5. As in the earlier case, previous observations of this star published by Khaliullin et al. (1985) are shown by circles. They are consistent with this period, as well as with the other one, incidentally. Data on the newly discovered variable HD 221142 are given in Table IV.

Young (1942) gave a radial velocity −12.9 km/sec, with a probable error 2.3 km/sec, for this star. Such a small error is consistent, however, with this being a pulsating star. For a photometric amplitude $0^m.03$ in the blue band, the expected amplitude of radial velocity is about 2 km/sec, according to the data of Frolov (1970, p. 247, Fig. 79). The observations of Voroshilov and Metlov (1989) in May 1989 confirm our data nicely.

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Evolution of initially highly eccentric orbits of the growing nuclei of the giant planets

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Computer modeling supports the suggestion of Zharkov and Kozenko (1990) that if the initial orbits of the nuclei of Saturn, Uranus, and Neptune with masses of several Earth masses had been highly eccentric, their eccentricities could have decreased with evolution to the present values. The orbital eccentricity of the nucleus of Saturn could have decreased due to gas accretion, while that of the nuclei of Uranus and Neptune could have decreased due primarily to gravitational interactions with planetesimals. The investigation of models that take into account the migration toward Jupiter of some bodies initially located beyond the orbit of Saturn has shown that the nearly formed Saturn could have migrated from the zone of Jupiter, while the nuclei of Uranus and Neptune, with masses of several Earth masses, could have migrated from the zone of Saturn, moving continuously in low-eccentricity orbits.

Introduction. Zharkov and Kozenko (1990) have suggested that bodies ejected by Jupiter from its supply zone at the concluding stage of its formation played a major role in the formation of Saturn, Uranus, and Neptune. They hypothesize that the forming Jupiter ejected a massive nucleus of $\sim 5M_\oplus$ ($M_\oplus$ is the mass of the earth) from its supply zone into the supply zone of Saturn. By accreting gas and planetesimals, this nucleus became the massive proto-Saturn. After the formation of Jupiter and Saturn, some of the planetesimals from the supply zones of these planets acquired highly eccentric orbits and became the nuclei of Uranus and Neptune. Zharkov and Kozenko (1990) believe that these nuclei could have acquired hydrogen envelopes with a mass $\sim(1\text{-}1.5)M_\oplus$ while still in the zones of Jupiter and Saturn. The mass of a nucleus able to accrete gas is at least $2M_\oplus$ (Safronov and Vityazev, 1983), so the mass of such a nucleus with a hydrogen envelope must exceed $3M_\oplus$. The orbits of the planetary nuclei formed according to this hypothesis would have had rather high eccentricities (≥ 0.6) at some time, and their perihelia would have been near or inside the orbits of Jupiter or Saturn.

Calculations by Ipatov (1987a, 1989a, b) have shown that bodies from the supply zone of one planet penetrated into the supply zones of other planets. These calculations support the suggestion of Vityazev and Pechernikova (1981) that besides the nuclei of Jupiter and Saturn, other planetary nuclei with masses possibly reaching several Earth masses accumulated in the zones of Jupiter and Saturn. The question examined in this work is whether the giant planets could have acquired their present orbital eccentricities, if they had been formed from nuclei moving initially in highly eccentric orbits. We also investigate other possibilities for orbital evolution of planetary nuclei located initially in the zones of Jupiter and Saturn.

Initial data for numerical calculations and the model algorithm. Our investigations of the possible migration of the nuclei of the giant planets are based on numerical calculations of models denoted below by A, B, C, and D. The initial data for the different series of calculations (N_s is the series number) are given in Table I.

Model *A*. In model A, the nucleus and N_0 identical bodies — planetesimals — move initially about the sun in a single plane. The mass, semimajor axis, and orbital eccentricity of the initial nucleus are denoted by m_e^0, a_e^0, and e_e^0. The initial planetesimals with masses $m_0 = M_\Sigma^0/N_0$ move in orbits with eccentricities e_0. The initial semimajor axis of the ith body is determined from the formula

$$a_i^0 = \sqrt{(a_{\min})^2 + [(a_{\max})^2 - (a_{\min})^2]\, i/N_0}, \qquad (1)$$

where $i = 1, ..., N_0$, and $a_{\min}$ and $a_{\max}$ are constants to be specified. Pseudorandom numbers were used to choose the initial orbital orientations. The initial positions of the nucleus and bodies in the orbits are not specified since they are not used in this model.

