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MULTIPLICITY AMONG M DWARFS

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ABSTRACT

We examine surveys of M dwarfs within 20 pc to determine the incidence of stellar companions. Observational data are drawn from high-quality surveys, including IR-array imaging, precise velocities, IR speckle interferometry, and visual imaging, and their respective incompleteness is determined. Each technique permits detection of companions down to the H-burning limit, and each is nearly complete (owing to the proximity of M dwarfs) within a specific range of separation, the farthest being $\sim 10^4$ AU. The number of companions per primary per AU of semimajor axis, dN/da, is computed and found to decline monotonically toward larger semimajor axes. The period distribution (in bins of $\Delta \log P$) exhibits a unimodal, broad maximum with a peak in the range $P = 9$–220 yr, corresponding to separations 3–30 AU, similar to that for G dwarf binaries. Integrating over all semimajor axes yields the average number of companions per primary, 0.55 ± 0.09, which includes those companions clumped in multiple systems. We determine the distribution of multiplicities (binary, triple, etc.) both empirically and hypothetically. The resulting multiplicity of M dwarf primaries is single:double:triple:quadruple = 58:33:7:1 for 100 primaries. *Thus, 58% of nearby M dwarf primaries are single and 42% ± 9% have companions*, similar to Henry & McCarthy's binary frequency of 34%. The M-dwarf binary frequency is lower than that for G dwarfs ($\sim$57%), owing partly to the smaller range of companion masses included, i.e., companions less massive than the M- or G-type primary. The mass function of companions to M dwarfs is roughly flat in shape, similar to the field mass function. Companions to G dwarfs also exhibit the field mass function, which provides support for protostellar "capture" as the dominant mechanism by which binaries form.

Subject headings: binaries: general — binaries: spectroscopic — stars: late-type — stars: statistics

1. INTRODUCTION

The frequency with which stars are found in binary and multiple systems bears upon many astrophysical issues, including processes of star formation, dynamical evolution of stellar systems, evolutionary mass transfer, interpretation of stellar luminosity functions, and stability of disks and planetary systems. The most robust determinations of binary frequency have come from studies of solar neighborhood stars for which a variety of high-quality data permit detection of low-mass companions even at small separation. Abt & Levy (1976, hereafter AL) examined 135 solar-type stars for spectroscopic binaries above a threshold of 2 km s^{-1}. They included visual binaries and common proper motion (CPM) pairs in order to determine the total number of multiple systems in their sample. The data analysis by AL has been reconsidered (Morbey & Griffin 1987; Branch 1976), and some of the suspected binaries have since been deleted (Abt 1987). However, a revision in the incompleteness factors by Abt leaves the original conclusions unchanged. AL found that 58% of the G dwarf primaries had at least one H-burning companion. Corrections for observational incompleteness led them to conclude that over 72% of the systems in their sample were binary or multiple star systems. In another study of solar-type stars, Duquennoy & Mayor (1991, hereafter DM), using CORAVEL, followed 164 primary G dwarf stars in the solar neighborhood. After correcting for observational incompleteness, they found that 57% of these stars had companions with a mass ratio $M_2/M_1 > 0.1$, i.e., H-burning.

Determinations of binary frequency for other spectral types are more difficult, owing to either their distance or their faintness. It is unknown whether duplicity is correlated with stellar mass; also unclear is whether the distribution of orbital parameters depends on the mass of the primary or only on the mass ratio (Bodenheimer, Ruzmaikina, & Mathieu 1991). Here we examine the extant information on M dwarf binaries. Although nearby M dwarfs are not as bright as solar-type stars, they numerically dominate other spectral types, and their proximity offers significant advantages. First, it permits the resolution of visual components which may go unresolved at distances typical of the G dwarfs. Second, CPM pairs are more easily detected among nearby stars. Third, M dwarfs have been extensively surveyed for astrometric perturbations caused by unseen companions (Lippincott 1978; van de Kamp 1986). Finally, there have been recent improvements in observational techniques, including precise radial velocity measurements (Marcy, Lindsay, & Wilson 1987; Marcy & Benitz 1989, hereafter MB), the infrared array (Skrutskie, Forest, & Shure 1989, hereafter SFS), and infrared speckle interferometry (Henry & McCarthy 1990, hereafter HM) which have targeted M dwarfs to search for brown dwarf companions. In addition, Henry (1991) has significantly improved the census of all stellar objects within 8 pc. HM found the multiplicity of systems in their sample to be single:double:triple = 19:8:2, thus yielding the remarkable result that only about one-third of the nearest and best-studied M dwarfs have known companions.

Here the original published samples from the radial velocity work (MB), IR speckle interferometry (HM), and the IR array project (SFS) will be examined in detail. In addition, an independent sample of M dwarfs drawn from the Gliese catalog will be used to evaluate the visual binaries and CPM pairs. For

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each of these observational techniques, a range of detectable separation will be defined in which the technique is most robust, and the detection incompleteness in that range will be evaluated. This will allow us to determine the occurrence of companions per AU in each range of detectable separations.

2. OBSERVATIONAL APPROACH

For the purposes of this paper, we consider an M dwarf "system" to be either a single M dwarf or a collection of two or more stars that are gravitationally bound *with an M dwarf primary*. M dwarfs that are companions to more massive primaries are excluded. The binary frequency, as usual, is the ratio of the number of binary systems to the total number of systems. However, we will consider systems having any number of companions and will designate those having more than one companion as "multiple."

There are two types of incompleteness which plague estimates of multiplicity. The first is due to the limits of detectable separation and companion brightness inherent in a particular technique. An observational technique sensitive to a given range of separations cannot be reliably extrapolated to provide the frequency with which companions occur at other separations. To minimize this type of incompleteness, we assemble several previously published samples observed by different techniques. For each sample we determine the companion frequency only in the range of separations where the observational technique is robust. When taken together, the observational techniques span all component separations from 0 to 10,000 AU. Because the luminosity limits for these techniques are generally substellar, essentially all H-burning components are detected within the range of separation addressed by that technique. The second type of incompletness is due either to the chance alignment of a companion within the unresolved image of a primary star or to an extreme value for an orbital angle, such as the inclination. In either case, such incompleteness can be determined by proper modeling of hypothetical binaries having the full distribution of possible orbital parameters. In §§ 3–6, we consider four different techniques used to detect binaries and determine the incompleteness for each.

3. PRECISE RADIAL VELOCITY MEASUREMENTS

In this section we examine a sample of M dwarfs for which radial velocity monitoring has provided robust detectability of companions within the inner few AU of each primary. Multiple precise velocity measurements (MB) have been acquired for a sample of 72 M dwarfs in the solar neighborhood in order to detect extremely low-amplitude single-line spectroscopic binaries. The M dwarfs were selected from the list of Joy & Abt (1974) that met the following criteria: spectral type later than M2, $V < 11.5$ mag, $-10° < +50°$, and an absence of companions within 10″. Typically, about 15 observations, spanning a 4 year interval, were obtained per star. The precision of each velocity measurement was typically 230 m s^{-1}, allowing detection of substellar companions down to 0.01 $M_\odot$. This project confirmed the presence of the 0.08 $M_\odot$ astrometric companion to GL 623 (Marcy & Moore 1989) and the spectroscopic companion to GL 570B (MB). Table 1 includes the known binary and multiple systems in the radial velocity sample (as well as those from other samples examined), along with relevant stellar data.

For binaries having well-determined orbits, the masses in Table 1 were obtained dynamically. For the remaining cases, photometric masses were determined. Calibrations of stellar mass as a function of absolute K-magnitudes have been constructed by Marcy & Moore (1989) and HM. Both empirical calibrations apply to the mass range from 0.1 to 1 $M_\odot$ and are based on nearby stars with high-precision stellar masses (i.e., uncertainty ~20%). The two relations agree to within 5% in mass. The photometric stellar masses listed in this paper are an average of the values obtained from these two relations. The values for the K-magnitudes were taken from Stauffer & Hartmann (1986), Liebert & Probst (1987), or Veeder (1974). The K-magnitudes have typical quoted uncertainties of ± 0.05 mag.

A superior set of photometric masses has recently been determined by Henry (1991) for many of the stars considered here. Those determinations are based on a new calibration using additional dynamical masses. A comparison of photometric mass determinations for individual stars common to both studies shows good agreement, typically within 10%. However, for some stars K photometry was unavailable, and for these cases we constructed an empirical relation between stellar mass and M_V. Only M dwarf binaries with well-known orbits and high-quality parallaxes were used in this calibration, yielding

$$\log (M/M_\odot) = -0.108 M_V + 0.681 \pm 0.053 . \quad (1)$$

This relation applies only to masses below about 0.5 $M_\odot$, and we estimate an uncertainty of ~30% for the masses so derived.

3.1. *Incompleteness and Analysis of Radial Velocity Measurements*

The detectability limits for the radial velocity measurements are a function of both secondary mass and component separation. For each mass and separation, the detailed orbital elements (most notably, inclination) may inhibit detection. Isoprobability curves were computed to find the (largest) limiting separation that corresponded to a 90% probability of detecting a 0.08 $M_\odot$ companion mass (i.e., H-burning) as a function of a_{rel}. At each a_{rel}, radial velocity curves were simulated with incremented combinations of e, i, ω, and M_2, and Gaussian noise was added to the curve in a Monte Carlo fashion to represent the actual experimental uncertainty. The simulated velocity variations were required to be detectable at the 95% confidence level, as defined by the F-ratio test (cf. MB). The velocity curves spanned 4 years, mimicking the interval of the radial velocity project. If the generated orbit had a period longer than 4 years, only a portion of the hypothetical velocity curve would have been observed. In that case, the curve was examined for many assumed starting phases, and at least 90% of these phases were required to show detectable velocity variation. Thousands of sets of orbital elements with minimum detectable companion masses were generated. A weighted average for the minimum detectable companion mass at each a_{rel} was then calculated as follows:

$$\langle M_2(a_{rel}) \rangle = \sum_{e=0}^{0.9} \sum_{i=0}^{\pi/2} \sum_{\omega=0}^{2\pi} M_2(a_{rel}, e, i, \omega) \Delta p_\omega \Delta p_i \Delta p_e . \quad (2)$$

Here the weights for M_2 are the normalized probabilities of observing the incremental set of orbital elements. Consequently, $\Delta p_i \equiv \sin i \, \Delta i$, and $\Delta p_\omega \equiv \Delta\omega / 2\pi$. The normalized eccentricity distribution for G dwarfs found by DM was initially used to obtain Δp_e. To test the sensitivity of our result to the eccentricity distribution, additional runs were made using both a fictitious, flat distribution and a distribution heavily weighted with large values of eccentricity (also fictitious). The same values for

