difficult to predict. The problem is similar to that encountered in studying dynamo action in the convective core of a massive star: the cooling time of the star greatly exceeds $\tau_{\rm con}$, and so the convection turns off adiabatically. Convection ends when the star becomes transparent to neutrinos. Detailed neutron star cooling models have not been extended to this point, but extrapolating the BL86 model from age $t = 20$ s, we deduce that convection ceases at $t \sim 30$ s, when the internal temperature is ~ 2 MeV. Since the energy density in the degenerate nuclear matter is proportional to T^2, the convective heat flux scales as $\rho V_{\rm con}^3 \propto T^2/t$. The final convective velocity (and the saturation magnetic field strength) is smaller by a factor $\simeq 20$ at $t = 30$ s than it is at $t = 1$ s. This corresponds to $B_{\rm sat} \simeq 3 \times 10^{14}$ G. At this stage, the convection zone is buried beneath a stably stratified layer (see Reisenegger & Goldreich 1992) of increasing depth.

The Maxwell stresses in the stellar interior will drop in proportion to the turbulent stresses if the field is highly intermittent (with most of the magnetic energy concentrated in a small fraction of the volume). Fields stronger than $B_{\rm sat} = V_{\rm con}(4\pi\rho)^{1/2}$ are forced to the surface by buoyancy. We expect that the flux ropes experience a net upward drift velocity at least as large as $\sim(\tau_{\rm con}/t_{\rm cool})V_{\rm con}$, since the strength of the ropes is determined by a dynamical balance with the ram pressure of the steadily weakening turbulence (see § 6). At minimum, flux ropes carrying energy comparable to the total magnetic energy of the star accumulate on a time scale $t_{\rm cool}$ in a layer below the stellar surface. So long as this layer remains convectively unstable, these ropes are capable of three-dimensional reconnection and merge into larger ropes with lower total energy (see § 14.4). Note that the field in these ropes is ohmically damped on a time scale exceeding $t_{\rm cool}$ if its strength is even a tiny fraction of $B_{\rm sat}$ (§ 7.3).

However, the outermost layers of the star do eventually become stably stratified. Flux ropes with magnetic pressure comparable to the total ambient pressure[11] can force themselves upward into this layer, and weaker ropes are carried into it by convective overshoot. Conversely, the downward motion of flux ropes is impeded by the stable stratification. This inherent asymmetry between upward and downward motions allows tangled magnetic fields to accumulate in the outermost stably stratified layers. Within the stably stratified layer, which eventually encompasses the whole neutron star, a horizontal flux tube which is in pressure equilibrium and initially in β equilibrium will rise a distance $\delta r \sim Y_e^{-1}(B_0^2/8\pi P)l_p \ll R_*$ before its density becomes equal to the ambient density. Thus, the field induces small departures from β equilibrium. Goldreich & Reisenegger (1992) show that further upward drift of the field can occur by ambipolar diffusion, but only on the long time scale over which weak interactions change Y_e. In most configurations the Parker buoyancy force is negligible compared to the Lorentz force $\boldsymbol{f_B} = \boldsymbol{j} \times \boldsymbol{B}$. Magnetostatic equilibrium implies $\boldsymbol{f_B} = n_e \nabla(\mu_p + \mu_e - \mu_n)$ (Goldreich & Reisenegger 1992).

A tangled magnetic field in a stably stratified fluid can still undergo a restricted form of three-dimensional motion, in which elements move horizontally but not vertically. The relaxation of the fluid to magnetostatic equilibrium generically creates discontinuities in the field, that is, current sheets (Parker 1972; Arnold 1974; Moffatt 1986). In this case, both horizontal and vertical discontinuities can be created. Reconnection at a horizontal discontinuity is clearly suppressed by the stable stratification. An example of a stable vertical discontinuity is provided by a slender, closed flux loop. The loop relaxes to a configuration which consists of two parallel, vertical flux ropes tied together at the bottom and top. Reconnection of the two ropes is prevented by the topology of the field.

