CLUMPY LANGMUIR WAVES IN TYPE III RADIO SOURCES: COMPARISON OF STOCHASTIC-GROWTH THEORY WITH OBSERVATIONS

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ABSTRACT

Detailed comparisons are made between the Langmuir-wave properties predicted by the recently developed stochastic-growth theory of type III source regions (Robinson 1992b; Robinson, Cairns, & Gurnett 1992) and those observed in situ by the joint TRW-JPL-Iowa Plasma Wave experiment on ISEE 3 (Scarf et al. 1978).

In the standard model of type III bursts (e.g., Ginzing & Zheleznnyakov 1958; Melrose 1990; Muschietti 1990; Robinson 1992b) Langmuir waves are generated via a beam instability of electrons streaming outward from the Sun. Most waves generated at the head of the beam are absorbed by electrons further back, thereby reducing the net energy loss to a level compatible with the observed propagation of beams to 1 AU or more from the Sun. When waves exceed the thresholds for three-wave decay and coalescence, secondary Langmuir and ion sound waves are generated, giving rise to radiation at the fundamental and second harmonic of the local plasma frequency. Inhomogeneous quasi-linear simulations (e.g., Crognard 1985) indicate that the beam propagates in a state close to marginal stability, in which wave growth just balances net losses. Such simulations also reproduce the approximate form of the electron distributions observed at 1 AU (Grossard 1985).

The standard model suffers from two shortcomings. First, observations have shown that ambient low-frequency density fluctuations in the solar wind have a level sufficient to scatter Langmuir waves out of resonance with the beam faster than they can grow (Celnikier et al. 1983; Celnikier, Muschietti, & Goldman 1987; Muschietti, Goldman, & Newman 1985). Second, the model does not explain the extreme clumpiness of the Langmuir waves, which show structure down to the shortest scales resolved (Gurnett & Anderson 1976, 1977; Lin et al. 1981, 1986) and vary by several orders of magnitude in energy density over periods of a second or less. A number of theories have been proposed to circumvent one or other of these problems, based on nonlinear instabilities (e.g., Lin et al. 1986, and the references cited therein), local suppression or alignment of density fluctuations (Muschietti et al. 1985; Melrose, Dulk, & Cairns 1986; Melrose & Goldman 1987), or fluctuation-dependent growth rates and growth lengths (Smith & Sime 1979). Robinson (1992b) incorporated some of these ideas into a new model in which the beam propagates in a state close to marginal stability, but both beam and waves are perturbed by ambient density fluctuations. This leads to stochastic growth in which the growth rate fluctuates between positive and negative values and the logarithmic wave density undergoes a random walk, with positive net growth in localized regions.

Robinson (1992b) showed that the stochastic growth model can account for the presence of Langmuir waves, despite the mean growth rate being negative, and (semiquantitatively in his work) for the clumpiness and field-strength distribution of the waves. More recently, Robinson et al. (1992) showed that the stochastic-growth model implies that the Langmuir clump-size distribution should be the same as that of the ambient density fluctuations and confirmed this prediction by spectral analysis of ISEE 3 wave-field data. It was also shown that several relationships between the variations of type III source parameters with distance from the Sun, predicted in part on the basis of stochastic-growth theory, are consistent with those actually observed (Robinson 1992a).

Given the initial successes of the stochastic-growth theory in accounting for the properties of type III sources, it is of considerable interest to test its predictions further. Data from the ISEE 3 spacecraft have been used to study the predicted clumping of the Langmuir waves (Robinson et al. 1992). These data can also be compared with the predicted distribution of field strengths. Such an analysis not only enables the stochastic-growth theory to be tested, but allows us to add to
the existing knowledge of bursts which have been studied from other perspectives.

In § 2 we briefly review Robinson’s (1992b) stochastic-growth model of Langmuir waves in type III source regions and generalize it to incorporate the existence of a nonlinear saturation level. In § 3 we outline the ISEE 3 plasma wave observations that are considered here, discuss our choice of the specific type III events for analysis, and analyze the main instrumental effects on the data. Detailed comparisons between theory and observation are then made in § 4, focusing on the distributions of field strengths and spatial scales.

2. STOCHASTIC GROWTH OF LANGMUIR WAVES

In this section we briefly review Robinson’s (1992b) stochastic-growth model and discuss its generalization to incorporate both long-wavelength ambient density fluctuations with a spectrum of length scales (Robinson et al. 1992) and saturation of the growth via nonlinear interactions involving the beam-driven waves (e.g., three-wave decay).

Ambient density fluctuations have been inferred to scatter Langmuir waves out of resonance with the beam at a typical rate of \( \geq (1-10) \) s\(^{-1} \), whereas the largest growth rates for the observed electron beams (time averaged over 64 s) are \( \leq (0.1-1) \) s\(^{-1} \) (Muschietti et al. 1985; Melrose et al. 1986; Robinson 1992b). Hence, under most if not all circumstances the waves cannot grow uniformly in space. Robinson (1992b) showed that the observed time-averaged growth rates are consistent with peak growth rates of a few times 10 s\(^{-1} \) in localized clumps of \( \sim 100 \) km in size. Hence, at least in regions where the scattering is not too large (e.g., where the gradient of the plasma density is approximately parallel to the beam direction), the beam-driven growth can overcome refraction of waves out of the growth region in wave vector space.