The values of M_Σ^0 are selected so that the mass of the planet formed from the initial nucleus is near the mass of the actual planet. The semimajor axis and orbital eccentricity of the planet in final form are denoted in Table I by a_e^f and e_e^f. We note that model A is used to investigate only the orbital evolution of the nucleus, and not the entire process of accumulation in the supply zone. Since the orbital eccentricity of the nucleus decreases with evolution and the interaction between the bodies is not taken into account, many of the initial bodies do not reach the nucleus, but remain in the initial orbits. Thus, M_Σ^0 is not the mass of the initial material in the supply zone of the planet.

It is assumed that all objects (bodies and nuclei) outside the sphere of influence of the nucleus move in unperturbed Keplerian orbits, and a body encountering the nucleus at a distance equaling the radius r_s of its sphere of influence is absorbed by the nucleus. In modeling the encounter of two objects, the position of the slow (overtaken) object in its orbit upon entry into the sphere of influence is determined with pseudorandom numbers in the region where the distance between the orbits of these objects is less than r_s. A faster (overtaking) object is taken in its orbit at a distance r_s from the slow object so that it overtakes the latter.

The cartesian coordinates $\mathbf{R}^+$ and velocity $\mathbf{V}^+$ of the nucleus after the fall of the kth body are determined by the equations for a perfectly inelastic collision: $\mathbf{R}^+ = \mathbf{R}^-\mu_e + \mathbf{R}_k\mu_k$ and $\mathbf{V}^+ = \mathbf{V}^-\mu_e + \mathbf{V}_k\mu_k$, where $\mu_e = m_e^-/(m_e^- + m_k)$, $\mu_k = m_k/(m_e^- + m_k)$, and $\mathbf{R}^-$, $\mathbf{V}^-$, m^-, $\mathbf{R}_k$, $\mathbf{V}_k$, m_k are the radius vector, velocity vector, and mass of the nucleus and kth body upon entry into the sphere of influence. The new orbital elements of the nucleus are determined from these values of $\mathbf{R}^+$ and $\mathbf{V}^+$.

The values of p_i — a quantity proportional to the probability of contact between the nucleus and ith body ($i = 1, ..., N_b$, where N_b is the number of bodies at the time under consideration) and $p_\Sigma = \sum_{i=1}^{i=N_b} p_i$ are calculated (see below) before each contact (encounter at the distance r_s). The number i of the body reaching the nucleus is determined from the condition $\sum_{k=1}^{k=i-1} p_k < \eta < \sum_{k=1}^{k=i} p_k$, where η is a pseudorandom variable uniformly distributed over the interval $[0, p_\Sigma]$. With this definition of i, the probability of selecting the ith body is p_i/p_Σ.

The value of p_i is determined from the formula $p_i = \Delta\varphi_i/2\pi T_s$, where $\Delta\varphi_i$ is the sum of the angles (in radians) with vertex at the sun within which the distance (along the radial line with vertex at the sun) between the orbits of the nucleus and ith body is less than r_s, and T_s is the synodic heliocentric period of the nucleus and ith body. This formula for p_i was determined from the following considerations. If the orbits of the body and nucleus do not vary, then the body and nucleus, after being on a radial line with vertex at the sun, will again lie on a radial line after the time T_s. The probability that this radial line lies within $\Delta\varphi_i$ is $\Delta\varphi_i/2\pi$, which is the probability of body–nucleus contact per time T_s. It is assumed in the investigation of the interactions of the two objects that the average time between successive contacts is $T_s(2\pi/\Delta\varphi_i) = 1/p_i$. It is assumed in modeling disk evolution that after contact of the nucleus and ith body, the disk evolution time is $t = t^- + 1/Sp_i$, where t^- is the evolution time after the preceding contact and S is the number of nonzero p_k ($k = 1, ..., N_b$) before the next contact.

To calculate $\Delta\varphi_i$ for $e_m = \max\{e_i, e_e\} < 0.5$, we investigate the relation $R_i - R_e = \pm r_s$, where $R_e = a_e(1 - e_e^2)/[1 + e_e\cos(\nu - \Delta\psi_i)]$, $R_i = a_i(1 - e_i^2)/(1 + e_i\cos\nu)$, and $\Delta\psi_i$ is the angle between the directions to the perihelia of the orbits of the nucleus (whose orbital elements are denoted by the letter e) and the ith body. To determine $\Delta\varphi_i$ when $e_m \geq 0.5$, the segments of the orbits of the nucleus and body near their intersections are replaced by straight lines.

