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ASTRONOMY AND ASTROPHYSICS

Groups of galaxies within 80 Mpc

I. Grouping hierarchical method and statistical properties

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Abstract. A hierarchical algorithm, similar to Tully's (1987) one, has been devised and applied to an all-sky sample of 4143 galaxies comprising all the objects with an apparent diameter D_{25} larger than 100 arcsec and having known recession velocities smaller than 6000 km s^{-1} (i.e. closer than 80 Mpc, with $H_0 = 75$ km s^{-1} Mpc^{-1}). This sample is at least 84% complete to these limits of diameter and redshift. The hierarchy is built on the mass density of the aggregates progressively formed by the method, corrected for the loss of faint galaxies with the distance; this correction represents the main improvement upon Tully's treatment. In the method, a group is defined as an entity having an average luminosity density higher than $8\ 10^9\ L_{B\odot}$ Mpc^{-3}, chosen as to ensure that the group is gravitationally bound and does not follow the Hubble expansion. 264 groups of at least three members have been identified in this way, among which 82 have more than five members and are located at distances lower than 40 Mpc. Our sample represents the deepest and richest collection of groups homogeneous over both hemispheres and whose global properties do not present significant biases with the distance; it can thus be used confidently for a variety of statistical studies. A first quick analysis of the sample leads to the following main conclusions: (i) almost all the crossing times are lower than H_0^{-1}, which confirms the bound nature of our groups; (ii) the median virial mass to blue luminosity ratio of the groups is $74\ M_\odot\ L_{B\odot}{}^{-1}$, a high value, but lower than those obtained in previous studies; (iii) we confirm clearly the increasing of the M/L ratio with the group size, a result which can be taken as an indication of the presence of dark matter around galaxies to a distance of 500 kpc.

Appendix A gives detailed definitions of the various group characteristic parameters used in the study, whereas new corrections for increasing incompleteness with the distance in Huchra & Geller's (1982) method are reported in Appendix B.

Key words: clusters: of galaxies – galaxies: general – numerical methods

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1. Introduction

The study of the groups of galaxies presents at least two major interests for extragalactic astronomy: first it displays the distribution of matter in the local Universe; second, since the majority of galaxies are within groups, the dynamical study of these small aggregates provides valuable figures for the local matter density and hence the cosmological parameter Ω.

In order to perform fruitful statistical studies of that kind, it is necessary to have at disposal a good list of groups, comprising as many objects as possible and determined according to well suited and objective clustering criteria.

Until now, several lists of groups have been issued, defined by a variety of methods. The first important list was given by de Vaucouleurs (1975), circulating in preprint form as early as in 1965. This list contained the 54 nearest groups identified mainly from their surface density contrast with their surrounding, in a partly subjective manner; additional criteria of group membership were similarity in redshifts, in magnitudes and in morphological types. Although the clustering criteria were mainly two-dimensional, the groups obtained in this way were generally real and are still currently in use. Later on, a grouping method which was purely objective and completely bidimensional was applied by Turner & Gott (1976). It defines groups as surface density enhancements higher than $10^{2/3}$ on the average galaxian surface density of these sample used. Of course, these groups suffer obvious foreground and background contaminations, but the effect is not very important because of their relative proximity. Although their definition is somewhat imperfect, Turner & Gott's and de Vaucouleurs' groups have allowed valuable statistical studies (Rood & Dickel 1978, and references therein).

However the definition of bona fide groups requires a complete three-dimensional treatment, therefore the redshifts for large samples of galaxies must be available. This has been possible since several years, after redshift surveys of large zones of the sky (in particular of CfA survey (Huchra et al. 1983) and the 21-cm line survey by Fisher & Tully (1981)). Nevertheless it must be noticed that classical methods of cluster analysis cannot be applied directly to the identification of groups of galaxies; indeed these methods obviously require the knowledge of the distances between all the objects of the sample analyzed. These distances can only be determined from the redshifts; but within a given group, because of the dynamical motions of its members, the distance between the galaxies derived in this way is meaningless, being generally much too large; this uncertainty results in an artificial elongation of the groups along the line-of-sight (the so-called "finger of God" effect), therefore to an artificial break of groups along these directions by the clustering techniques. That problem is well known and has been overcome in various ways, the most satisfactory one being the probabilist treatment (Press & Davis 1982; Materne 1979).

Essentially two large lists of groups have been obtained using three-dimensional clustering methods: Geller & Huchra's (1983) list of 176 groups, identified from the CfA survey data using a companionship method, and the complementary Huchra & Geller's (1982) list of 57 nearby groups (92 are listed, but 35 are common with the CfA list); Tully's (1987) list of 179 groups, determined from an H I line survey by a hierarchical method.

For reasons which will be developed in the following, we have adopted Tully's method in our determination of groups, after having brought several improvements to it. In particular, we have taken advantage of the quasi-completeness of our sample in apparent magnitudes to make appropriate corrections for the loss of galaxies with the distance, in order to avoid any bias in the group properties.

The sample of galaxies is presented in Sect. 2, the method is discussed in Sect. 3, and the statistical properties of the 264 groups obtained are analyzed in Sect. 4, the Sect. 5 giving a few concluding remarks. Definitions of various group characteristic parameters useful for the algorithm and the dynamical analysis are given in Appendix A and a discussion of the correction for increasing incompleteness with the distance in Huchra & Geller's (1982) method is presented in Appendix B.

2. The sample of galaxies

Our initial sample is made of the 5554 galaxies extracted from the PGC catalogue of galaxies (Paturel et al. 1989a, b) and having blue isophotal diameters D_{25} at 25 mag arcsec^{-2} larger than 100 arcsec. The sample is thought to be complete to this limiting diameter, since the catalogues used for the realization of the PGC reach completeness limits in diameters clearly lower than this value, i.e. $D_{25} = 1'.0$, $0'.9$, $1'.3$ and $1'.4$ for UGC (Nilson 1973), ESO (Lauberts 1982), MCG (Vorontsov-Velyaminov & Arkhipova 1964) and ESGC (Corwin & Skiff 1990), respectively (Paturel et al. 1991). Our diameter limit corresponds roughly to a limit in apparent magnitude: $m_l = 14.2$, from the average relation $m_l = 15.3 - 5 \log D_{25}$.

The completeness of our sample to $D_{25} = 100''$ is indirectly confirmed by a plot of $\log N(\geq D_{25})$ versus D_{25}, where $\log N(\geq D_{25})$ is the number of galaxies of our sample having a diameter larger than D_{25}. The plot is perfectly linear to $D_{25} = 100''$, whereas any incompleteness to the lowest diameters would lead to a significant departure from linearity. Note that the slope of the line is 2.70 ± 0.01, which is clearly different from the slope of 3 expected for a homogeneous distribution; this is due to the local excess of galaxies within and around the Virgo cluster and also probably to the flattening of the Local Supercluster. Such a departure from a homogeneous distribution is also apparent in a plot $\log N(\geq m)$ (Pellegrini et al. 1990).

972 galaxies of our sample (18%) have no redshift measurement. Our redshift compilation is based on RC3 (de Vaucouleurs et al. 1991) and several unpublished measures carried out by us

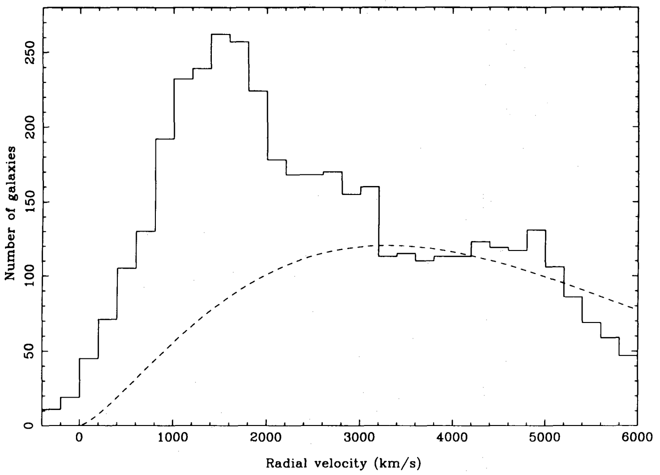

Fig. 1. Radial velocity distribution of the 4143 galaxies of the sample. The dashed curve corresponds to the distribution expected for a spatially homogeneous sample of galaxies following the Schechter luminosity function and complete to a limiting magnitude $m_l = 14.2$. The two curves are drawn such as to give the same integrated number of galaxies for $v_r > 3000$ km s^{-1} (in order to avoid the inhomogeneities at small distances, especially the Virgo cluster)

and colleagues. The recession velocities have been referred to the galactocentric reference frame (see RC3 for details); the histogram of the corrected velocities is shown in Fig. 1. In this figure, we have also drawn the distribution of the recession velocities expected for a homogeneous distribution of a sample of galaxies complete to $m_l = 14.2$ and accomodating Schechter (1976) luminosity function. The difference between the two curves is due to the inhomogeneity of the actual distribution of galaxies, resulting in a large excess of redshifts around 1500 km s^{-1} (Virgo cluster and surroundings) and another one around 4500 km s^{-1}. At velocities higher than 5000 km s^{-1}, one can note a deficiency in galaxies in the histogram, which can be partly due to our redshifts incompleteness.

In our sample limited in apparent diameter, the proportion of catalogued galaxies decreases with distance; so we have to limit our group study to a distance at which the loss of objects is still acceptable. Since the Schechter luminosity function of galaxies drops suddenly at $M^* = -20.4$, a natural choice is to take for the maximum distance that one at which a galaxy having $M = M^*$ just reaches the inclusion condition in our sample, which corresponds to 6000 km s^{-1} for the limiting magnitude $m_l = 14.2$.

Consequently our working sample comprises all the 4143 galaxies of our initial sample having a recession velocity $v_r \leq 6000$ km s^{-1}.

3. The grouping method

3.1. The hierarchical algorithm

As noted in the introduction, two three-dimensional clustering techniques have been mainly used to produce large catalogues of groups of galaxies: these are the companionship method and the hierarchical method.

In the companionship method, developed by Huchra & Geller (1982) (hereafter HG), each sample galaxy is searched for companions close both in projected distance and in velocity; then companions are searched for second order companions, and so on until no more companions can be found; the set of companions obtained in this way forms a group with the sample galaxy initially chosen. Then one examines the next sample galaxy not belonging to this group, and so on until all the sample galaxies have been considered. As an inconvenience of this method, note that the companions are picked up only in reference to their closest neighbour in the group, and not to the whole set of the group galaxies gathered at the previous steps, a fact which can lead to sort out non-physical groups; for instance, a filament of galaxies 20 Mpc long aligned close to the line-of-sight with an average separation of 1 Mpc between neighbouring galaxies will be identified as a definite group according to the companionship method, although the filament members are not physically related and just follow naturally the Hubble expansion.

This inconvenience does not exist in the hierarchical method, first introduced by Materne (1978), and that is the reason of our choice of this technique, in Tully's (1987) version, with slight modifications.

In the hierarchical clustering methods, the galaxies are merged successively, forming various units which constitute the seeds of the final groups. More precisely, suppose one starts with a sample of N objects. At the beginning of the procedure, each galaxy constitutes a unit by itself, i.e. one starts with N independent units. Then one has to define an affinity parameter u between the galaxies, which will condition the merging operation (for instance their distance). In the first step, the two objects presenting the highest affinity are merged, and are replaced by a single unit: so one is left with $N-1$ units. The merging procedure is then repeated, in such a way that after $N-1$ mergings, one obtains a single unit comprising all the N objects of the sample. One can see that a hierarchical sequence of units organized by decreasing affinity appears in the method, justifying its name. On the other hand, the merging of a galaxy in a given unit involves the whole unit, and not only the last object merged in this unit, which removes the inconvenience underlined about HG's method.