In the stochastic-growth model, the beam propagates in a state close to marginal stability, as implied by quasilinear calculations (which are unaffected on average by clumping; Melrose & Cramer 1989). Density fluctuations perturb the waves, allowing them to grow effectively only in localized clumps, while clumps of waves corresponding to unfavorable density fluctuations are damped. This in turn leads to perturbations of the beam (corresponding to spatial inhomogeneities in the beam distribution function) about its average state of marginal stability on time scales determined by the rate of change of the density fluctuations (see below). The growth rate thus fluctuates, while the space- and time-averaged effective growth rate is zero at marginal stability. Under these conditions waves in a given clump (defined to be simply a region of space of characteristic scale size of order that of the density fluctuations) \( C \) undergo a random walk in the logarithm of their energy density until either they leave the clump or wave growth saturates via nonlinear processes; this gives a characteristic interaction time \( t_c \approx l/v, \) where \( v \) is the group velocity of the Langmuir waves and \( l \) is the clump size. (In terms of the electron speed \( v_e \) and the beam speed \( v_b \) one has \( v = 3V^2/v_b. \) During a time \( t_c \) the beam travels at velocity \( v_b \) reach the clump \( C \) from distances up to \( v_b t_c \) away and traverse \( t_c \) clumps in transit, where \( \langle t_c \rangle \) is the mean clump size. Assuming these clumps have a characteristic lifetime \( t_g \approx \langle 1/G \rangle /v, \) where \( v \) is the ion-sound speed, a typical upstream clump forms and disperses \( \sim t_g \) times near a particular location during the time \( t_c. \) Each upstream clump formation and dispersal “injects” a corresponding inhomogeneity into the part of the beam that interacts with the given clump \( C \) during \( t_c. \) Hence, a total number \( n_y(l) \) of beam inhomogeneities cross \( C \) during \( t_x, \) with

\[
\frac{n_y(l)}{\langle 1/G \rangle} = \frac{v_b t_y}{v_s}\frac{\langle 1/G \rangle}{t_x} = \frac{v_b t_y}{v_s} \frac{1}{t_x}.
\]  

(1)

A time \( t \leq t_y \) into the evolution of the clump, a typical number \( n(t) \) of inhomogeneities will have passed by, with

\[
n(t) = \frac{n_y(t)}{t_y} = \frac{t}{t_y},
\]  

(2)

\[
t_i = \frac{\langle 1/G \rangle}{v_b v_s}.
\]  

(3)

These estimates improve those of Robinson (1992b) and imply \( n_y(l)/l \approx 1 \) for typical parameters from Table 1, with \( v_s \approx (1 + 3T_3/T_2)^{1/3}(m_e/m_i)^{1/3}Y \) in terms of the electron and ion temperatures and masses.

The total number of e-foldings \( G \) undergone by a clump of waves is the sum over individual increments \( G \approx G_t \) as the waves are encountered by each inhomogeneity in the beam, where the net growth rate \( \Gamma \) varies between inhomogeneities. The mean and variance of \( G \) are thus related to the corresponding quantities for \( \Delta G \) by

\[
\langle G \rangle = \langle \Delta G \rangle \langle n(t) \rangle = \langle G \rangle t_y,
\]  

(4)

\[
\sigma_G^2 = \sigma_\Delta G^2 \langle n(t) \rangle = \frac{\Gamma t_y^2}{2},
\]  

(5)

for \( t \leq t_y \). Typical values are obtained by setting \( t = t_y \) and \( n_y = n_y(l) \) in equations (4) and (5), which gives

\[
\langle G \rangle = \frac{\langle 1/G \rangle}{v_b},
\]  

(6)

\[
\sigma_G^2 = \frac{\langle 1/G \rangle^2}{v_b v_s}.
\]  

(7)

Robinson et al. (1992) result \( G_t \) \sim \Gamma \sim 1 \) has been used in obtaining the rightmost members of equations (6) and (7).

In terms of \( \langle G \rangle \) and \( \sigma_G \) the spatially averaged probability distribution of \( G \) is approximately

\[
P(G) = \frac{1}{\sigma_G (2\pi)^{1/2}} \exp \left[ -\frac{(G - \langle G \rangle)^2}{2\sigma_G^2} \right],
\]  

(8)

via the Central Limit Theorem. The mean energy density in the waves is obtained by averaging \( W \sim e^G \) over the distribution (8), using equations (4) and (5). This gives

\[
\langle W \rangle = \exp \left[ \langle G \rangle + \frac{1}{2} \sigma_G^2 \langle G \rangle \right],
\]  

(9)

for \( t \leq t_y \) (Robinson 1992b). Hence, the effective growth rate is

\[
\Gamma_{eff} = \langle G \rangle + \frac{1}{2} \sigma_G^2 \langle G \rangle t_y.
\]  

(10)

This result resolves the paradox of wave growth being observed in the presence of a negative mean growth rate \( \langle G \rangle \) (Robinson 1992b) because \( \Gamma_{eff} \) can be positive even for \( \langle G \rangle < 0. \) The distribution \( P(E) \) of electric field strengths \( E \) is obtained from equation (8) by writing \( G = 2 \ln (E/E_0), \) where \( E_0 \) is the characteristic field; this gives

\[
E P(E) = \frac{1}{\sigma_G (2\pi)^{1/2}} \exp \left[ -\frac{[2 \ln (E/E_0) - \langle G \rangle]^2}{2\sigma_G^2} \right].
\]  