Model *B*. In model B, the evolution of the orbit of a planetary nucleus interacting with planetesimal bodies is examined in the same manner as in model A. However, besides the bodies combining with the nucleus, the gravitational interactions of the bodies with the nucleus are taken into account, and a three-dimensional model, rather than the two-dimensional, is examined. The evolution of three-dimensional disks has been investigated by an appropriate reduction of the three-dimensional evolution problem to the two-dimensional on the basis of computer models of the evolution of flat disks around arbitrary bodies of high density (Ipatov, 1987b). The density of the arbitrary bodies is taken such that the changes in the orbital eccentricities between collisions of the arbitrary bodies are approximately the same as in the evolution of the three-dimensional disk under consideration. The differences between the probabilities of contact for the two- and three-dimensional models has been analyzed by Ipatov (1988).

The density of the objects (bodies or nuclei) is assumed close to that of the present planets, and the gravitational effect is taken into account by the sphere-of-influence method. The positions of the objects in their orbits at contact (encounter at r_s) are chosen as in model A. Relative cartesian coordinates are then determined, and the relative motion of the objects within the sphere of influence is modeled as a two-body problem. During this time, the center of mass of the objects moves about the sun in an unperturbed Keplerian orbit. If the minimum distance between the centers of mass of

TABLE I. Initial Data for Calculation Series

Model *A* (body–nucleus mergers only)

N_s	a_{min}	a_{max}	e_0	M_Σ^0	N_0	e_e^0	a_e^0	m_e^0	a_e	e_e
1	5	20	0	400–999	999	0.6–0.65	13–14	10	0.03–0.05	9–10
2	5	20	0	400–999	999	0.6–0.65	12–14	5	0.05–0.06	8–9
3	10–15	40–45	0	50	999	0.67–0.75	30–40	3	0.11–0.12	20–24
4	10–15	40–45	0	75	999	0.67–0.75	30–40	2	0.08–0.09	22–23
5	10–15	40–45	0	100	999	0.67–0.75	30–40	1	0.04–0.05	18–26
6	15	45	0	100–150	999	0.75	40	0.5	0.03	23–24
7	25	45	0	50	999	0.75	40	1	0.05	31
8	25	45	0	50	999	0.75	40	0.5	0.03	30

Model *B* (mergers and gravitational effect of nucleus)

N_s	a_{min}	a_{max}	e_0	M_Σ^0	N_0	e_e^0	a_e^0	m_e^0
9	15	45	0	10^{-7}–50	100	0.67	30	3–10
10	15	45	0.4	0.05	100	0.05–0.67	30	3
11	15–19	21–45	0	0.05	100	0.05–0.67	30	3
12	15	45	0	5	100	0.67	30	0.05–0.5

Model *C* (mergers and gravitational effect of nucleus and bodies)

N_s	a_{min}	a_{max}	e_0	M_Σ^0	N_0	e_e^0	a_e^0	m_e^0
13	15	45	0	0.5–5	100	0.67	30	3

Model *D* (proto-Jupiter, proto-Saturn, and two nuclei)

N_s	a_{min}	a_{max}	e_0	M_Σ^0	N_0	e_{e1}^0	e_{e2}^0	a_{e1}^0	a_{e2}^0	m_e^0
14	15	45	0.02	100	100	0.75–0.8	0.82–0.85	30	40	3–10
15	15	45	0.02	100	300	0.75	0.82	30	40	3–10
16	15	45	0.02	100	100	0.02	0.02	6–7	7–7,5	10
17	8	32	0.02	180	450	0.02	0.02	7	7,5	3–5
18	8	32	0.02	180	450	0.02	0.02	8	10	10
19	8	32	0.02	135	675	0.02	0.02	8	10	10
20	8	32	0.02	150	750	No nuclei				

Note: Masses expressed in Earth masses, distances in AU.

the two objects becomes equal to the sum of their radii, then it is assumed that these objects merge in a perfectly inelastic collision. If a collision does not occur, then the coordinates $\mathbf{R}_i^r$ and velocities $\mathbf{V}_i^r$ of the objects emerging from the sphere of influence relative to their center of mass are determined. The coordinates $\mathbf{R}^c$ and velocity $\mathbf{V}^c$ of the center of mass upon emergence of the objects from the sphere of influence are then calculated, and the elements of the new heliocentric orbits of the objects are determined from their new coordinates $\mathbf{R}_i = \mathbf{R}_i^r + \mathbf{R}^c$ and velocities $\mathbf{V}_i = \mathbf{V}_i^r + \mathbf{V}^c$.