It remains now to define the groups, which are obtained by cutting the hierarchy at some chosen value u_0 of the affinity parameter; so the groups are those units present at the step corresponding to $u=u_0$.

Another advantage of the hierarchical methods is the easy visualisation of the whole merging procedure under the form of a hierarchical arborescence, the dendrogram. On such a graph, not only the groups appear, but also the local concentrations within the groups and the superstructures including several groups. No such features can be immediately discerned in the companionship method.

There are several possible choices for the merging parameter (see Materne 1978, Tully 1980). The most physical one is certainly the gravitational force between entities i and j used by Tully (1980). However Tully cuts the hierarchy according to the density of the entity. Here we prefer to use the same parameter for the two operations, namely the density. So at each step of the hierarchical procedure, one merges the two units i and j forming the pair having the highest density ρ_{ij}, with $\rho_{ij}=3(m_i^c+m_j^c)/(4\pi R_{ij}^3)$, where m_i^c and m_j^c are the corrected masses of i and j (as defined below), and R_{ij} their mutual distance. After the merging, units i and j are replaced by a single unit whose position, velocity and mass are those of their barycenter.

3.2. Practical computation of ρ_{ij}

3.2.1. Corrected masses

The masses m_i of the galaxies are directly computed from their luminosities using different values of the mass-to-luminosity ratios for spirals and elliptical/S0 galaxies. As a matter of fact, in a purpose of simplification, we use pseudo-luminosities m_i^* proportional to the m_i's instead of the masses (cf. Appendix A). That does not change anything in the hierarchical procedure.

The m_i^*'s have then to be corrected for increasing incompleteness with the distance. Indeed, the samples used for group determinations have generally a galaxy completeness which decreases with the distance. That is notably the case for the samples complete in apparent magnitude (CfA sample) or nearly complete as ours. That variable incompleteness has to be corrected already at the level of the grouping criterion, otherwise the definition of the groups could be distance-dependent. More precisely, the grouping method has to fulfil a necessary condition which can be expressed in the following way: let us consider a group identified by the method and located at a distance D; if we carry away this group at a distance $D'>D$, only a part of its members found at the distance D will be present in the sample (namely the most luminous); the necessary condition we require is that these remaining galaxies are recognized by the method as an independent group – and of course the same must occur if the group is put closer at a distance $D''<D$. If this condition is fulfilled, then it can be hoped that the usual global properties of the group will not be found distance-dependent. Indeed those properties involve mainly the luminosity, the velocity dispersion, the dimension and the galaxy distribution of the group. And none of those quantities is expected to change at different distances for the test-group defined under the above condition (after appropriate corrections of the total luminosity for incompleteness). This is so because positions and peculiar velocities of galaxies in a group are uncorrelated with their luminosities; thus samples of group galaxies limited to some luminosity, as for the test-group at a given distance, are just random samples of group members for those global properties[1].

Conversely, a dependence of the group properties with the distance indicates a lack of correction or an inappropriate correction for the distance effect. As a matter of fact, this is the case for HG's corrections; despite the incompleteness correction they have made, there is a clear increase of their group velocity dispersions as a function of the distance (Magtesyan 1988), which points out towards something wrong in their correction. Due to the interest and the frequent use of this method, we establish the right corrections in Appendix B.

In the hierarchical method we use, the seeds of the various groups are determined at each step by the maximum value of ρ_{ij}, which represents the mean density of the group seed formed by the units i and j. If the group seed is at the distance D, ρ_{ij} is computed only from the galaxies of the group seed which are present in the sample at that distance; so if the unit (i,j) is put at a distance $D'>D$, ρ_{ij} will be diminished, resulting in a group dimension which will decrease with distance when one cuts the hierarchy at a constant density value. Therefore, in order to make

[1] In fact, a slight distance dependence cannot be avoided at the largest distances, due to the minimal number of galaxies required to form a group ($N=3$) (see Figs. 11 and 12)

the correction, we have just to account for the group galaxies which are too weak to appear in the sample. Since the densities are computed from the luminosities, this can be simply done by correcting the luminosities themselves. For that purpose, one uses the fact that our sample is nearly complete in apparent magnitudes to $m_1=14.2$ (see Sect. 2). In a group of galaxies identified at a distance D, the only members appearing in a sample complete to $m=m_1$ are those having luminosities $L>L_1(D)$, where $L_1(D)$ is the luminosity of a galaxy with an apparent magnitude m_1 located at the distance D. Thus the total luminosity of the group is derived from the luminosity of its members present in the sample by multiplication by the factor:

$$\beta(D)=\frac{\int_0^{+\infty} L\Phi(L)\mathrm{d}L}{\int_{L_1(D)}^{+\infty} L\Phi(L)\mathrm{d}L} \tag{1}$$

where $\Phi(L)$ is the luminosity function of the galaxies. For $\Phi(L)$, we take the Schechter's (1976) luminosity function. Except for a constant multiplicative term, this function is the same for spirals and ellipticals. If we assume now that the relative proportion of spirals and ellipticals is the same whatever the luminosity may be, the correction for the progressive loss of galaxies with the distance is simply obtained by replacing the mass m_i^* of any galaxy located at a distance D by the corrected mass

$$m_i^c=\beta(D)m_i^*. \tag{2}$$

Explicitly, one has:

$$\beta(D)=\frac{1.23}{\int_{L_1(D)/L^*}^{+\infty} x^{-1/4}\mathrm{e}^{-x}\mathrm{d}x} \tag{3}$$

where L^* is the reference luminosity in the Schechter function, taken as $2.05\ 10^{10}\ L_\odot$ in the B band. For $m_1=14.2$, $\beta(D)$ amounts to 1.2, 1.5, 2.3 and 3.9 at 20, 40, 60 and 80 Mpc, respectively.

At this stage it is interesting to estimate the influence of the slight incompleteness of our sample in redshifts. Obviously the proportion of galaxies without measured redshits increases with m. Now a given group is defined by a corrected observed mean density higher than ρ_1, which corresponds to a true mean density higher than $\rho_g=\rho_1/f$, where f is the proportion of the total mass present in the group galaxies having a known redshift. Let us consider a nearby group; the main part of its mass stands in galaxies with low apparent magnitudes, having generally redshift measurements, so $f\sim 1$; if this group is put at a large distance, f will be lowered, since its members will have higher m-values. Thus, for the fixed ρ_1 of the hierarchical method, the group will be defined by true densities higher at large distances than at lower ones, resulting generally in a decrease of its radius with the distance. Figure 12 shows on the contrary a slight increase of our group radii with their distances, notably beyond 40 Mpc; so the effect of the redshift incompleteness is certainly weak, if not negligible, and one can believe it does not affect significantly our group statistics.

To end this part, note that Tully (1987) has made the correction for increasing incompleteness with the distance only for the total luminosities of the groups, but not for the limit densities involved in the hierarchical method. As a matter of fact, the correction developed hereabove represents the major improvement upon his treatment.

3.2.2. Separation between the galaxies

Another sensitive point concerns the derivation of the separation R_{ij} between galaxies i and j from their angular distance θ_{ij} and their radial velocities v_{ri} and v_{rj}. Indeed we look for groups of galaxies, and in the groups, the derivation of the radial component of the separation between two members i and j from their radial velocity difference $V_{ij}=|v_{ri}-v_{rj}|$ by application of the Hubble law is meaningless; in fact, the V_{ij} value is essentially given by the dynamical motions of i and j within the group, which are uncorrelated with R_{ij}. In order to overcome this problem, we adopt Tully's approach, distinguishing two cases according to the value of V_{ij} compared to a transition velocity V_1.

1) For the small values of V_{ij}, with $V_{ij}\le V_1$, no information is available about the line-of-sight component of the vector $\overrightarrow{ij}$ joining i and j, and R_{ij} is simply obtained from the projection of $\overrightarrow{ij}$ on the sky plane by use of the average projection factor for a random position of $\overrightarrow{ij}$ given by Eq. (A3a), i.e.:

$$R_{ij}=\frac{8}{\pi}D_{ij}\sin\frac{\theta_{ij}}{2} \tag{4}$$

where $D_{ij}=(D_iD_j)^{1/2}$, with $D_i=V_{ri}/H_0$.

2) For $V_{ij}>V_1$, we use a transition formula which connects smoothly to the previous one at $V_{ij}=V_1$, and which reaches asymptotically the triangle formula for $V_{ij}\gg V_1$, namely:

$$R_{ij}=\left(\left(\frac{8}{\pi}D_{ij}\sin\frac{\theta_{ij}}{2}\right)^2\left(1-\left(1-\frac{\pi^2}{16}\right)\frac{f(x_{ij})}{1+x_{ij}}\right)+\left(\frac{V_1}{H_0}f(x_{ij})\right)^2\right)^{1/2} \tag{5}$$

where

$$x_{ij}=\frac{V_{ij}-V_1}{V_1}\quad\text{and}\quad f(x)=\frac{x^3}{1-x+x^2}. \tag{6}$$

For $V_{ij}=V_1$, the relations (4) and (5) are indeed identical, and for $V_{ij}\gg V_1$, Eq. (5) is equivalent to:

$$R_{ij}=\left(4D_{ij}^2\sin^2\frac{\theta_{ij}}{2}+\frac{V_{ij}^2}{H_0^2}\right)^{1/2}. \tag{7}$$

Equation (7) expresses the distance between i and j when the individual motions are negligible compared to the difference in recession velocities (simple triangle formula).

The formula (5) is more complicated than the transition formula used by Tully (1987), but it insures a smoother transition with the case $V_{ij}\le V_1$; indeed, for fixed values of D_{ij} and θ_{ij}, the derivative of R_{ij} at $V_{ij}=V_1$ computed from Eq. (5) is:

$$\frac{\mathrm{d}R_{ij}}{\mathrm{d}V_{ij}}(V_1)=0,$$

which insures the continuity with the derivative of the formula (4). This is not the case in Tully's formula, for which the derivative of the transition formula at $V_{ij}=V_1$ is generally very large. As a consequence, for a given V_1 value, R_{ij} computed from Eq. (5) increases more slowly with V_{ij} near V_1 than when derived from Tully's formula. In other terms, in order to obtain R_{ij} values similar to Tully's ones, we can choose a V_1-value lower than his.

3.2.3. Choice of the transition velocity V_1

As discussed by Tully (1987), the choice of V_1 is a compromise between too low values which would lead to rejection of group members with large peculiar velocities and too high values which would allow inclusion into the groups of unrelated galaxies appearing along the same line-of-sight. Tully's choice was $V_1 = 300\ \mathrm{km\,s^{-1}}$, and Tully indicates that groups are generally unaltered by a $100\ \mathrm{km\,s^{-1}}$ change around this value. After the previous remark, we can choose a lower V_1 value than his, which has the advantage that there are less cases where $V_{ij} \leq V_1$, i.e., where little information is known about R_{ij}; a second advantage is that we can pick groups with low velocity dispersions without adding to them spurious members. It can be seen that any V_1-value around $200\ \mathrm{km\,s^{-1}}$ is convenient in our case; since we look for a unique value for our whole sample, we constrain the final value using two well known group cases: (i) despite a $250\ \mathrm{km\,s^{-1}}$ velocity difference between the galaxies M 81 and M 82, it is evident that they belong to the same group. If V_1 is smaller than $150\ \mathrm{km\,s^{-1}}$, they are separated. (ii) IC 239, whose velocity differs by $270\ \mathrm{km\,s^{-1}}$ from that of NGC 1023, is projected onto the center of the NGC 1023 group, but does not belong to it (Tully 1980). If V_1 is larger than $190\ \mathrm{km\,s^{-1}}$, it is included into the group. So our final choice is $V_1 = 170\ \mathrm{km\,s^{-1}}$.