(11)
TABLE 1
PARAMETERS OF 1979 FEBRUARY 8, FEBRUARY 17, AND MARCH 11 EVENTS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Feb 8</th>
<th>Feb 17</th>
<th>Mar 11</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_e$</td>
<td>$7 \times 10^6$</td>
<td>$1.2 \times 10^7$</td>
<td>$2 \times 10^6$</td>
<td>m$^{-3}$</td>
</tr>
<tr>
<td>$v_{th}$</td>
<td>$3.5 \times 10^7$</td>
<td>$5 \times 10^7$</td>
<td>$3.5 \times 10^7$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>$6 \times 10^4$</td>
<td>$2.1 \times 10^4$</td>
<td>$4 \times 10^4$</td>
<td>K</td>
</tr>
<tr>
<td>$v_p$</td>
<td>$1.7 \times 10^8$</td>
<td>$1.5 \times 10^8$</td>
<td>$2 \times 10^8$</td>
<td>K</td>
</tr>
<tr>
<td>$v_\phi$</td>
<td>$1.6 \times 10^8$</td>
<td>$1.5 \times 10^8$</td>
<td>$1.7 \times 10^8$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$v_p$</td>
<td>$2.2 \times 10^9$</td>
<td>$1.4 \times 10^9$</td>
<td>$2.6 \times 10^9$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>$1.5 \times 10^3$</td>
<td>$2.0 \times 10^3$</td>
<td>$8.2 \times 10^3$</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\Delta v_{nu}$</td>
<td>$3.5 \times 10^{6}$</td>
<td>$3.5 \times 10^6$</td>
<td>$4.8 \times 10^6$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$E_{\phi,\nu}$</td>
<td>$2.5 \times 10^{-5}$</td>
<td>$4.0 \times 10^{-5}$</td>
<td>$5.0 \times 10^{-3}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$E_{\phi,\nu}$</td>
<td>$2.7 \times 10^{-5}$</td>
<td>$4.8 \times 10^{-3}$</td>
<td>$5.3 \times 10^{-3}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$\eta_\nu$</td>
<td>38</td>
<td>100</td>
<td>26</td>
<td>...</td>
</tr>
<tr>
<td>$t_{\phi,\nu}$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.3</td>
<td>s</td>
</tr>
<tr>
<td>$t_{\phi,\nu}$</td>
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<td>0.006</td>
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<td>s</td>
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<tr>
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<td>4.5</td>
<td>...</td>
</tr>
<tr>
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<td>0.3</td>
<td>0.9</td>
<td>...</td>
</tr>
<tr>
<td>$\langle \gamma \rangle$</td>
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<td>37</td>
<td>68</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\langle \Delta \gamma \rangle$</td>
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<td>-3.8</td>
<td>-10</td>
<td>...</td>
</tr>
<tr>
<td>$\langle \gamma \rangle$</td>
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<td>-0.04</td>
<td>-0.4</td>
<td>...</td>
</tr>
<tr>
<td>$\langle \Gamma(0) \rangle$</td>
<td>-10</td>
<td>-3</td>
<td>-13</td>
<td>...</td>
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<td>$E_{\nu}$</td>
<td>$3 \times 10^{-3}$</td>
<td>$4 \times 10^{-3}$</td>
<td>$6 \times 10^{-3}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$E_{\phi}$</td>
<td>$3 \times 10^{-3}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$7 \times 10^{-3}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$E_{\phi,\nu}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$8.5 \times 10^{-6}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$E_{\phi,\nu}$</td>
<td>$3.5 \times 10^{-3}$</td>
<td>$2.8 \times 10^{-3}$</td>
<td>$2.4 \times 10^{-5}$</td>
<td>V m$^{-1}$</td>
</tr>
<tr>
<td>$a$</td>
<td>1.7</td>
<td>1.5</td>
<td>1.7</td>
<td>...</td>
</tr>
<tr>
<td>$b$</td>
<td>2.3</td>
<td>3.0</td>
<td>2.5</td>
<td>...</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>0.4</td>
<td>0.8</td>
<td>0.6</td>
<td>s</td>
</tr>
</tbody>
</table>

Notes.— Most parameters are from Lin et al.'s (1981, 1986) papers, or from the National Space Science Data Center's OMNI database. No estimate of $T_e$ for the 1979 February 17 event was available, so we have used a typical value of 1 AU. Note that $I = 200$ km is assumed in the evaluation of $\langle \gamma \rangle$ and $\langle \gamma \rangle$, and $I = 90$ km is assumed in evaluating $t_{\phi}$ and $t_{\phi}$.

The stochastic growth theory therefore predicts a parabolic shape for $\ln [E(P(E))]$ when plotted as a function of $\ln E$. In § 4 we show that this theoretical prediction is consistent with the observational data. (Note that the form of the distribution [11] is valid only for fields above the thermal level. This implies that the normalization should be adjusted accordingly. However, the thermal level, estimated below, turns out to be far below the observed fields, implying that this is a relatively small correction. Since, in addition, the absolute normalization is not investigated in the present work, we do not make this refinement here.)