TABLE 1

COLLECTED MULTIPLE SYSTEMS

Gliese Number	Number of Objects	Parallax (mas)	a_{rel} (AU)	Period (yrs)	M_1 ($M_\odot$)	M_2 ($M_\odot$)	M_2/M_1	References	Notes
15AB	2	290	140	2600	0.33	0.14	0.42	1	
65AB	2	367	5.2	26.5	0.10	0.10	1.00	2	
150.1AB	2	052	2400	115000	0.57	0.46	0.81		1, 2, 3
195AB	2	074	40	300	0.53	0.19	0.36		1, 2, 3
228AB	2	104	16	90	0.29	0.20	0.67		1, 2, 3
234AB	2	252	3.8	16.6	0.19	0.11	0.57	3, 4	
277AB	2	088	550	16000	0.39	0.27	0.71		1, 2, 3
400AB	2	082	14.5	60	0.57	0.26	0.46	1, 2, 3, 4	
412AB	2	186	190	3820	0.37	0.10	0.27	1, 2, 3, 4	
473AB	2	233	3.1	16.2	0.14	0.12	0.86	4, 5	
537AB	2	089	44	300	0.44	0.43	0.98		1, 2, 3
568AB	2	096	6	20	0.27	0.21	0.78		1, 2, 4
569AB	2	096	52	625	0.41	0.09	0.22	6	2, 3
589AB	2	093	240	6200	0.25	0.11	0.44		1, 2, 4
623AB	2	132	2.1	3.73	0.32	0.08	0.25	4, 7	
644AB	5	161	1.4	1.7	0.33	0.31	0.94	4, 8	
644AC	...	161	1730	80000	0.33	0.09	0.22	4	
661AB	2	158	4.5	13	0.32	0.29	0.91	9	
669AB	2	095	210	4370	0.30	0.20	0.67		1, 2, 4
720AB	2	068	2075	110000	0.53	0.22	0.42		1, 2, 3
725AB	2	282	49	408	0.29	0.22	0.75	10	3
735AB	2	093	0.07	0.027	0.32	0.30	0.95	11	
745AB	2	113	1300	62000	0.27	0.27	1.00		1, 2, 4
752AB	2	171	545	18000	0.42	0.08	0.19	4	
799AB	3	113	38	300	0.33	0.32	0.97		1, 2, 4
815AB	3	063	12	29	0.42	0.30	0.72	11	
815AC	...	063	...	3.3[a]	0.42	0.27	0.64	11	
829AB	2	150	~1	~1	0.27	0.27	1.00	12	3
831AB	2	134	~1	1.9	0.22	0.12	0.55	4, 13	
852AB	2	104	110	2100	0.16	0.14	0.88		1, 2, 4
860AB	2	253	9.4	45	0.25	0.15	0.64	4, 14	
866AB	2	290	1.2	2.2	0.15	0.12	0.85	4, 15	
867AB	3	109	175	2750	0.49	0.27	0.55		1, 2, 4
896AB	2	155	44	360	0.32	0.21	0.65	4, 8	
1116AB	2	192	30	360	0.12	0.11	0.91	4	1, 2
1245AB	3	211	48	675	0.14	0.13	0.93	4	1
1245AC	...	211	1.4	15	0.14	0.10	0.70	4, 16	

NOTES.—(1) Observed separation converted to $\langle a_{rel} \rangle$ using eq. (7). (2) Approximate period calculated with Kepler's law from mass and separation. (3) Photometric mass using M_K relation. (4) Photometric mass using M_V relation.

[a] Period in days.

REFERENCES.—(1) Lippincott 1972; (2) Geyer, Harrington, & Worley 1988; (3) Probst 1977; (4) Henry 1991; (5) Heintz 1989; (6) Forrest, Skrutskie, & Shure 1988; (7) Marcy & Moore 1989; (8) Heintz 1984; (9) Heintz & Borgmann 1984; (10) Heintz 1987; (11) Duquennoy & Mayor 1988; (12) Marcy, Lindsay, & Wilson 1987; (13) Lippincott 1979; (14) Heintz 1986; (15) Leinert et al. 1990; (16) McCarthy et al. 1988.

$\langle M_2 \rangle$ were obtained, showing that although a certain eccentricity might facilitate or hinder the detection of a companion in an individual case, the effect is canceled out statistically.

In summary, we have established the thresholds in such a way that 90% of the hypothetical velocity curves would have exhibited variations at the 95% confidence level. Thus, the actual probability of detection is $0.90 \times 0.95 = 0.86$. We thank T. Henry for pointing this out. The 86% probability curve showing the minimum detectable $\langle M_2 \rangle$ for each a_{rel} with the radial velocity technique is plotted in Figure 1. This curve shows that 86% of 0.08 $M_\odot$ companions with an a_{rel} of 4 AU could be detected with radial velocity measurements. Companions at smaller separations or with greater mass will have a higher probability of being detected. However, our estimate of the completeness out to 4 AU is taken to be 86%, understood to be an underestimate, since many companions are significantly more massive than 0.08 $M_\odot$.

The selection criteria for the radial velocity project contain biases which affect a determination of the rate of duplicity. First, a magnitude-limited sample favors the inclusion of double-lined spectroscopic binaries (Branch 1976). The combined luminosity of equal-mass components is brighter by 0.75 mag than that of the individual components. To correct for this effect, three SB2's (GL 206, GL 268, GL 289) were thrown out because the luminosity of the individual components did not meet the selection criteria.

Of greater concern is a second bias: stars with known companions closer than 10″ were excluded from the MB sample. This criterion was imposed to eliminate stars which would exhibit velocity variations due to known companions. This bias produces an anomalously low number of visual binary and multiple systems in the sample. A search turned up 10 systems which were excluded on this basis. Also, another three CPM systems which met all the selection criteria were over-

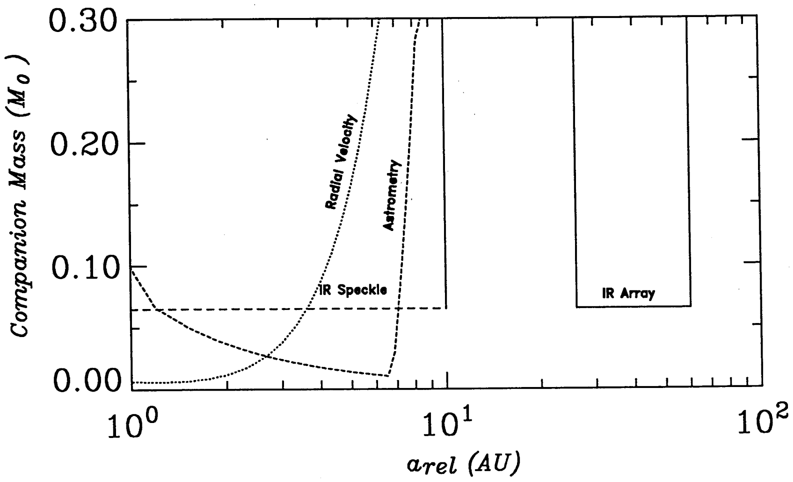

FIG. 1.—Detectable separation and companion mass ranges for the observational techniques examined in this work. Not included are the visual observations, which are effective between 10 and 10,000 AU.

looked when the sample was constructed by MB. Three of these 13 additional systems (GL 250B, GL 527B, GL 695BC) have primaries that are not M dwarfs and may therefore be excluded from consideration here. This leaves 10 systems for which we would like information regarding the absence or presence of companions within 4 AU.

One of the 10 systems (GL 644) has a known companion within the detectable 4 AU separation range. This system will be added to the present velocity sample, since its companion would have been detected had it been observed by MB. Two other systems are CPM systems (GL 150.1, GL 277) with separations over 1000 AU. We will assume that the presence of a widely separated companion does not affect the possible existence of an addtional component in the observable 0–4 AU range. Therefore, the probability of observing a close companion in these two systems is assumed to be the same as for the rest of the sample, and excluding them will have no net statistical impact. The remaining seven systems (GL 195, GL 228, GL 400, GL 537, GL 661, GL 815, GL 896) are more problematic. These systems have known companions with moderate separations (between 4 and 40 AU). Based on dynamical stability arguments for multiple systems, the presence of an additional close stellar companion (between 0 and 4 AU) may be unfavorable (Bailyn 1987). The resulting bias in these seven stars is unknown. However, GL 1245AB (separation = 48 AU) was recently found to have an additional component (GL 1245C) separated by 1.4 AU from component A (HM). We therefore adopt the assumption that the frequency with which close companions (separations of 0–4 AU) would occur is only weakly dependent on the presence of companions having wider separation. Thus, these seven stars will have close companions with a frequency representative of the well-observed larger sample, and we exclude them from the present sample on that basis.

Five stars in the radial velocity sample (GL 179, GL 402, GL 486, GL 875.1, GL 905) had less than three observations. These stars are deleted here because the number of observations was too low to draw any conclusions regarding duplicity. Three additional stars, GL 107B, GL 570B, and GL 820A, are deleted from the original MB sample because the primary star is not an M dwarf.

3.2. *Results from the Radial Velocity Sample*

The final sample of stars contains 62 M dwarf primaries, all of which were observed for velocity variations by MB, except for GL 644, whose close companion was already known. We now wish to determine dN/da, the number of companions per AU per primary. As previously discussed, this velocity sample will only provide dN/da in the separation range 0–4 AU where the velocities are robust. We arbitrarily adopt two decade ranges in semimajor axis, namely, 0.04–0.4 AU and 0.4–4.0 AU, within which to determine dN/da. In the range 0.04–0.4 AU, one of the 62 sample stars, GL 735, has a known companion. Correcting for the 86% completeness, discussed above, yields $dN/da = 0.051$. In the range 0.4–4.0 AU, three of the 62 sample stars, namely, GL 623, GL 644, GL 829, have companions. After correcting for the 86% completeness, one finds $dN/da = 0.015 \pm 0.009$. The uncertainty is due to $(N_{obs})^{1/2}$

TABLE 2

SUMMARY OF RESULTS

Range of a_{rel} (AU)	Sample	Number of Primary Stars	Number of Companions in Range	Detection Probability (%)	dN/da (AU^{-1})	$dN/d \log P$
0.04–0.4	MB	62	1	86	0.051	0.013
0.4–4.0	MB	62	3	86	0.015	0.039
1.0–10.0	HM	31	6	95	0.0226	0.138
8.5–13.0	SFS	28	1	73	1.33×10^{-3}	0.091
10–100	Table 3	58	6	86	1.34×10^{-3}	0.080
10^2–10^3	Table 3	58	4	66	1.15×10^{-4}	0.069
10^3–10^4	Table 3	58	3	Uncertain	5.7×10^{-6}	0.003

(Poisson statistics), where N_{obs} is the number of detected binaries. The results of this analysis (and those of the following techniques) are distilled in Table 2.

4. INFRARED SPECKLE INTERFEROMETRY

We now consider a separate sample of M dwarfs, namely, those studied carefully via speckle interferometry for companions having separations from 1 to 10 AU. HM selected every known M dwarf within 5.2 pc and north of $-30°$ in a search for brown dwarf companions. Their infrared speckle interferometry search covered a region from 0″.2 to 5″ around 27 target stars and was sensitive to companions down to infrared limits of about $M_K = 11.5$ and $M_H = 12.0$. This corresponds to a mass limit extending to the H-burning threshold (~ 80 Jupiter masses) for the typical ages of stars in their sample. HM list five binary systems which were not observed for additional components because of the difficulty in extracting a second set of visibility fringes from one-dimensional speckle data. These binaries were (properly) included in their count of binary systems within 5.2 pc. HM detected new low-mass stellar companions to GL 1245A (G208−44) and GL 866. All known multiple systems in the IR speckle sample are contained in Table 1. One star, GL 166C, is an M dwarf component of a triple system with an earlier type primary; this star will be excluded for the purposes of this paper.

4.1. *Incompleteness and Analysis of IR Speckle Interferometry Technique*

Since the infrared speckle interferometry used by HM is sensitive to the substellar threshold, the detectability of bona fide stars is only limited by the separation of the components. The range of detectable projected angular separations corresponds to linear separations that depend on the distances to the individual stars, ranging from 1.8 to 5.2 pc. We therefore propose a restricted range of detectable separations, 1–10 AU, within which every HM target star (except GL 699) was fully examined. Because of its proximity, GL 699 would have been observed only out to 9 AU. Thus 99% of the sum of all detectable separation ranges were examined by HM.