Note that with such a low V_1-value, the clusters of galaxies are split into various subunits, due to their large velocity dispersions leading to a general overestimate of the separation between their members by our formula (triangle formula). For instance, for $V_1 < 450\ \mathrm{km\,s^{-1}}$, the Virgo cluster is split into groups located at about the same position, but with different average velocities. We have called *clusters* such cases as Virgo, where the aggregate is split into several parts at about the same position on the sky. 18 such clusters have been found in this way, by collecting the various pieces into one aggregate. We have preferred to proceed this way rather than to remove the clusters before running the algorithm, as done by Tully (1987), because in this latter case the list of members has to be determined a priori. Whatever it may be, this subjective intervention is certainly a clear inconvenience of our method when adopting a universal V_1-value which is relatively low for the whole sample.

3.3. *Choice of the limiting density*

As already noticed by Tully (1987), the use of the mean density to cut the hierarchy is particularly relevant because one can think that there is a density threshold under which aggregates are not gravitationally bound and will never contract.

A necessary condition to be fulfilled by a galaxy collection to be called a group is *not* to be an unbound aggregate expanding with the Universe. Let us examine this latter double condition. First, if the galaxy collection is unbound, its total energy must be positive. Using notations of Appendix A, we may write:

$$E \equiv T + U = \frac{1}{2}\, M V_\mathrm{V}^2 - \frac{GM^2}{R_\mathrm{G}} > 0. \tag{8}$$

Second, if the galaxy collection is expanding with the universe, it can easily be shown (Jackson 1975) that the virial velocity satisfies:

$$V_\mathrm{V} = H_0 R_\mathrm{I} \tag{9}$$

where R_I is the inertial radius of the considered galaxy collection defined by Eq. (A7).

Equations (8) and (9) are equivalent to the following relation:

$$\frac{1}{2}\, M H_0^2 R_\mathrm{I}^2 - \frac{GM^2}{R_\mathrm{G}} > 0. \tag{10}$$

Let us express Eq. (10) as a condition on the mean mass density ρ_M of the collection, which is defined by:

$$\rho_\mathrm{M} = \frac{3M}{4\pi R_\mathrm{max}^3}.$$

The use of the group radius R_max in this expression corresponds to the definition of the density ρ_{ij} used in the hierarchical algorithm, because for the last galaxy j joined to the group i, we have $R_{ij} \simeq R_\mathrm{max}$.

So, if we put:

$$\lambda = \frac{R_\mathrm{G}}{R_\mathrm{max}} \quad \text{and} \quad \mu = \frac{R_\mathrm{I}}{R_\mathrm{max}},$$

Equation (10) is equivalent to

$$\rho_\mathrm{M} < \lambda \mu^2 \rho_\mathrm{c} \tag{11}$$

where ρ_c is the closure density of the Universe, defined as:

$$\rho_\mathrm{c} = \frac{3H_0^2}{8\pi G} = 1.6\ 10^{11}\ M_\odot\ \mathrm{Mpc}^{-3}.$$

For a homogeneous spherical group, the factor $\lambda\mu^2$ equals unity [cf. Eq. (A9a) and Eq. (A9b)].

As we want to define a group as a galaxy collection which is not an unbound one expanding with the Universe, we impose a group to satisfy the opposite of Eq. (11), with the geometrical factor $\lambda\mu^2$ taken as one (homogeneity and spherical symmetry assumption), i.e.:

$$\rho_\mathrm{M} \geq \rho_\mathrm{c}. \tag{12}$$

But the densities used in the hierarchical algorithm are pseudo-luminosity densities, as seen hereabove. The accurate M/L values for groups are unknown; reasonable values would be about $100\ M_\odot/L_\odot$; so our limiting density would be of the order of $10^9\ L_{\mathrm{B}\odot}\ \mathrm{Mpc}^{-3}$. This uncertainity enables us to choose a value within this range so as to reproduce the well known nearby groups. We took finally:

$$\rho_\mathrm{L} = 8\ 10^9\ L_{\mathrm{B}\odot}\ \mathrm{Mpc}^{-3} \tag{13}$$

Note that this value is not very critical and is not constrained within a factor of 2. Tully's value is $\rho'_\mathrm{L} = 2.5\ 10^9\ L_{\mathrm{B}\odot}\ \mathrm{Mpc}^{-3}$, significantly lower than ours due to an absence of incompleteness corrections, the use of only the luminosity of the brighter component in the computation of ρ_{ij}, and the same mass-to-luminosity ratio for ellipticals and spirals.

3.4. *Definition of the associations*

Noticing that many groups present a core-halo structure, Tully (1987) calls "associations" the collections of galaxies corresponding to overdensities 10 times lower than those defining the groups. As noted hereabove, those superstructures sort out naturally in the hierarchical method, simply by looking at the whole dendrogram. Here we define the associations by a density threshold 5 times lower than ρ_L, i.e.:

$$\rho_\mathrm{L}^\mathrm{ass} = 1.6\ 10^9\ L_{\mathrm{B}\odot}\ \mathrm{Mpc}^{-3}. \tag{14}$$

4. Results

4.1. *The list of groups*

Running the hierarchical algorithm described in Sect. 3 on the sample of galaxies defined above and adding the 18 clusters indicated in Sect. 3.2.3 results in a list of 264 groups (by *group* we mean a unit with at least 3 galaxies) involving 1729 galaxies (41% of the total sample). Allowing for galaxy pairs, we would obtain 580 groups involving 2361 galaxies (57% of the total sample). 466 galaxies belong to associations surrounding the groups. The total fraction of galaxies involved in a group or associated to a group is then 53%.

The list of groups, associations and of their members is detailed in the Supplement Series of this journal (Fouqué et al. 1991), as well as their distribution into regions defined in Tully's NBG Atlas.

The characteristic parameters of each group are given in Table 1.

In order to perform some statistical analysis of the groups properties, we have defined two samples of groups:

— *sample* (*a*): The whole sample, i.e. groups with more than 3 galaxies and located at a distance lower than 80 Mpc (264 groups, involving 1729 galaxies).

— *sample* (*b*): A selected sample made of groups with more than 5 galaxies and located at a distance lower than 40 Mpc (82 groups, involving 973 galaxies).

Sample (b) is expected to give more reliable statistics. The median values as well as the dispersions of the different dynamical quantities for these two samples are given in Table 2.

4.2. *Group sizes*

The distribution of the number of galaxies in each group is presented on the histogram of Fig. 2. 158 groups (60%) have less than 5 members 30 (11%) have more than 10 members, the largest one being the Virgo Cluster with 177 members.

The histogram of the groups virial radii is presented on Fig. 4. For the selected sample, there is no virial radius greater than 2.8 Mpc and the median value is 0.7 Mpc, which also corresponds to the peak value.

The comparison with Tully's (1987) median value (0.34 Mpc) has to take account that the two definitions of the virial radius (Tully (1987), footnote 2, and our Eq. (A5)) are quite different, resulting in the formula: $R_V^{\rm Tully} \simeq 0.85 \times 8/\pi^2 R_V^{\rm us} \simeq 0.69\ R_V^{\rm us}$. The factor 0.85, stated in footnote 2 of Tully (1987), stands for the passage from the theoretical definition of our virial radius as given by Eq. (A5) (and which is what Tully called "the harmonic radius") and Tully's one. The second factor, $8/\pi^2$, comes from a different statistical "deprojection" factor used by Tully and us. As it may be seen by comparison of formula (6) of Tully (1980) (which involves a projected radius) and formula (4) of Tully (1987) (which involves a deprojected radius), Tully seems to use the factor $4/\pi$ given by Eq. (A3a), while we take the factor $\pi/2$ given by Eq. (A3b). This latter seems preferable since the virial radius is defined by a weighted mean of the *inverse* of the intergalatic distances. Thus, with Tully's definition, we would have found a median value of the virial radius of 0.48 Mpc, which is not too far from Tully's value (0.34 Mpc).

4.3. *Velocity dispersions*

The histogram of the 1D weighted velocity dispersions, V_V^1, (which is the virial velocity defined by Eq. (A10) divided by $\sqrt{3}$) is shown in Fig. 3. This histogram displays a sudden fall at $V_V^1 = 140\ {\rm km\,s^{-1}}$. More precisely, all the groups formed from the hierarchical method have $V_V^1 < 150\ {\rm km\,s^{-1}}$. The only aggregates with $V_V^1 > 150\ {\rm km\,s^{-1}}$ are 17 of the 18 clusters which have been split by the method and have been constituted by hand (see

Table 1. Group properties. The columns are as follows: Col. 1: Common name of the group, with the convention of naming the group from its brightest member unless there exists an unambiguous literal name or a name from Abell's (1958) catalogue. The suffix 'cl' means cluster and refers to those cluster split by the algorithm and reconstructed by hand (see Sect. 3.2.3). Col. 2: Number of galaxies within the group. Col. 3: Distance D in Mpc (assuming $H_0 = 75\ {\rm km\,s^{-1}\,Mpc^{-1}}$). Col. 4: Total luminosity L_B in the B band in units of $10^9\ L_\odot$. Col. 5: 1D weighted velocity dispersion V_V^1 in ${\rm km\,s^{-1}}$. Col. 6: Virial radius R_V in Mpc. Col. 7: Virial mass to blue luminosity ratio M_V/L_B in $M_\odot\ L_{\odot B}^{-1}$. Col. 8: Crossing time t_{cr} in units of 10^9 yr

Name (1)	N (2)	D (Mpc) (3)	L_B ($10^9\ L_\odot$) (4)	V_V^1 (${\rm km\,s^{-1}}$) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
Abell 262 cl	15	66	1465	418	2.1	344	2.5
Abell 347 cl	5	72	654	294	0.5	91	0.6
Abell 1060 cl	23	44	748	421	1.0	339	2.0
Antlia cl	22	38	833	392	1.0	253	2.8
Cancer II cl	17	56	871	552	3.1	1515	2.4
Centaurus cl	28	43	1364	652	1.1	500	1.0
CVn I	10	4	7	33	0.3	72	3.8
CVn II	22	7	53	73	0.6	84	3.7
Dorado cl	19	13	145	195	0.3	125	1.1
Eridanus	34	20	436	132	1.8	98	6.4
Fornax I cl	49	19	624	287	0.8	152	2.0
Leo I	14	9	125	75	0.4	22	5.8
Pavo I	4	51	330	114	0.1	5	1.0

Table 1 (continued)