Stochastic growth leads to arbitrarily high wave levels in a vanishingly small proportion of clumps, a situation that is prevented by the existence of a nonlinear saturation mechanism. If the highest wave levels saturate via three-wave decay (or, more generally, via some other nonlinear process [see, e.g., Cairns 1984, 1986; Lin et al. 1986; Robinson, Willes, & Cairns 1993]), the distribution of $G$ will fall off rapidly beyond the point $G_e$ corresponding to the threshold level $E_0$ for decay. Dynamically, the energy density of those waves that exceed the threshold decreases suddenly as product waves are generated, falling to an asymptotic level that is an exponentially decreasing function of the extent to which the threshold was exceeded (Robinson 1992c). In order to calculate $P(G)$ near the three-wave threshold we approximate the threshold as an absorbing boundary (Robinson 1992c) and solve the diffusion problem in $G$ for the half-interval $G < G_e$. This is most easily done by the method of images in which a symmetrically placed distribution of opposite sign, centered at $2G_e - \langle G \rangle$, diffuses in the half-interval $G < G_e$. After normalizing the unit total probability the result is

$$P(G) = \left[ \frac{\sigma(G)2\pi^{1/2}}{\sigma(G)j/2} \right]^{-1} \left\{ \exp \left[ -\frac{(G - \langle G \rangle)^2}{2\sigma(G)^2} \right] - \exp \left[ -\frac{(2G_e - \langle G \rangle - G)^2}{2\sigma(G)^2} \right] \right\},$$

for $G < G_e$, and $P(G) = 0$ for $G > G_e$. Figure 1 shows the distributions (8) and (12) and illustrates the effect of the decay threshold. The electric-field distribution is obtained from distribution (12) by using $G = 2\ln(E/E_0)$, as before.

Robinson et al. (1992) found that values of $G$ can be described as being spatially distributed in clumps of width $l$ and (without loss of generality) unit height. The probability distribution $P_c(l)$ of the clump size was found to have the approximate form

$$P_c(l) \sim \left\{ \begin{array}{ll} l^{a-1} & l < l_{\text{max}}, \\ l_{\text{min}} & l_{\text{min}} < l < l_0, \end{array} \right.$$  

which gives the mean values

$$\langle l \rangle \approx \frac{b - 1}{b - 3} l_{\text{min}},$$

$$\langle l^2 \rangle \approx \frac{b - 1}{3 - a} \left( \frac{b - 1}{b - 3} \right)^2 l_{\text{min}},$$

where some small terms have been neglected for the case relevant to the solar wind, which has $a \approx 1.5$ and $b \approx 2.5$ (Robinson et al. 1992). In the solar wind $l_{\text{min}}$ is the smallest fluctuation size consistent with observation (Celniker et al. 1983, 1987) and with moderate to weak dissipation of Langmuir waves. These constraints give 0.4 km $< l_{\text{min}} < 30$ km in general and 2 km $< l_{\text{min}} < 30$ km for beam-driven waves, for which $l_{\text{min}}$ must also exceed the wavelength $2\pi v_\phi/\omega_0$ (Robinson et al. 1992). The maximum scale $l_{\text{max}}$ is determined by the
distance that the Langmuir waves can remain in resonance
with the marginally stable beam, giving

$$l_{\text{max}} \approx \frac{3r v^2}{2v_b^3},$$  

(16)

where $r$ is the distance from the Sun and $\Delta v_b$ the spread in
beam velocities (Melrose et al. 1986; Robinson et al. 1992).
Using $l_{\text{max}} = 5 \times 10^4$ km, $l_0 \approx 4 \times 10^3$ km, and $l_{\text{min}} = 30$ km
for $a = 1.5$ and $b = 2.5$, we find

$$\langle l \rangle \approx 90 \text{ km},$$  

(17a)

$$\langle P^2 \rangle \approx (700)^2 \text{ km}^2.$$  

(17b)

Robinson et al. (1992) found the result $G \propto n$, where $n$ is the
density fluctuation and, hence, predicted the distribution $P_d(l)$
of ambient density fluctuations to satisfy

$$P_d(l) = P_d(l).$$  

(18)

This result was verified to within the observational uncertain-
ties in their work.

The time $t$ for which a clump is detected at a spacecraft is
proportional to its width, with $t = l/v_{sw}$, where $v_{sw}$ is the solar
wind speed. The observed time history of $G$ (or of log $E$) is the
sum of contributions from individual clumps. If we assume
these clumps are uncorrelated, the total power spectrum $S(\omega)$
is the integral, weighted by $P_d(t)$, over the spectra $S(\omega)$ of
individual clumps of characteristic duration $t$. This gives
(Robinson et al. 1992)

$$S(\omega) \propto \begin{cases} \omega^{\alpha - 3}, & \omega \ll \omega_0, \\ \omega^{\beta - 3}, & \omega_0 \ll \omega, \end{cases}$$

(19)

with $\omega_0 t_0 = \omega l_0/v_{sw} \approx 1$; this result is independent of
the shape of the clumps. The spectrum plateaux for $\omega \lesssim 1/l_{\text{max}}$ and
depends on the shape of the clumps for $\omega \gtrsim 1/l_{\text{min}}$.

3. OBSERVATIONS AND INSTRUMENTAL EFFECTS

In this section we briefly discuss the choice of type III events
analyzed in this work. Instrumental effects resulting from filter
responses, antenna projection effects, and other sources are then
discussed.

3.1. Data Selection

The data used in this work were obtained from the joint
TRW-JPL-Iowa plasma wave instrument on ISEE 3 (Scarf et al.
1978), which was located in the solar wind $1.6 \times 10^6$ km
($259R_E$) upstream of Earth where $R_E$ is the radius of the Earth.
The data consist of time series of electric field strengths mea-
sured over 0.5 s intervals and sampled simultaneously in each of
16 frequency channels. These channels cover the frequency
range 17.8 Hz to 100 kHz, 4 per frequency decade. Each
channel up to 5.6 kHz has a characteristic full bandwidth of
30%, while the bandwidths of the remainder are 15%.