Let us now summarize. Convection in a young neutron star is expected to impart small scale structure to the surface field of a pulsar. The field in a stably stratified layer retains most of its small structure after achieving magnetostatic equilibrium. The mean field on length scale l at the surface of a pulsar may

[11] Note that the flux ropes at the surface of the Sun have this property (e.g., Stenflo 1991).

approach the dynamical saturation strength B_{sat} in the young neutron star, at a time when the depth of the stably stratified layer at the stellar surface was of order l. For $l \sim 1$ km, the corresponding upper bound to the field is $\sim 1 \times 10^{15}$ G.

14.2. *Stability of the Magnetic Field in a Quiescent Neutron Star*

A neutron star can support a field *much* stronger than the largest observed pulsar dipole fields—in principle, a field as strong as $(8\pi P)^{1/2} \sim 10^{18}$ G. The minimum field that can overcome the stable stratification of the star is somewhat smaller, by a factor $\sim Y_e^{1/2} \sim 10^{-1}$ (Goldreich & Reisenegger 1992).

Consider a highly conducting, purely liquid star—that is, a star composed of material without tensile strength. Some fraction of the stellar material is threaded with a magnetic field. A field configuration is stable so long as there is no neighboring configuration with lower total energy. This is true if the field is in magnetostatic equilibrium and is not subject to an interchange instability. The star will retain the field for a long time whether or not it is stably stratified, due to its high conductivity. However, we expect that the geometry of the field, especially the geometry exterior to the star, will depend sensitively on the presence or absence of stable stratification.

The simplest field configuration, which is purely poloidal inside the star, and dipolar outside, *is* unstable (Flowers & Ruderman 1977). The field energy *exterior* to the star may be reduced by picking a great circle which intersects both magnetic poles and rotating the two resultant hemispheres relative to each other until the dipole moment is set to zero. Continued rotation of quasi-conical slices of the star (whose surfaces run parallel to the internal poloidal field) will further reduce low-order moments of the exterior field, without changing the energy of the internal field (Ray 1980). Note that this instability is not impeded by stable statification (Eichler 1982). However, it would be impeded if the action of differential rotation on the field before the onset of convective instability left behind a stabilizing internal toroidal field (see Flowers & Ruderman 1977; Tayler 1979; § 7.4). More importantly, we have argued (§ 6) that turbulent convection will give rise to flux ropes and impart a complicated topology to the field on scales smaller than the stellar radius. The assumption of axisymmetry, which has been made in almost all analyses of this problem (Tayler 1979; Roberts 1981; Wang & Eichler 1988, and reference therein), is therefore strongly violated. The question of stability becomes much more delicate when the rms surface field greatly exceeds the dipole field, and the presence or absence of stable stratification can have a pronounced effect on the result.

To proceed further, it is best to visualize the field as being composed of a discrete set of continguous flux ropes. Consider the simplest case of a single flux rope which enters and leaves the star at widely separated points, and weaves some arbitrary path through the star between these points. To begin, we treat the rope as having negligible width. Any motion of the flux rope which preserves the radius of each fluid particle on the rope is allowed. In particular, the field energy outside the star is minimized by moving the entry and exit points side by side. The field energy inside the star is minimized by making the rope point in a purely radial direction (except at points where the radial direction of the rope changes sign). Now let us suppose that the rope has a finite width. The field energy outside the star can be further reduced by breaking the rope in two, and then rotating the two ropes relative to each other, until the exterior dipole field vanishes. (One may imagine that the entry and exit points of each rope mark the poles of a magnet. The exterior field energy is minimized when the two magnets are antialigned.) Such a fissioning would be prevented by twisting of the rope, which creates an energy barrier between the two local energy minima defined by the one rope and two rope states. We should note in this regard that the external field of a flux loop relaxes to a force-free equilibrium which *is* twisted, because outside the star magnetic forces cannot be compensated by material forces (Sakurai 1979).

Can the field relax to a configuration where the entry and exit points of each flux rope are contiguous, and the external field energy is minimized? We conjecture that stable stratification will prevent the field in a liquid star from attaining such a configuration, for two reasons. First, the field lines which enter the star together in a given flux rope will leave the star separately in more than one flux rope. That is, components of a single flux rope will split off it and merge with other flux ropes in the stellar interior. Second, two nonradial flux ropes which intersect can be prevented from reconnecting by the stable stratification. (While reconnection can occur in a thin ohmic layer at the surface of the two ropes, the impossibility of vertical motions prevents most of the field lines in the two ropes from coming into physical contact.) If our conjecture is correct, then the small-scale field of a neutron star can be stabilized by topology. The field will exhibit discontinuities where reconnection would be favored if the stellar material were free to move vertically. The persistance of white dwarf magnetic fields suggests that realistic magnetic field geometries in liquid stars are not susceptible to an instability which significantly reduces the scale of the external field (Eichler 1982).