Name (1)	N (2)	D (Mpc) (3)	L_B (10^9 $L_\odot$) (4)	V_V^1 (km s^{-1}) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
Pegasus cl	14	53	746	478	0.9	365	2.8
Sculptor	8	2	22	73	0.9	285	3.1
Telescopium cl	6	38	265	158	0.6	74	2.3
UMa I N	28	15	172	105	0.9	81	4.3
UMa I S	33	12	151	99	1.0	89	5.1
Virgo I cl	177	12	888	575	1.0	528	1.1
Virgo M	14	30	621	102	2.0	47	7.0
Virgo W cl	15	30	280	209	0.7	144	1.9
NGC 45	3	7	5	57	0.4	396	1.9
NGC 128	4	58	173	67	2.1	77	3.2
NGC 134	5	21	55	65	0.6	61	3.1
NGC 173	3	59	195	74	0.5	19	5.7
NGC 266	6	65	459	100	2.1	64	6.4
NGC 383	5	71	910	136	1.7	48	3.4
NGC 439	3	77	740	86	3.0	42	7.8
NGC 467	3	74	436	78	2.8	54	9.2
NGC 484	3	67	540	125	1.6	65	2.8
NGC 488	4	32	179	60	0.5	15	6.0
NGC 507 cl	12	65	1218	343	0.8	101	1.6
NGC 524	3	34	117	32	1.2	15	2.9
NGC 541	6	73	712	107	0.6	13	5.6
NGC 584	8	25	123	44	0.5	11	5.8
NGC 628	5	11	35	74	1.1	245	1.9
NGC 672	3	6	6	94	0.1	155	1.2
NGC 676	3	21	105	59	1.3	60	2.4
NGC 681	3	25	35	98	0.4	141	1.1
NGC 691	5	38	167	85	0.3	17	2.2
NGC 720	4	23	40	34	4.3	176	4.7
NGC 825	3	46	48	25	0.3	5	8.8
NGC 841	3	63	134	40	1.7	28	5.5
NGC 864	3	22	27	19	2.5	50	6.8
NGC 877	3	53	173	65	0.1	5	3.4
NGC 908	6	20	57	71	0.7	83	2.2
NGC 925	4	9	14	15	1.8	42	5.4
NGC 936	4	19	34	52	1.1	128	1.7
NGC 945	4	61	288	126	1.5	119	4.2
NGC 973	5	63	503	133	0.4	19	2.2
NGC 1023	8	10	39	44	1.1	77	6.0
NGC 1052	9	19	113	73	0.6	38	3.7
NGC 1060	3	71	357	16	2.0	2	16.3
NGC 1068	6	15	86	64	0.5	35	2.5
NGC 1097	3	16	48	64	1.4	173	3.5
NGC 1167	3	67	287	41	1.7	14	6.4
NGC 1186	3	38	196	37	0.6	6	5.1
NGC 1209	3	35	70	33	0.4	8	3.7
NGC 1255	7	22	76	26	1.2	15	15.0
NGC 1359	4	26	37	105	0.5	199	2.3
NGC 1376	3	54	170	67	1.5	56	4.9
NGC 1417	6	53	282	115	1.1	72	3.3
NGC 1433	10	12	48	79	1.0	186	4.8
NGC 1519	4	24	12	20	0.3	15	5.9
NGC 1532	3	15	44	85	0.7	154	1.4
NGC 1589	5	48	270	111	0.7	44	3.8
NGC 1667	3	61	308	129	2.8	214	4.5
NGC 1672	4	16	44	84	0.6	128	1.8

Table 1 (continued)

Name (1)	N (2)	D (Mpc) (3)	L_B ($10^9\,L_\odot$) (4)	V_V^1 (km s^{-1}) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
NGC 1721	3	59	304	25	0.2	1	14.5
NGC 1725	3	53	181	57	1.7	44	6.3
NGC 1779	4	45	150	119	0.9	113	3.2
NGC 1792	4	13	36	105	0.3	136	0.6
NGC 1800	3	10	3	78	0.3	985	1.1
NGC 1832	3	24	42	36	0.9	37	4.9
NGC 1947	3	14	14	81	0.8	524	1.9
NGC 1961	5	55	466	107	1.5	52	3.6
NGC 2207	3	34	175	33	0.1	0	7.8
NGC 2217	4	19	27	47	1.0	114	2.1
NGC 2273	4	28	50	127	0.8	340	1.7
NGC 2276	3	34	99	98	1.7	232	2.7
NGC 2280	4	23	155	64	0.8	30	3.5
NGC 2300	4	28	54	53	1.3	92	3.4
NGC 2403	3	3	7	17	0.7	41	5.3
NGC 2417	4	40	150	41	1.3	19	9.6
NGC 2427	5	11	22	53	1.1	191	5.9
NGC 2442	3	16	51	49	0.2	13	2.0
NGC 2541	3	8	4	16	0.4	33	7.6
NGC 2559	3	19	39	119	0.2	107	2.0
NGC 2633	4	31	44	66	0.1	14	1.5
NGC 2655	5	20	75	50	0.6	30	4.2
NGC 2663	3	26	31	93	2.4	928	1.8
NGC 2768	3	19	56	17	0.9	6	12.3
NGC 2775	3	16	20	23	2.0	72	3.1
NGC 2781	3	24	28	62	0.7	142	2.7
NGC 2782	3	34	42	20	2.1	27	6.3
NGC 2805	3	23	58	89	0.4	78	2.9
NGC 2815	3	32	77	52	1.9	92	6.8
NGC 2835	4	7	12	94	0.7	717	1.1
NGC 2836	3	18	24	107	0.7	487	1.8
NGC 2855	3	23	15	11	0.7	8	6.3
NGC 2859	3	22	19	13	1.3	15	1.9
NGC 2962	3	25	17	29	0.8	56	4.7
NGC 2964	3	18	28	77	0.2	59	2.7
NGC 2967	6	23	37	29	1.2	36	11.4
NGC 2985	3	18	34	117	0.3	185	0.8
NGC 2986	3	27	70	123	1.0	289	2.0
NGC 2997	8	11	53	78	0.7	118	2.9
NGC 3031	11	2	8	92	0.1	137	0.8
NGC 3054	6	30	119	58	1.0	38	5.8
NGC 3087	9	33	313	110	1.9	102	5.5
NGC 3091	3	49	192	59	3.3	85	8.5
NGC 3115	3	7	8	12	0.3	8	1.0
NGC 3166	3	15	31	80	0.1	24	0.4
NGC 3175	4	12	15	44	0.6	101	3.5
NGC 3190	5	17	37	71	0.1	19	1.5
NGC 3203	3	30	38	81	1.3	329	3.6
NGC 3227	3	15	18	82	0.0	24	0.4
NGC 3245	4	17	28	25	0.5	16	5.0
NGC 3250	4	34	113	96	0.8	90	1.4
NGC 3256	4	32	176	116	0.7	73	2.6
NGC 3263	4	37	334	58	2.1	29	9.5
NGC 3338	3	16	23	15	0.8	11	13.2
NGC 3347	6	37	186	71	0.5	19	4.9

Table 1 (continued)

Name (1)	N (2)	D (Mpc) (3)	L_B ($10^9\ L_\odot$) (4)	V_V^1 (km s^{-1}) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
NGC 3370	6	15	12	87	0.2	216	1.7
NGC 3396	5	21	46	44	0.1	8	3.9
NGC 3557	5	37	214	83	0.3	14	2.0
NGC 3585	3	16	28	34	4.5	255	2.0
NGC 3607	11	13	45	111	0.2	67	2.6
NGC 3613	3	28	51	47	0.8	47	2.6
NGC 3626	3	18	19	86	1.1	586	1.4
NGC 3640	5	17	30	106	0.4	206	1.1
NGC 3642	5	24	75	122	0.7	190	2.0
NGC 3665	3	28	60	15	0.4	2	8.9
NGC 3672	3	22	28	65	0.2	37	2.5
NGC 3675	3	10	14	23	1.4	75	6.9
NGC 3780	3	33	61	6	1.3	1	42.3
NGC 3798	3	47	68	77	0.3	35	1.0
NGC 3813	3	20	16	57	0.9	240	2.3
NGC 3892	4	21	22	16	1.1	17	14.4
NGC 3923	7	20	113	89	0.6	62	1.6
NGC 3941	4	13	12	12	1.3	21	8.5
NGC 4008	3	47	96	74	0.7	58	2.3
NGC 4038	14	20	152	45	0.2	5	5.8
NGC 4062	4	10	5	38	0.9	356	2.6
NGC 4105	6	23	63	66	0.7	64	3.2
NGC 4125	8	19	106	79	1.4	112	6.3
NGC 4128	3	33	31	33	1.2	57	4.2
NGC 4151	6	14	28	28	0.4	15	6.8
NGC 4169	4	51	160	51	0.3	7	5.0
NGC 4179	3	16	25	34	0.4	29	5.4
NGC 4256	4	38	92	149	0.5	152	0.9
NGC 4274	7	13	38	51	0.4	41	2.8
NGC 4291	5	24	44	105	0.3	92	1.9
NGC 4303	15	21	212	104	1.1	77	4.9
NGC 4304	3	33	76	58	1.3	78	4.8
NGC 4521	3	40	71	120	0.2	62	0.9
NGC 4546	3	13	13	22	2.1	110	5.5
NGC 4565	6	17	159	50	1.0	22	7.6
NGC 4594	8	13	104	40	2.7	58	7.2
NGC 4631	12	9	81	72	0.5	41	4.3
NGC 4643	3	17	14	71	1.6	787	1.4
NGC 4658	4	30	47	35	0.9	33	7.3
NGC 4666	4	20	47	70	1.0	144	4.2
NGC 4697	25	16	200	119	1.3	131	4.1
NGC 4750	5	24	30	47	0.8	80	4.8
NGC 4751	3	25	33	77	2.3	591	1.9
NGC 4753	9	14	56	104	0.9	254	2.9
NGC 4756	3	52	77	99	1.7	296	3.1
NGC 4856	4	16	24	74	1.3	413	3.1
NGC 4902	4	33	95	31	0.7	10	5.4
NGC 4936	3	42	181	103	1.9	155	2.6
NGC 4965	3	28	34	42	0.4	27	2.2
NGC 4976	4	17	65	34	0.7	17	2.3
NGC 4995	4	22	33	65	0.8	148	2.4
NGC 5018	5	36	148	72	0.4	22	3.2
NGC 5033	9	13	49	47	0.4	28	3.7
NGC 5044	10	34	186	80	0.8	38	4.7
NGC 5064	4	38	195	85	1.3	65	5.3

Table 1 (continued)