Three type III events are considered below: (1) 1979 Feb-
uary 8 from 06:30 UT to 08:00 UT, (2) 1979 February 17
from 19:30 UT to 22:00 UT, and (3) 1979 March 11 from
11:00 to 12:00 UT. The reason for the choice of these particu-
lar events is that they have previously been studied in detail
from other perspectives involving both particle and wave mea-
surements (the first and third by Lin et al. 1986, and the second
by Lin et al. 1981). Hence, the present study can both make use
of an extend existing knowledge of these events.

3.2. Filter Rolloff Effects

In § 4 we are concerned with data from pairs of channels
bracketing the plasma frequency during type III events, either
the 10 kHz and 17.8 kHz channels, or the 17.8 kHz and 31.6
kHz channels. These channels respond both to Langmuir
waves near the plasma frequency and to the electromagnetic
waves that form the type III radio burst.

For narrow-band waves at a frequency $f$, located between the
central frequencies $f_a$ and $f_b$ of the channels immediately above
and below $f$, respectively, the measured fields in the two channels
are related to the true field $E_d$ on the antenna by

$$E_a = E_d r^{f/f_a},$$  

(20a)

$$E_b = E_d r^{f/f_b},$$  

(20b)

where $r(\rho)$ is the filter response function. Thus, if $E_a$, $E_b$ and $r$
are known, $f$ and $E_d$ can be calculated. Figure 2 shows the
measured form of $r(\rho)$ for filters used in the ISEE 1, ISEE 2,
and CRRES plasma wave instruments (R. R. Anderson 1991,
private communication); filters of the same design were used in
the ISEE 3 instrument (E. W. Greenstadt & R. R. Anderson
1991, private communication), but calibration data for the
actual filters flow on ISEE 3 are currently unavailable (E.
Greenstadt 1991, private communication). The true field and
frequency can be approximated by

$$E_d \approx 13(E_a E_b)^{1/2},$$  

(21)

$$f f_a \approx 1 - 0.043 \ln (180 E_d / E_a).$$  

(22)

Figures 3a and 3b show $E_d/E_a$ and $f f_a$, respectively, as
functions of $E_d/E_a$. Solid curves are obtained numerically from
the measured response function, while dashed curves are from the
approximations (21) and (22).

3.3. Projection Effects

The field $E_d$ detected by the ISEE 3 antenna is the com-
ponent of the true field $E$ projected onto the antenna direction
$a$. Since the exact direction of $E$ is unknown, it is not possible
to correct individual data for this effect. However, it is possible
to estimate the effect on the overall statistical distribution of
fields and show that it can be neglected in the cases of interest

![Figure 2](image-url)

**Fig. 2.**—Measured filter response (rolloff) function $r(\rho)$
During its rotation (period \( \approx 3 \) s) the antenna's angle \( \theta_A \) relative to the interplanetary magnetic field \( \mathbf{B} \) sweeps through almost the full range \( 0^\circ \leq \theta_A \leq 360^\circ \), because the field lies close to the (ecliptic) plane in which the antenna rotates. Since the strongest Langmuir fields lie approximately in the direction of \( \mathbf{B} \), the measured field will underestimate the true ones by large factors when \( |\cos \theta_A| \) is small. If we approximate the relative directions of \( \mathbf{B} \) and \( \mathbf{a} \) as being distributed uniformly in \( \theta_A \) between fixed values of azimuth relative to \( \mathbf{B} \), the probability distribution \( P_A(E_A) \) of \( E_A \) is determined by the distribution \( P(E) \) of true fields having \( E > E_A \). This gives

\[
P_A(E_A) = \int_0^1 \frac{d\mu}{\mu} \left( \frac{E_A}{\mu} \right),
\]

where \( E_A = \mu E \).

The integral in equation (23) can be evaluated analytically for the predicted distribution (11) by making the change of variable \( u = 2 \ln \mu \). This yields

\[
E_A P(E_A) = \frac{1}{2} \exp \left[ \frac{4x + \sigma^2(G)}{8} \right] \text{erfc} \left[ \frac{2x + \sigma^2(G)}{2\sigma(G)\sqrt{2}} \right],
\]

\[\approx \frac{2\sigma^2(G)}{2x + \sigma^2(G)} \left( \frac{1}{\sigma(G)(2\pi)^{1/2}} \right) \exp \left[ -\frac{x^2}{2\sigma^2(G)} \right],\]

with

\[x = 2 \ln \left( \frac{E_A}{E_0} \right) - \langle G \rangle.\]

The approximation (25) is valid for

\[2x + \sigma^2(G) \gtrsim 2\sigma(G)\sqrt{2},\]

a condition that is found to be satisfied by the data discussed in §4 [\( \sigma(G) \approx 4, x \gtrsim 0 \)]. Comparison of equation (25) with equation (11) shows that the two distributions are the same except for the factor \( 2\sigma^2(G)/(2x + \sigma^2(G)) \), which is of order unity for the parameters of interest in §4. Thus \( \sigma(G) \) will be essentially unaffected, a result that is valid even in the case in which there is no correlation between the antenna direction and \( \mathbf{B} \). Similarly, the number of observations at high \( E \) will not be significantly reduced, because the change in normalization is of order unity. One effect that should be noted and corrected for when analyzing data is the offset of the peak toward the left as \( \sigma(G) \) increases, as seen in Figure 4, which shows \( E_A P(E_A) \) for \( \sigma(G) = 0.5, 1, 2, \) and 4. The distribution \( E P(E) \) for \( \sigma(G) = 2 \) is also shown (dashed) for comparison, demonstrating that the shape of the high-\( E \) distribution is unchanged by projection effects and that the approximation (25) is adequate where (27) is satisfied.