Incompleteness between 1 and 10 AU is caused by the chance alignment of the companion within the 0″.2 diffraction image of the primary star. To estimate the incompleteness in HM's observations, thousands of orbits were simulated through loops of e, i, ω, Ω, and a_{rel}. The three-dimensional orbits were projected onto the plane of the sky, and the standard relationships for $r(\theta)$ and $t(\theta)$ were used:

$$r = \frac{a_{rel}(1 - e^2)}{1 + e \cos v} \quad (\text{where } v = 0 - \omega) , \tag{3}$$

$$\int dt = \frac{P}{2A} \int r^2 \, d\theta . \tag{4}$$

A spline interpolation was used to invert these relationships numerically from $t(\theta)$ to $r(t)$, and 40 position vectors at evenly spaced intervals of time were calculated for each orbit. A hypothetical companion with a particular set of orbital elements will be observable for a fraction of time found by

$$f(a, i, \omega, \Omega, e) = \frac{\text{number of projected separations between 0″.2 and 5″}}{\text{total number of separations calculated}} . \tag{5}$$

Not all orbits are observed with equal probability. The value for the observational incompleteness for a particular orbit must be weighted by the probability of observing that orbit. The observed number of binaries, $N_{obs}(a)$, is equal to the true number of binaries, $N_{true}(a)$, times the probability of observing a binary system:

$$N_{obs}(a) = N_{true}(a) \times \sum_{e=0}^{0.9} \sum_{\Omega=0}^{2\pi} \sum_{\omega=0}^{2\pi} \sum_{i=0}^{\pi/2} f(a, i, \omega, \Omega, e) \Delta p_i \Delta p_\omega \Delta p_\Omega \Delta p_e . \tag{6}$$

The Δ quantities give the normalized probabilities of observing each orbital element as described in § 3.1. The summation represents the total probability of observing a hypothetical companion as a function of a_{rel}. This probability was found to rise steeply from about 80% for the closest linear separations to over 99% for separations greater than 10 AU. Averaging over all values of a_{rel} yielded an overall probability of detection between 1 and 10 AU of 95%. Note that this averaging technique implicitly assumes that the companion rate is constant from 1 to 10 AU, since no weighting was applied to account for the as yet unknown distribution with a_{rel}. The small incompleteness of 5% for this technique can be credited to the high resolution of the primary image afforded by the speckle technique.

4.2. *Results from the IR Speckle Interferometry Sample*

This sample of 31 primaries contains six stars (GL 65AB, GL 234AB, GL 473AB, GL 860AB, GL 866AB, GL 1245AB) with companions between 1 and 10 AU. So, after correcting for the 5% incompleteness, in a sample of 100 primaries we expect a total of 20.4 to have companions with separations (a_{rel}) between 1 and 10 AU. This yields $dN/da = 0.0226 \pm 0.0065$ companions per AU per primary (Table 2).

5. IR ARRAY

SFS examined 55 stars in the solar neighborhood in the K band (2.2 μm) with an IR array to search for substellar companions. To minimize background contamination, the target stars were required to be at least 20° out of the Galactic plane. In addition, the primary stars were required to be fainter than $m_K = 6.5$ to reduce scattered light. Their sample consisted almost entirely of M dwarfs, and only these are used here. SFS detected one new object: a companion to GL 569 at a separation of 5″ (Forrest, Skrutskie, & Shure 1988; Henry & Kirkpatrick 1990).

5.1. *Incompleteness and Analysis of the IR Array*

The faint luminosity limit for the IR array corresponds to a 0.06 $M_\odot$ substellar object for distances and ages typical of the stars in this sample (cf. Burrows, Hubbard, & Lunine 1989; Stringfellow, Black & Bodenheimer 1991). This eliminates the incompleteness due to missed low-luminosity *stellar* companions. The spatial regime for detecting a stellar companion with this technique is dictated by the point-spread function (PSF) of the primary and by the size of the array, providing the detectable angular separation limits of 2″–7″. Because the distances to stars in this sample vary widely (from 1.8 to 14.7 pc), the observed ranges of linear separations are very different. The common range of observed linear separations for all of the stars in this sample is too small to be statistically useful. Therefore, we restrict our attention to a subset of the SFS sample stars that have similar parallax values. The limits for the parallax, chosen to maximize both the number of stars and the size of the observed linear separation range, were defined to be 0.″077 and 0.″118 (8.5–13.0 pc). The resulting separation range considered here is 26–60 AU, corresponding to the 2″ limit at 13.0 pc and to the 7″ limit at 8.5 pc.

The incompleteness in this technique, as with IR speckle interferometry, is due to the chance that the hypothetical companion will be aligned with the image of the primary at the time of observation. The observational incompleteness for the IR array was calculated by the same method described for IR speckle interferometry in § 4.1, namely, by Monte Carlo simulations of thousands of orbits spanning all orbital parameters. The larger image of the primary star with the IR array results in a larger fraction of unobservable space and a larger value for the incompleteness. The probability of detecting a hypothetical companion was again calculated as a function of a_{rel} for separations between 26 and 60 AU. The unweighted average of these probabilities for all values of a_{rel} yields a completeness of 73% for the IR array survey.

5.2. *Results from the IR Array Sample*

There are 28 separately imaged M dwarfs between 8.5 and 13.0 pc, and, of these, only GL 569 has a companion between 26 and 60 AU. After correcting for the observational completeness of 73%, one expects 4.5 companions with separations of 26–60 AU in a sample of 100 M dwarf primaries. Thus, $dN/da = 0.0013 \pm 0.0013$ companions per AU per 100 systems (Table 2). It is unfortunate that the number of detections is so small that Poisson errors cannot be meaningfully assigned. Nonetheless, this result places useful upper limits on the binary frequency in this separation range, and we will compare this small-number result to that obtained from visual binaries having slightly larger separations.

6. VISUAL BINARIES

In order to obtain information about visual binaries and CPM pairs, *an independent, complete sample of M dwarfs was established.* A set of stars from the Gliese catalog (Gliese 1969; Gliese & Jahreiss 1979) was selected according to the following criteria: spectral type later than M2 (according to Joy & Abt 1974), $V < 11.0$, and $-10° < \delta < +50°$. The selections were made without regard to the presence of companions of any sort, to yield a sample which is unbiased in visual binary frequency. Table 3 lists all 58 primaries of this independent sample along with their characteristics. As usual, only systems in which the primary was an M dwarf were included, i.e., M dwarfs as companions to more massive stars were rejected.

The CPM stars and many of the visual binaries are too widely separated to allow a determination of the orbital elements. However, the semimajor axis, a_{rel}, of such wide binaries can be estimated from the projected physical separation, derived from the parallax and observed angular separation, α. AL constructed a scheme to convert α to a_{rel} based on the assumption of circular, edge-on orbits, giving $a_{rel} = (\pi/2)d\alpha$, where d is the distance to the system. This scheme yielded values of a_{rel} which were statistically useful even though they were inaccurate for each individual systen.

We have refined this conversion scheme with a Monte Carlo simulation of visual binaries having all possible orbital parameters, to determine more accurately the statistical relation between α and a_{rel}. Thousands of elliptical orbits with incremented combinations of all orbital parameters were projected onto the plane of the sky, with randomly oriented inclinations assumed. For each orbit the time-averaged projected separation, $\langle\alpha\rangle$, was determined and assigned a weight based on the probability of observing that set of orbital elements. For example, the inclination distribution was assumed proportional to sin i, for random orientations. These weights permitted determination of a grand average, over all orbits, for the ratio of observed separation to the semimajor axis, $\langle\alpha\rangle/a_{rel}$, for visual binaries having full distributions of orbital parameters:

$$\langle a_{rel}\rangle = 1.26d\langle\alpha\rangle \ . \quad (7)$$

The conversion constant here, 1.26, is different from AL's $\pi/2$, owing to the inclusion of all possible orbital elements, most notably inclination. For wide binaries that have not executed an appreciable fraction of their orbit, this relation was used to give both statistically valid semimajor axes and the corresponding orbital periods from Kepler's law. When more than one value of the angular separation was available for a system, an average angular separation was used.

6.1. *Incompleteness in the Visual Observations*

AL derived substantial correction factors for missed visual companions to G dwarfs. The incompleteness is significantly less in a sample of M dwarfs, owing to their relative proximity. Historically, intense scrutiny has been applied to nearby (<20 pc) stars in order to determine their proper motions and parallaxes. Liebert & Probst (1987) review the observational methods applied to low-mass stars. They point out that the faint limiting magnitude of $m_R \approx 20$ (Luyten red plate survey) provided near-completeness for companions to the substellar regime.

Gliese, Jahreiss, & Upgren (1986) examined the space distribution of nearby stars, and found the number of red dwarfs closer than 13 pc and brighter than $M_V = 13.5$ to be nearly complete. Visual companions to these stars are observed when photographic plates are blinked or measured astrometrically to determine parallax and proper motions. Because of the large parallaxes of these nearby stars, a fainter companion would probably be identified even at $m_V = 16$, corresponding to 0.1 $M_\odot$ at 10 pc. Identification of such low-luminosity companions is significantly more difficult for G dwarf primaries, since the difference in brightness between the primary and secondary can be as much as 10 mag. Furthermore, the average distance to the nearest 100 G dwarfs is about twice that of the nearest M dwarfs because of their lower space density. Thus, G dwarfs do not permit the same level of scrutiny by photographic and astrometic studies, leaving a greater incompleteness in the visual detection of low-mass companions.

Detection probabilities were derived for visual binaries in the G dwarf sample of DM. They based their incompleteness upon the assumptions of AL and upon the method outlined in Halbwachs (1987). Specifically, for primaries fainter than $m_1 = 6.5$ (as in the case for all M dwarfs), their adopted detection probability was 100% for $\Delta m_V < 5$ and was 50% for $5 < \Delta m_V < 7$.

As usual, all known binary and multiple systems in the present M dwarf visual sample are listed in Table 1. To evaluate the incompleteness in the number of observed binary systems, we have binned the known binary systems according to the angular separation (α) and the magnitude difference of the components (Δm). This distribution is shown in Table 4.

In the first separation range $\alpha < 1''$ there is an apparent bias toward equal luminosity components. We prefer to rely upon the more robust methods of IR speckle interferometry and precise velocities to determine the occurrence of companions at