Model *C*. An encounter of bodies with the planetary nucleus is calculated in model *C* in the same manner as in model *B*. In addition, interactions between bodies are examined. The mutual gravitational effect of bodies is taken into account by the sphere-of-influence method. Colliding objects always merge. A deterministic algorithm (see below) is used to select the pairs of contacting objects.

Model *D*. Besides the initial disk of bodies, model *D* examines the two planetary nuclei, Jupiter and Saturn in nearly final form, and the terrestrial planets. For $N_s \geq 17$, there are small bodies in the zone of the asteroid belt. The initial orbits of the planets are circular. The initial values of the semimajor axis and mass are 5.5 AU and $300 M_\oplus$ for proto-Jupiter and 6–6.5 AU and $85 M_\oplus$ for proto-Saturn. The masses of the nuclei are the same — m_e^0. The semimajor axes and eccentricities of their orbits are denoted by $a_{e1}{}^0$, $a_{e2}{}^0$, $e_{e1}{}^0$, and $e_{e2}{}^0$. The total mass $M_\Sigma{}^0$ of the bodies in the zones of Uranus and Neptune corresponds to the initial mass of the solid material in these zones. Calculations show that the mass of the bodies ejected into hyperbolic orbits is an order of magnitude greater than the mass of the bodies reaching the planets, so it is assumed that $M_\Sigma{}^0 \geq 100 M_\oplus$. As in model *C*, a three-dimensional model is examined, and the interactions (gravitational effect and mergers) of all the objects under consideration (nuclei, planets, and bodies) are taken into account.

Deterministic algorithm for selecting of pairs of contacting objects. In the deterministic algorithm, the time of contact of the two objects (*i*th and *j*th) $\tau_{ij} = \Delta\tau_{ij} + t_b$, where $t_b = \max\{t_i, t_j\}$ and t_i is the time at which the *i*th object last participated in a contact, is calculated instead of the probability p_{ij} of contact of these objects. A precise determination of $\Delta\tau_{ij}$ would require a very large number of calculations, so approximate formulas are used. It is assumed in the two-dimensional case that

$$\Delta\tau_{ij} = 2\pi T_s \xi / \Delta\varphi_{ij} = \xi / p_{ij}, \tag{2}$$

where T_s is the synodic period of the *i*th and *j*th objects, ξ is a pseudorandom variable uniformly distributed over the interval [0, 1], and $\Delta\varphi_{ij}$ is the sum of the angles (in radians) with vertex at the sun within which the distance between the orbits of the *i*th and *j*th objects is less than the radius r_s of their sphere of influence. The formula for the calculation of $\Delta\tau_{ij}$ is found from the same arguments as the formula derived above for the calculation of p_i. In the special case of circular orbits, it can be shown that if the initial angle between the directions to the objects (from the fast object to the slow) is $2\pi\xi$, the objects will lie on a radial line with vertex at the sun after the time ξT_s. The determination that $\Delta\tau_{ij} = \infty$ requires much less time than the calculation of finite $\Delta\tau_{ij}$.

A large computer memory is required in the general case of the probability algorithm for which the interactions of all *N* objects is taken into account (Ipatov, 1978). In particular, the $N(N-1)/2$ elements of the matrix $\{p_{ij}\}$, for which $i = 1, \ldots, N-1$ and $j = i+1, \ldots, N$ (*N* is the number of objects in the disk at the time under consideration are stored, along with the $(N-1)$ values of $F_i = \sum_{j=i+1}^{j=N} p_{ij}$ and the value of $F_{\Sigma} = \sum_{i=1}^{i=N-1} F_i$.