The fact that the crust of a neutron star has a finite tensile strength has not entered into this argument. Indeed, the saturation field strength (32) is larger than the maximum field which the inner crust can support *outside* of magnetostatic equilibrium. This maximum field is given approximately by $B_{max}^2/4\pi = \epsilon\mu$, where $\epsilon \sim 10^{-4}$–10^{-2} is the dimensionless yield strain, and the shear modulus is given by (Ruderman 1972)

$$\mu = 4 \times 10^{29}\left(\frac{\rho}{10^{14}\ \mathrm{g\ cm^{-3}}}\right)^{4/3}\left(\frac{Z}{40}\right)^2 \times \left(\frac{A}{1000}\right)^{-4/3} \mathrm{dyne\ cm^{-2}}\,, \tag{42}$$

where A amd Z are the mass number and charge of the nuclei. Equivalently,

$$B_{max} = 2 \times 10^{14}\left(\frac{\epsilon}{10^{-2}}\right)^{1/2}\left(\frac{\rho}{10^{14}\ \mathrm{g\ cm^{-3}}}\right)^{2/3} \times \left(\frac{Z}{40}\right)\left(\frac{A}{1000}\right)^{-2/3} \mathrm{G} \tag{43}$$

(Flowers & Ruderman 1977; Romani 1990). This means that when a tangled field stronger than B_{max} in the inner crust is moved out of magnetostatic equilibrium, it breaks the crust. We emphasize that *the field strength is not limited by the tensile strength of the crust, since the turbulent stresses during the early convective epoch greatly exceed* $\epsilon\mu$.

14.3. *Structure of Pulsar Magnetospheres*

One consequence of an early neutron star convective episode, is that higher multipoles should make a large contribution to the field close to a young pulsar, at $R \lesssim 2R_*$. It is less clear whether these higher multipoles should significantly affect the rate at which the pulsar spins down, since the pulsar magnetosphere is filled with plasma (Goldreich & Julian 1969). In other words, the pulsar's spin-down rate is determined by MHD torques near the light cylinder, where the field is essentially dipolar. The dipole field is related to an integral quantity, the total flux through the star, whereas the higher multipoles are not. We conclude that the influence of higher multipoles on the spin-down rate is probably smaller than the vacuum approximation (Gunn & Ostriker 1969) would indicate. Measurements of $\ddot{P}$ probably do not usefully constrain the multipolar structure of a pulsar's field unless its spin period is in the millisecond range (see Krolik 1991).

Irrespective of whether the dipole field originates before or after the collapse, the *direction* of the dipole should not be correlated with the rotation axis once a star undergoes a period of high Rossby number convection. This contrasts with the approximate—but not precise—alignment of magnetic and rotation axes in low Ro dynamos (e.g., Earth, Sun, and planets) as predicted by conventional dynamo theory. There is some evidence that the alignment angle is indeed random in young pulsars (Lyne & Manchester 1989).

14.4. *Pulsar Field Decay?*

The question as to whether the dipole field of a pulsar decays is closely connected to the question as to whether a high Ro dynamo operates in a young, convective neutron star.

The magnetic energy stored in the crust *does* decay. A strong field anchored in the lower crust decays on a reasonably short time scale via turbulent Hall drift (Goldreich & Reisenegger 1992):

$$\tau_{\rm Hall} \simeq 5 \times 10^8 \left(\frac{l_{\rm loop}}{1\ {\rm km}}\right)^2 \left(\frac{B}{10^{12}\ {\rm G}}\right)^{-1} \left(\frac{\rho_{14}}{3}\right) {\rm yr}\ . \quad (44)$$

A flux loop of strength 3×10^{14} G and size $l_{\rm loop} \sim 1$ km decays in $\sim 1.7 \times 10^6$ yr.