TABLE 3

VISUAL BINARY SAMPLE

Gliese Number	R.A. (1950)	Decl. (1950)	Spectral Type	m_V	Parallax (mas)	Distance (pc)	Notes
15A	0h15m31s	43°44′	M2.5	8.07	290	3.4	1, 2
70	1 40 46	4 04	M2	10.91	114	8.8	1, 2
87	2 09 51	3 22	M2.5	10.04	100	10.0	1, 2
104	2 33 03	20 00	M2	10.80	069	4.5	3, 4
109	2 41 18	25 19	M3.5	10.57	129	7.1	1, 2
150.1A	3 41 02	16 31	M2	10.00	052	9.2	3, 4
176	4 39 58	18 52	M2.5e	9.98	104	9.6	1, 2
195A	5 13 42	45 47	M2.5	10.11	076	13.2	1
205	5 28 55	−3 41	M3	7.97	170	5.9	2
228A	6 08 09	10 20	M2.5	10.40	168	9.6	1
251	6 51 35	33 20	M3.5	10.00	100	6.0	1, 2
272	7 19 37	46 11	M2	10.52	060	16.7	1, 4
273	7 24 43	5 22	M4	9.87	266	3.8	1, 2
277A	7 28 40	36 19	M3.5e	10.56	082	12.2	1
353	9 28 54	36 32	M2	10.20	066	15.2	1, 2
361	9 38 30	13 26	M2.5	10.36	076	13.2	1, 2
378	9 59 14	48 21	M2	10.07	064	15.6	1, 4
382	10 09 46	−3 29	M2	9.25	117	8.5	1, 4
388	10 16 54	20 07	M3.5e	9.37	206	4.9	1, 2
393	10 26 23	1 06	M2.5	9.64	130	7.7	1, 2
400A	10 42 30	38 46	M2	9.25	082	12.2	1
408	10 57 25	23 06	M3	10.02	151	6.6	1, 4
411	11 00 37	36 18	M2	7.50	397	2.5	2
412A	11 03 00	43 47	M2	8.77	186	5.4	2
436	11 39 31	26 59	M3.5	10.66	110	9.1	1, 2
459.3	12 16 56	28 39	M2	10.61	052	19.2	1, 2
461	12 17 52	0 51	M2	10.20	060	16.7	2
464	12 21 21	12 51	M2	10.37	050	20.0	1, 2
507.1	13 17 22	33 36	M2	10.60	052	19.2	1, 2
521	13 37 20	46 26	M2	10.23	092	10.9	1, 2
526	13 43 12	15 09	M3	8.50	192	5.2	2
537A	14 00 32	46 34	M2	9.85	089	11.2	
552	14 27 11	15 44	M2.5	10.66	062	16.2	1, 2
569	14 52 08	6 18	M2	10.20	096	10.4	2
581	15 16 50	−7 32	M4	10.55	153	6.5	1, 2
623	16 22 39	48 28	M3	10.26	132	7.6	1, 2
644A	16 52 48	−8 14	M3.5e	9.76	161	6.2	
649	16 56 07	25 49	M2	9.68	092	10.9	1, 2
661A	17 10 40	45 44	M4	9.96	158	6.4	
694	17 42 25	43 24	M3.5	10.48	092	10.9	1, 2
699	17 55 23	4 33	M4.5	9.54	545	1.8	2
701	18 02 28	−3 01	M2	9.38	136	7.4	2
720A	18 33 50	45 41	M2	9.82	068	14.7	2
735	18 53 03	8 20	M3e	10.10	088	11.4	1, 2
745A	19 04 58	20 48	M2	10.76	113	8.8	1, 2
745B	19 05 05	20 48	M2	10.75	113	8.8	1, 2
752A	19 14 29	5 05	M3	9.12	171	5.8	2
806	20 43 18	44 18	M3	10.77	085	11.8	1, 2
815A	20 59 09	39 52	M3e	10.26	063	15.9	5
829	21 27 12	17 25	M3.5	10.31	150	6.7	1, 2
844	21 59 24	16 13	M2	10.64	071	14.1	1, 4
849	22 07 00	−4 53	M3.5	10.42	112	8.9	2
851	22 09 05	18 10	M2.5	10.21	083	12.0	1, 2
863	22 30 31	9 07	M2	10.37	054	18.5	2
873	22 44 40	44 04	M4.5e	10.26	200	5.0	1, 2
880	22 54 10	16 17	M2.5	8.68	146	6.8	2
896A	23 29 20	19 39	M4e	10.38	155	6.5	
908	23 46 36	2 08	M2.5e	8.98	180	5.6	2

NOTES.—(1) V taken from Stauffer & Hartmann 1986. (2) Precise velocity measurements for this star (MB). (3) Apparent magnitude from Gliese not photoelectrically observed. (4) Spectral type taken from Gliese catalog. All others from Joy & Abt 1974. (5) Mariotti et al. (1990).

TABLE 4

ANGULAR SEPARATIONS AND Δm_V FOR VISUAL BINARY SAMPLE

α_{obs}	Δm_V (mag) <1	1–2	2–3	3–4	>4	TOTAL
<1″	2	1	1	0	0	4
1″ → 10″	1	1	1	1	0	4
10″ → 100″	0	1	1	0	2	4
>100″	2	0	0	1	0	3
Total	5	3	3	2	2	...

$\alpha < 1''$. We therefore adopt $\alpha > 1''$ as the threshold for detection of visual companions.

The second separation range, $1'' < \alpha < 10''$, is void of companions for $\Delta M_V > 4$, as shown in Table 4. Worley (1962) conducted a visual survey of 700 M dwarf stars (561 systems) to search for companions between 0\".2 and 15\". A tally of the binary systems in Worley's sample shows a similar paucity of low-luminosity companions. Worley's sample contains only two systems with $\Delta M_V > 3$, and no systems with $\Delta M_V > 4$. HM observed a separation range $0''.2 < \alpha < 5''$ and found the infrared luminosity function to be rising slightly until $M_K = 10.0$, corresponding to the H-burning limit of 80 Jupiter masses. They noted that this trend was especially noticeable for the multiple systems in their sample. To forcibly construct a flat luminosity function among our sample of visual binaries from 1″ to 10″, we would need to add one statistical companion, presumably missed because of its low luminosity.

The next separation range, $10'' < \alpha < 100''$, contains a fairly constant number of companions in each bin of Δm_V, i.e., the apparent luminosity function is nearly flat. In this separation range, Table 4 shows two companions having $\Delta m_V > 4$, a magnitude difference for which DM and AL suggest that completeness is only 50%. We adopt their correction factors for this separation range, resulting in the addition of two statistical companions in the computation of $dN/da(a_{rel})$. We are currently carrying out a CCD survey in $R-I$ to provide more information about companions in this range of separations.

We now consider the possibility of constructing another bin of separations for $100'' > \alpha > 1000''$. Because contamination from field stars becomes significant at wide separations, this range is less secure. In addition, it is more likely that faint companions having $\alpha > 100''$ are missed in surveys. Nonetheless, following DM, we have counted the binaries having $\alpha > 100''$, and applied a completeness correction identical to theirs. However, all three companions having $\alpha > 100''$ have $\Delta m_V < 4$, so no correction was applied, as dictated by DM. There is a serious danger that extremely low mass companions having $\alpha > 100''$ and $\Delta M_V > 4$ are missed in CPM surveys. However, we have no way to estimate the incompleteness in this region of parameter space, and there are no such binaries in our sample.

Finally, we check for incompleteness as a function of distance by segregating the 58 primaries into two groups according to whether they are closer or farther than 10 pc away. The concern is that companions of more distant primaries may suffer greater incompleteness. Closer than 10 pc, 7/32 = 22% of the primaries have known companions. Between 10 and 20 pc, 7/26 = 27% have companions. Thus, the observed rate of multiplicity is apparently independent of distance for companions detected visually. This consistency suggests that the incompleteness in the visual binaries is not a strong function of distance out to 20 pc.

6.2. *Results from the Visual Observations*

In the visually observed sample in Table 3, there are 58 primaries. To find the occurrence of companions, $dN/da(a_{rel})$, as a function of a_{rel}, the observed angular separations, α, are converted to a_{rel} from equation (7).

The angular separations $1'' < \alpha < 10''$ correspond to linear separations of 10–100 AU at the average distance of 10.0 pc for this sample. There are six known binary systems in this separation range, and, as discussed above, one statistical companion is included to correct for the incompleteness in the observations. The occurrence of companions is thus $dN/da = 1.34 \times 10^{-3} \pm 0.5 \times 10^{-3}$ AU^{-1} per primary in this separation range, with the uncertainty derived from Poisson statistics. In a sample of 100 systems, one would expect to find 12.1 systems with a_{rel} between 10 and 100 AU. The results for each separation range of the visual binary sample are summarized in Table 2.

In the separation range $10'' < \alpha < 100''$ (corresponding to 100–1000 AU) there are four known binary systems. After adding two statistical companions to correct for incompleteness, there are six systems in this range. The resulting occurrence of companions is $dN/da = 1.15 \times 10^{-4} \pm 0.5 \times 10^{-4}$ AU^{-1} per primary. In a sample of 100 systems, one would expect 10.3 systems to have components with separations between 100 and 1000 AU.

In the separation range $100'' < \alpha < 1000''$ (corresponding to 1000–10,000 AU at the average distance of 10 pc) there are three known binary systems. Following DM, no correction for completeness is appropriate here, as all three companions have $\Delta m_V < 4$. We suspect that past surveys have missed some components in this range, but no estimate of this incompleteness is possible. The resulting occurrence of companions is $dN/da = 5.7 \times 10^{-6} \pm 3 \times 10^{-6}$ AU^{-1} per primary.

7. THE MULTIPLICITY OF M DWARFS

We now integrate $dN/da(a_{rel})$ over a_{rel} to determine the average number of companions per primary, Φ_{tot}, at all separations. We carried out the integration two ways. First, we divided $dN/da(a_{rel})$ into 1 AU intervals. For some of these intervals, more than one value of dN/da was available from overlapping ranges. For example, the range 2–4 AU was observed with both high-precision velocities and IR speckle interferometry. For such cases, an average value of dN/da was adopted. The overlapping results all agree within the uncertainties. One region from 13 to 26 AU was not well covered by any observational technique; for this interval, dN/da was determined by a linear interpolation between the two neighboring values. Numerical integration of dN/da from 0 to 10^4 AU yields

$$\Phi_{tot} \equiv \sum_{i=0}^{10,000} (dN/da)_i = 0.55 \pm 0.09 . \quad (8)$$

It is conceptually helpful to normalize Φ_{tot} per 100 primaries instead of per primary. Then Φ_{tot} represents the total number of companions harbored on average by a random sample of 100 M dwarf primaries and has a value of 55 ± 9. The uncertainty was determined from the Poisson errors in each $dN/da(a_{rel})$ in Monte Carlo fashion.

Second, we carried out the integration by fitting a parabola to log (dN/da) versus log (a_{rel}), which yielded 58 companions per 100 primaries. The parabolic fit was

$$\log\left(\frac{dN}{da}\right) = -1.545 - 0.499(\log a) - 0.1333(\log a)^2 .$$

The discrepancy between the two integration approaches (55 versus 58) illustrates the uncertainty in this quantity owing to the scatter in the points themselves. We choose to adopt the former, more straightforward integration approach, which provides an uncertainty encompassing the two estimates: In a sample of 100 M dwarf primaries, we expect 55 ± 9 companions to exist out to 10^4 AU.

We emphasize that this value of 55 ± 9 represents the total number of companions per 100 primaries and not the number of primaries that have companions, since some systems will harbor more than one companion. For example, if all the companions were members of binary systems (not triple, quadruple, etc.), then only 45% of the primaries would be single. However, if triple and quadruple systems exist (which of course they do), then some of the 55 companions per 100 primaries would be clumped into these larger systems. Therefore, a calculation of the occurrence of single stars is dependent upon how the companions are partitioned among the binary, triple, quadruple (etc.) systems.

To evaluate this partitioning of the various multiple systems, we define

$$f_i \equiv \text{the fraction of primaries having } i \text{ companions} .$$

The total number of stellar companions per 100 primaries, Φ_{tot}, previously found to be 55 ± 9, is composed as follows:

$$\Phi_{tot} = \sum_{i=1}^{\infty} i f_i \times 100 = 55 \pm 9 . \qquad (9)$$

We first estimate the values of f_i empirically from the multiple systems used in this paper, all of which are listed in Table 1. We discuss the demerits of this sample of *M* dwarf multiples below. Of the 33 multiple systems, 27 are binary, four are triple, none are quadruple, and one is a quintuple system. These provide the relative magnitudes of the f_i, which, when coupled with the constraint in equation (9) permits the calculation of each f_i individually. The result is $f_0 = 0.55$, $f_1 = 0.38$, $f_2 = 0.056$, $f_3 = 0.0$, $f_4 = 0.0141$, $f_5 = 0.0$. The value of $f_0 = 0.55$, which is the fraction of primaries that are single, follows directly from the constraint that the sum of all f_i must be unity. Thus, the empirically determined partitioning of multiple systems yields the result that 55% of all M dwarfs are single and 45% have at least one companion.