With our deterministic algorithm, at most $(3N - 2)$ numbers are stored: the values of $\tau_i^* = \min_{j=i+1,N} \{\tau_{ij}\}$, k_i, t_i, and t_N, where $i = 1, ..., N - 1$, and k_i is the number j corresponding to the minimum τ_{ij} (for $j > i$). At the initial time, all values of τ_{ij} for $j > i$ are calculated, and the values of τ_i^* are determined from them. After each contact, the new values of τ_i^*, τ_j^*, k_i, and k_j are calculated for the separation of the ith and jth objects ($j > i$) along the new orbits. The quantities t_i and t_j then assume the value of the current time, which equals the time τ_i^* of the contact under consideration of the ith and jth objects. If these objects merge or the jth object is ejected into a hyperbolic orbit, only τ_i^* and k_i are calculated, and k_j assumes the value -1. Moreover, the new values of τ_{nj} are calculated for $n = 1, ..., j - 1$, $n \neq i$ with $k_n \geq 0$, as well as the new values of τ_{ni} for $n < i$. If the value of $\tau_n' = \min\{\tau_{nj}, \tau_{ni}\}$ ($\tau_n' = \tau_{nj}$ for $i < n < j$) obtained after the contact is less than τ_n^{*-} (where τ_n^{*-} is the value of τ_n^* calculated before the contact under consideration), then τ_n' assumes the new value τ_n^*. If $\tau_n^* = \tau_{ni}$, then $k_n = i$, or if $\tau_n^* = \tau_{nj}$, then $k_n = j$. If $\tau_n' \geq \tau_n^{*-}$ and k_n calculated before the contact is i or j, k_n assumes the value 0. The minimum value of τ_i^* among the $(N - 1)$ values of τ_q^* for which $k_q \geq 0$ is determined to select the pairs of contacting objects. If the corresponding $k_i = 0$, then τ_i^* and k_i are calculated, and the minimum among the $(N - 1)$ values of τ_q^* is again found. If $k_i \geq 1$, this means that the ith and $j = k$th objects contact. After $\sim 10^4$ contacts are modeled, the enumeration of the objects is changed, and the information about the objects for which $k_q = -1$ is removed from the data blocks τ_q^*, k_q, and t_q and from the data blocks characterizing the objects.

It has been proven that if pseudorandom numbers are not used to calculate the $\Delta\tau_{ij}$, the same pairs of contacting objects are determined in this algorithm as in the "complete sorting" method in which the entire matrix $\{\tau_{ij}\}$ is determined before each contact and the minimum element among the N^2 elements of this matrix is found. Computer calculations of the evolution of a number of disks consisting initially of a thousand identical bodies have shown that our algorithm is a factor of 1.5–2 faster than the best modifications of the virtual-contact method developed by Éneev and Kozlov (1982). In these tests, the calculation rate with this deterministic algorithm is at least 10% greater than with the probability algorithm used previously.

With the deterministic algorithm, the contacting objects are those for which τ_{ij} is the minimum value, not the average. Numerical calculations have shown that if the number of objects is not small, the rate of evolution of the disk with this algorithm is significantly greater than with the probability algorithm (by an order of magnitude for the evolution of a disk of identical particles, and by three orders of magnitude when the bodies fall onto a nucleus). The pseudorandom choice of ξ in the formula for calculation of $\Delta\tau_{ij}$ is related to the assumption of approximately random relative orientations of the orbits of the two objects in the disk. The calculations show that if the average value of ξ (i.e., ½) is taken in Eq. (2), rather than the random value, the evolution time increases by a factor of 3–10. For model C ($N_s = 13$), the ratio of the number of body–body contacts to the number of body–nucleus contacts is larger with $\xi = ½$ than with a random choice of ξ.

A detailed description of the algorithm for the evolution of a disk of bodies is given in Report N-1211 of the Institute of Applied Mathematics, USSR Academy of Sciences, for 1985. The feasibility and properties of this algorithm on a multiprocessor computer have been discussed in Appendix 3 of Ipatov (1989c).

Possible orbital evolution of the nucleus of Saturn. *Highly eccentric initial orbit of the nucleus.* Saturn accreted primarily gas, not solid objects. In moving through the gas, the planet reduced its eccentricity not only due to gas accretion, but also because of the gas drag. The planet probably acquired a lower eccentricity with gas accretion than with the fall of bodies of the same total mass, and model A can be used to evaluate the orbital evolution of the nucleus of Saturn. These calculations ($N_s = 1$ and 2) show that if Saturn formed from a nucleus moving initially in a highly eccentric orbit, then it could have acquired its present eccentricity of 0.056 with $m_e^0 \leq 10M_{\oplus}$. In particular, in one of the versions examined, the initial nucleus with mass $m_e^0 = 5M_{\oplus}$ moved in an orbit with parameters $a_e^0 = 14$ AU and $e_e^0 = 0.65$. The semimajor axes of the orbits of the bodies took values from $a_{min} = 5$ AU to $a_{max} = 20$ AU (bodies with values of a_i^0 outside this range could not collide with the nucleus). The mass of each body was $M_{\oplus}$. Upon evolution of the disk, the mass of the nucleus increased to $95M_{\oplus}$, and the semimajor axis and eccentricity of its orbit decreased to 8.4 AU and 0.034, respectively.