No matter what the origin of the dipole field, the strong, small-scale field generated by the convective motions will make some contribution $\Delta B_{\rm dipole}$ to the total dipole. As neighboring flux loops reconnect and merge, this contribution decreases only slowly. Since the density in the crust scales with depth z as $\rho(z) \propto z^3$, a dipole formed from the merger of two equal dipoles has physical dimensions larger by a factor $2^{1/6}$ than each of the original two. The flux threading the merged dipole is equal to that threading each of the original dipoles, so the moment of the merged dipole is smaller only by a factor $2^{-1/3}$ than the incoherent sum of the original moments. We conclude that $\Delta B_{\rm dipole}$ decreases only by a factor $\sim (N_{\rm loop}/4\pi)^{1/3} = (l_p/R_*)^{2/3} \sim 0.2$ as the scale of the field increases from $l_{\rm loop} \sim l_p$ to $l_{\rm loop} \sim R_*$. At this point, the rms surface field has decreased from perhaps 1–3 $\times 10^{14}$ G to $\sim 3 \times 10^{12}$ G, in a time scale $\sim 5 \times 10^6$ yr. We conclude that if a large fraction of the dipole field is generated in a high Rossby number dynamo operating in the postcollapse convection zone, then $B_{\rm dipole}$ does not decrease much as the surface field is smoothed out.

We note that observations of cyclotron lines indicate fields in the range 10^{12} G for some neutron stars in X-ray binaries. Neutron stars in *low*-mass X-ray binaries are old enough that any very intense, higher multipole fields initially entrained in the crust are expected to have decayed. For example, the neutron stars in both Her X-1 and 4U 1626−67 are almost certainly older than 10^7 yr and probably older than 10^8 yr (Sutanyo, van der Linden, & van den Heuvel 1986; Verbunt, Wijers, & Burm 1990). Stronger constraints on small-scale magnetic fields come from high-mass X-ray binaries, which have ages of order 10^6 yr. The most interesting object, in this regard, is the 7 s soft X-ray pulsar 1E 2259+586, which resides in the 1×10^4 yr old supernova remnant CTB 109. This pulsar has no detected binary periodicity or optical companion (Davies & Coe 1991). Its spectrum may exhibit cyclotron lines corresponding to a 5×10^{11} G field (Iwasawa, Koyama, & Halpern 1992), which is the dipole field required to produce the observed X-ray period derivative via disk-magnetosphere coupling, if the star is accreting at the rate implied by its X-ray luminosity (Iwasawa et al. 1990). It is also possible that 1E 2259+586 has spun down to its present period via MHD radiation, and that the X-rays are powered by the decay of a strong internal magnetic field. The required (initial) surface dipole field and internal field are 4×10^{14} G and 4×10^{15} G, respectively (Thompson & Duncan 1993).

The question as to whether the surface field can decay completely depends, in part, on whether this field also threads the core. (It is not clear whether flux can diffuse out of the core in less than a Hubble time; e.g., Goldreich & Reisenegger 1992.) The field must thread the core if the dipole originates prior to the collapse in an $\alpha - \Omega$ dynamo (DT; § 12), but not necessarily if the dipole is generated after the collapse by a high Ro dynamo (§ 12). In this second case, the field is amplified on the scale of an individual convective cell, which is comparable to the thickness of the crust which forms later, and so most of the exterior field may not be connected to the core.

14.5. *Magnetic Triggering of Glitches*

As the magnetic field lines undergo Hall drift in the crust, the field moves out of magnetostatic equilibrium. We have argued that the strength of the crustal field is likely to exceed the critical strength (43) at which anisotropic magnetic stresses can break the crust, especially if ϵ is as small as 10^{-4}. If the crustal material is sufficiently brittle, the evolving field will rupture it from time to time. This rupture should occur in a region of size comparable to the coherence length $l_B \sim (10^{-1} - 1)l_p$ of the field.

