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THE CONTRIBUTION OF QUASARS TO THE ULTRAVIOLET EXTRAGALACTIC BACKGROUND

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ABSTRACT

The quasar luminosity function and its evolution, as recently derived from multicolor optical surveys, is used to estimate the integrated UV background from observed QSOs as a function of redshift. We compute the spectrum of the diffuse radiation field taking into account the opacity of H and He associated with intervening Lyα clouds and Lyman-limit absorption systems. We show that J_{912} increases by a factor of 25–35 between the present epoch and $z \simeq 2$. The attenuation at 912 Å due to the accumulated absorption is a factor of 4–6 over the redshift range $2 < z < 4$: at this epoch, the background flux is found to decrease slowly from a peak value of $J_{912} \simeq 1 - 2 \times 10^{-22}$ ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1}. The estimated ionizing flux is consistent with 2 σ upper limits to the Gunn-Peterson optical depth at high redshifts only for $\Omega_{\rm IGM} h_{50}^{3/2} \lesssim 0.03$. Also, the UV radiation from the observed QSOs falls slightly short of the amount needed to satisfy the "proximity effects." We discuss the redshift evolution of absorption systems in terms of such a QSO-dominated background.

Subject headings: cosmology: observations — intergalactic medium — quasars: general

1. INTRODUCTION

The application of the Gunn-Peterson (1965) constraint on the amount of neutral hydrogen in the intergalactic medium (IGM) to QSO absorption spectra requires the universe to have been highly ionized by $z \simeq 4.9$ (Schneider, Schmidt, & Gunn 1991; Jenkins & Ostriker 1991). Much recent discussion has focused on the source of the required metagalactic flux (Bechtold et al. 1987; Shapiro & Giroux 1987; Donahue & Shull 1987; Steidel & Sargent 1989; Songaila, Cowie, & Lilly 1990; Miralda-Escudé & Ostriker 1990; Madau 1991, hereafter Paper I; Lin & Phinney 1991), with QSOs and young, star-forming galaxies as the most plausible candidate sources of photoionization.

The "proximity effect," which is the measured decrease in the number of Lyα-absorbing clouds (LCs) induced by the UV radiation field of a QSO in its neighborhood, provides an approximate lower limit to the intensity of the metagalactic flux at the hydrogen Lyman edge (912 Å), $J_{-22} \gtrsim 3$ (where $J_{912} = J_{-22} \times 10^{-22}$ ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1}), over the redshift range $1.7 < z < 3.8$ (Bajtlik, Duncan, & Ostriker 1988; Lu, Wolfe, & Turnshek 1991). As spectroscopic deep searches (Osmer 1982; Schmidt, Schneider, & Gunn 1986) have suggested a decline in the comoving space density of quasars at $z \gtrsim 3$, the general conclusion of many of the investigations has been that the observed quasars cannot provide the required number of ionizing photons at early epochs. Recently, however, the results of several multicolor photographic surveys have greatly advanced our knowledge of the QSO luminosity function (LF) and evolution at redshifts $2 < z < 4.5$ (see, e.g., Irwin, McMahon, & Hazard 1991; Warren, Hewett, & Osmer 1991). In particular, it has been shown by Boyle (1991) that, in the limit of negligible intergalactic absorption, QSOs could account, after all, for the bulk of the diffuse ionizing flux at high redshift. In light of the implications for the current debate, we believe that a reexamination of this scenario, with the inclusion of absorption, is needed. In this *Letter*, we describe new calculations of the ionizing metagalactic flux as a function of redshift. We will limit our discussion to the contribution of objects actually detected in optical surveys; the possibility that dust in foreground galaxies could obscure quasars at cosmological distances (Heisler & Ostriker 1988) will be ignored (but see Fall & Pei 1991). In computing the integrated radiation field, the attenuation due to the accumulated absorption by H and He in intervening clouds will be fully taken into account.

