The history of the tropical year

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When we look at different books on fundamental astronomy, we are surprised to find that the definition of the tropical year varies from one author to the other.

For instance, Smart[1] writes that 'the tropical year is defined to be the interval between two successive passages of the Sun through the vernal equinox'. Bouasse[2] distinguishes two types of tropical years, the true and the mean one, and he gives the following definitions: 'the true tropical year is the time interval between two successive passages of the Sun through the vernal equinox', and 'the mean tropical year is the mean of a large number of true tropical years'.

Andoyer[3] writes that 'the tropical year at epoch t is the time needed for the secular part of the Sun's longitude or, what amounts to the same thing, of its right ascension, to increase by 2π [this is 360°]'.

Finally, according to Danjon[4] the tropical year is the time needed for the Sun's mean longitude to increase by 360°. This is the definition which is adopted nowadays.

As we shall see, the definition of the tropical year did change in the course of the centuries, and presently it has nothing to do with the beginning of astronomical spring.

Antiquity

The oldest references to the Greek word τροπή [turn, solstice] go back to Hesiod and to Homer (9th century BC?). It is evident that from the earliest times the Chinese, the Hindus, the Greeks, and others did measure more or less approximately the length of the tropical year, also called the seasonal year. According to Delambre[5] this year is so called 'because the first astronomers did deduce it from the return of the Sun to the same tropic'.

However, many sources which reached us remain more or less vague, and one should be cautious when stating that this or that people knew accurately the length of the year.

On the other hand, in the earliest times one did not make a distinction between the tropical and the sidereal year. It was generally known that the length of the year was sensibly 365 days 6 hours.

It is only since Ptolemy that we can examine the methods used for the precise determination of the length of the tropical year. Ptolemy, indeed, did report in his *Almagest* (*Mathèmatikè Suntaxis*) the works of Hipparchus, who had lived three centuries earlier and had written a (now lost) book entitled *On the Length of the Year*.[6]

The work of Hipparchus constitutes a decisive turning in the concept itself of the definition of the year. Hipparchus used, as principal instrument, meridian armillae for the determination of the times of the equinoxes and the solstices. He compared his own observations with those of earlier astronomers, in particular Euctemon and Meton (5th century BC) and Aristarchus (310–230).

He was, however, confronted with several difficulties. First, of course, was the trust he should give to ancient observations. Secondly, he suspected his own solsticial observations to be somewhat uncertain, because the variation of the Sun's declination near these times is practically zero. The use of the armillae thus appeared to be difficult when the declination of the Sun reached a maximum. For this reason, Hipparchus applied himself more to the equinoxes.

According to Ptolemy, Hipparchus wrote: 'I composed a book about the length of the year, in which I show that this length is the time required for the Sun to travel from a tropic to this same tropic again, or from an equinox to that same equinox, and that it is equal to 365.25 days minus 1/300 of a day-and-night, and not to a fourth of a day as the mathematicians believed'.[7]

In modern notations this means that, according to Hipparchus, the length of the year was 365 days 05 hours 55 minutes 12 seconds. Ptolemy, almost three centuries after Hipparchus, tried to improve that approximation of 1/300 day, but he was unsuccessful. According to the modern planetary theory VSOP 87 (see later in this article), at the time of Hipparchus, the second century BC, the length of the tropical year was 365 days 05 hours 48 minutes 56 seconds. However, the unit of time here is the so-called ephemeris day, which is practically equal to the mean solar day at the epoch 1900. At the time of Hipparchus, the Earth's rotation was very slightly more rapid, so there were approximately 13 more 'instantaneous' mean seconds than ephemeris seconds in the tropical year.

Thanks to his discovery of precession, Hipparchus, for the first time, made the distinction between the sidereal year and the tropical year. He evaluated that the equinoctial points moved over a distance of one degree per century along the ecliptic, in a direction opposite to that of the annual revolution of the Sun. This value of 1° per century has been retained for a long time. One had to wait till the 9th century before Arabian astronomers did change that value according to their observations.

From the known values of the precession rate and the length of the tropical year, it was possible to deduce the length of the sidereal year, and vice versa. Hipparchus found the length of the sidereal year to be 365 days 06 hours 10 minutes, that is, a difference of 14 minutes 48 seconds with respect to the tropical year due to the precession. However, after comparing the length of the tropical year deduced from a comparison of his observations with those of the astronomers of the fifth and third centuries BC, with the length he found from his own equinox observations over a time span of 30 years, Hipparchus doubted that the seasonal year had a constant length. Later, Copernicus and Kepler had the same doubt. Ptolemy, however, contrary to Hipparchus, thought the length of the tropical year to be invariable.

So, from the works of Hipparchus and Ptolemy, it appears that the year was based on the time interval between two equinoxes (or two solstices) taken sufficiently distant from each other.