Let us examine what happens if this rupture is violent enough to unpin the vortex lines in the crustal neutron superfluid from the crustal lattice. So long as the vortex lines remain pinned, the common angular velocity Ω of the crustal lattice and the superconducting interior drops below the angular velocity Ω_n of the crustal superfluid as the star spins down (e.g., Cheng et al. 1989). We assume that the unpinning occurs in a region of size $\sim l_B$. The moment of inertia of the crustal neutron superfluid in this region is a fraction $I_s/I \sim 10^{-2}(l_B^2/4\pi R_*^2) \simeq 10^{-5}(l_B/1\ {\rm km})^2$ of the total moment of inertia of the star. Balancing the angular momentum lost by the unpinned superfluid, with that gained by the rest of the star, and assuming that the superfluid achieves an angular velocity comparable to the rest of the star when it repins, one finds that the surface angular velocity increases by the amount $\Delta\Omega \simeq (I_n/I)(\Omega_n - \Omega)$. Now, the maximum angular velocity difference between superfluid and lattice, above which the vortex lines become unpinned spontaneously, is not well determined, but for illustration we take the value $\Omega_n - \Omega \sim 5\ {\rm s}^{-1}$ (Ruderman 1991a). The glitch must be smaller than

$$\frac{\Delta\Omega}{\Omega} \lesssim 8 \times 10^{-7} \left(\frac{P_{\rm rot}}{10^{-1}\ {\rm s}}\right) \left(\frac{I_n/I}{10^{-5}}\right) . \quad (45)$$

In this picture, the vortex lines become unpinned sequentially in small patches of the crust, rather than in the crust as a whole as has been argued by Ruderman (1991b).

We have also argued in § 14.1 that the crustal field has numerous discontinuities, at which reconnection is temporarily suppressed by the stable stratification of the star. Eventually, however, the field will evolve to the point where reconnection is possible with purely horizontal movements. This will cause a second, potentially more energetic type of rupture of the crust. The energy released in such an event will be of order the magnetic energy in a single small flux loop, $\Delta E \simeq 4 \times 10^{42} (B_{\rm loop}/3 \times 10^{14}\ {\rm G})^2(l_{\rm loop}/1\ {\rm km})^3$ ergs. This is to be compared with the rotational energy released energy released when the vortex lines unpin, $\Delta E \simeq (I^2/I_n)(\Delta\Omega)^2$, which is $5 \times 10^{41}[(\Delta\Omega/\Omega)/10^{-6}]^2$ ergs for the Vela pulsar, with $P_{\rm rot} = 0.089$ s.

In this picture, a pulsar such as the Crab, which suffers much smaller amplitude glitches than Vela, is endowed with a crustal field of comparable strength. In order to explain the difference in glitch amplitudes, one might invoke a transition of the crust from a plastic state to a brittle state at a critical temperature $T \sim 10^{-1} T_{\rm melt}$, as advocated by Ruderman (1991a, b) in a different glitch model.

The above model for glitches predicts concurrent X-ray bursts. Indeed, observations of thermal X-radiation from neutron star surfaces constrains the small-scale fields of pulsars. Since magnetic structures of dimension ~ 1 km and strength $B \lesssim 10^{15}$ G decay due to Hall drift in $\tau_{\rm Hall} \gtrsim 10^6$ yr, the strongest constraints on the upper strength of small-scale pulsar fields come from $\sim 10^6$ year old neutron stars. Upper bounds to the thermal X-ray flux at this age are typically $\sim 3 \times 10^{33}$ ergs s^{-1}, with a large scatter (e.g., Van Riper 1991). Given that a fraction ϵ_x of the energy released by magnetic reconnection or crustal fracture is radiated thermally, this implies an upper bound on a scale l of

$$B(l) \lesssim 3 \times 10^{14} l_5 \left(\frac{\epsilon_x}{0.3}\right)^{-1/3} \text{ G}, \tag{46}$$

a factor ~ 3 smaller than the upper bound estimated in § 14.1.

The dipole moment of a young pulsar will shift suddenly if the surface field is rearranged at a glitch. If the glitch occurs without magnetic reconnection (that is, if the rupture of the crust is of the first type which we have described), then the change in the dipole moment should be small. If, however, the glitch is due to a reconnection event, then the dipole will change by a fractional amount $\sim 10^{-3}(l_B/1\ {\rm km})^2$. The 1975 Crab glitch (Demianski & Proszynski 1983) in which $\dot{P}$ increased discontinuously and permanently may be an example of such a shift in the surface field (see also Link, Epstein, & Baym 1992).