The possible demerits in this empirical analysis of f_i arise from incompleteness. A known binary system might actually be triple, quadruple, etc., owing to undetected companions. Once one companion is detected, the detectability of an additional fainter companion is inhibited. This will generally occur because the signal from the second, less massive companion may be embedded in the signal from the more massive companion. Note that these incompleteness issues do not affect the concrete result that 100 M dwarf primaries will have 55 ± 9 companions on average, since this was computed in §§ 3–6 from samples suffering little incompleteness. Only the distribution among various multiplicities is affected by the concerns above. If, for example, the estimate of f_3 (fraction of primaries having three companions) were increased relative to f_1 and f_2, more of the 55 companions (per 100 primaries) would be locked into those large systems. This leaves more primaries with no companions, thus increasing the estimated fraction of primaries that are single.

We may estimate f_i entirely differently by assuming that the presence of one companion does not affect the occurrence of additional companions. We assume that companions are drawn independently without regard to dynamically unacceptable orbits that would occasionally occur in such a scenario. If the probability of a primary having one companion is p, then the probability that a primary has i companions is given by p^i. Thus, $f_i = p^i$, and equation (9) becomes

$$\Phi_{tot} = \sum_{i=1}^{\infty} i p^i \times 100 = 55 \pm 9 . \qquad (10)$$

Again we define Φ_{tot} to be the number of companions per 100 primaries for ease of visualization.

One may solve equation (10) for p numerically to derive the f_i. This yields $f_0 = 0.61$, $f_1 = 0.28$, $f_2 = 0.080$, $f_3 = 0.023$, $f_4 = 0.006$, $f_5 = 0.002$.

These hypothetical values of f_i are quite similar to those determined empirically above. Thus, the observed distribution of the number of companions per primary is consistent with a model in which companions are acquired independently, with some fixed probability, p.

We therefore adopt the average of the two sets of f_i obtained above:

$$(f_0, f_1, f_2, f_3, f_4, f_5) = (0.58, 0.33, 0.068, 0.011, 0.010, 0.001) . \qquad (11a)$$

We adopt uncertainties in all f_i as the difference in the two determinations:

$$(\Delta f_0, \Delta f_1, \Delta f_2, \ldots) = (0.06, 0.10, 0.013, 0.024, 0.008, 0.002) . \qquad (11b)$$

These final values for f_i represent the fraction of local M dwarfs that have i companions. *Therefore, 58% of all local M dwarf primaries are single and 42% ± 9% are in binary or multiple systems.* The uncertainty follows from the Poisson statistics of the finite number of stars in each sample (i.e., there are 55 ± 9 companions per 100 primaries) and from the error in f_0 (0.58 ± 0.06).

8. DISCUSSION

8.1. *Multiplicity of M Dwarfs*

The preceding analysis of companions to M dwarfs shows that 42% ± 9% of nearby M dwarfs have companions from 0 to 10^4 AU, including masses down to 0.08 $M_\odot$, the H-burning limit. This 42% includes corrections for the small observational incompleteness. The survey of M dwarfs within 5.2 pc by HM revealed a multiplicity rate of 34% ± 9%, in rough agreement with the 42% found here. The smaller multiplicity of HM probably arises either because of the small number of stars in their sample (29 M dwarf primaries) or because they did not concern themselves fully with incompleteness at all separations. The 8 pc survey of Henry (1991) shows a multiplicity of 31%, which is also low, perhaps owing to incompleteness in various ranges of separation. For example, less than half of their stars have been subjected to multiple precise velocity measurements to detect the closest companions, and few of them have been surveyed for companions with deep CCD images. The HM survey is not entirely independent of the analysis here, since some of the stellar systems are in common. However, the overlap is small; only about 20% of the 166 M dwarf primaries considered here were also in the HM sample.

Reid (1991) has conducted a literature search for companions to all G, K, and M dwarfs north of declination $-30°$ within 10.5 pc, yielding a sample of 77 primaries that exhibit a binary frequency of no more than 45%. This sample is heavily weighted toward M dwarfs. Reid notes that the true binary fraction is likely less than 45%, owing to the possible inclusion of some optical doubles. This conclusion is clearly consistent with the multiplicities found by HM and here, 34%–42%.

The derived multiplicity depends on the outer limit of companion separation included in the counting. Since our analysis was truncated at 10^4 AU (0.02 pc), there may be stellar companions left unaccounted because of their wide separation. Close, Richer, & Crabtree (1990) compiled a sample of 1073 stars brighter than $M_V = 9.0$ and closer than 25 pc. They found that 3% of the stars were expected to be members of systems wider than 2000 AU (0.01 pc) and noted a broad range in the spectral types of the companions. The primaries considered by Close et al. were earlier in spectral type than the M dwarfs considered here. However, if their results hold for later type stars, then we expect at most a 3% increase in the multiplicity frequency above 42% found here for a restricted outer limit of 10^4 AU.

8.2. *Comparison of G and M Dwarf Binaries*

Surveys of the multiplicity of G dwarfs reveal higher frequencies of occurrence of companions. Abt's (1987) reanalysis of his G dwarf survey suggested that over 70% of G dwarf primaries had companions above the H-burning limit. The excellent survey by DM of 164 G primaries revealed 57% to have H-burning companions. These two G dwarf surveys include companions at all separations, and each includes a careful assesment of incompleteness.

In the present sample of M dwarfs, the incompleteness is smaller than for the G dwarf surveys because of the relative proximity of M dwarfs, one-half the average distance. The faintness of M dwarfs also aids detectability of the lowest mass companions because of the reduced "glare" of the primary. The G dwarf surveys, particularly that of DM, cover a larger sample of stars at all separations with uniform detection techniques (primarily velocities and direct resolution), thereby providing smaller Poisson errors and more uniform error assessment.

We estimated the error in the G dwarf binary frequency from the Poisson statistics in each of DM's mass bins, giving 57% $\pm$ 9% for the final binary frequency. This uncertainty is similar to that derived here for the M dwarfs because the total numbers of stars included in the two surveys are nearly equal. The incompleteness corrections in the G dwarf surveys are sufficiently small and accurate to warrant a clear conclusion: *The multiplicity of G dwarfs, ~57%, is definitely greater than that of the M dwarfs, ~42%, given the uncertainties in each.* Specifically, we consider it highly unlikely that the true binary frequency of M dwarfs could be as high as 57%, and unlikely that the G dwarf binary frequency could be as low as 42%.

A meaningful comparison of the binary frequencies of G and M dwarfs is not straightforward because the ranges of included companion masses are different. The surveys around both types of stars include only companions having masses less than the primary mass, extending to the H-burning limit, 0.08 $M_\odot$. For G dwarfs, this encompasses companions from ~1.0 to 0.08 $M_\odot$, while for M dwarfs the range of companion masses is about ~0.4–0.08 $M_\odot$. *Clearly, the lower multiplicity of M dwarfs results, in part, from this decreased companion mass range.*

A meaningful comparison of G and M dwarf binary frequencies requires that we consider equal ranges of companion masses or equal ranges of mass ratios. To accomplish this, we integrate DM's mass function of companions to G dwarfs (their Fig. 10) within a restricted mass range, namely, for M_2 between 0.08 and 0.4 $M_\odot$. The result is that 29% $\pm$ 7% of surveyed G dwarfs have a companion between 0.08 and 0.4 $M_\odot$, which may be directly compared with 42% $\pm$ 9% for M dwarfs. The error bars for these two binary frequencies overlap. However, the incompleteness in DM's mass function at the lowest masses, most notably for $q = 0.1$–0.2, is very uncertain. Thus, it appears that the G dwarf binary frequency is at most equal to the M dwarf binary frequency, for equal companion mass ranges. A more detailed comparison of the binary frequencies for G and M dwarfs is carried out in § 8.6 for a range of companion masses 0.1–0.3 $M_\odot$. That analysis yields nearly equal binary frequencies for G and M dwarfs for that fixed range of companion masses.

Another meaningful comparison of G and M dwarf binary frequencies can be obtained by including equal ranges of *mass ratios*, $q = M_2/M_1$. To accomplish this, we integrate the DM companion mass function between restricted limits, $q = 1.0$–0.2, so that the G dwarf range of mass ratios is equal to that in the present M dwarf survey (0.08–0.4 $M_\odot$ implies $q \approx 0.2$–1.0). The result is that 47.5% $\pm$ 7% of surveyed G dwarfs have companions in the range $q = 1.0$–0.2, which may be directly compared with 42% $\pm$ 9% for M dwarfs. These are equal within errors. Apparently, the G dwarf binary frequency is nearly equal to that for M dwarfs, for equal ranges of mass ratios. Note, however, that many G dwarf binaries have mass ratios below 0.2, and indeed DM show a peak at $q = 0.2$. But mass ratios of less than 0.2 for M dwarf primaries correspond to substellar companions, i.e., brown dwarfs. Thus, if M dwarfs continue to maintain a binary frequency equal to that of the G dwarfs at such low mass ratios, then these companions would be substellar. If so, we estimate that ~15% of M dwarfs harbor brown dwarf companions that have masses between 0.04 and 0.08 $M_\odot$. However, in § 8.4 we show that the distribution of mass ratios for G and M dwarf binaries is markedly different, thus casting doubt on the assumption that binaries of low mass ratios are as prevalent for M dwarf primaries as for G dwarfs.

8.3. *The Binary Period Distribution*

The distribution of semimajor axes of M dwarf binaries, compiled here, is shown in Figure 2*a*, spanning 0–10^4 AU. This distribution was constructed from four distinct stellar samples (§§ 3–6), each having been well observed in a particular range of semimajor axes. The incompleteness is small and well determined for all points in Figure 2*a* (except the last point representing the widest binaries). Thus, Figure 2*a* represents the distribution of semimajor axes for M dwarf binaries in the solar neighborhood and is free of any substantial systematic errors. Indeed, the Poisson errors shown are probably several times larger than any possible systematic error.

The values of dN/da (number of companions per primary per AU) decrease with semimajor axis, from $dN/da = 0.051$ AU^{-1} at $a = 0.2$ AU to $dN/da = 6 \times 10^{-6}$ AU^{-1} at 5000 AU. Figure 2*a* clearly shows a monotonic decline in dN/da with

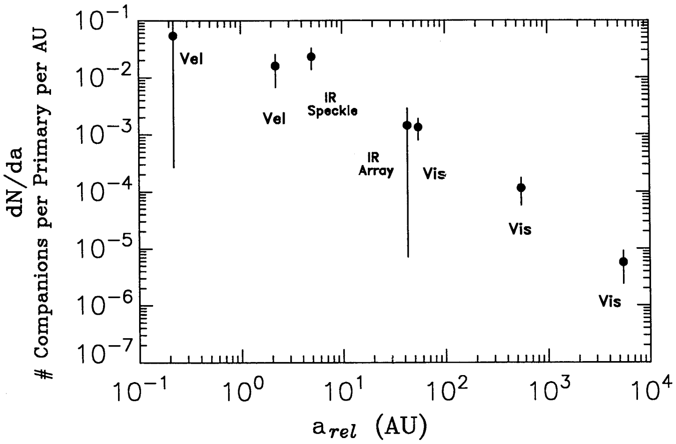

FIG. 2a

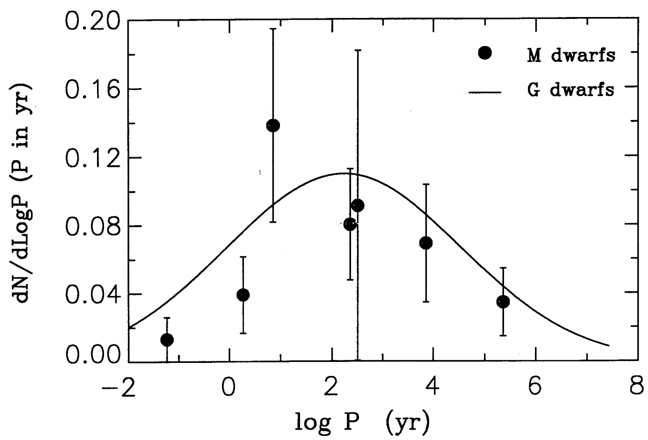

FIG. 2b

FIG. 2.—Number of companions per M dwarf primary as a function of separation in two representations: (*a*) by AU in semimajor axis (dN/da) and (*b*) by equal intervals in log P_{yr}, log orbital period. Points represent the value dN/da or $dN/d \log P$ (corrected for incompleteness) at the midpoint of the range in each stellar sample, labeled by the technique most robust in detecting companions. Error bars show the uncertainty due to Poisson statistics. In (*b*) the solid line shows the results for companions to G dwarfs from DM. The shapes of the two period distributions are similar, with a peak near 9–30 AU.

semimajor axis, indicating that the frequency of occurrence of companions in equal bins of separation declines outward. We emphasize that in this representation there is no peak in the distribution or any preferred semimajor axis at which binaries occur. Further, there is no large break in the slope of $dN/da(a_{rel})$, and thus no clear evidence that two or more mechanisms are responsible for the formation of M dwarf binaries.