2. THE INTEGRATED UV BACKGROUND

The *mean* specific intensity J_ν of the diffuse radiation field at wavelength $\lambda_{\rm obs}$, as seen by an observer at redshift $z_{\rm obs}$, is given by (Bechtold et al. 1987)

$$J_\nu(\nu_{\rm obs}, z_{\rm obs}) = \frac{c}{4\pi H_0} \int_{z_{\rm obs}}^{z_{\rm max}} \frac{(1+z_{\rm obs})^3}{(1+z)^3} \times \frac{\epsilon(\nu, z) \exp[-\tau(\nu_{\rm obs}, z_{\rm obs}, z)]}{(1+z)^2(1+2q_0 z)^{1/2}} dz , \quad (1)$$

where $\epsilon(\nu, z)$ is the proper volume emissivity (ergs cm^{-3} s^{-1} Hz^{-1}) of QSOs at frequency $\nu = \nu_{\rm obs}(1+z)/(1+z_{\rm obs})$ and redshift z, $H_0 = 50\ h_{50}$ km s^{-1} Mpc^{-1}, and $\exp[-\tau(\nu_{\rm obs}, z_{\rm obs}, z)]$ is the average transmission over all lines of sight of Poisson-distributed clouds.

2.1. *The Luminosity Function and Evolution of Quasars*

The global properties of the quasar population are well described by a "pure luminosity evolution" model, in which QSOs statistically conserve their number after $z \simeq 2$, but fade by a factor $\sim$40 on average (Boyle, Shanks, & Peterson 1988). Beyond $z \simeq 2$, no decrease in the comoving space density of quasars is seen up to a redshift of $\sim$3 (see, e.g., Koo & Kron 1988; Boyle, Jones, & Shanks 1991). More precisely, the best fit to the QSO LF for $0.3 < z < 2.9$ is given by Boyle (1991) as a "broken" power law:

$$\phi(L, z) = \frac{\phi^*}{L^*(z)} \left\{ \left[\frac{L}{L^*(z)} \right]^{\beta_1} + \left[\frac{L}{L^*(z)} \right]^{\beta_2} \right\}^{-1} . \quad (2)$$

The entire LF shifts along the luminosity axis as the position of the break L^* evolves with redshift:

$$L^*(z) = (1 + z)^{k_L} , \quad z \le z_c ; \qquad L^*(z) = L^*(z_c) , \quad z > z_c . \quad (3)$$

Here z_c is the redshift at which the luminosity evolution switches off. In a cosmological model with $q_0 = 0.5$, $h_{50} = 1$, the best-fit values for the model parameters are $\phi^* = 7.1 \times 10^{-7}$ Mpc^{-3}, $\beta_1 = 3.9$, $\beta_2 = 1.5$, $k_L = 3.45$, and $z_c = 1.9$; the B-magnitude at the break is $M_B^*(0) = -22.4$ at the present epoch. For $q_0 = 0.1$, we have $\phi^* = 3.8 \times 10^{-7}$ Mpc^{-3}, $\beta_1 = 3.8$, $\beta_2 = 1.6$, $k_L = 3.55$, $z_c = 2.1$, and $M_B^*(0) = -22.6$ (Boyle 1991). In the simple case of a power-law spectrum, we can then write

$$\epsilon(\nu, z) = \epsilon(\nu_B, 0)(1 + z)^{3 + k_L}(\nu/\nu_B)^{-\alpha} , \quad (4)$$

where $\epsilon(\nu_B, 0)$ is the extrapolated $z = 0$ emissivity at the reference frequency $\nu_B = c/4400$ Å (the center of the B-magnitude band):

$$\epsilon(\nu_B, 0) = \int_{L_{\rm min}}^{\infty} \phi(L, 0) L \, dL . \quad (5)$$

We assume $L_{\rm min} = 0.017L^*$ at every epoch. For $q_0 = 0.5$, this corresponds to $M_{B_{\rm max}}(0) = -18$, which yields $\epsilon(\nu_B, 0) = 6.7 \times 10^{23}\, h_{50}$ ergs s^{-1} Hz^{-1} Mpc^{-3}. At $z = 3$, $M_B^* = -26.4$ and $M_{B_{\rm max}} = -22$; thus we do not include quasars fainter than $B_{\rm max} \simeq 22.5$. Given the shape of the LF, it is clear that bright objects at L^* are the dominant contributors to ϵ; in the limit $L_{\rm min} \to 0$, although the number density of QSOs diverges, the emissivity increases by only 10%–20%. A larger increase should be expected if the LF actually steepens at the faint end.