15. APPLICATION TO GAMMA-RAY BURSTS: DYNAMOS IN RAPIDLY ROTATING NEUTRON STARS

15.1. *Bursts at Cosmological Distances*

A neutron star with a short rotation period, $P_{\rm rot} \sim 1$ ms, is formed in the merger of a neutron star binary, or when a white dwarf undergoes accretion-induced collapse (Narayan & Popham 1989). (In the first case, the star is likely to collapse to a black hole in a few milliseconds; Rasio, Shapiro, & Teukolsky 1991.) The magnetic transport mechanisms discussed above (along with nonmagnetic processes such as $\nu + \bar{\nu} \rightarrow e^+ + e^-$; Goodman, Dar, & Nussinov 1986; Haensel, Paczyński, & Amsterdamski 1991) will deposit energy outside the neutron star in the form of electron pairs and gamma rays. Since the free energy in differential rotation is $\sim 10^{52}$ ergs, enough energy could plausibly be released in this manner to create a gamma-ray burst detectable at cosmological distances ($\sim 10^{51}$ ergs at a redshift $z \sim 1$). Related models for cosmological GRBs involving strong magnetic fields in rapidly rotating compact objects have been proposed by Usov (1992) and Narayan, Paczyński, & Piran (1992).

Such a burst model might seem promising at first sight, for several reasons: the star is not clothed by a large column of supernova ejecta, the magnetic field can dissipate energy above the neutrinosphere, and the star is convective only for ~ 30 s, not much longer than the mean duration of a classical gamma-ray burst. Unfortunately, the required energy flux in electron pairs and gamma rays lies well above the Eddington limit. The expanding wind must carry less than a fraction $\sim 1 \times 10^{-3}$ of its energy in baryon rest mass, if the temperature of the emergent photons is not to be degraded below 10^{10} K by adiabatic expansion (Paczyński 1990). This may be difficult to achieve if, for example, the pair-photon plasma is created via the reaction $\nu \rightarrow \nu + e^+ + e^-$. In this case, the energy deposition rate is proportional to B^2 and decreases outward from the neutrinosphere. We discuss this issue further in TD2.

It is easier to circumvent the effects of baryonic pollution if most of the gamma rays are generated far from the stellar surface, which might happen as follows. A very efficient $\alpha - \Omega$ dynamo can operate in such a star, because the Rossby number is close to unity. Scaling from the various dynamo mechanisms discussed in § 12, one finds that the resulting dipole field is plausibly as large as $\sim 10^{15}$ G (DT). Such a rapidly rotating, highly magnetized star will spin down quickly, releasing $\sim 10^{50}(B_{\rm dipole}/10^{15}\ {\rm G})^2$ ergs to magnetic torques in the first 10 s. Even with a high efficiency of conversion into gamma rays, this energy is not quite sufficient to create a detectable gamma-ray flux from a redshift $z \sim 1$ unless the resulting magnetic wind is collimated into a jet. Observations of star-forming regions and active galactic nuclei suggest that this will be the case, especially if excess angular momentum is deposited in a disk orbiting the (otherwise naked) star. Turbulence in such a jet is capable of generating nonthermal particles and thence gamma rays via a number of mechanisms, such as pion production and Compton upscattering of soft photons. However, the spin-down time scale is $\simeq 0.6(B_{\rm dipole}/10^{15}\ {\rm G})^{-2}(P_{\rm rot}/1\ {\rm ms})^2$ hr, significantly longer than the ~ 10 s duration of a typical burst. In this model, the correct burst lifetime could be achieved only if the dipole field were extremely strong, $B_{\rm dipole} \sim 10^{16}$ G, or if the formation of the jet were tied in some way to the presence of convection in the star.

15.2. *Bursts from Galactic Neutron Stars*

Very strongly magnetized neutron stars formed in this manner will spin down to periods much longer than those of ordinary pulsars at the same age. Several pieces of evidence connect these "magnetars" with the soft gamma repeaters, in particular with the source of the 1979 March 5 event, whose error box overlaps with the LMC supernova remnant N49. First, the 8 s periodicity of this source could be achieved by neutron star with the same age ($\sim 10^4$ yr) as the remnant only if its surface dipole field were $\sim 5 \times 10^{14}$ G (DT). Second, the soft repeat bursts exhibit a well-defined maximum luminosity, which greatly exceeds the standard Eddington luminosity at the distance of the LMC, but which is equal to the magnetic Eddington luminosity if the surface field strength is $\sim 3 \times 10^{14}$