In a distribution like (24) that decreases steeply at large \( E \), projection effects do not make much difference to the highest field \( E_{\text{max}} \) observed in a set of \( N \) observations. This field is determined by the condition that less than one even higher field is statistically likely to be observed in the \( N \) measurements, i.e. by

\[N \int_{E_{\text{max}}}^{\infty} dE_A P(E_A) = 1.\]

The exponentially rapid fall-off in \( P(E) \) large \( E \) implies that the change of order unity in normalization between the high-\( E \) distributions (11) and (25) has only a slight effect on \( E_{\text{max}} \) (reducing it \( E_{A_{\text{max}}} \)) because even a large fractional reduction in

![Figure 3](image_url)

**Figure 3.** Parameters corrected for rolloff effects. Solid curves show results using the measured response, while dashed curves show approximate results. (a) \( E_A/E_a \) vs. \( E_a/E_a \), with the dashed curve from eq. (21). (b) \( f_{(b)} \) vs. \( E_a/E_a \), with the dashed curve from eq. (22).

![Figure 4](image_url)

**Figure 4.**—Projection effects on the distribution, showing \( E_A P(E_A) \) vs. \( x \) (with \( x = 2 \ln \left( \frac{E_A}{E_0} \right) - \langle G \rangle \)) from eq. (24) for \( \sigma(G) = 0.5, 1, 2, \) and 4, as labeled. The true distributions are Gaussians of the form (11) centered at \( x = 0 \), one of which is shown (dashed curve) for \( \sigma(G) = 2 \).
the probability of observing a very high field is compensated for by a slight decrease in the lower bound of the integral in (28) where \( P(E_A) \) is relatively large. We can estimate this effect by noting
\[
\int_{E_A_{\text{max}}}^{\infty} dE_A P(E_A) = \int_{x_{\text{max}}}^{\infty} dE P(E).
\]
(29)

Upon substituting equations (11) and (25) and using the asymptotic form for the error functions that result from the integrals, equation (29) yields
\[
\frac{E_{\text{max}}}{{E_A}_{\text{max}}} \approx \left[ \frac{\sigma^2(G) + 2x_{A_{\text{max}}}}{2\sigma^2(G)} \right]^{s^2(G)/(2s_{A_{\text{max}}})},
\]
(30)

where \( x_{A_{\text{max}}} \) is the value of \( x \) corresponding to \( E_{A_{\text{max}}} \). The estimate (30) is verified for \( s(G) = 2 \) by comparing the dashed curve in Figure 4 with the corresponding solid curve.

3.4. Other Effects

The automatic gain control (AGC) used for each of the 16 channels in the main \( \text{ISEE} \) 3 filter bank has a rise time \( \delta t \approx 10 \) ms, a decay time of order 50 ms, and a mean settling time comparable to the sampling interval of 0.5 s. Clumpy fields with scales small compared with the distance the solar wind travels during the rise time (i.e., with \( t \ll v_{sw} \delta t \approx 4 \) km) will therefore be seriously underestimated. If a significant fraction of fields have scales smaller than 4 km, this will result in an underestimation of the highest fields and a reduction in their apparent probability; however, this appears unlikely since the shortest scale length for beam-driven waves is at least 2 km, as discussed following equation (15). Moreover, since each AGC ideally settles during the 0.5 s sampling interval and the rise and decay times are much smaller than 0.5 s, the electric field and clump distributions derived here should be unaffected by AGC effects. Our analyses are, in any case, necessarily restricted to scales greater than the resolution limit of 200 km (i.e., \( l/v_{sw} \approx 0.5 \) s). Information on scales \( \leq 200 \) km must await analyses of data from the Ulysses, Galileo and ISTP missions.

4. COMPARISON WITH OBSERVATIONS

In this section we analyze the data from the 1979 February 8, February 17, and March 11 events and compare the results with the theoretical predictions from § 2. In particular, we are concerned with the statistical distributions of field strengths and Langmuir clump sizes.

4.1. Field-Strength Statistics

Before commencing detailed analysis of fields corrected for instrumental effects, we first consider the raw data from one event to provide a baseline against which to judge the effects of rolloff corrections. Figures 5a and 5b show the time series from the 17.8 and 31.6 kHz channels, respectively, for the 1979 Feb-

Fig. 5.—Raw data from the 1979 February 8 event. (a) \( E_a \) vs. \( t \) in the 17.8 kHz channel. (b) \( E_a \) vs. \( t \) in the 31.6 kHz channel. (c) \( E_a P(E_a) \) for the 17.8 kHz channel. (d) \( E_a P(E_a) \) for the 31.6 kHz channel.
Figure 6.—Data from the 1979 February 8 event, corrected for rolloff effects. (a) $E_A$ vs. $t$. (b) $E_A P(E_A)$ (solid curve) and fits from eqs. (11) and (12) (upper and lower dashed curves, respectively). (c) Cumulative distribution function.

January 8 event. The characteristic highly impulsive nature of the Langmuir waves is visible in both channels, indicating that the plasma frequency $f_p$ lies between them, in accord with the estimate $f_p \approx 24$ kHz obtained from the in situ density measurement (Lin et al. 1986). Smoothly rising electromagnetic emission is seen in the 31.6 kHz channel, beginning at approximately 06:55 UT. The probability distributions of raw electric field strengths $P(E)$ in the 17.8 and 31.6 kHz channels ($E_8$ and $E_{31}$, respectively) are shown in Figures 5c and 5d, respectively, in the form $E P(E)$. Both plots show large peaks toward the left, corresponding to signals at or near the instrumental threshold and, in Figure 5d, to electromagnetic waves. The remainder of the curves, which show the statistics of Langmuir waves detected above both the threshold and the electromagnetic background, have the characteristic approximately parabolic shape expected from (11) and (25).