What was the advantage of comparing observations over long time intervals? For Hipparchus and Ptolemy it allowed them to reduce observational errors, or the uncertainties, by spreading them over a large number of years. Moreover – but Hipparchus was unaware of this – it also allowed them to average the periodic terms and, so, to decrease their effect (nutation and planetary perturbations).

In fact, the tropical year is not constant. However, to discover the very slow variation, one needs very accurate observations, or an elaborate theory of the motion of the Sun.

Here it is necessary to give a precision which is often overlooked. Many authors define the tropical year as the time interval between two successive spring equinoxes. However, it is easy to realize that such an interval (for example, from the March equinox of 1990 to that of 1991) is not equal to the tropical year, and that it is not constant. This results from the fact that the instant of the equinox is connected to the vernal point, which is subject to periodic variations due to the nutation, and to the Sun which undergoes planetary perturbations.

The following examples illustrate the differences between the tropical year and the time interval between successive March equinoxes:

Interval between equinoxes

1985–1986	365d	05h	48m	58s
1986–1987	365	05	49	15
1987–1988	365	05	46	38
1988–1989	365	05	49	42
1989–1990	365	05	51	06

Middle Ages and Renaissance

From the Middle Ages to the Renaissance, the most evident interest for the measure of the length of the tropical year is its direct relation with the calendar. The slow drift of the date of the March equinox, which the astronomers observed until the Gregorian calendar reform, was the principal factor for the quest of a more accurate value of the seasonal year. But this period of the history of astronomy was also decisive as for the approach of the problem.

In 1252 the famous Alfonsine Tables were published. These tables pass for being the work of king Alfonso X of Castile (1223–1284), of whom Delambre said he had an unhappy reign. (It is said that Alfonso X said, 'if God had consulted him at the time of creation, he would have given Him good advice').

Alfonso X of Castile had conceived the plan to reform the tables of Ptolemy, which were in disagreement with the observations. The Alfonsine Tables, in use from AD 1315 and printed under a pompous name at Venice in 1483, are in fact perhaps even not Spanish. It was two Frenchmen, Pierre de St Cloud and Jean de Murs,[8] who constructed them in Paris at the beginning of the 14th century and who attributed the work to Alfonso X. The Tables were periodically brought up to date; many editions exist, the last one being that of the CNRS (Centre National de la Recherche Scientifique, in France), published in 1978 by E. Poule. The Tables actually use the theories of Ptolemy described in the *Almagest*, and their success (principally among the astrologers) is due to the simplicity of their use.

The length of the tropical year, as given in the Alfonsine Tables, is 365 days 05 hours 49 minutes 16 seconds. The Alfonsine Tables were used for more than two centuries, and it was the length of the tropical year as derived from them that was used for calendar reform.

Copernicus, too, was interested in the measurement of the length of the tropical year. At Frauenburg he observed an equinox on the 18th Calendae of October [14 September] 1515. (The old Roman manner to reckon the days was still in use in the 16th century!) He then measured the lapse of time between his measurement and an observation made by Albategnius, a great authority of the Arabian world in astronomy (middle of the 9th century). The result obtained by Copernicus was remarkably accurate. The great astronomer was convinced that the seasonal year had no constant length, and he ascribed this variation to the motion of the vernal point. However, Copernicus resumed the idea that one should deduce the length of the tropical year not from observations of the equinoxes but from the sidereal year and the rate of precession. For this purpose, between 1515 and 1525 he made observations in order to improve the constant of the precession.

In spite of the accuracy of his measurements, nobody thought of using them for the calendar reform. This was essentially due to Copernicus' heliocentric ideas, which, as we know, the Church condemned.

In 1551, Erasmus Reinhold (born at Saalfeld in 1511) published the *Tabulae Prutenicae*, or 'Prussian Tables', which were calculated from the theories of Copernicus. These Tables contributed to the diffusion of the heliocentrism, although it seems that Reinhold himself did not believe the ideas of Copernicus! In fact, Reinhold improved the numerical values of the theories of Copernicus; for the length of the year he adopted the value of 365 days 05 hours 55 minutes 58 seconds.

Finally, in 1627 Kepler published the famous *Rudolphine Tables*, the result of a labour of more than 25 years. These Tables, which included the results of Tycho Brahe, surpassed in accuracy all that had been in use so far. Just as Copernicus had, Kepler thought that the length of the year was variable and that it should not be determined with respect to the equinoctial points. By a comparison of his own observations with those of the astronomer Waltherus, he obtained the value of 365 days 05 hours 48 minutes 45 seconds.

The 18th and 19th centuries

In the course of the next centuries, almost every astronomer performed his own determination of the length of the tropical year: Flamsteed, Newton, La Caille, Maskelyne, von Zach, and others. For example, Lalande in his *Astronomie* reports how Cassini found several values of the tropical year by comparing his observations of equinoxes with those of ancient astronomers:[9]

with Hipparchus:	365d	05h	48m	49s
with Albategnius:	365	05	48	49
with Copernicus:	365	05	48	43
with Tycho Brahe:	365	05	48	47

Lalande himself found 365 days 05 hours 48 minutes 45.5 seconds. It should be noted that, with Lalande, one begins to take an interest in the precautions to be taken when one compares two equinoxes, precautions already mentioned by Copernicus who distinguished four of them. Lalande mentioned three corrections: the motion of the Earth's perihelion, which according to him amounts to 65.5 arcseconds per year with respect to the mean equinox of the date (the modern value, for the epoch 2000, is 61″.9 per year); the secular increase of the rate of precession; and the periodic actions of the Moon, Venus, and Jupiter on the motion of the Earth and, hence, on the longitude of the Sun.