Above $z = 3$ the situation is more controversial. Within the uncertainties of the individual surveys, it appears that a model with a constant comoving space density of QSOs is consistent with the number of bright ($M_B < -26$) quasars at high z (Boyle 1991; Irwin et al. 1991). A decline in the LF is only seen at the low luminosities ($M_B \gtrsim -26$) probed by the Schmidt, Schneider, & Gunn (1991) grism survey, and might be suggestive of a luminosity-dependent evolution. However, the amount of decrease which is required is highly uncertain because of corrections for incompleteness and the adopted continuum emission spectrum. In the following, we will assume that the comoving number density of QSOs is constant, and the $z = z_c$ LF provides an adequate description of the global properties of the quasar population for $z_c \le z \lesssim 5$: if the decline at the faint end of the LF is indeed severe, this will provide an upper limit for the quasar contribution to the UV background. A lower limit to the integrated flux can be obtained by imposing, for $z \gtrsim 3$, a low-luminosity cutoff at $M_{B_{\rm max}} = -26$; in this case the resulting radiation intensity decreases by a factor of 3.

We shall adopt the following model for the quasar spectral energy distribution:

$$F_\nu \propto \nu^{-0.7} \quad (\lambda > 1216 \text{ Å}) ; \qquad F_\nu \propto \nu^{-1.5} \quad (\lambda < 1216 \text{ Å}) . \quad (6)$$

This is based on observations reported by O'Brien, Gondhalekar, & Wilson (1988) and Sargent, Steidel, & Boksenberg (1989), and mimicks fairly well the "medium" QSO spectrum used in Bechtold et al. (1987) and "model QS2" of Miralda-Escudé & Ostriker (1990); its extrapolation to the soft X-rays is consistent with $\alpha_{\rm ox} \simeq 1.4$ (Zamorani et al. 1981).

2.2. *Continuum Absorption by Intervening Systems*

In order to proceed further, we need to estimate the opacity of intervening material. We will only include photoelectric absorption by H I and He II in the following, since the He I contribution to the attenuation is negligible (Miralda-Escudé & Ostriker 1991). Optically thin, low-density clouds at $T = 2 \times 10^4$ K, in ionization equilibrium with a power-law metagalactic flux, have a column density ratio $N(\text{He II})/N(\text{H I}) \simeq 1.8(J_{912}/J_{228})$, with $N(\text{He}) \approx N(\text{He III})$. Because of self-shielding, the He II column is larger in clouds which are optically thick at 228 Å; however, the correct value of N(He II) is not important in this regime, since most of the photons are removed anyway.

The "effective" optical depth of a cloudy IGM, for a photon emitted at redshift $z_{\rm em}$ and observed at redshift $z_{\rm obs}$ with wavelength $\lambda_{\rm obs}$, is (Paresce, McKee, & Bowyer 1980; Miralda-Escudé & Ostriker 1990; Møller & Jakobsen 1990; Paper I; Lin & Phinney 1991)

$$\tau(\lambda_{\rm obs}, z_{\rm obs}, z_{\rm em}) = \int_{z_{\rm obs}}^{z_{\rm em}} \int_0^\infty \frac{d^2N}{dN(\text{H I})dz} (1 - e^{-\Delta}) dN(\text{H I}) dz , \quad (7)$$

where

$$\Delta = N(\text{H I})[\sigma_{\rm H\,I}(\lambda/912 \text{ Å})^3 H(912 \text{ Å} - \lambda) + 1.8(J_{912}/J_{228})\sigma_{\rm He\,II}(\lambda/228\text{Å})^3 H(228 \text{ Å} - \lambda)] \quad (8)$$

is the optical depth of a cloud at redshift z with column density N(H I), $\sigma_{\rm H\,I}$ and $\sigma_{\rm He\,II}$ are the photoionization cross sections, $\lambda = \lambda_{\rm obs}(1 + z_{\rm obs})/(1 + z)$, and H is the step function. When $\lambda_{\rm obs} \le 912$ Å (228 Å), H I (He II) absorbs photons all the way down to $z_{\rm obs}$. As in Paper I, we will adopt "model A2" from Miralda-Escudé & Ostriker (1990) for the distribution $d^2N/dN(\text{H I})dz$ of LCs and Lyman-limit systems (LLSs) in redshift and neutral column density along a given line of sight. In this model, optically thin LCs evolve rapidly with redshift between $z = 1.7$ and $z = 3.8$ (Murdoch et al. 1986), while the distribution of the optically thick LLSs is consistent with no evolution over the range $0.7 < z < 3.6$ (Sargent et al. 1989). Photons observed at the hydrogen Lyman edge will not undergo He II absorption on their way if $(1 + z_{\rm em}) < 4(1 + z_{\rm obs})$, and in this limit we obtain (Paper I)