Name (1)	N (2)	D (Mpc) (3)	L_B ($10^9 L_\odot$) (4)	V_V^1 ($\mathrm{km\,s^{-1}}$) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
NGC 5084	5	22	82	71	1.2	99	3.1
NGC 5090	4	44	242	88	0.2	10	3.9
NGC 5101	5	25	160	110	0.7	76	2.5
NGC 5121	4	18	27	27	0.6	23	5.1
NGC 5128	15	5	147	44	1.9	35	8.6
NGC 5135	6	55	431	110	0.3	10	4.5
NGC 5194	17	6	67	117	0.5	138	3.0
NGC 5248	3	15	21	34	0.9	70	2.2
NGC 5322	6	28	113	88	0.8	74	2.2
NGC 5333	8	37	374	147	1.9	151	4.5
NGC 5364	5	16	40	80	0.2	45	1.8
NGC 5365	3	32	59	110	0.4	119	0.6
NGC 5371 cl	17	33	392	178	0.4	41	3.1
NGC 5395	4	48	156	61	0.2	7	4.8
NGC 5419	4	54	456	107	3.0	103	5.2
NGC 5427	4	35	142	71	0.2	10	6.2
NGC 5457	7	5	23	27	0.7	30	3.3
NGC 5485	7	27	76	113	0.5	128	1.6
NGC 5557	4	43	180	143	1.3	209	2.2
NGC 5566	5	20	55	78	0.2	31	1.1
NGC 5668	4	23	32	103	0.5	252	2.0
NGC 5676	7	31	137	98	0.7	65	3.3
NGC 5678	3	28	46	30	1.1	30	8.6
NGC 5746	11	23	155	92	0.9	67	4.5
NGC 5775	4	21	30	89	0.1	48	0.7
NGC 5791	4	44	96	91	1.2	146	2.2
NGC 5796	4	39	89	37	3.1	66	8.1
NGC 5846	9	25	170	106	1.1	104	3.9
NGC 5866	5	12	35	50	0.6	59	2.6
NGC 5898	3	31	115	148	0.2	40	2.4
NGC 5908	5	46	172	97	0.7	54	3.7
NGC 5930	3	37	45	51	1.1	84	4.3
NGC 5962	3	27	47	73	0.8	119	3.5
NGC 6070	3	28	39	19	1.3	16	11.9
NGC 6340	3	18	23	30	0.6	32	6.3
NGC 6500	3	41	83	69	0.1	7	3.6
NGC 6585	3	40	39	5	0.9	1	38.4
NGC 6744	6	10	60	33	1.4	35	5.0
NGC 6753 cl	12	42	652	451	1.7	723	1.9
NGC 6769 cl	12	54	1365	213	0.8	39	5.2
NGC 6907	3	43	164	35	1.6	17	7.7
NGC 7014	5	66	607	124	1.1	38	5.2
NGC 7079	6	34	104	101	1.3	174	4.0
NGC 7144	5	25	57	49	0.6	34	5.5
NGC 7154	3	35	45	35	0.6	22	4.5
NGC 7166	4	32	55	77	0.3	41	1.1
NGC 7172	5	36	111	113	0.5	76	3.3
NGC 7179	4	38	97	49	0.8	28	4.2
NGC 7331	3	14	50	11	1.1	4	3.4
NGC 7368	3	31	32	102	0.7	305	1.5
NGC 7377	3	45	150	80	2.2	132	4.2
NGC 7421	4	26	34	98	0.5	191	1.8
NGC 7424	4	13	27	106	0.6	339	1.3
NGC 7448	5	30	70	97	0.7	138	2.8
NGC 7507	3	21	53	9	0.3	1	11.7

Table 1 (continued)

Name (1)	N (2)	D (Mpc) (3)	L_B ($10^9 L_\odot$) (4)	V_V^1 (km s^{-1}) (5)	R_V (Mpc) (6)	M/L ($M_\odot/L_\odot$) (7)	t_{cr} (10^9 yr) (8)
NGC 7582	13	21	203	72	0.6	21	7.6
NGC 7640	3	8	6	22	0.4	42	2.5
NGC 7711	4	56	185	11	2.0	2	36.3
NGC 7713	3	9	10	14	0.6	17	8.1
NGC 7714	3	39	44	47	0.1	4	2.3
NGC 7721	3	29	38	44	1.7	123	4.6
NGC 7814	5	15	18	91	1.0	603	1.7
IC 342	5	2	53	30	2.5	61	1.2
IC 438	3	39	91	92	0.9	120	3.1
IC 520	3	49	137	72	0.9	46	3.2
IC 764	6	27	65	97	1.1	225	4.0
IC 1459	8	22	115	95	0.7	79	3.7
IC 3370	4	38	151	76	0.8	45	4.4
IC 4296	7	48	455	89	1.4	34	7.5
IC 4329 cl	4	60	626	117	3.1	96	8.4
IC 4351	3	33	71	56	1.3	82	5.6
IC 4682	3	47	142	43	0.7	12	6.2
IC 4765	10	61	1044	131	2.4	56	7.5
IC 4956	3	70	289	72	2.4	60	7.6
IC 5181	3	27	38	80	0.2	41	0.9
IC 5250	4	45	167	104	0.7	61	2.6
UGC 2800	4	18	31	39	0.7	50	5.1
UGC 3426	3	55	105	25	0.4	3	12.6
UGC 3697	5	42	161	109	0.4	40	2.1
UGC 7170	3	32	10	87	0.5	500	1.9
ESO 185-54 cl	8	59	836	227	0.9	78	2.2
ESO 221-26	5	18	44	112	1.6	630	3.0
ESO 501-23	3	12	4	56	0.4	483	2.4
ESO 505-3	3	21	9	48	0.2	62	0.8
ESO 507-25	6	41	176	66	0.9	31	6.5
ESO 563-16	3	21	5	76	0.3	544	1.0

Table 2. The median values and the first-to-third quartiles[a] of different group parameters for two groups samples: *Sample (a)*: Groups with more than 3 galaxies and located at a distance lower than 80 Mpc (264 groups) (the whole sample). *Sample (b)*: Groups with more than 5 galaxies and located at a distance lower than 40 Mpc (82 groups)

Group parameters	Median (a)	1–3 quartile (a)	Median (b)	1–3 quartile (b)
1D virial velocity (km s^{-1})	73	59	80	55
Virial radius (Mpc)	0.77	0.83	0.69	0.57
Inertial radius (Mpc)	0.40	0.39	0.48	0.36
Gravitational radius (Mpc)	2.03	2.16	1.65	1.38
Maximal radius (Mpc)	0.87	0.74	1.14	0.79
M_V/L_B ($M_\odot L_\odot^{-1}$)	62	108	74	93
Crossing time (Gyr)	3.4	3.4	3.3	2.7
Collapse time (Gyr)	48	87	35	33

[a] The *first-to-third quartile* is the length of an interval around the median containing half of the sample members.

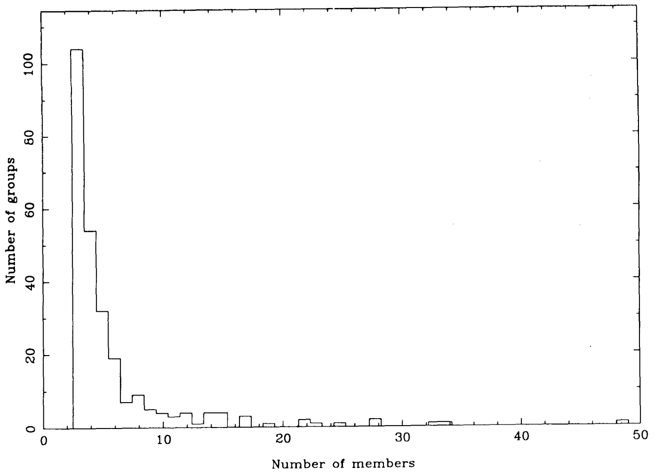

Fig. 2. Histogram of the number of members in each of the 264 groups. 60% of the groups have less than 5 members and 11% have more than 10 members

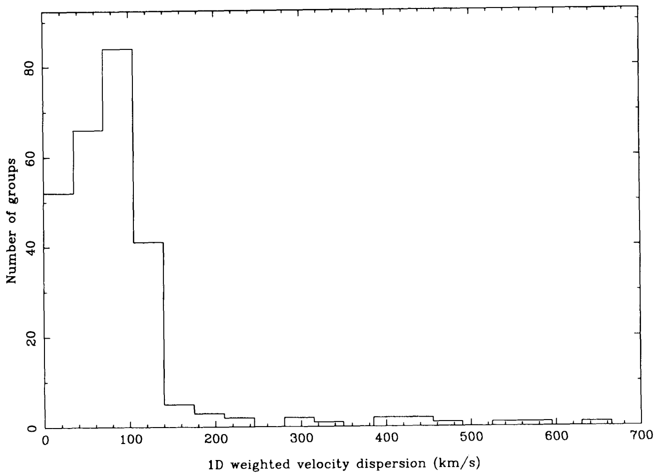

Fig. 3. Histogram of the 1D weighted velocity dispersions, V_V^1: the median value is 73 km s^{-1}

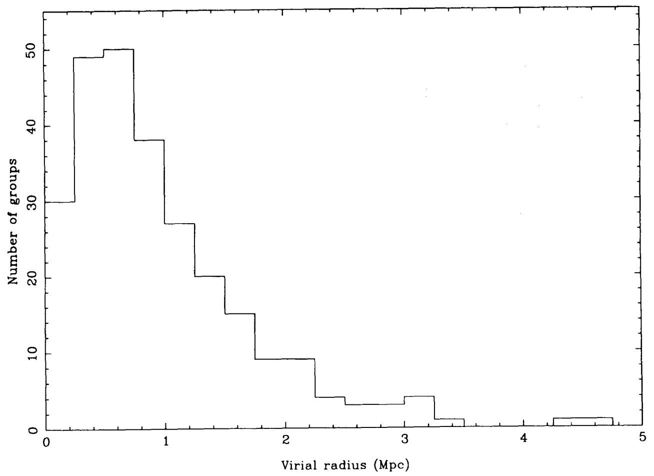

Fig. 4. Histogram of the virial radii, R_V: for the selected sample (b) ($N \geq 5$ galaxies, $D \leq 40$ Mpc), there is no virial radius greater than 2.8 Mpc; the median value is 0.77 Mpc

Sect. 3.2.3). Thus one can wonder whether this 150 km s^{-1} limit is generated by the method itself through the value adopted for V_1.

In order to examine that point, let us consider a group formed by our method, which will be assumed to have a significant number of members ($N \geq 5$) of roughly equal masses and some central concentration (just for simplification). In such a group, the first steps of the hierarchical procedure will result in a merging for central galaxies, so that the radius of the group $R_{\max}$ will be the distance R_{ij} obtained for the last galaxy i merged, since entity j will be nearly at the center of mass of the group. As a consequence, only galaxies presenting a velocity difference with the central velocity of the group lower than some limit will be merged in the course of the procedure. Indeed the merging of a galaxy i to the entity j implies: $\rho_{ij} > \rho_L$; since we look for mergings occuring before the last one, this leads to $R_{ij} < R_{\max}$ from the definition of ρ_{ij} and the fact that the masses of the group seed increase in each of the successive mergings. Now, when the velocity difference V_{ij} between two galaxies of a group increases (with the same projected distance), R_{ij} is first constant for $V_{ij} \leq V_1$ [Eq. (4)], then increases [Eq. (5)]. Equation (5) comprises two terms: the first one is positive and decreases with V_{ij}, whereas the second one, $[V_1/H_0 f(x_{ij})]^2$, increases with V_{ij} or x_{ij}. So a necessary condition for merging is:

$$\frac{V_1}{H_0} f(x_{ij}) < R_{\max},$$

i.e.:

$$V_{ij} < V_{\lim}(V_1, R_{\max}). \tag{15}$$

The histogram of the virial radius R_V shows that $R_V < 1.2$ Mpc for most of the sample (b) groups, i.e., from Eq. (A9c), $R_{\max} < 1.4$ Mpc. For $V_1 = 170$ km s^{-1}, the necessary condition for merging is then: $V_{ij} < 300$ km s^{-1}, i.e., practically: $|v_{ri} - v_{rG}| < 300$ km s^{-1}, where v_{rG} is the radial velocity of the center of mass of the group.