Slightly eccentric orbit of the nucleus. The nucleus of Saturn, initially moving along a highly eccentric orbit could have been ejected by Jupiter into a hyperbolic orbit before it acquired a low-eccentricity orbit. The migration of Saturn from the zone of Jupiter could also have occurred with an orbit of low eccentricity. The calculations with model D show that some of the bodies in the disk initially beyond the orbit of Saturn migrated toward Jupiter, which ejected most of them into hyperbolic orbits. It is found that when $M_{\Sigma}^0 \geq 100M_{\oplus}$, the semimajor axis of the nearly formed Saturn could have increased from 6–6.5 AU to 9–10 AU under the influence of these bodies. Even larger changes of the semimajor axes of Jupiter and Saturn occur for low eccentricities of their orbits, and are found for high eccentricities of the initial orbits of the nuclei of Uranus and Neptune ($N_s = 14$ and 15), for low eccentricities ($16 \leq N_s \leq 19$), and even without these nuclei ($N_s = 20$). The masses of the actual planetesimals were less than the masses of the bodies in the calculations, so their mutual gravitational effect, and consequently the change in the orbit of Saturn could have been somewhat smaller.

Possible orbital evolution of the nuclei of Uranus and Neptune. *Orbital evolution of nuclei moving initially in highly eccentric orbits under the influence of mergers.* The results of the investigation of model A are given in this section. The initial orbital eccentricities of the nuclei $e_e^0 > 0.67$, since the required values of a_e^f for the semimajor axes of the planets in final form cannot be obtained with smaller values of e_e^0. The perihelia of the initial orbits of the nuclei lie near the orbit of

Saturn. If they are taken within the orbit of Saturn, then even larger values of e_e^0 must be examined. The calculations show that with the fall of bodies onto the nucleus of Uranus with $m_e^0 = 3M_\oplus$, the forming planet acquires an eccentricity e_e^f more than twice the present value of 0.047 for Uranus. Applegate et al. (1986) have shown that the maximum values reached by the eccentricities of Uranus and Neptune over 200 Myr because of the gravitational effect of the other planets are 0.076 and 0.023, respectively. It has been found with model A for Uranus that $e_e^f \le 0.047$ for $m_e^0 \le M_\oplus$ and $e_e^f \le 0.076$ for $m_e^0 = 2M_\oplus$ ($3 \le N_s \le 5$).

Let us discuss the calculations characterizing the possibility of the formation of Neptune from a nucleus moving in a highly eccentric orbit. The required value of the semimajor axis of Neptune cannot be obtained for disks simultaneously encompassing the supply zones of Uranus and Neptune. For example, $a_e^f \le 20$ AU is obtained for a disk with $a_{min} = 10$ AU and $a_{max} = 40$ AU, while $a_e^f \le 25.5$ AU is obtained for a more remote disk with $a_{min} = 15$ AU and $a_{max} = 45$ AU. Most cosmogonists believe that the distance from the sun to the outer boundary of the bulk of the protoplanetary nebula was no more than 40–45 AU. Thus, $a_{max} = 45$ AU may be considered an upper limit. Nearly the same values of a_e^f are found in calculations with a perihelion distance of the nucleus $r_\pi \approx 10$ AU, e_e^0 varying from 0.67 to 0.95, and m_e^0 varying from $0.5M_\oplus$ to $3M_\oplus$. For a disk with $a_{min} = 15$ AU and $a_{max} = 45$ AU, we have 21.7 AU $\le a_e^f \le$ 25.5 AU. The choice of a_{min} and a_{max} has the most significant effect on the value of a_e^f. One must take $a_{min} \ge 25$ AU to obtain $a_e^f \approx 30$ AU. We note that $a_e^f \le 27$ AU is found for $a_{min} = 25$ AU and $a_{max} = 35$ AU, and $a_e^f \approx 30$ AU is found for $a_{min} = 25$ AU and $a_{max} = 45$ AU. These results indicate that to form Neptune in model A, Uranus must be nearly formed, with almost no material remaining in its supply zone, by the time of formation of the eccentric orbit of the nucleus of Neptune. It is unlikely that the large body which would become the nucleus of Neptune could have remained in the zones of Jupiter and Saturn during the formation of Uranus. It is found (N_s = 7 and 8) that $e_e^f \le 0.03$ only for $m_e^0 < M_\oplus$ in the formation of Neptune.