Delambre gave another method, which he found 'more sure and easier'. He proceeded from the motion of the Sun in longitude (according to his tables) in 36525 days and found a year of 365 days 05 hours 48 minutes 51.6 seconds.

Until the beginning of the 19th century, the comparison of dates of equinoxes was practically the only way to find the length of the tropical year. It was then almost impossible to discover whether this year was constant or variable in the course of the centuries. To settle the matter, it would have been necessary to dispose of extremely accurate measures made at the time of Hipparchus, in order to be able to detect the extremely small decrease of 0.5 seconds per century.

The definitive progress was achieved in the 18th century thanks to specialists in celestial mechanics such as Laplace and Lagrange. Indeed, when one succeeds in expressing the mean longitude of the Sun as a function of the form

$$L_0 = A_0 + A_1T + A_2T^2,$$

where T is the time in Julian centuries, it is sufficient to take the derivative of L_0 with respect to the time and then to inverse the result (dT/dL_0) in order to obtain the expression of the length of the tropical year with a term in T.

For instance, Leverrier gives for the length of the tropical year

$$365.242\ 196\ 47 - 0.000\ 006\ 24\ T \text{ days}$$

while Newcomb's well-known expression, derived from his solar theory, is

$$365^d\ 05^h\ 48^m\ 46^s.0 - 0^s.530\ T$$

In these two expressions, T is the time in Julian centuries of 36525 days measured from 1900 January 0.5 Ephemeris Time.

Present astronomical considerations

It should be noted that the tropical year is *not* equal to the (mean) time interval between two successive spring equinoxes. In present-day astronomy, the tropical year is defined as the time interval needed for the mean tropical longitude of the Sun to increase by 360 degrees. Tropical longitudes are measured from the vernal equinox, this point being in motion due to the precession of the Earth's rotational axis. Because presently the precessional rate is slowly increasing, the length of the tropical year is *decreasing*. Presently, this decrease amounts to 0.53 second per century. At the start of the Christian era, the length of the tropical year was 365.24231 days. In AD 2000, the length will be 365.24219 days.

The (mean) time interval between two successive spring equinoxes is not, rigorously speaking, equal to the tropical year, for the following reason. The vernal equinox slowly regresses along the ecliptic (precession). When, starting from the vernal point, the Sun has described its annual path along the ecliptic and returns to the vernal point, it has not made a complete circuit with respect to the stars. Hence, it has made a complete circuit less a 'small bit'. This small arc is described with a somewhat different speed, according to its position with respect to the perihelion of the Earth's orbit. Depending on the starting point adopted for the 'year', one complete circuit in tropical longitude will have a somewhat variable length.

Near AD 2000, the mean time interval between two successive March equinoxes is 365.242 374 days; this is a little longer than the tropical year. On the other hand, the mean time interval between two successive June solstices is a little shorter than the tropical year. Besides, these lengths vary in the course of the centuries, by reason of the motion of the line of apsides (perihelion–aphelion) of the Earth's orbit with respect to the vernal point. These durations, for the years 0 and AD 2000, are as follows (these values have been deduced by author J.M. from the planetary theory VSOP 87 of P. Bretagnon and G. Francou, of the Bureau des Longitudes of Paris):

	Mean time interval (days)	
	Year 0	Year 2000
Between two March equinoxes	365.242 137	365.242 374
Between two June solstices	365.241 726	365.241 626
Between two September equinoxes	365.242.496	365.242 018
Between two December solstices	365.242 883	365.242 740

Again according to the VSOP 87 theory, the length of the tropical year, in days, is given by

$$365.242\ 189\ 623 - 0.000\ 061\ 522\ T$$
$$- 0.000\ 000\ 0609\ T^2 + 0.000\ 000\ 265\ 25\ T^3$$

where T is measured in Julian *millennia* (of 365 250 days) from the epoch J 2000.0.

The variation of the length of the tropical year is as follows:

year −1000	−0.469 second/century
0	−0.503
+1000	−0.524
+2000	−0.532
+3000	−0.526
+4000	−0.505

Finally, it is important to note that, in what precedes, we understand by 'day' the mean solar day at the epoch 1900, which is also known as 'ephemeris day'. However, the Earth's rotation is gradually slowing down. Consequently, there will be less and less 'instantaneous' mean solar days in a year. In the far future, there will be only 365 days exactly in a year, and still later only 364 days, and so on. But this is another story . . .

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