Taking a uniform distribution for the v_{ri}'s in [0, 300 km s^{-1}] leads to: $V_V^1 < 170$ km s^{-1}.

Accounting for the first term of Eq. (5) lowers slightly that limit to: $V_V^1 < 155$ km s^{-1}, which is exactly the figure found in our sample. For $V_1 = 100$ km s^{-1} and 300 km s^{-1}, we should obtain respective limits of 140 and 280 km s^{-1} (with the same $R_{\max}$).

Note that in Tully's treatment, the limit for V_V^1 computed from his Eq. (1b) is about 180 km s^{-1} (with $V_1 = 300$ km s^{-1} used in that study), corresponding exactly to his histogram Fig. 6 when the clusters (picked out by hand) are removed.

So the hierarchical method cannot form aggregates of galaxies with large velocity dispersions: such entities are automatically split into various subunits; we hope to have correctly recognized them and gathered them into the corresponding clusters. But conversely, as a positive point of our method, interloper contamination is practically absent in our treatment. The situation is opposite in HG's method, which can accomodate such clusters with high velocity dispersions as the Virgo one, but where the interloper contamination is far from negligible. Those effects account for the large difference between the median velocity dispersions found: 80 km s^{-1} for our selected sample (and 100 km s^{-1} in Tully's one) and 180 km s^{-1} for HG's groups.

4.4. Characteristic time scales

Most of the crossing times obtained here are lower than $H_0^{-1} = 1.3\ 10^{10}$ yr (see Fig. 5); this result is consistent with the choice of the limiting density ρ_L (cf. discussion in Sect. 3.5) and indicates that our groups are not spurious concentrations following the general Hubble expansion, but probably gravitationally bound entities.

In turn, most of the groups have a collapse time greater than H_0^{-1} (see Fig. 6), the median value for the selected sample (b) being 3.5 10^{10} yr. The condition for a group to be collapsed or being in a process of collapse is: $t_{col} < 2H_0^{-1}$. Less than 30% of our groups satisfy this condition. Tully (1987) found the opposite result on his groups but he used a collapse time formula (his Eq. (4)) different from ours [Eq. (A15)] and which we think to be somewhat incorrect. Indeed (A15) and Tully-(4) converted in years have the same numerical factor but the "virial radii" used in the two formulae are different, as explained in Sect. 4.2: our virial radius defined by Eq. (A5) is exactly the quantity which arises in the theoretical collapse time formula Eq. (A15); it is not the virial radius $R_V^{\rm Tully}$ defined by Tully (1987) in his footnote 2. As a consequence, in his collapse time formula, Tully should have used what he called the "harmonic radius", $R_H^{\rm Tully}$, and which is our virial radius. The relation between $R_V^{\rm Tully}$ and $R_H^{\rm Tully}$ depends on the distribution of matter in the group, a mean relation stated by Tully being: $R_V^{\rm Tully} \approx 0.85\ R_H^{\rm Tully}$.

Another difference arises with Tully's collapse time calculation because of the difference in the virial masses used. Tully makes use of an *unweighted* virial mass which gives higher values because, in a given group, the velocities of the smallest objects are more scattered than those of the largest ones. He argues in favor of unweighted virial mass estimates for they are less noisier than weighted estimates (see discussion in Tully (1987)). As for us, we prefer the *weighted* virial velocity V_V defined by Eq. (A10) for it is directly related to the kinetic energy of the group [Eq. (A11)] and, consequently, the virial mass defined from V_V by Eq. (A12) coincides with the *actual* mass in equilibrium. On the other hand, there is no physical argument to identify the unweighted virial mass and the actual mass. This difference may be quantified by the comparison of the two M_V/L_B median values: 74 $M_\odot\ L_{B\odot}^{-1}$ in our case compared to 94 $M_\odot\ L_{B\odot}^{-1}$ for Tully.

The net result of these two differences on the collapse times leads to: $t_{col}^{\rm Tully} \simeq 0.51\ t_{col}^{\rm us}$. Thus, with our formulae, the median value of the collapse time of Tully's groups becomes 1.5 10^{10} yr instead of 7.8 10^9 yr. Since we have found 3.5 10^{10} yr, a discrepancy remains between Tully's groups and ours. At this point, let us emphasize that t_{col} is not a fully reliable quantity since it has a meaning (i.e. it coincides with the actual collapse time) only under the assumption that the group is virialized, contrary to the previous quantities, R_V, V_V and t_{cr}. But the criterion for a group to be virialized is to have $1.5 t_{col} < H_0^{-1}$ (Peebles 1970); so it depends on t_{col}. This means that we do not have a fully consistent criterion to decide whether a group is collapsed and virialized or not. All we can say is that in the cases where we find $1.5 t_{col} < H_0^{-1}$, the groups are *probably* virialized. In our sample, only 8.3% of the groups satisfy that condition.

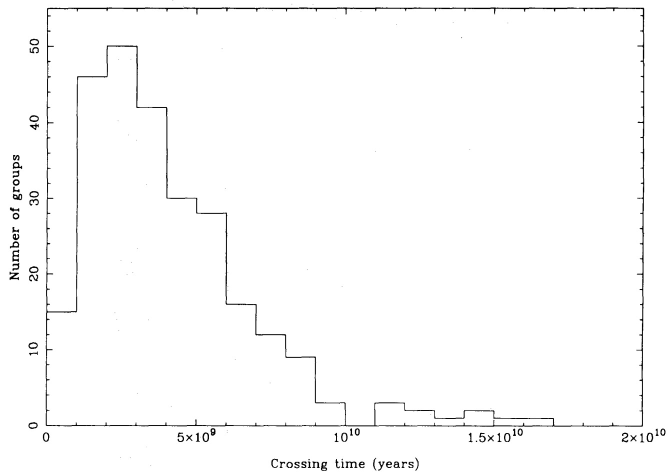

Fig. 5. Histogram of the crossing times, t_{cr}: the fact that, in most cases $t_{cr} < H_0^{-1}$ is consistent with our choice of the limiting density ρ_L (see Sect. 3.3)

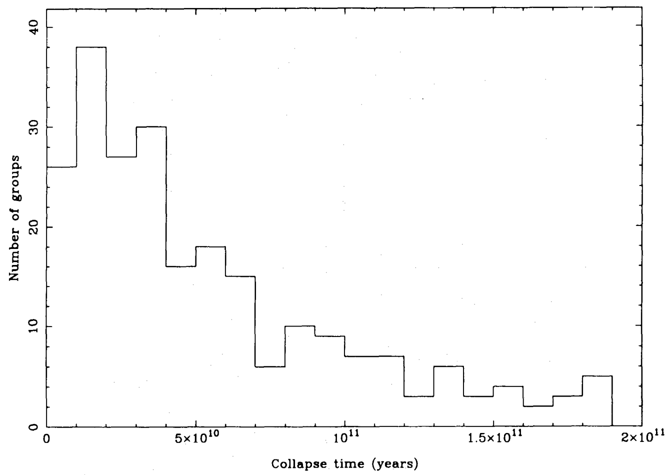

Fig. 6. Histogram of the collapse times, t_{col}. Only 24% of the groups have a collapse time lower than the age of the Universe. For the others, the expression t_{col} is meaningless and does not coincide with the actual collapse time

4.5. Mass-to-light ratios

The M_V/L_B values for the different groups are presented in Fig. 7. The median value for the selected sample (b) is 74 $M_\odot L_{B\odot}^{-1}$ which is somewhat lower than Tully's value (94 $M_\odot L_{B\odot}^{-1}$). This difference is undoubtly related to the difference in the velocity dispersions (cf. formula (A12)) as mentioned above.

A necessary condition for the absence of distance-bias in the results is that M_V/L_B should be independent of the group distance. This is clearly the case, as shown in Fig. 8.

The dispersion of the log (M_V/L_B) values measured as the first-to-third quartile scatter is about 0.6 for the selected sample (b). From N-body simulations, Heisler et al. (1985) have found an observational scatter of the same quantity of 0.5 for groups of five members. So a part of the scatter of M/L obtained in groups of our sample (b) would be intrinsic; this scatter could be about a factor of two, as found by Tully (1987).

To end this section, note the correlations between M_V/L_B and the virial radius R_V (Fig. 9) and between M_V/L_B and V_V (Fig. 10). These correlations have been already found previously, and Rood & Dickel (1978) have shown convincingly that they are not

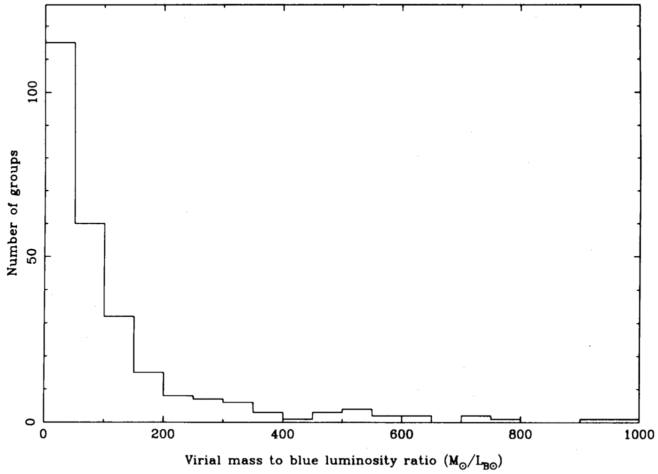

Fig. 7. Histogram of the virial mass to blue luminosity ratios: for the selected sample (b) there is no value greater than 620 $M_\odot\, L_{B\odot}^{-1}$ and only three values greater than 300 $M_\odot\, L_{B\odot}^{-1}$. The median value is 74 $M_\odot\, L_B^{-1}$.

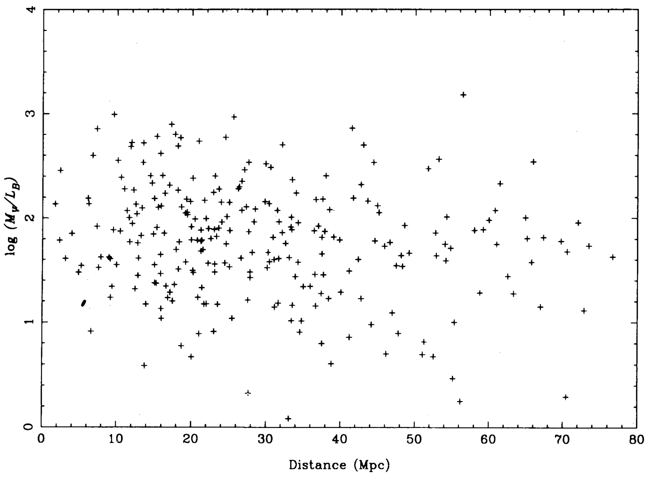

Fig. 8. Virial mass to blue luminosity ratio as a function of the distance. This diagram shows that our computed M/L ratios are not distance-dependent