These calculations show that consideration of body–body mergers increases the orbital eccentricity of the forming planet. By the time of the formation of the nuclei of Uranus and Neptune, little gas remains in the supply zones of these planets, and it probably could not significantly reduce the eccentricities of the nuclei. Thus, the model investigations, which take only mergers into account, indicate that if the nuclei of Uranus and Neptune had moved initially in highly eccentric orbits, they could have acquired the present orbits and masses only if their initial masses had been no greater than $2M_\oplus$ and $M_\oplus$, respectively.

We note that in the models considered, when a body of mass $m \approx M_\oplus$ collides with a nucleus of mass $m_e \approx 3M_\oplus$ with $e_e \approx 0.6$, the kinetic energy of the nucleus with respect to the sun is about 3% less on the average after the collision with the body than the sum of the kinetic energies of the nucleus and body before their collision. For smaller values of m, the energy loss is proportional to m. The lower the orbital eccentricities of the nucleus and body, the smaller the energy loss during collision and, accordingly, the smaller the breakup and heating of the nucleus.

Orbital evolution of nuclei initially moving in highly eccentric orbits under the influence of gravitational interactions and mergers. For the three-dimensional model, no less than several thousand encounters at r_s occur per collision of objects beyond the orbit of Saturn. The calculations show that the changes in the orbital eccentricities are determined primarily by the gravitational interactions of the objects. The investigations of model B ($9 \le N_s \le 11$) have shown that in gravitational interactions between a nucleus moving in a highly eccentric orbit and bodies of significantly lesser masses, the orbital eccentricity e_e of this nucleus usually decreases, although it may increase at particular stages of evolution. It has been found that for an initial mass of the nucleus $m_e^0 = 3M_\oplus$, masses m_0 of the identical bodies from $10^{-9}M_\oplus$ to $0.5M_\oplus$, and eccentricities e_0 of the initial orbits of these bodies from 0 to 0.4 (other initial data were not investigated), $\Delta e_e^1 = \Delta e_e M_\oplus / N_{en} m_0 \sim 10^{-3}$, where Δe_e is the decrease in orbital eccentricity of the nucleus after N_{en} encounters with bodies ($N_{en} \sim 10^4$). For small values of e_0 and e_e^0, the values of Δe_e^1 may be several times larger than for high eccentricities. For $m_e^0 = 10M_\oplus$, Δe_e^1 is about two times smaller, but the rate of change of e_e is about the same as for $m_e^0 = 3M_\oplus$. The value e_* to which e_e decreases stably is smaller, the smaller the value of m_0. It has been found that $e_* \approx 0.15$ for $m_0 \sim 0.5M_\oplus$ and $e_* \approx 0.05$ for $m_0 \sim 0.05M_\oplus$. Similar results have been obtained with the additional consideration of body–body interactions (model C). The fact that Neptune has the lowest eccentricity of the giant planets may be related to the masses of the largest planetesimals in its supply zone being smaller than in the supply zones of the other giant planets. We note that in those versions of model D in which Jupiter and Saturn do not encounter nuclei whose masses are several Earth masses, the eccentricities of the final orbits of Jupiter and Saturn are no more than 0.01.

If the mass of the nucleus is taken close to the mass of the bodies (N_s = 12), its orbital eccentricity may increase, and with consideration of the long time interval, the average value of this eccentricity approaches the average eccentricity of the orbits of the bodies, which usually increases with evolution.