Correction of the data in Figures 5a and 5b for filter rolloff effects yields the time series of $E_A$ shown in Figure 6a, where all fields below the bound imposed by the instrumental threshold and electromagnetic background have been omitted. Figure 6b shows the corresponding function $E_A P(E_A)$ together with least-squares fits to the theoretical expressions (11) and (12). Figure 6c shows the cumulative distribution function

$$
cdf(E_A) = \int_{E_A}^{\infty} P(E')dE',
$$

as a function of $\log E_A$. This function gives the fraction of observations with fields greater than $E_A$. In terms of the cumulative distribution function, equation (28) becomes $cdf(E_{A,max}) = 1/N$ where $N$ is the number of data points.

From Figure 6 we find the following main results, which are summarized in Table 1:

1. The parabolic shape of the function $E P(E)$ is strong evidence that the stochastic growth theory correctly describes the generation and characteristics of type III Langmuir waves.

2. The relevant values $\sigma(G) = 4.2$ (from the numerical fits) and $0 \leq x \leq 5.3$ imply that the inequality (27) is satisfied in the region shown and, hence, that projection effects do not significantly affect the shape of the tail of $P(E)$.

3. If we choose $l = 200$ km to correspond to the resolution of the ISEE 3 measurements and use the other parameters listed in Table 1, equations (6) and (7) imply $\sigma(G) \approx 0.7$ and $\sigma(T) \approx 0.7$. This value of $\sigma(T)$ is consistent with the observed values of the mean slopes of type III beams, because the latter, averaged over 64 s, do not reflect the instantaneous values of $T$. Robinson (1992b) showed that the observed mean slopes are consistent with $\sigma(T)$ of a few times $10^{-3}$ for $l \sim 100$ km, as found here.

4. If marginal stability is assumed on average ($\Gamma_{eff} = 0$), equation (10) implies $\langle G \rangle = -\sigma^2(G)/2 \approx -8$. Using equation (6) we find $\langle T \rangle \approx -10$ s$^{-1}$, and $\langle \Delta G \rangle \approx -0.2$ for the same parameters as in point 2, above. Since $\sigma(T) \approx 0.7$, from point (3) above, we find $\sigma(T) \approx 0.7$, which implies that there is instantaneous growth of Langmuir waves in nearly half the plasma. (Note that the value $\sigma(T)/\langle T \rangle \approx 3/2$ is independent of $l$.) However, clumps in which net growth is positive for more than a few $t_i$ are rare, consistent with the highly inhomogeneous qualitative nature of the observed waves. The waves in a given clump typically grow or diminish by less than one $e$-folding for each beam inhomogeneity that passes by. For $\sigma(T) = 0.7$, and $\langle T \rangle = -10$ s$^{-1}$, the maximum instantaneous growth rate is $\sim 100$ s$^{-1}$, consistent with Robinson (1992b) and MeBrose & Goldman (1987).

5. The characteristic measured field strength is the field at the peak of the distribution, $E_{40}$.$\dollar$ The numerical fit gives $E_{40} \approx 13 \mu$V m$^{-1}$. After correction for the offset illustrated in Figure 4 with $\sigma(G) = 4.2$, the corresponding true field is $E_{40} \approx 35 \mu$V m$^{-1}$. For comparison, the thermal level of Langmuir waves with
CLUMPY LANGMUIR WAVES

...
Fig. 8.—Data from the 1979 March 11 event, corrected for rolloff effects. (a) $E_A$ vs. t. (b) $E_A P(E_A)$ (solid curve) and fits from eqs. (11) and (12) (upper and lower dashed curves, respectively). (c) Cumulative distribution function.

7b and 8b show the corresponding probability distributions, and Figures 7c and 8c show the cumulative distribution functions. The parameters obtained from fitting this data with the theoretical expressions are given in Table 1. In most respects the results for the third events in Figures 6–8 are similar. Several points worth noting are:

1. The peak of the parabolic distribution in Figure 7b lies well clear of the lower bound imposed by the instrumental threshold and electromagnetic background. This confirms the predicted parabolic form of the statistical distribution more strongly than the results in Figure 6, but the cutoff is too high to reveal the relatively flat behavior seen at small $E_A$ in Figure 4.

2. The values of $E_{A_{\text{max}}}$ and $E_{A_{\text{c}}}$ are similar for all three events, at a few mV m$^{-1}$, despite $E_{A_{\text{pk}}}$ being a factor of 10 higher in Figure 7b. This implies that the saturation mechanism cuts in at a characteristic level, independent of the strength with which the Langmuir waves are driven. The consequences of this are investigated in detail elsewhere (Robinson et al. 1993). Figure 7b shows the effects of the nonlinear cutoff most clearly with a ~5 fold difference between the simple parabolic fit (11) and the actual distribution at $E_{A_{\text{max}}}$. The cutoff field is not well determined from Figure 8b and may be somewhat higher than our estimate of 7 mV m$^{-1}$.