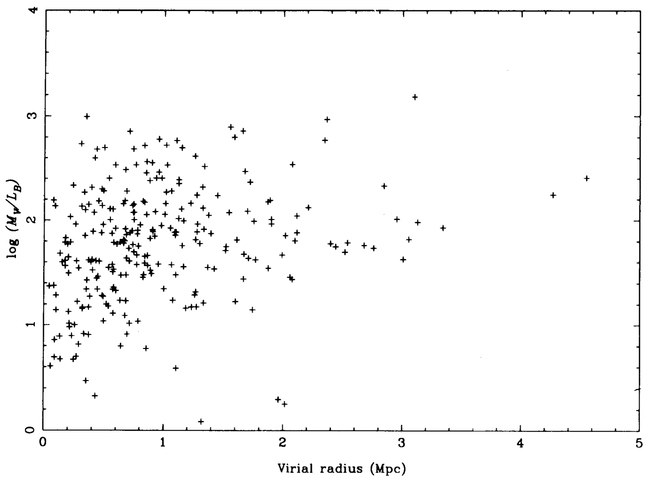

Fig. 9. Virial mass to blue luminosity ratio a a function of the virial radius. Clearly, M/L grows with R_V

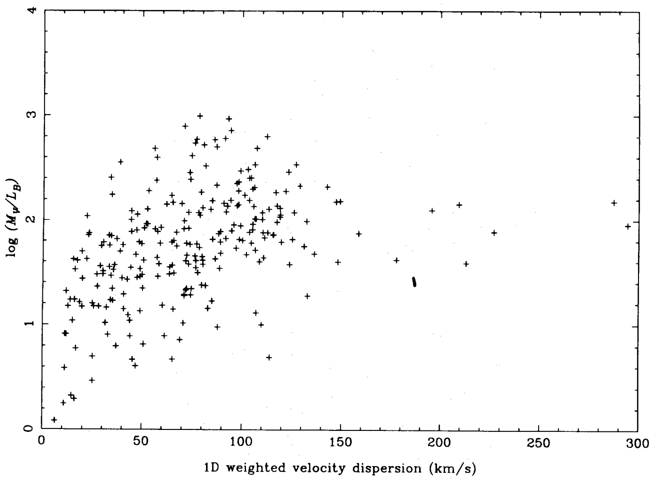

Fig. 10. Virial mass to blue luminosity ratio as a function of the 1D weighted velocity dispersion. The observed correlation could be due to the massive groups, which have important virial velocities and high M/L ratios as well

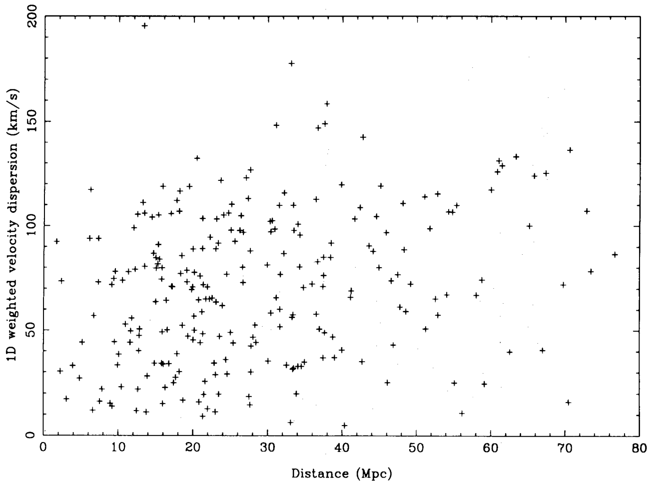

Fig. 11. 1D weighted velocity dispersion as a function of the distance. At large distances, only the groups with at least three bright members can be picked; these aggregates are massive and have consequently large velocity dispersions. This results in an artificial distance effect clearly apparent on this figure beyond 40 Mpc

artefacts due to measurement uncertainties. For our sample, these correlations are particularly clear: M_V/L_B increases from 30 to 100 when R_V increases from 50 to 400 kpc, and increases more slowly beyond 500 kpc. The most straightforward interpretation of that result is the presence of dark matter around galaxies to a distance of about 500 kpc.

5. Concluding remarks and perspectives

To conclude, we would like to propose some general comments about the grouping methods. Clearly there is no such thing as a definitely best method to make groups of galaxies. This is so

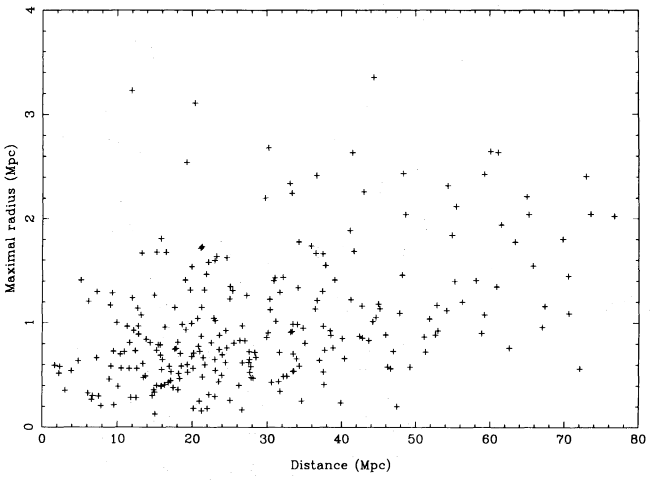

Fig. 12. Maximal radius as a function of the distance. For the same reason as explained in caption of Fig. 11, only the largest groups are picked at large distances; hence the lack of low $R_{\max}$ values beyond 40 Mpc

because any method needs the knowledge of sufficiently accurate distances between the galaxies, and such a requirement cannot be fulfilled for objects located within a given group. This lack of information introduces necessarily some uncertainty in the group membership, and the main problem for the different methods is then to keep such an uncertainty to a minimum. For the hierarchical method, a partial solution of this problem has been provided by Tully's (1987) ingenious way of estimating the distances between galaxies within or outside groups, we have used in this study in a slightly modified form. But, in turn, this procedure introduces an artificial upper limit for the velocity dispersions of the groups, as it has been pointed out in the previous section. Aggregates having velocity dispersions higher than that limit are split by the method in several parts and have then to be restored in some way. In Tully's work and in the present study, this restoration has been done by hand. However, in order to improve the hierarchical method in the future, it would be interesting to treat apart the groups found with a velocity dispersion close to the theoretical upper limit, by using less restrictive grouping parameters for them; we could then sort out automatically the possible clusters they are part of, which would evitate any subjective intervention and any possible bias towards low velocity dispersions. Doing that, one would still keep the main advantage of the method upon HG's one i.e. the absence of any interloper contamination. This characteristic, joined to the physical significance of the grouping procedure and to the easy visualisation of the groups would make of the hierarchical method a clustering technique especially attractive and efficient.

There would be three other improvements to bring to our study:

(i) A systematic study of the effect of varying the V_1 and ρ_L parameters on the whole galaxy sample. Since the hierarchical method is quite computer-time consuming, such a study has not been conducted yet.

(ii) The obtention of a complete sample of galaxies with measured redshifts, although such an improvement is not thought to change significantly the properties of the groups.

(iii) A substantial increase of the number of group members, which is still relatively limited. For this purpose, a complementary study, presently in progress thanks to allocated observing time at the European Southern Observatory and at the Nançay radiotelescope, is centered on a few selected groups, where the limiting magnitude will be pushed up. This will lead to a significant increase of the known group members, allowing a more detailed analysis.

Acknowledgements. The initial sample of galaxies has been extracted from the Base de Données Extragalactiques of Lyon Observatory, developed by George Paturel. Without his work, this study could not have been possible and we wish to tell him our warm gratitude.

Appendix A: group characteristic parameters

Various quantities have been introduced in the literature to characterize groups of galaxies and are used in the present study. To make reading easier, their accurate operational definitions are reminded here. They concern the three quantities linked by the virial theorem, namely mass, radius and velocity dispersion, and the time scale of evolution of the group, as well.

Let N be the number of group members. Each galaxy i is given a weight m_i^* proportional to its luminosity l_i in B band corrected for absorptions, and to its assumed mass-to-luminosity ratio:

$$m_i^* = l_i f(T_i) \tag{A1}$$

where $f(T_i)=1$ for spirals and $f(T_i)=2$ for ellipticals and lenticulars, in order to take into account that the ellipticals and lenticulars have generally a mass-to-luminosity ratio in B band about twice that of spirals (since they are underluminous in B band).

Let $M^* = \sum_{i=1}^{N} m_i^*$ and D the distance to the group, defined as:

$$D = \frac{1}{H_0 M^*} \sum_{i=1}^{N} m_i^* v_{ri} \tag{A2}$$

where H_0 is the Hubble constant ($H_0 = 75\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is adopted here), and v_{ri} is the radial velocity of the galaxy i, corrected to the center of our Galaxy.

In order to compute the group characteristic parameters one needs quantities like the 3*D separation* R_{ij} between two members i and j or the 3*D velocity* v_i of each galaxy. Since the observable quantities are not these but the *angular separation* θ_{ij} and the *radial velocity* v_{ri}, we use the following statistical transformation relations between the two respective sets:

$$\langle R_{ij} \rangle = \frac{4}{\pi} d_{ij} \tag{A3a}$$

$$\left\langle \frac{1}{R_{ij}} \right\rangle = \frac{2}{\pi} \frac{1}{d_{ij}} \tag{A3b}$$

$$\langle R_{ij}^2 \rangle = \frac{3}{2} d_{ij}^2 \tag{A3c}$$

$$\langle v_i^2 \rangle = 3 v_{ri}^2 \tag{A3d}$$

where $d_{ij} \equiv 2D \sin \theta_{ij}/2$ is the projected distance on the plane of the sky between the galaxies i and j.

A.1. Radius of the group

a) Virial radius R_V

R_V is defined so that the gravitational potential energy U of the group can be written:

$$U \equiv -G \sum_{i<j} \frac{m_i m_j}{R_{ij}} = -G \frac{N(N-1)}{2} \frac{\left(\frac{M}{N}\right)^2}{R_V} \tag{A4}$$

where M is the total mass of the group.

In term of observed quantities, one has:

$$R_V = \frac{N-1}{N} \frac{\pi}{2} D \frac{M^{*2}}{\sum_{i<j} \frac{m_i^* m_j^*}{\sin \theta_{ij}/2}}. \tag{A5}$$

b) Gravitational radius R_G

R_G is defined from the gravitational potential energy by:

$$U = -\frac{GM^2}{R_G}$$

so:

$$R_G = 2 \frac{N}{N-1} R_V. \tag{A6}$$

Thus R_G is nearly twice the virial radius.

c) Inertial radius R_I

R_I is defined from the total moment of inertia I with respect to the group barycenter G by:

$$I \equiv \sum_{i=1}^{N} m_i R_{iG}^2 = M R_I^2.$$

In terms of observed quantities, this parameter can be written:

$$R_I = D \left(\frac{6}{M^*} \sum_{i=1}^{N} m_i^* \sin^2 \theta_{iG}/2 \right)^{1/2}. \tag{A7}$$

d) Maximal radius R_{max}

R_{max} is defined by:

$$R_{max} = \max_{1 \le i \le N} \{R_{iG}\}.$$

The resulting operational expression is:

$$R_{max} = \frac{8}{\pi} D \sin \left(\frac{1}{2} \max_{1 \le i \le N} \{\theta_{iG}\} \right). \tag{A8}$$

In the case of a homogeneous distribution within a sphere of radius R_{max}, these various radii are related by

$$R_I = 0.77 R_{max} \tag{A9a}$$

$$R_G = 1.67 R_{max} \tag{A9b}$$

$$R_V = 0.83 \frac{N-1}{N} R_{max}. \tag{A9c}$$

A.2. Velocity dispersion of the group

In order to measure the velocity dispersion of the group, we choose the virial velocity V_V which is the *weighted* velocity dispersion:

$$V_V = \left(\frac{3}{M^*} \sum_{i=1}^{N} m_i^* (v_{r_i} - H_0 D)^2 \right)^{1/2}. \tag{A10}$$

We prefer this expression to the unweighted velocity dispersion because V_V is directly related to the total kinetic energy of the group, T, by:

$$T = \frac{1}{2} M V_V^2. \tag{A11}$$

A.3. Mass of the group

This important quantity is unknown. The most usual estimate is the virial mass, defined as:

$$M_V = \frac{R_G V_V^2}{G}. \tag{A12}$$

When the group is virialized, its mass equals the virial mass. Some authors define an unweighted virial mass (Tully 1987), arguing that it is a less noisier mass estimate.