$$\tau(912, z_{\rm obs}, z_{\rm em}) \simeq 0.24x_{\rm obs}^3(x_{\rm em}^{0.4} - x_{\rm obs}^{0.4}) + 2.3x_{\rm obs}^{1.5} \ln\left(\frac{x_{\rm em}}{x_{\rm obs}}\right) - 0.8x_{\rm obs}^3(x_{\rm obs}^{-1.5} - x_{\rm em}^{-1.5}) , \quad (9)$$

where $x \equiv 1 + z$. The first term on the right-hand side represents the contribution of Lyα clouds to the total opacity; the others are due to Lyman-limit systems. The accumulated absorption of Lyα clouds provides 40% (64%) of the differential optical depth, $d\tau/dz$ at $z = 2$ (4). At the He II Lyman edge, we can neglect H I absorption as $\Delta_{\rm He\,II}/\Delta_{\rm H\,I} \simeq 28.8J_{912}/J_{228}$, and derive, in the limit $y/x_{\rm em}^3 < 2.2$,

$$\tau(228, z_{\rm obs}, z_{\rm em}) \simeq 0.1y(x_{\rm em}^{0.4} - x_{\rm obs}^{0.4}) + 1.5y^{1/2} \ln\left(\frac{x_{\rm em}}{x_{\rm obs}}\right) - 0.3y(x_{\rm obs}^{-1.5} - x_{\rm em}^{-1.5}) , \quad (10)$$

where $y \equiv (J_{912}/J_{228})x_{\rm obs}^3$. Whenever $y/x_{\rm em}^3 > 2.2$, equation (5) of Paper I should be used instead. Since the attenuation due to He II is an explicit function of the field J_ν, equation (1) must be solved by iteration. Note that a constant ratio J_{912}/J_{228}, independent of redshift, has been assumed here; we have verified through a few numerical trials that this approximation introduces only a modest error in the calculations. One caveat is that the computed attenuation is sensitive to uncertainties in the column density distribution of absorption clouds. In particular, there is very limited information on this distribution for $16 \lesssim \log N(\text{H I}) \lesssim 17$. If we neglect the opacity of LCs in this column density range, the total optical depth decreases by 30% (48%) at $z = 2$ (4). At the same time, the absorption at $z \gtrsim 2.5$ could be much larger than in our model if the rate of incidence of the LLSs evolves very strongly, as recently suggested by Lanzetta (1991). We also notice that rms fluctuations away from the *average* background field might be significant at low redshifts, where the attenuation is dominated by the much scarcer LLSs.

3. RESULTS

3.1. *A QSO-dominated Background*

Figure 1 shows the spectrum of the background radiation field which results from the numerical integration of equation (1), with $q_0 = 0.5$ and $z_{\rm max} = 5$. The attenuation by the intervening clouds significantly reduces the intensity of the metagalactic flux and hardens its spectrum (Miralda-Escudé & Ostriker 1990; Paper I). As suggested by Lin & Phinney (1991), the qualitative behavior can be readily obtained by expanding τ in z around $z_{\rm obs}$. At $z_{\rm obs} = 3$, $\tau(912, 3) = 1$ for $\Delta z \simeq 0.2$; this corresponds to a comoving absorption length $\Delta l \sim 150$ Mpc. The ionizing radiation is therefore largely "local" at this frequency, since sources at higher redshifts are severely absorbed. Note that in the "attenuation sphere" associated with Δl, the quasar LF predicts ~ 40 objects with $L > L^*$. Fifteen times more QSOs would be detected at 500 Å, as $\tau(500, 3) = 1$ for $\Delta z \simeq 0.5$: the observed spectrum is flatter than the local emissivity because higher energy photons arrive unabsorbed from a larger number of sources. It is worth remarking that this spectral hardening is not peculiar to a cosmological setting, since it would occur wherever the attenuation sphere is a small fraction of the volume filled by the ionizing sources. At $z_{\rm obs} = 4$, as $\tau(4, 912) = 1$ for $\Delta z \simeq 0.14$, only about five QSOs can be seen unattenuated, and the radiation field becomes highly anisotropic.