3. The results in Figures 7 and 8 confirm $|\langle \Delta G \rangle| < 1$ and imply that many beam inhomogeneities must interact with a given clump of Langmuir waves for significant net growth to occur. Large values of $|G - \langle G \rangle|$ are correspondingly rare because of fluctuations in the sign of $G$. However, in each case the ratio $\sigma[T(0)]/\langle T(0) \rangle$ is large, implying that instantaneous positive and negative growth rates are approximately equally common, regardless of the scale size $l$.

4.2. Clump-Size Distribution

The time series in Figures 6a, 7a, and 8a are extremely impulsive and clumpy (Gurnett & Anderson 1976, 1977; Lin et al. 1981; 1986). Robinson et al. (1992) argued that the length scales of the fluctuations in $G$ should have the same probability distribution as the underlying density fluctuations (eq. [18]). In order to test this prediction they Fourier transformed the time series from individual channels to obtain the power spectra of $G$, using a Monte Carlo method to remove instrumental effects. Their method is

1. Construct a Monte Carlo distribution of the form (13) using rectangular clumps of unit height.

2. Calculate the mean and standard deviation of the Monte Carlo distribution.

3. If the observational cutoff occurs a certain number of standard deviations from the observational mean of $\log E$, cut off the Monte Carlo distribution at the same point in terms of its own mean and standard deviation. (N.B. This step must be iterated until it converges, with the mean and standard deviation being those of the cutoff distribution itself.)

4. Fourier transform both the observational and (cutoff) Monte Carlo time series and adjust the Monte Carlo parameters $a$, $b$, and $\omega_0$ until the two spectra match when equivalently normalized. Uncertainties in the parameters of the best fit can be determined at this point.

5. Determine the true underlying spectrum by Fourier transforming the original Monte Carlo time series (without instrumental cutoffs) for the best-fit cases.

Here we carry out this procedure for time series that have been corrected for rolloff effects in order to obtain improved estimates of the parameters $a$, $b$, and $\omega_0$ in equation (19).

Figures 9a–9c show the raw spectra and corresponding Monte Carlo fits for the 1979 February 8, February 17, and March 11 events. The parameters of the fits, listed in Table 1, are consistent with those found by Robinson et al. (1992), with $a = 1.5–1.75$, $b = 2.3–3.0$, and $\omega_0 = 0.4–0.8$ (with uncertainties of $\pm 0.1$ in $a$ and $b$ and $\pm 0.2$ s$^{-1}$ in $\omega_0$). The corresponding
CLUMPY LANGMUIR WAVES

5. DISCUSSION AND SUMMARY

We have extended the stochastic-growth theory of Langmuir waves in type III source regions (Robinson 1992b; Robinson et al. 1992) to incorporate the existence of a nonlinear saturation level and have compared its predictions in detail with ISEE 3 observations corrected for the main instrumental and selection effects. The predictions and observations are found to be in good accord, thereby confirming the main results of the theory for the field-strength distribution and clumping of the Langmuir waves.

Analysis of observed field-strength statistics indicates:

1. The predicted function form of the field-strength probability distribution implied by the stochastic-growth theory result (12) is correct.

2. The mean wave growth/damping per interaction between a Langmuir wave clump and an inhomogeneity in the electron beam is $\langle G \rangle \approx 0.1$, and the standard deviation satisfies $\sigma(G) < 1$. Many interactions are thus required to achieve significant growth or damping, and this only occurs in a small fraction of clumps, consistent with the highly impulsive nature of the waves observed. Nonetheless, the result $\sigma(G) \gg 0.1$ implies that growth is positive in almost half the plasma volume at any instant. Energy lost by damped waves is approximately balanced by energy gained by growing waves to yield approximate marginal stability overall.

3. A nonlinear cutoff exists at a field strength of several mV m$^{-1}$, apparently independent of the characteristic Langmuir field $E_k$.

4. The estimated fields after correction for rolloff effects are necessarily higher than the raw single-channel fields. This implies that previous theoretical estimates of fundamental and harmonic electromagnetic emissivities based on raw fields must be revised upward and that higher energy densities than previously thought are available to drive other nonlinear processes such as wave collapse.

Remaining open questions regarding the field strengths include determining the form of the probability distribution at low $E_k$ and using a higher sampling rate to resolve short-scale events...
wave clumps. In addition, further generalization of stochastic-growth theory to incorporate the dynamics of nonlinear processes is necessary to determine the saturation mechanism responsible for the nonlinear cutoff mentioned in point 3 above and to calculate fundamental and harmonic emissivities. It is also of interest to determine why the magnitudes of $\sigma(\Gamma)$ and $\langle \Gamma \rangle$ have the observed values and whether these values are typical of weaker type III events than those considered here.

Analysis of wave clumping using rolloff-corrected fields here has refined the earlier analysis by Robinson et al. (1992). The results are in accord with that earlier work and imply that the Langmuir clump-size distribution is consistent with the size distribution of ambient density fluctuations in the solar wind, as predicted by the stochastic-growth theory (Robinson et al. 1992). Specifically, the probability distribution is found to be of the form (13) with $a = 1.5 - 1.75$, $b = 2.3 - 3.0$, and $\rho_0 = 0.4 - 0.8 \, s^{-1}$.

Open questions regarding wave clumping chiefly involve determining the clump-size distribution at the shortest scales, down to $l_{\text{min}}$. A sampling rate of $\sim 10^4 \, s^{-1}$ would be necessary to be sure of resolving all structures that are theoretically possible. More observations of the density scale-size distribution, particularly at short scales and within type III sources, would also be useful to improve our knowledge of the density spectrum, its variation with time, and its degree of correlation with the Langmuir clump-size distribution.

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