A.4. Time scale of evolution of the group

a) Crossing time t_{cr}

The crossing time of the group has been defined by Jackson (1975) as:

$$t_{cr} = \frac{R_I}{V_V}. \tag{A13}$$

If an unbound collection of galaxies expands with the Universe, its crossing time is H_0^{-1}.

b) Collapse time t_{col}

The collapse time can be obtained from the simple model of a spherical homogeneous ball initially in expansion with the Universe and which evolves subsequently according to its own gravity only (in accordance with the Gauss Theorem). The details can be found in Gun & Gott (1972); their Eq. (26) explicitly gives:

$$t_{col} = \pi \left(\frac{6}{5}\right)^{3/2} \left(\frac{R_G^3}{2GM_V}\right)^{1/2}. \tag{A14}$$

To derive this expression, one assumes that the group is virialized; so contrary to the previous quantities, t_{col} may not always have a direct meaning (in case the group is not virialized). Numericaly, using (A6) leads to:

$$t_{col} = 1.24\ 10^{17} \left(\frac{N}{N-1}\right)^{3/2} \left(\frac{R_V^3}{M_V}\right)^{1/2}, \tag{A15}$$

where t_{col} is in years, R_V in Mpc and M_V in solar masses.

Peebles (1970) has shown numerically that the group may be considered as virialized after $1.5 t_{col}$. But let us emphasize that in case the group is not virialized – that is $M \neq M_V$ – the expression (A15) is meaningless.

Appendix B: correction for increasing incompleteness with the distance in Huchra & Geller's (1982) method

As indicated in the main text, in the galaxy samples which are complete in apparent magnitudes, there is a loss of objects increasing with the distance. Any grouping criterion has to take this effect into account, otherwise biases would appear in the group properties as a function of the distance. HG's criterion corrects for such an effect, but, despite the correction, distance dependent bias remains in the properties of their groups (Magtesyan 1988), which points towards an inappropriate correction. So we reexamine this question here.

The condition for an appropriate correction to a grouping criterion is the following: suppose a group picked by the criterion is at a distance Δ_1. Let us move this group to a distance $\Delta_2 > \Delta_1$. If the galaxy sample is complete in apparent magnitude, less galaxies of the group will then appear in the sample. Quite naturally, the searched condition is that those remaining galaxies are recognized by the criterion as an independent group (and of course the same is true if the group is put at a distance $\Delta_3 < \Delta_1$).

So let us consider a group $\mathscr{G}$ at the distance Δ_1; be i a given galaxy of $\mathscr{G}$, assumed to be still included in the sample when $\mathscr{G}$ is carried away at Δ_2. In HG's method, the membership of i to $\mathscr{G}$ means that i has at least one companion p in $\mathscr{G}$, i.e. such that:

$$d_{ip} \leq D_L(\Delta_1)$$

$$|v_{ri} - v_{rp}| \leq V_{\mathrm{L}}(\Delta_1)$$

where d_{ip} is the distance between i and p projected on the sky plane, the v_{rj} are the radial velocities and $D_{\mathrm{L}}(\Delta_1)$ and $V_{\mathrm{L}}(\Delta_1)$ are limiting fixed distances and velocities which may depend on Δ_1 in order to get the appropriate distance corrections.

Now we put the group $\mathscr{G}$ at a distance $\Delta_2 > \Delta_1$. In order that i pertains to $\mathscr{G}$ at Δ_2, it is necessary that i has still at least one companion in $\mathscr{G}$. More generally, if i has n companions in $\mathscr{G}$ when $\mathscr{G}$ is at a distance Δ_1, it is sufficient that it has still n companions when $\mathscr{G}$ is at a distance Δ_2, and one will use that condition from now on. Let us consider now $\mathscr{G}$ at a distance Δ, and let us call neighbours of i the galaxies in $\mathscr{G}$ located at a projected distance from i lower than $D_{\mathrm{L}}(\Delta)$. The number of these neighbours is:

$$p_i(\Delta) = \rho_i(\Delta) \pi D_{\mathrm{L}}^2(\Delta) L \tag{B1}$$

where L is the group dimension along the line-of-sight $\mathscr{D}_i$ passing through i and $\rho_i(\Delta)$ is the average density in sample galaxies within the part $\mathscr{R}_i$ of the group intercepted by the cone having $\mathscr{D}_i$ for axes and for solid angle $\Omega = \pi D_{\mathrm{L}}^2(\Delta)/\Delta^2$.

Among those $p_i(\Delta)$ neighbours, there are $n_i(\Delta)$ companions, i.e. having velocities v_{rn} satisfying: $|v_{rn} - v_{ri}| < V_{\mathrm{L}}(\Delta)$.

Our purpose is to find $D_{\mathrm{L}}(\Delta)$ and $V_{\mathrm{L}}(\Delta)$ such that $n_i(\Delta)$ does not depend on Δ. We show now that a sufficient condition for that property is that the number, $p_i(\Delta)$ of neighbours of i is independent of Δ.

Indeed, let us consider the velocity distribution of the $p_i(\Delta)$ neighbours; if we change Δ, the neighbours of i within $\mathscr{G}$ will be the same if $\mathscr{G}$ is a true group (i.e. not a filament gravitationally unbound). This is so since the positions and velocities of the galaxies in a group are uncorrelated. In the same way, since the velocity v_{ri} of i is fixed, the distribution of $|v_r - v_{ri}|$ for the neighbours of i will not change with Δ, therefore the proportion of neighbours having $|v_r - v_{ri}| \leq V_{\mathrm{L}}$ will not depend on Δ. Thus, there will be the same number of companions of i for $\mathscr{G}$ at different distances if there is the same number of neighbours.

Finally, in order that HG's criterion fulfils the condition imposed at the beginning, it is sufficient that $D_{\mathrm{L}}(\Delta)$ is such that the number of neighbours of any galaxy i in $\mathscr{G}$ does not depend on Δ, the limiting velocity V_{L} being independent of Δ.

From Eq. (B1), the condition can be written:

$$D_{\mathrm{L}}(\Delta) \propto (\rho_i(\Delta))^{-1/2}.$$

Now, at a distance Δ, the galaxies of the sample (which is complete to $m = m_1$) are those brighter than $M_1 = m_1 - 5 \log \Delta - 25$. The density $\rho_i(\Delta)$ of those galaxies in the region $\mathscr{R}_i$ of $\mathscr{G}$ defined hereabove is:

$$\rho_i(\Delta) = \alpha \int_{-\infty}^{M_1} \phi(M) \mathrm{d}M$$

where ϕ is the galaxy luminosity function, and α is the over-density in $\mathscr{R}_i$ compared to the average general galactic density.

Thus, finally:

$$D_{\mathrm{L}}(\Delta) \propto \left(\int_{-\infty}^{m_1 - 25 - 5 \log \Delta} \phi(M) \mathrm{d}M \right)^{-1/2} \tag{B2}$$

$$V_{\mathrm{L}} \text{ independent of } \Delta. \tag{B3}$$

In HG, $D_{\mathrm{L}}(\Delta)$ is taken proportional to $\rho_i(\Delta)^{-1/3}$, but, above all, V_{L} is taken dependent on Δ in the same way. This erroneous dependence is likely to fully account for the increase of the velocity dispersion σ_V of HG's groups with their distance found by Magtesyan (1988). Indeed V_{L} expresses more or less the highest difference one can admit between two members of a same group; so one expects that σ_V and V_{L} are proportional on an average, the proportion coefficient depending on the number of group members among others. And as a matter of fact, σ_V increases by the same factor 3 as $V_{\mathrm{L}}(\Delta)$ (scaled as in HG) in HG's groups when Δ increases from 1000 to 4000 km s^{-1}.

Now, when use is made of Eqs. (B2) and (B3) in HG's method, there is no significant dependence of σ_V with Δ, which confirms the correctness of our treatment.

References

Abell G.O., 1958, ApJS 3, 211
Bothun G.D., Geller M.J., Beers T.C., Huchra J.P., 1983 ApJ, 268, 47
Corwin H., Skiff B., 1991, Extension to The Southern Galaxy Catalogue (E.S.G.C.) (in preparation)
Fisher J.R., Tully R.B., 1981, ApJS 47, 139
Fouqué P., Gourgoulhon E., Chamaraux P., Paturel G., 1991, A&AS (in press)
Geller M.J., Huchra J.P., 1983, ApJS 52, 61
Gunn J.E., Gott J.R., 1972, ApJ 176, 1
Heisler J., Tremaine S., Bahcall J.N., 1985, ApJ 298, 8
Huchra J.P., Davis M., Latham D., Tonry J., 1983, ApJS 52, 89
Huchra J.P., Geller M.J., 1982, ApJ 257, 423
Jackson J.C., 1975, MNRAS 173, 41P
Lauberts A., 1982, The ESO-Uppsala Survey of the ESO (B) Atlas, European Southern Observatory
Magtesyan A.P., 1988, Astrophysics 28, 150
Materne J., 1978, A&A 63, 401
Materne J., 1979, A&A 74, 235
Nilson P., 1973, Acta Univ. Uppsala, Ser. V, Vol. 1

Paturel G., Fouqué P., Bottinelli L., Gouguenheim L., 1989a, A&AS 80, 299

Paturel G., Fouqué P., Bottinelli L., Gouguenheim L., 1989b, Catalogue of Principal Galaxies. In Monographies de la base de données extragalactiques n° 1. Observatoires de Lyon et de Paris-Meudon, Lyon (PGC)

Paturel G., Fouqué P., Buta R.J., Garcia A.-M., 1991,A&A 243, 319

Peebles P.J.E., 1970, AJ 75, 13

Pellegrini P.S., da Costa L.N., Huchra J.P., Latham D.W., Willmer C.N.A., 1990, AJ 99, 751

Press W.H., Davis M., 1982, ApJ 259, 449

Rood H.J., Dickel J.R., 1978, ApJ 224, 724

Schecter P., 1976, ApJ 203, 297

Tully R.B., 1980, ApJ 237, 390

Tully R.B., 1987, ApJ 321, 280

Turner E.L., Gott J.R., 1976, ApJS 32, 409

de Vaucouleurs G., 1975, in: Stars and Stellar Systems, Vol. 9, Galaxies and the Universe, ed. A. Sandage, M. Sandage, J. Kristian, Univers. of Chicago Press, Chicago, p. 557

de Vaucouleurs G., de Vaucouleurs A., Buta R.J., Corwin H.G., Fouqué P., Paturel G., 1991, Third Reference Catalogue of Bright Galaxies. Springer, New York Berlin Heidelberg (RC3)

Vorontsov-Velyaminov B.A., Arkhipova V.P., 1964, Proc. Sternberg State Astron. Inst 34, Vol. II