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DISK-SHOCKING OF GLOBULAR CLUSTERS AS A TEST FOR THE MASS DISTRIBUTION IN THE GALACTIC DISK.

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ABSTRACT We show that the disruption of globular clusters under the action of compressive gravitational shocks is a good test for chosing between different models of the mass distribution in the galactic disk.

INTRODUCTION

Among the strongest dynamical effects influencing globular cluster masses in the inner part of the Milky Way are compressive gravitational shocks, i.e. gravitational perturbations resulting from the rapid passage of a cluster through the Galactic disk (Ostriker et al. 1972). Disk-shocking determines not only the dynamical evolution of globular clusters, but also their survival probability (Fall and Rees 1977, Surdin 1979, Aguilar et al. 1988). Actually, this is an important evolutionary factor, in so far as it allows us to understand the relation between the galactocentric distances (R_{gc}) of globular clusters and the lower boundary of their King model concentration parameter $c = \log r_t/r_c$, where r_t is the tidal (limiting) radius, and r_c is the core radius of a cluster (Surdin 1979). The location of the boundary in the c versus R_{gc} plane depends on the mass distribution in the Galactic disk. Therefore, we may use this property of the globular cluster system as a tool for the determination of the characteristic parameters of the mass distribution.

DYNAMICAL EFFECTS

The characteristic time for the disk-shocking disruption of a globular cluster in the impulsive approximation (Ostriker et al. 1972) is

$$t_{sh} = \frac{3GMPV_z^2}{20r_h^3g_m^2} , \qquad (1)$$

where M, r_h, and P are, respectively, the mass, spatial half-mass radius, and the orbital period of a cluster; V_z is the Z-component of the cluster's velocity when it approaches the galactic plane, and g_m is the maximum value of the gravitational Z-acceleration due to the Galactic disk.

To determine P, V_z and r_h we can use a model of the Galaxy with constant circular velocity V_c, i.e. a singular isothermal model: $M_G(R) = RV_c^2/G$. Using the empirical relation between r_h, r_c, and r_t (Fall and Rees 1977):

$$r_h \cong 0.7\sqrt{r_c r_t} = 0.7 r_t 10^{-c/2} \ , \tag{2}$$

we may suppose that the limiting radius is equal to the tidal radius of the cluster at the pericenter distance of its orbit (R_p), which for an isothermal potential (Rastorguev and Surdin 1978, Oh et al., 1992) is

$$r_t = R_p \left[\frac{M}{(1+\nu)M_G(R_p)}\right]^{1/3} = \left[\frac{GMR_p^2}{(1+\nu)V_c^2}\right]^{1/3} \tag{3}$$

where

$$\nu = \frac{(1+e)^2}{2e} \ln\left(\frac{1+e}{1-e}\right), \tag{4}$$

$e \equiv (R_a - R_p)/(R_a + R_p)$ is the eccentricity, and R_a is the apocenter distance of the cluster orbit. To preserve the analytical nature of our calculations we use, in some cases, the relations obtained for Keplerian orbits in a Newtonian potential, provided, of course, the relations are essentially the same as the results of numerical calculations for the isothermal potential. For example, there is no analytical expression for the orbital period in an isothermal model, but it is easy to show that the Keplerian expression:

$$P = 2\pi \left[\frac{R_a^3}{GM_G(R_a)(1+e)^3}\right]^{1/2} = \frac{2\pi R_a}{V_c(1+e)^{3/2}} \tag{5}$$

approximates this case with an error of no more than 11%. To compare theoretical and observational results we must use as an argument in our formulae the maximum-likelihood galactocentric distance of a globular cluster:

$$\langle R_{gc}\rangle = \frac{1}{P}\int_0^P R_{gc} dt \ . \tag{6}$$

To simplify the notation we will write R instead of $\langle R_{gc}\rangle$. Our numerical calculation has shown that for a wide range of eccentricity ($0 \leq e \leq 0.85$) the value of R for the isothermal model is the same as the Keplerian value:

$$R = R_a \frac{(1+e^2/2)}{1+e} \tag{7}$$

to an accuracy of better than 5%. From equations 1-5 and (8) we can obtain a more suitable expression for the time of disk-shocking disruption:

$$t_{sh} = \frac{3V_c V_z^2}{Rg_m^2} \varphi(e) 10^{1.5c}, \tag{8}$$

where

$$\varphi(e) = \frac{(1+\nu)(1+e^2/2)}{\sqrt{1+e}(1-e)^2} \ . \tag{9}$$

Formulae 8-9 can be used to evaluate the disk-shocking time for individual clusters with an arbitrary mass distribution in the Galactic disk.

It must be noted that the value of t_{sh} does not depend on the mass of a cluster. Therefore the disk-shocking of globulars cannot be used to determine the "survival triangle" on the mass (M) versus radius (r_h) plane (see: Fall and Rees 1977). But, of course, this dynamical effect can be used to determine a "survival region" in another set of coordinates, for example, c versus R.

DISK-SHOCKING IN THE MILKY WAY

Let us consider a more limited case: a thin exponential disk with a surface density $\sigma(R_d) = \sigma_0 \exp\{(R_0 - R_d)/h\}$ and an acceleration $g_m = 2\pi G\sigma(R_d)$, where σ_0 is the surface density near the Sun at a distance R_0 from the Galactic center, h is the scale parameter, and R_d is the galactocentric distance of the crossing point of the disk and the cluster orbit. Since for highly inclined orbits the value of R_d is not very different from R_p, we can write: $R_d = \alpha R_p$ and take $\alpha = 1$ for the simple case.