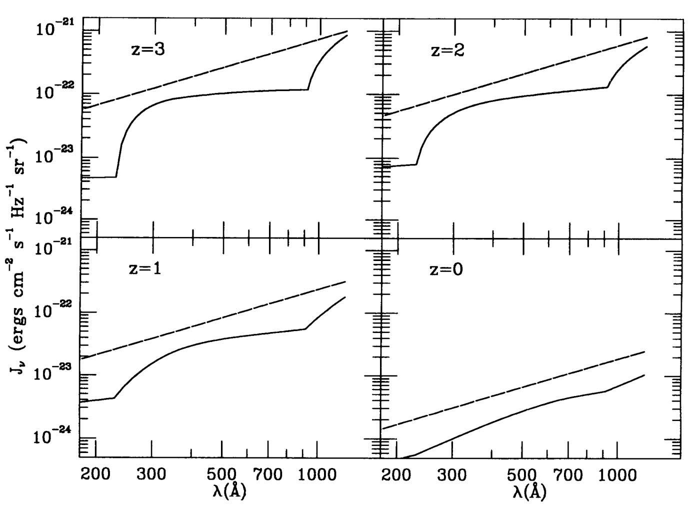

FIG. 1.—Spectrum of a QSO-dominated background at different redshifts for $q_0 = 0.5$ and $z_{\rm max} = 5$, as modified by the absorption of intervening clouds (*solid line*). If we fit a power law in frequency between 900 and 400 Å ($J_\nu \propto \nu^{-\alpha}$), the background spectral index is found to evolve from $\alpha = 1.4$ at the present epoch to $\alpha = (0.8, 0.6, 0.3)$ at $z = (1, 2, 3)$. The spectrum ($J_\nu \propto \nu^{-1.5}$) in the limit of a perfectly transparent ($\tau = 0$) medium is also shown for comparison (*dashed line*).

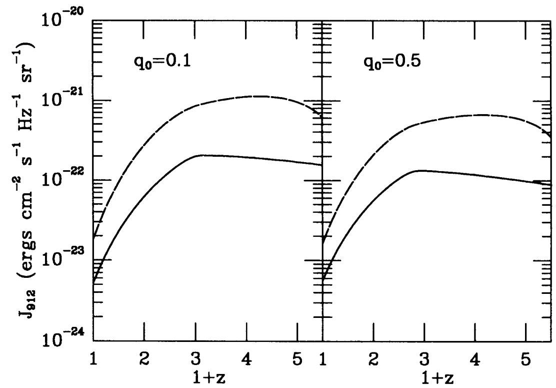

FIG. 2.—Intensity of the diffuse field at 912 Å as a function of redshift for $q_0 = 0.1$, 0.5 and $z_{\rm max} = 5$. The solid line depicts the attenuated flux due to QSOs; the dashed line is derived in the $\tau = 0$ limit.

The integrated intensity at the hydrogen Lyman edge is plotted in Figure 2 as a function of redshift. In a cosmological model with $q_0 = 0.5$, J_{912} increases by a factor ~ 20 between $z = 0$ and $z = 1.9$, where it peaks at a value of $J_{-22} \simeq 1.3$. The peak value for $q_0 = 0.1$ is $J_{-22} \simeq 2.0$ at $z = 2.1$. If we fit a power law in redshift, $J \propto (1 + z)^\sigma$, the background flux is found to evolve with $\sigma \simeq 3$ for $q_0 = 0.5$ ($\sigma \simeq 3.2$ for $q_0 = 0.1$). Over the range $2 < z < 4$, J_{912} decreases with $\sigma \simeq -0.58$ for $q_0 = 0.5$ ($\sigma \simeq -0.3$ for $q_0 = 0.1$): this decline is the consequence of the increasing Lyα forest absorption at early epochs. Because of the high optical depth, our estimate of the ionizing flux at, say, $z = 2.5$ depends only on the local number of sources and would not be affected by, e.g., a cutoff in the QSO space density at $z \gtrsim 3$. We stress that the observed QSOs fail to provide the UV flux which is required to satisfy the proximity effect by only a small factor. Fall & Pei (1991) have recently argued that dust in damped Lyα systems may obscure 10%–70% of the bright quasars at $z \sim 3$. As shown by these authors, the inclusion of obscured QSOs could help to account for the proximity effect.