Alternatively, Figure 2*b* shows the same data as in Figure 2*a*, but converted to the period distribution for M dwarfs, $dN/d \log P$, representing the frequency of occurrence of companions in bins of equal decades in period. This distribution exhibits a unimodal peak, though the Poisson errors are large for each point. This peak is not an artifact of incompleteness, since each point represents a survey having high completeness, with the exception of the point at $\log P_y = 5.3$, which represents the widest binaries. The existence of a peak in the period distribution results from the binning in log P and would also occur if the bins were equal in log a_{rel}. The two highest points lie at periods of 9 and 270 yr ($\approx$4 and 30 AU), so the true peak probably lies between those two. This agrees with Henry's (1991) peak in the M dwarf period distribution in which the two most populous bins corresponded to 1–10 AU and 10–100 AU.

The M dwarf period distribution may be compared to that of the G dwarfs, found by DM. Figure 2*b* shows superposed (*solid line*) the Gaussian fit to their G dwarf period distribution. The median orbital period for the G dwarfs is found to be about 180 yr ($a \approx 30$ AU), to be compared with 9–270 yr (4–30 AU) for M dwarfs. Apparently the period distributions of G and M dwarfs are not greatly different within the errors. The similarity in the median orbital periods shows that the median semimajor axes for G and M dwarf binaries are also similar. (Note that the difference in primary mass between M and G dwarfs causes only a slightly relative displacement of the distributions of period and semimajor axis, from Kepler's third law). Apparently, the distributions of binary separation for G and M dwarfs have about the same shape and have similar median separations. However, we note that the greatest number of companions per decade in period occurs at $P = 9$ yr, sampled with IR speckle by HM and Henry (1991). Thus there is a suggestion (though Poisson errors are large) that the peak period for M dwarf binaries is shorter ($\sim$9 yr) than that for G dwarfs ($\sim$180 yr) found by DM. This prospective shorter period preferred by M dwarfs is also indicated by Henry's (1991) peak at $P = 1$–10 yr, though the sample numbers are small.

It is also apparent from Figure 2*b* that the M dwarf binary frequency is less than that of the G dwarfs at all periods except at $\log P_{yr} = 1$. Apparently the shape of the period distributions for G and M dwarfs are similar, but the normalization (the average number of companions per primary) is systematically less for the M dwarfs, as discussed in § 8.2.

8.4. *Companion Mass Distribution*

The masses of all companions considered in this work are compiled in Table 1. These masses were derived by a variety of techniques, including dynamical (orbital), photometric, and spectroscopic methods, as described in § 3. The references in the last column of Table 1 indicate the source of information about each binary. This compilation of companions may be subtly biased because unequal numbers of M dwarf primaries were observed by each detection technique considered here. If the companion mass distribution depends on separation, then this compilation of companion masses will carry a weighting toward the more heavily observed separation ranges. In particular, all known visual companions are included in Table 1, along with the companions detected by IR speckle, velocities, and IR arrays, each derived from different sample sizes.

A histogram of mass ratios must be constructed carefully to avoid biases caused by the substellar limit, $\approx 0.08\ M_{\odot}$, at which mass the detection of companions is severely limited. In particular, M dwarf primaries that have low masses (say 0.08–0.2 $M_{\odot}$) will exhibit mass ratios biased toward unity, since low mass ratios cannot be efficiently detected photometrically. We therefore restrict our attention to M dwarf primaries having masses between 0.3 to 0.55 $M_{\odot}$ (found in Table 1) and consider only mass ratios $M_2/M_1 = 0.4$–1.0, so that the lowest actual companion mass considered is 0.12 $M_{\odot}$. We augment our sample of M dwarf binaries with those listed by Henry (1991), and we adopt his excellent mass determinations for those systems. Only those binaries having a primary between 0.3 and 0.55 $M_{\odot}$ were included. For binaries common to both Table 1 and Henry (1991), we use our masses, except for GL 644AB,

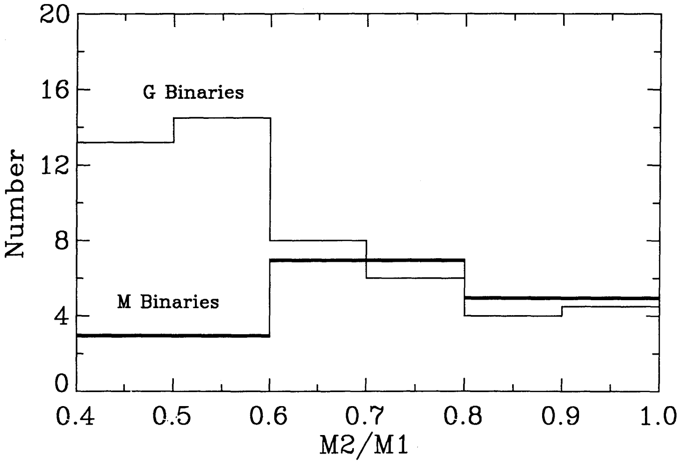

FIG. 3.—Mass-ratio distribution of companions in G and M dwarf binaries. At modest mass ratios, ~0.5, there is a deficiency of companions to M dwarfs. The effect is real, with little bias due to incompleteness. *Heavy line*: companions to M dwarfs from both Henry (1991) and this work. *Light line*: Companions to G dwarfs from DM.

GL 644AC, and GL 752AB, for which we consider Henry's masses to be more reliable.

Figure 3 shows the resulting histogram of mass ratios (M_2/M_1) including only systems having $M_1 = 0.3$–$0.55\ M_\odot$, and for M_2/M_1 extending from 0.4 to 1.0. Only 15 M dwarf binaries satisfy the above constraints, but we consider the restrictions necessary to minimize biases against low-mass companions. The histogram contains only three bins, for $M_2/M_1 = 0.4$–0.6, 0.6–0.8, and 0.8–1.0, which contain 3, 7, and 5 binaries, respectively. No significant trend is seen in these three bins, though the Poisson errors are large.

The bin corresponding to $M_2/M_1 = 0.4$–0.6 is important, since it represents the companions of lowest mass, as low as 0.12 $M_\odot$ for primaries of 0.3 $M_\odot$. The incompleteness of this bin is worthy of scrutiny because of the faintness of these companions. Stars having 0.12 $M_\odot$ have $M_V \approx 15.5$ (eq. [1]). At a distance of 20 pc, the farthest considered here, they will have $m_V = 17$. Such low-mass companions will be detected, even by historical photographic techniques. The typical companion in this bin of $M_2/M_1 = 0.4$–0.6, for a typical primary of 0.4 $M_\odot$, will have $M_2 = 0.2$, corresponding to $M_V = 13$, which is easily detectable at 20 pc. This detectability is enhanced by the fact that historical searches have concentrated on the target M dwarfs for faint companions. Thus, we conclude that the mass-ratio bin in Figure 3 for $M_2/M_1 = 0.4$–0.6 carries no significant incompleteness relative to the other two bins. Therefore, Figure 3 shows that the distribution of mass ratios is flat and perhaps declines at low mass ratios.

The mass-ratio distribution of G dwarf binaries is also shown in Figure 3, from DM. The number of G dwarf companions in each mass-ratio bin of size 0.1 is taken directly from their survey of 164 G dwarfs (their Fig. 10), and no renormalization has been done. This G dwarf distribution of mass ratios shows a monotonic rise toward low mass ratios. Coincidentally, the number of G dwarf companions at high mass ratios (0.8–0.9 and 0.9–1.0) is nearly equal to that for the M dwarfs from 0.8 to 1.0, thereby resulting in a match of the two distributions at that region. However, at the low–mass-ratio end, the two distributions differ greatly. In particular, at the lowest mass ratios of 0.4–0.6, the G dwarf distribution has risen by a factor of 3, with no corresponding rise in the M dwarf distribution. *Thus the fraction of M dwarf binaries having low mass ratios of 0.4–0.6 is deficient relative to the G dwarfs by a factor of 3–4.*

We are unaware of any theoretical explanation for this difference between G and M dwarf binaries. It is noted (e.g., AL) that secondary mass distributions must contain information regarding the formation mechanism. Simple notions of "capture" or "fragmentation" are often considered. DM suggest that the G dwarf companions are preferentially captured, since their secondary distribution increases toward lower masses, similar to the field mass function. Indeed, they note that the G dwarf binary mass function may be explained by random association of stars from the field initial mass function.

To test this "capture" suggestion for the M dwarfs, we construct the distribution of masses of companions (not mass ratios) from the same M dwarf binaries that were used in Figure 3. This "mass function" of companions to M dwarfs (including only primaries having $M = 0.3$–$0.55\ M_\odot$) is shown in Figure 4. Extending from 0.075 to 0.375 $M_\odot$ in bins of 0.075 $M_\odot$, this companion mass function is roughly flat, with no apparent systematic slope, though Poisson errors are large. In this case, the bin of lowest mass, 0.075–0.15 $M_\odot$, may be subject to incompleteness errors. However, even ignoring this bin, there is no apparent slope to the mass function.

For comparison, the mass function of primaries (field mass function) within 8 pc is overplotted in Figure 4, taken from Henry (1991). As Henry notes, the lowest mass primaries out to 8 pc probably have not all been discovered. So the bin of lowest mass again may be underestimated. However, Figure 4 shows that the field mass function of primaries is roughly flat, with a possible increase in the bin of lowest masses, especially considering the incompleteness there. Therefore, the mass function of the M dwarf companions is not significantly different from that of the primaries. *It appears that for both M dwarf and G dwarf binaries, the companion mass function is similar to the field mass function.*

Henry (1991) shows that the combined mass function of primaries and secondaries within 8 pc actually rises toward lower masses. The reason that Figure 4 does not show this effect is

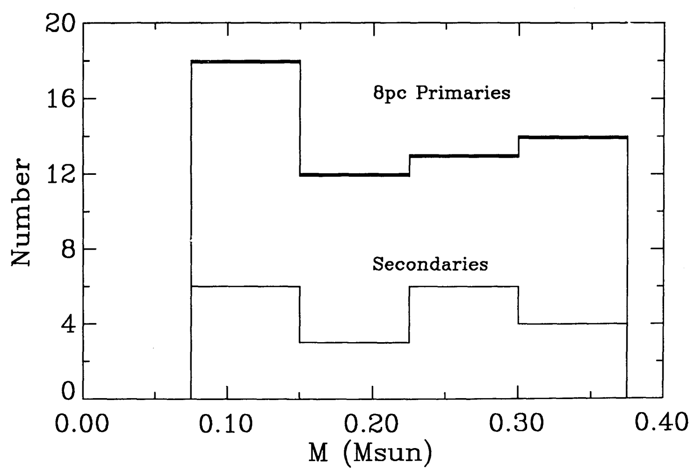

FIG. 4.—Mass function of companions to M dwarfs (*light line*) and the mass function of primaries within 8 pc (Henry 1991). The companion mass function is not significantly different from the primary (field) mass function, similar to the finding for companions to G dwarfs (DM).

that the companion mass function was constructed by using only primaries having masses between 0.3 and 0.55 $M_\odot$. If we had included binaries having primaries below 0.3 $M_\odot$, their companions would, of course, all have even lower masses. It is these companions to very low mass primaries that systematically enhance the number of very low mass objects within 8 pc. Of the 99 objects in Henry's 8 pc survey, 12 are companions to primaries that have $M < 0.3\ M_\odot$. None of these were included in Figure 4 here, because they would artifically bias the companion mass function toward lower masses.