G (Paczyński 1992). Third, the proper motion of the N49 neutron star inferred from the angular displacement of the gamma-ray error box from the center of the remnant is $\sim 920 \pm 530$ km s^{-1}; there are a number of mechanisms which could impart a large (~ 1000 km s^{-1}) kick to a very highly magnetized, rapidly rotating neutron star (DT). We have suggested that some classical gamma-ray bursts originate from a population of magnetars streaming out of the Galaxy or residing in a weakly bound Galactic corona (DT; Duncan et al. 1993). Finally, the stellar field contains sufficient free energy to power detectable bursts at a distance ~ 100 kpc or greater, and in particular to power the 1979 March 5 event at the distance of the LMC. Note that magnetars might form in core collapse events, in addition to binary neutron star coalescence and accretion-induced collapse of white dwarfs. One amusing possibility is that some gamma-ray bursts are produced in the births of naked magnetars at cosmological distances, while other gamma-ray bursts, including the soft-gamma repeater bursts, are magnetic reconnection events in magnetars associated with our Galaxy.

Of course, a $\sim 10^{15}$ G field, if purely poloidal inside and outside the star, would be unstable to the Flowers-Ruderman mode on a very short (Alfvén crossing) time scale (§ 14.2). Crustal lattice stresses could not suppress this instability in such a strong field. However, a neutron star born with $B_{\rm dipole} \sim 10^{15}$ G would probably contain a stronger toroidal field (which is generated in $\alpha - \Omega$ dynamo action; see § 12.2) as well as a very strong, disordered, spatially intermittent field on a scale ~ 1 km, or smaller. (The strongest possible field in a star born with a period $P_{\rm rot}$ is $\sim 3 \times 10^{17} (P_{\rm rot}/1 \text{ ms})^{-1}$ G.) We have conjectured (§ 14.2) that an interchange instability analogous to the Flowers-Ruderman instability, which could significantly reduce the field energy outside the star, is at first suppressed by a combination of the stable stratification of the stellar material, and the nontrivial topology of the field. Such an instability eventually *does* occur as the internal field of the star is rearranged by ambipolar diffusion and Hall drift. One observational signature of this instability would be a diminishment of the spin-down rate of the star. Note, in this respect, that the soft X-ray pulsar 1E 2259+586, which has a possible identification as a magnetar (Thompson & Duncan 1993; § 14.4), has a spin-down age $P/2\dot{P}$ which is ~ 10 times (Davies, Wood, & Coe 1990) the age of 1.5×10^4 yr estimated for the surrounding supernova remnant CTB 109 (Sofue, Takahara, & Hirabayshi 1983). These two characteristic ages could be reconciled if the initial dipole field were larger by a factor $\gtrsim 3$ than the present dipole field, which is inferred from $\dot{P}$ to be $\sim 0.7 \times 10^{14}$ G (assuming vacuum magnetic braking). In the simplest case, $B_{\rm dipole}$ decreased to its present strength at time $\tau_{\rm dipole}$ and was constant before this time. The initial field strength required to spin down to the observed value of P is, then, $B_{\rm dipole} \sim 3 \times 10^{14} (\tau_{\rm dipole}/1.5 \times 10^4 \text{ yr})^{-1/2}$ G.

The same comparison cannot be made for the N49 neutron star, as its $\dot{P}$ is not known, but the same instability could also have occurred in this star if $B_{\rm dipole}$ were of the strength suggested by DT. The onset of this instability would trigger one or more major reconnection events in the magnetosphere and would provide enough energy to power the 1979 March 5 burst (DT). The strength of the *surface* field (which can suppress Thomson scattering and thereby raise the limiting radiative flux from the star up to the level required to match the observed brightness of the N49 bursts; Paczyński 1992; Thompson & Duncan 1993) is less affected by this instability. The partial decay of lower-order moments of the field on such a short time scale would not preclude the possibility that these stars emit some gamma-ray bursts on much longer time scales.