The absence of an absorption trough on the blue side of the Lyα emission line in the spectra of high-redshift QSOs represents the best constraint on the density of a neutral, smoothly distributed IGM (Gunn & Peterson 1965). Assuming ionization equilibrium by a diffuse UV flux with $\alpha \simeq 0.5$, the H I optical depth is given by

$$\tau_{\rm GP} \simeq 0.31(1 + z)^{4.5} h_{50}^3 \Omega_{\rm IGM}^2 J_{-22}^{-1} , \qquad (11)$$

where a gas temperature of 2×10^4 K has been taken with cosmic abundances. Conservative 2 σ upper limits on $\tau_{\rm GP}$ have been set recently by Jenkins & Ostriker (1991) (cf. Steidel & Sargent 1987). For $J_{-22} = 1$ they translate to

$$\Omega_{\rm IGM}\, h_{50}^{3/2} < (0.043, 0.026) \quad \text{at } z = (2.6, 4.2) . \qquad (12)$$

Nucleosynthesis constraints on the cosmological baryon density require $\Omega_b\, h_{50}^2 \geq 0.04$ (Walker et al. 1991), and would be consistent with the limits above only if less than $\sim 65\%$ of the baryons were in a uniformly distributed medium, the rest having collapsed into galaxies and discrete absorption systems.

3.2. *The Lyman-α Forest at $z \simeq 0$*

At the present epoch equation (7) yields $\tau(912, 0) = 1$ for $z_{em} = 0.83$. In fact, the local universe might be even more opaque than this, since UV spectra of 3C 273 taken by the *Hubble Space Telescope* (Bahcall et al. 1991a; Morris et al. 1991) have revealed a larger number of LCs than predicted by extrapolating from the high-z regime. In the model adopted here, one expects 0.57 systems with an equivalent width limit of 0.36 Å for 3C 273, while two lines are actually seen (both in Virgo; see Bahcall et al. 1991b). Bearing in mind the large statistical uncertainties, if we portray the number density evolution of the Lyα forest in the conventional way, we obtain $dN/dz \simeq 12(1 + z)$ over the redshift range $0 < z < 1.7$.[1] Given the same column density distribution, the larger number of clouds results in a larger effective opacity. This, however, does not affect the total optical depth much at low z, as it similarly yields $\tau(912, 0) = 1$ for $z_{em} = 0.73$, the dominant contribution being due to optically thick systems.

It has been recently suggested by Ikeuchi & Turner (1991) that the excess of LCs seen at $z \simeq 0$ by *HST* might be related to the sharp drop expected for $z < 2$ in a QSO–dominated UV background. (Note that such an effect had been predicted by Bechtold et al. 1987.) We are now in a position to quantify the expected trend better. The H I column density of a thin cloud changes as J^{-1}, and for a distribution $dN/dN(\text{H I}) \propto N(\text{H I})^{-1.5}$ (Tytler 1987), the number of lines above a given column evolves as $J^{-0.5}$. The number of systems seen per unit redshift is then $dN/dz \propto (1 + z)^{\delta - 0.5\sigma}$, where the factor $(1 + z)^{\delta}$ incorporates all other unknown effects responsible for the cloud evolution, such as, for example, radial expansion. Consider now our results for $q_0 = 0.5$: at $z \gtrsim 2$, the exponent $\gamma_{LC} \equiv \delta - 0.5\sigma$ is observed to be $\simeq 2.4$, which implies $\delta \simeq 2.1$. If there is no compensating change in the evolution of cloud properties, we would therefore predict a much flatter redshift distribution for $z < 2$, $dN/dz \propto (1 + z)^{0.6}$, which is consistent (note the conspiracy here) with no evolution in the properties of the absorbing objects. The implied $\Delta\gamma_{LC} \simeq 1.8$ agrees, within the uncertainties, with the *HST* observations quoted above.

3.3. *The Evolution of the Lyman-Limit Systems*

At the same time, as J drops from $z \simeq 2$ to the present epoch, LCs with $16 \lesssim \log N(\text{H I}) \lesssim 17$ should become optically thick and increase the number of LLSs seen per unit redshift. The magnitude of the effect would be larger than estimated in the previous section, since for optically thick LLSs, $N(\text{H I}) \propto J^{-\eta}$, with $\eta \gtrsim 1$ (Steidel 1990); this would imply $\Delta\gamma_{LLS} \gtrsim 1.8$. (We notice that such a trend would not be seen if there is a deficit of LCs in the column density range of interest here.) A strong increase in the rate of incidence of LLSs at $z \gtrsim 2.5$ has actually been claimed by Lanzetta (1991). A detailed quantitative assessment of this evolution awaits the construction of a self-consistent model.

[1] This must be compared with $dN/dz \simeq 3(1 + z)^{2.4}$ valid for $z > 1.7$ (Bajtlik et al. 1988).

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