8.5. *Brown Dwarf Companions?*

There is currently no firm evidence regarding the incidence (or existence) of brown dwarf companions to M dwarfs. However, many efforts have been made to detect them, including MB, HM, SFS, Latham et al. (1989), and Zuckerman & Becklin (1992), the latter search being around white dwarfs. All were sensitive to brown dwarfs, but none was detected. Indeed, the MB search was sensitive to companions down to 0.010 $M_\odot$, with periods up to 4 yr, but none was found around 70 targets. Taken together, these nondetections by all groups constitute a qualitative suggestion that the mass function for companions declines beyond 0.08 $M_\odot$. This suggestion can be made more quantitative only by extrapolation of the mass function *of companions* (not the field mass function), to determine whether the searches to date would have been expected to detect one. Such a quantitative assessment is currently underway by Laughlin (1992).

We carry out a simple analysis here of the nondetections of brown dwarfs by MB. That radial velocity survey was sensitive to brown dwarf companions from 0.01 to 0.08 $M_\odot$, at all orbital distances, 0.2–2 AU. From Table 2 we find $dN/da = 0.015\ \mathrm{AU}^{-1}$ per primary at $a = 1$ AU, so we expect 0.03 companions between 0.2 and 2 AU per primary. Only some fraction of those companions will have brown dwarf masses. Assuming that the mass function of companions is flat from 0.01 to 0.4 $M_\odot$ (i.e., extrapolating from Fig. 4), there would be one brown dwarf companion for every four known stellar companions. Thus, the expected number of brown dwarf companions between 0.2 and 2 AU is $\frac{1}{4} \times 0.03 = 0.012$ per primary, suggesting that MB should have detected 0.84 brown dwarfs in their sample of 70 primaries. Therefore, their nondetection of brown dwarfs represents, at best, weak evidence for a drop in the companion mass function. Similar analyses of the nondetections by HM, SFS, and Zuckerman & Becklin (1992) should be carried out to determine whether a statistically significant deficit of brown dwarf companions really exists.

8.6. *Speculation about Binary Formation*

The characteristics of binaries must be understood in terms of their early formation because the acquisition and loss of companions is negligible during main-sequence lifetimes. Evolution of orbital parameters is important only for close binaries with periods under 20 days (Mathieu & Mazeh 1988). Bodenheimer et al. (1991) review the proposed origins of multiple systems and emphasize several processes, including fragmentation of the protostellar cloud, instabilities in the protostellar disk, and capture of other protostellar objects. The precise admixture of these processes in nature is not known, nor are the conditions under which each becomes dominant.

The following binary facts seem secure, as derived by AL, DM, HM, Henry (1991), and this work: (1) The fraction of primaries that have companions is about 57% for G dwarfs and 42% for M dwarfs, where the only companions included are H burning and those having masses less than the primary mass. (2). The mass function of companions to both G and M dwarfs is indistinguishable from the field mass function. (3) The distribution of mass ratios is different for G and M binaries, with a marked deficiency at low mass ratios for M dwarf binaries. This effect is real and not due to incompleteness or the onset of brown dwarf status. (4). The distribution of semimajor axes declines monotonically toward greater separations (for equal bins of a). But when plotted in bins of equal log a or log P, the distributions exhibit a roughly Gaussian form, with a peak at ~30 AU (G dwarfs) and 4–30 AU (M dwarfs).

We speculate here about the formation of binaries based on the above facts. As is often noted, the mass distribution of companions offers clues to their formation. AL suggested that G dwarf wide binaries ($P > 100$ yr) show a mass distribution similar to the field, while close binaries favor equal masses. However, the AL short-period binaries have been retracted, and so must be their conclusions about the formation of short-period binaries. DM find that the mass distribution of companions in short-period binaries is not distinguishable from that in long-period binaries, both showing a distribution similar to the field mass function. Since the companions to M dwarfs also mimic the field mass function, the constraint to theory is clear. *Any theory of binary formation must predict a companion mass function that has the same shape as the mass function of the primaries.*

The semimajor axis distribution shows that binaries form with a wide range of separations, with no strongly dominant separation. In particular, the FWHM of the period distributions for G and M binaries, shown in Figure 2*b*, extends from $P = 1$ to 10^5 yr. Clearly, the binary formation process(es) must yield companions at not just one predominant distance scale. Of course, it is possible that two or more formation mechanisms contribute significant numbers of binaries. But, if so, the period distribution (Fig. 2*b*, especially for G dwarfs) and the semimajor axis distribution (Fig. 2*a*) show no evidence of a bimodality or a discontinuity in the slope. The DM period distribution is remarkably unimodal. In addition, the mass distribution of companions (Figs 3 and 4) shows no bimodality, as might occur if two binary formation mechanisms were important. *Thus, all observational evidence to date is consistent with the hypothesis that only one formation mechanism yields the preponderance of binary systems.*

We now consider the fragmentation picture of star formation in light of the above discussion. Boss (1988, 1991) has shown that fragmentation can occur on a wide variety of spatial scales, consistent with the wide period distribution mentioned above. (See Zinnecker 1990 for an alternative picture of fragmentation.) Also, various perturbations ($m = 1$, $m = 2$) imposed on the protostellar cloud appear capable of yielding a range of companion masses, not necessarily favoring equal masses. By these measures, fragmentation is not inconsistent with observed binary characteristics. However, it appears that fragmentation faces several severe challenges. First, the observed companion mass function mimics the field mass function. It would appear to be a remarkable coincidence if fragmentation yielded a mass function similar to that which results from the formation of single stars. Second, the mass-ratio distribution is different for G and M dwarfs, as mentioned above. If fragmentation were the dominant formation mechanism for both G and M binaries, the theory must predict a difference in the resulting spectrum of fragment mass ratios.

This may be difficult, unless the original protostars have different characteristics (energy budget, perturbation) depending on initial mass. Fragmentation theories also have difficulty explaining the shortest period binaries, notably periods less than 1 year (Boss 1986), though subsequent dissipation may promote orbital decay.

Another possible binary formation mechanism involves instabilities in a protostellar disk (Yang et al. 1991). Disk instabilities, if dominant in binary formation, must also predict a companion mass function that mimics the field mass function, as observed in G and M dwarf binaries. Such a prediction seems particularly difficult for a disk instability which takes place under very different physical conditions from those that apply to the formation of single stars. In addition, mass ratios near unity seem less likely within the framework of disk instabilities. Also, wide binaries ($a > 1000$ AU) seem difficult to form in protostellar disks that have radii of typically 100 AU (Mathieu, Walter, & Myers 1989).

Finally, we consider the "capture" hypothesis for binary formation, as considered by van Albada (1968) and by Clarke & Pringle (1991). Boss (1988) has emphasized that three-body encounters and tidal capture occur too infrequently to be important. However, in a dense molecular cloud core, dissipative interactions among the "growing density enhancements" permit capture, augmented perhaps by the dissipative effects of the associated disks (Larson 1990). Capture theories clearly predict that the companion mass function will be similar to the field mass function, because the captured companions are simply objects that would otherwise have become single stars. Since both the G and M dwarf binaries indeed exhibit a companion mass function similar to the field mass function, the capture theory is supported. Further support for the capture mechanism comes from Clarke & Pringle (1991), who show that the capture mechanism can yield a wide range of semimajor axes, as seen in observed binaries here (Fig. 2*a*). The capture scenario also explains why the distribution of mass ratios for G and M binaries are different (Fig. 3): captured companions have masses completely unrelated to the primary mass.

The capture mechanism suggests that the deeper potential wells of G dwarf primaries may be able to capture more companions (of specified mass range) than M dwarf primaries. The raw binary frequency of G dwarfs is indeed larger than that of M dwarfs, but the companion mass ranges are not equal, as discussed in § 8.2. To test this prediction properly, we adopted two well-defined samples of G and M dwarf primaries, namely, those of DM and Henry (1991), respectively. We extracted an M dwarf sample consisting of all 27 primaries that have masses between 0.30 and 0.55 $M_\odot$. The fraction of these M dwarfs that harbor companions between 0.10 and 0.3 $M_\odot$ was $6/27 = 22\%$. (A companion mass range of 0.1–0.3 was chosen because it is the largest possible range common to both the M and G binaries.) For comparison, $38/164 = 23\%$ of the G dwarfs had companions in the same companion mass range. It appears that the binary frequencies of G and M dwarfs are nearly equal, within this range of companion masses. These nearly equal binary frequencies are consistent with capture, but the expected enhancement of the G binary frequency was not seen.

Alternatively, the suggestion of capture into the deeper potential wells of G primaries can be tested by determining the fraction of M dwarfs that have companions up to 1 $M_\odot$. This fraction is to be compared with the total G dwarf binary frequency, which also includes companions up to 1 $M_\odot$. We considered all M dwarfs within 5 pc and counted the number of companions that had masses from 0.08 to 1.0 $M_\odot$. Here we explicitly included "companions" to M dwarfs that are K or G dwarfs, as well as M dwarfs. The result is that the binary frequency increases by 6% when one includes these higher mass companions. So the multiplicity of M dwarfs, including companions from 0.08 to 1.0 $M_\odot$, is revised upward from 42% to 45%. This is less than the G dwarf binary frequency of 57% (DM). Apparently, the binary frequency of M dwarfs is lower than that for G dwarfs when considering the full range of companions masses. This lower binary frequency for M dwarfs is certainly consistent with capture into potential wells of two different depths.

9. SUMMARY

We have determined the frequency of multiplicity of M dwarfs using an analysis that incorporated several outstanding surveys for companions in the solar neighborhood. Each survey provided a stellar sample for which a range of separations had been observed with little incompleteness. These surveys provide estimates of the companion frequency per AU per M dwarf primary. Incompleteness was computed for each sample and was found to be small, both because of the proximity of M dwarfs and because of the advent of new observational techniques, namely, IR arrays, IR speckle interferometry, and precise radial velocities. The resulting number of companions per AU per primary, dN/da, shown in Figure 2*a*, exhibits a monotonic decline outward, over the entire range out to 10^4 AU.

Integrating over all semimajor axes shows that one expects 55 ± 9 companions per 100 M dwarf primaries. This quantity carries no information about how those companions are clumped into multiple systems of varying sizes. We estimate this relative occurrence of binaries, triples, etc., in two ways: empirically (based on nearby M dwarf multiples) and theoretically (by assuming that companions are accrued in a probabilistic fashion). These give similar results and when averaged provide the final distribution of the multiplicity of M dwarfs: single:binary:triple:quadruple = 58:33:7:1. The fraction of M dwarfs that are single is 58%, and the fraction that are multiple is $42\% \pm 9\%$. The uncertainty comes mainly from the Poisson statistics of the finite stellar samples. This multiplicity agrees well with Henry & McCarthy (1990), who find 34% for M dwarfs within 5.2 pc. Therefore, for M dwarfs, binary formation is *not* the main branch in the star formation process.