15.3. *Formation of Very Strongly Magnetized Neutron Stars*

We conclude with a few general points about rotational collapse and magnetar formation. When the ratio β of rotational to gravitational energy in an object on the verge of collapse (the evolved core of a massive star, or a white dwarf accreting beyond the Chandrasekhar limit) exceeds a critical value $\beta_{\rm crit} \sim 10^{-2}$, then centrifugal forces avert dynamical collapse to nuclear density (e.g., Tohline 1984; Mönchmeyer & Müller 1989). Such collapsed objects, termed "fizzlers" by Shapiro & Lightman (1976), must shed angular momentum if they are to become true neutron stars. When $\beta < \beta_{\rm crit}$ (the rotation period of the neutron star is > 1 ms), the mixing length theory of convection discussed in § 2 can be applied. Near the edge of this domain (i.e., $P_{\rm rot}$ near but above 1 ms), postcollapse entropy-driven convection is characterized by Ro ~ 1. This is the regime in which DT argued that magnetars would form. In faster-rotating stars (true "fizzlers"), the convection is strongly modified by rotation, but is probably still sufficient to engender efficient dynamo action and to transfer angular momentum from the contracting star to a surrounding disk. Magnetars may also form in this regime, although details of the interaction between the star and disk must be considered in determining the outcome.

We should emphasize that, insofar as $\tau_{\rm con} > 1$ ms or $[\mathrm{Ro}]_{\rm crit} > 1$, a successful postcollapse $\alpha - \Omega$ dynamo does not require critical rotation. The maximum field allowed energetically scales as $P_{\rm rot}^{-1}$ at fixed Rossby number, and so the transformation of the star into a "magnetar" with $B_{\rm dipole} > B_{\rm crit} = 4.4 \times 10^{13}$ G may be possible when $P_{\rm rot}$ and/or $\tau_{\rm con}$ are as long as ~ 10 ms. The rotational energy available for energizing the supernova remnant is less than 10^{51} ergs when $P_{\rm rot} \gtrsim 3$ ms. In near-critical rotators and "fizzlers," a substantial amount of rotational energy could be lost to gravitational radiation, collimated emission in a jet, or magnetic coupling to a surrounding disk.

An alternative mechanism for forming neutron stars with large $B_{\rm dipole}$ is simply via flux conservation in a highly magnetized white dwarf that undergoes accretion-induced collapse (Usov 1992). However, there are a number of impediments to producing neutron stars with dipole fields as strong as 10^{15} G in this way. The maximum fields observed in white dwarfs (Schmidt 1989), and the dynamical saturation field strength during core carbon burning (the last possible stage of nuclear burning in the progenitor of a white dwarf) both indicate a neutron star dipole field no stronger than 10^{14} G. (We calculate $B_{\rm sat}$ from the 8.8 $M_\odot$ model of Nomoto 1987.) Even if a sufficiently strongly magentized white dwarf were to exist, it would be spun down during the accretion phase, and the resulting neutron star would have a spin period well in excess of 1 ms (according to the model of Narayan & Popham 1989), which would greatly reduce the rotational energy available for cosmological bursts (cf. Usov 1992). Nevertheless, efficient α-Ω dynamo action is probably inevitable in *any* rapidly rotating newborn neutron star, and the resulting neutron star field probably does not depend strongly on the seed field bequeathed by the progenitor star (DT).

16. CONCLUSIONS AND UNRESOLVED QUESTIONS

We have examined the possibility of dynamo action in a newborn neutron star, as well as in the progenitor star. Fields as strong as 10^{16} G are generated by convection in a young neutron star, and much of the energy in these fields is probably retained by the star. The magnetic field of a pulsar should have a multipolar structure, and may contain enough energy to power the largest observed glitches.

It is not clear whether the dipole field of a pulsar originates before or after the formation of the neutron star. The key unsolved problem concerns the viability of a high Rossby number stellar dynamo, or equivalently, the viability of a dynamo in mirror-symmetric turbulence.

Magnetic fields can deposit an enormous amount of energy outside a young neutron star, and can catalyze the conversion of energy from neutrinos to electron pairs. This energy can revive a stalled supernova shock, but only under optimistic assumptions regarding the strength of the amplified magnetic field.

In conclusion, a neutron star offers a novel setting in which to test and extend the theory of fast dynamos. The study of a neutron star dynamo gives a fresh perspective on more conventional stellar dynamos.

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