The Galactic disk does not have a strong influence on the velocity of rapidly moving objects like globular clusters. Therefore we may connect the values of V_z and V_c by a simple relation: $V_z^2 = \beta V_c^2/2$. The value of β depends on the shape and orientation of the orbit of a particular cluster. As a first step we may consider $\beta = 1$ for the total sample of globular clusters with isotropically distributed velocities. For this case we obtain from equation (9) a critical value of the parameter c_{sh}, the concentration of surviving clusters. Let $t_{sh} = 16$Gyr, $R_0 = 8.5$kpc, $V_c = 220$km/s, and $\sigma_0 = 65 M_\odot pc^{-2}$. Then

$$c_{sh} = 0.36 + \frac{4.9kpc}{h} - \frac{1.2R(1-e)}{h(2+e^2)} + \frac{2}{3}\log\left(\frac{R/1kpc}{\varphi(e)}\right). \qquad (10)$$

There currently is only one method for evaluating the eccentricities of globular clusters: the tidal radii method, first used by Peterson (1974). Our calculations for 104 Galactic globulars, using the modified King formula (see eq's (4) and (5)), give us the mean relation between the minimum value of the eccentricity and the galactocentric distance of the clusters (Rastorguev and Surdin 1978):

$$\langle e_{min} \rangle = \begin{cases} 0.7\log(R_{gc}/1kpc), & \text{if } 1\text{kpc} \le R_{gc} \le 10\text{kpc}; \\ 0.7, & \text{if } R_{gc} > 10\text{kpc}. \end{cases} \qquad (11)$$

We can use this relation, supposing $e = \langle e_{min} \rangle$ and $R = R_{gc}$, to compare the observed distribution of globular clusters with our theoretical prediction (eq. (10)) for their lower boundary in the "c-R_{gc}" plane (Fig. 1). Since there is uncertainty in the value of the disk scale length (van der Kruit 1987), the values of c_{sh} are calculated for the two different scales: $h = 3.5$kpc and $h = 5.0$kpc .

CONCLUSION

Theoretical and empirical relations between the dynamical parameters of Galactic globular clusters allow us to understand the lower boundary of their distribution in the concentration versus galactocentric distance plane in terms of disk-shocking disruption. We have found an especially good fit between the calculated and observed lower boundary in the range 3kpc$\le R_{gc} \le$ 15kpc for

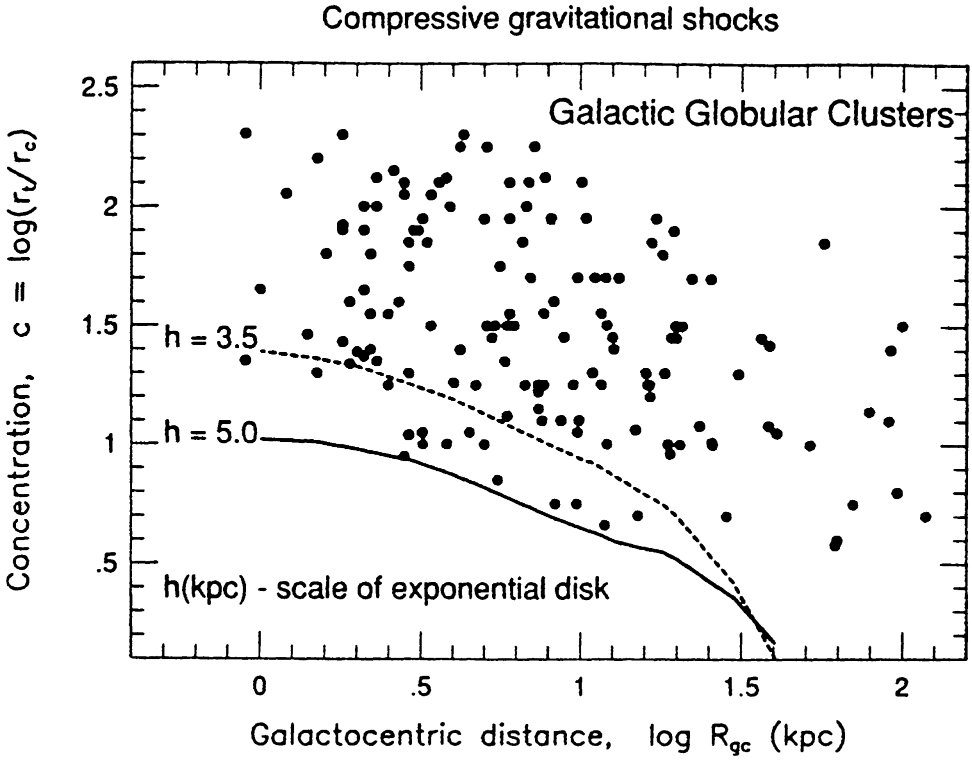

FIGURE 1. Theoretical prediction for the lower boundary of survival clusters (lines; $\beta = \alpha = 1$) in comparison with distribution of individual Galactic globular clusters (dots; data by Peterson and Reed 1987, Chernoff and Djorgovski 1989).

the longer disk scale length ($h = 5$kpc). Obviously the longer scale of a Galactic disk is more appropriate for the requirements of globular cluster dynamical evolution. In principle, an analysis of the globular cluster distribution in the "c-R_{gc}" plane can be a useful tool for determining the mass distribution in the disk of our (and not only our) Galaxy.

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