The M dwarf multiplicity frequency is lower than that for G dwarfs; see Abt (1987) and DM, who find about 57%. The lower multiplicity for M dwarfs may be explained by the smaller range of companion masses (up to the mass of the primary) included in the definition of "M dwarf binary." However, an accounting of M dwarf binary frequency, including companions up to 1 $M_\odot$, increases the total M dwarf binary frequency only to 45%. Mass ratios below 0.2 are missed in M dwarf binaries because the companions would be brown dwarfs which are difficult to detect. If the distributions of mass ratios were the same for G and M binaries, then $\sim 15\%$ of all M dwarfs harbor brown dwarfs, undetected given current dynamical detection techniques.

Importantly, the mass distribution of companions to M dwarfs is roughly flat, similar to the field mass function at low masses. This agrees with the companion mass function of G dwarf binaries, which also show a mass function similar to that in the field (DM). The two distributions of mass ratio for G and

M binaries are different, with a large deficiency at modest mass ratios (0.5) for the M dwarf binaries. Thus, theories of binary formation must explain this significant difference for 1.0 and 0.4 $M_\odot$ primaries. Finally, the period distributions of G and M binaries are similar, both Gaussian-like (for bins of equal log *P*), with a peak period at 30 AU (G dwarf) and 9–30 AU (M dwarf). Both distributions are broad, with a FWHM spanning three orders of magnitude in semimajor axis: 1–1000 AU.

There is no evidence for multiple formation mechanisms for G and M dwarf binaries, since all distributions of binary characteristics are smooth and unimodal. Any theory for binary formation of G and M dwarfs must predict that the companion mass function is similar to the field mass function, as suggested by DM. This fact supports capture as the dominant mechanism. Capture may be dynamically possible among extended, young protostars in the dissipative environment of molecular cloud cores with the possible aid of dissipative disks. The modest binary frequency for low-mass stars aggravates the difficulty in resolving the discrepancy between observed luminosity functions in the Galaxy and that determined locally (Kroupa, Tout, & Gilmore 1991; Reid 1991).

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APPENDIX

NOTES ON INDIVIDUAL STARS

The following section includes references and notes accumulated on the individual stars in the sample.

GL 2.—Rejected here because primary (GL 4) is not an M dwarf.

GL 14.—Single. Possible velocity variation (Gliese), not seen by MB.

GL 15A.—Visual binary. GL 15A shows no velocity variation (MB; Petterson & Griffin 1980; Young, Sadjadi, & Harlan 1987, hereafter YSH) and no IR speckle close companion (Henry 1991). We conclude that 15A is single. The provisional orbit for AB is uncertain. Lippincott (1972) gives $a_{\rm rel} = 140$ and $P = 2600$ yr.

GL 109.—Single. Marcy et al. (1987, hereafter MLW) note broad lines on the spectra, probably due to rotation. YSH find no velocity variation.

GL 150.1A.—Visual binary. Separation of AB: ≈ 100″ corresponding to 1900 AU. The photometric masses together with Kepler's law give $P = 4300$ yr.

GL 195A.—Visual binary; $\alpha = 1\farcs8$ (1935) and $\alpha = 3\farcs0$ (1958). $P = 300$ yr, estimated using Kepler's law with photometric masses. CPM and parallax with GL 194AB. Separation of these two stars given as 723″ = 9510 AU. It is not clear whether this is a bound system.

GL 205.—Single. YSH and MB note constant velocity. Possible detection of companion with IR speckle (Henry 1991).

GL 206.—SB2 according to MLW. After correction for the combined brightness, this star exceeds the magnitude limit for sample selection.

GL 228A.—Visual binary. Separation of AB: 0″.9 (1945) and 2″.4 (1961). Photometric masses used to estimate *P*.

GL 268.—This star is an SB2, and exceeds the magnitude limit for sample selection.

GL 272.—Single (Heintz 1986).

GL 273.—Single. No velocity variation (MB) and no stellar companions were observed (HM; Henry 1991).

GL 277A.—Triple. The visual binary, GL 277AB, has an observed separation of $\alpha = 38\farcs6$. Gliese also notes that GL 277A is a spectroscopic binary with velocity variation of 51 km s^{-1}. YSH obtained constant velocities but took only two measurements.

GL 289.—Absorption lines found to be consistent with $v \sin i = 0 \pm 1$ km s^{-1} on 1992 January 23. Probably not SB2. No reliable information on velocity variations known.

GL 353.—Single. No velocity variation observed (MB). Heintz & Borgmann (1984) give no mention of duplicity.

GL 378.—Single. Heintz (1987) gives no mention of duplicity.

GL 382.—Single. The Gliese catalog mentions an uncertain companion at $\alpha = 25''$. Worley (1962) calls this a probable optical companion.

GL 388 (=AD Leo).—Single. No radial velocity variations observed by MB or YSH. The Gliese catalog notes possible unseen companion: $\alpha = 0\farcs110$ and $P = 26.5$ yr. Lippincott (1978) finds no astrometric perturbation, and Henry (1991) finds no speckle companion.

GL 400A.—Visual binary. Separation of AB: $\alpha = 1\farcs2$ (1896) → $\alpha = 0\farcs7$ (1965).

GL 402.—Reported unseen companion, $P = 3.8$ yr (Gliese), but single by IR speckle (Henry 1991).

GL 411.—Single. Astrometric orbit with $P = 8.0$ yr (Gliese). The supposed astrometric companion would have been detected by radial velocity variation, but none detected (by MB or YSH). No nebula exists (Henry 1991).

GL 412A.—Visual binary. GL 412B has spectral type M8 (Gliese) and is a flare star, WX UMa. Separation of AB is 28″. Photometric masses used in Kepler's law imply $P = 2600$ yr. No velocity variation of A seen by MB or YSH.

GL 459.3.—Single. No radial variation observed (MB). Note distance of 20 pc, so astrometry not robust.

GL 521.—Single. Possibly a low-amplitude substellar spectroscopic binary according to MLW.

GL 526.—Single. Gliese notes spectral variability possibly due to flare. No radial velocity variation seen (MB; YSH), and no speckle companion (Henry 1991).

GL 537AB.—Visual binary; $\alpha = 2\farcs4$ (1889) → $\alpha = 3\farcs8$ (1965). Photometric masses yield $P = 265$ yr.

GL 569.—Binary. Companion with separation of $\alpha = 5''$ discovered by FSS. Companion mass is 0.06–0.09 $M_\odot$, and spectral type is M8.5 (Henry & Kirkpatrick 1990).

GL 570BC.—Orbit given by MB and Mariotti et al. (1990). AB is hyperbolic.

GL 623.—Complete orbital analysis by Marcy & Moore (1989); $a_{rel} = 2.1 \pm 0.2$ AU, $M_2 = 0.08\ M_\odot$, $P = 3.73$ yr.

GL 628.—Gliese reports that this star is a spectroscopic binary; however, HM see no evidence of a companion.

GL 644AB.—Quintuple system. GL 644AB are visual binaries with $\alpha = 0\farcs218$, $P = 1.7$ yr. GL 644C is VB 8. Separation of AC: 221″. This triple system also has CPM with GL 643, which is also a spectroscopic binary. Separation of GL 643 and GL 644 is 72″.

GL 649.—Single. No radial velocity variation (MB).

GL 654.—Deleted from sample because the primary is not an M dwarf.

GL 661AB.—Visual binary; $\alpha = 0\farcs71$, $P = 12.98$ yr (Gliese). Using these in Kepler's equation, $(M_1 + M_2) = 0.54\ M_\odot$. Photometric masses yield 0.61 $M_\odot$.

GL 695BC.—Visual triple. Deleted from sample because the primary is a G dwarf.

GL 699.—Single. Barnard's star has been extensively studied, and no H-burning or brown dwarf companion has been observed, planetary companions aside.

GL 701.—Single. Constant radial velocity (MB and YSH).

GL 720A.—Visual binary. GL 720B is VB 9. No perturbations of A seen by MB. Separation of AB is 112″.

GL 725.—Gliese notes radial velocity variation in GL 725B, but none observed in either component by MB. No close companions seen by HM.

GL 735.—Spectroscopic binary (SB2). MLW note lines double. Duquennoy & Mayor (1988) give $P = 10.32$ days, eccentricity = 0.2, $K_1 = 21.89$ km s^{-1}, $K_2 = 23.29$ km s^{-1}, $a_1 \sin i = 0.0203$ AU, $a_2 \sin i = 0.216$ AU, inclination = 32°, $M_1 = 0.32\ M_\odot$, $M_2 = 0.30\ M_\odot$.

GL 745AB.—Visual binary. Gliese lists $\alpha = 115''$. No radial velocity variation noted in either component. Using photometric masses: $M_1 \sim M_2 = 0.27\ M_\odot$. Using Kepler's law, $P \sim 60{,}000$ yr.

GL 752A.—Visual binary; GL 752B is VB 10. Gliese lists $\alpha = 74\farcs0$, and photometric masses total 0.36 $M_\odot$, giving $P = 15{,}000$ yr.

GL 806.—Single. Very low amplitude radial velocity variation (MB), which may indicate a *substellar* companion. Gliese notes magnitude variability of 1.1 mag. Lippincott (1979) fitted orbital elements: $P = 6.3$ yr, inclination = 78°, $\tau = 1968.1$, eccentricity = 0.5, $\Omega = 90°$, $\omega = 172°$.

GL 815A.—Triple. GL 815AB are visual binaries in rapid motion: $\alpha = 0\farcs9$ (1934), $\rightarrow \alpha = 0\farcs1$ (1946). Gliese notes that A is a flare star; Duquennoy & Mayor (1988) observe spectral companion to A with $P = 3.3$ days. AB: $P = 29.3$ yr, $\alpha = 0\farcs78$ $M_{Aa} = 0.42\ M_\odot$, $M_{Ab} = 0.27\ M_\odot$, $M_B = 0.30\ M_\odot$, inclination = 57°.

GL 816.—Single. There is an optical companion to this star.

GL 820.—Single. The Gliese catalog reports that GL 820A has an astrometric companion with $P = 4.8$ yr. A stellar companion with this period would have been detectable with the precise radial velocity measurements, but none was seen by MB.

GL 829.—Possible SB2 (MLW).

GL 831.—Binary (Lippincott 1979; MacNamara et al. 1987; Henry 1991). $P = 1.92$ yr, $a_{rel} \approx 1.1$ AU.

GL 873.—Single. Previous orbital elements by Lippincott (1979). $P = 30$ yr, $\alpha = 1\farcs3 = 6$ AU. No radial velocity variation observed by MB; however, the period and separation exceed their limits of detectability. HM will continue to observe ("suspicious scan") for possible substellar companion. The direct observations did not reveal a companion, so this star will be classified as single. There is an optical companion at 5″.

GL 875.1.—Gliese reports that this star is variable, with $P = 4.65$ days.

GL 880.—Reported radial velocity range of 33 km s^{-1} (Gliese 1969), but no radial velocity variation was observed by MB and there was no speckle detection by Henry (1991).

GL 896A.—Visual binary. Gliese gives $\alpha = 3\farcs5$ (1941) → $\alpha = 3\farcs7$ (1962) and notes an uncertain $P = 177$ yr.

GL 908.—Single by IR speckle (Henry 1991).

GL 1111.—Single. No close companions observed by HM.

GJ 1245.—Triple system; orbital elements given by Henry (1991).

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