One can conclude from analysis of the results of model C, that for $M_\Sigma^0 = 100M_\oplus$, the orbital eccentricity of the nucleus decreases over a $\sim 10^6$ yr, while the average eccentricity of the orbits of the bodies increases by 0.05 through their interactions with the nucleus. In model D (N_s = 14 and 15), a decrease in orbital eccentricity is found for those nuclei which ceased to approach Jupiter and Saturn before they could be ejected into hyperbolic orbits. The major changes in the average orbital eccentricities of the bodies then occur because of their mutual gravitational effect, and not through interactions with the nuclei. After a decrease in eccentricity of the nucleus to 0.1, about twice the time is required for its semimajor axis to increase from 20 AU to 30 AU. Over the time during which the major changes in the orbit of the nucleus occur, its mass increases by less than $M_\oplus$.

Orbital evolution of planetary nuclei moving in slightly eccentric orbits. Investigations with model D have shown that if the orbit of the nucleus intersects the orbit of Jupiter or Saturn, the probability of ejection of the nucleus into a hyperbolic orbit is an order of magnitude greater than the probability that the nucleus becomes an accumulation center for Uranus or Neptune. Thus, the probability that both these planets formed from nuclei moving initially in highly eccentric orbits is small. Nuclei moving in slightly eccentric orbits beyond the orbit of Saturn have the greatest chance to increase their semimajor axes to 20–30 AU and to become Uranus or Neptune. The results of model *D* show that this increase in the semimajor axes of nuclei could have occurred at low eccentricities under the influence of bodies that have migrated from the supply zones of Uranus and Neptune toward Jupiter. It is found in the three-dimensional models examined that the total mass of the bodies falling onto the nucleus is usually no more than $M_\oplus$, and to form the present masses of Uranus and Neptune, one must examine nuclei with initial masses $\lesssim 10 M_\oplus$ or assume that the bodies merge in encounters at distances at least an order of magnitude greater than the sum of their radii.

Let us stress again the results found with model *D*. The major changes in the orbits of the nuclei occur over $\sim 10^6$ yr, with the increase in the semimajor axis a_e of the nucleus from 20 AU to 30 AU requiring twice the time as for the increase to 20 AU. The total time for the evolution of the disk is $\sim 10^8$ yr. The total mass of the bodies ejected into hyperbolic orbits is an order of magnitude greater than the mass of the bodies remaining in the solar system. A body or nucleus ejected by Jupiter is usually located within the orbit of Jupiter for some time before ejection, with aphelia lying near the orbit of Jupiter. The gravitational effect of these bodies could have reduced the number of asteroids by an order of magnitude. The total mass of the bodies falling onto Jupiter and Saturn is an order of magnitude greater than that onto the planetary nuclei, with several times more falling onto Jupiter than onto Saturn.

Orbital evolution of nuclei with a large number of planetesimals. In the investigation of model *D*, the initial number of bodies beyond the orbit of Saturn was no more than 750, significantly less than the actual number of planetesimals, and the masses of the bodies were significantly greater than the average masses of the planetesimals. Given the smaller masses of the bodies and considering the impossibility of mergers of bodies of similar mass at the high speeds of collisions, the formation of additional planets in the region of the asteroid belt as found in the calculations can probably be avoided. The evolution of the semimajor axes of the planets or their nuclei depends primarily on the total mass of the bodies migrating toward Jupiter. The orbital eccentricities acquired by the planets or their nuclei with evolution are smaller, the smaller the masses of the bodies. For smaller masses of the planetesimals, the rate of growth of their average eccentricity e_{av} because of their mutual gravitational effect is smaller. In analyzing computer models of the evolution of a number of disks around identical bodies of mass m, Ipatov (1989b) found that the average change in the eccentricity per encounter $\Delta e \propto m^{1/2}$. Taking into account that $p_{ij} \propto r_s^2 \propto m^{4/5}$ in the three-dimensional case, and the probability of the selection of the ith object $p_i \propto p_{ij}/m \propto m^{-1/5}$, we have that in the evolution of a three-dimensional disk consisting of N identical bodies, the rate of growth of e_{av} is proportional to $N^{3/10} \approx N^{1/3}$, i.e, for an increase in N by a factor of 10^6, this rate decreases by a factor of 100. The evolution of a disk of differing bodies is determined by the masses of the largest bodies, and the dependence of e_{av} on N is weaker. One would expect that the evolution times of the orbits of the nuclei and the total mass of the bodies falling onto them could have actually been an order of magnitude greater than in the calculations, and the initial masses of the nuclei of Uranus and Neptune could have been ~ 3–$5 M_\oplus$. It is found in the calculations that the time over which the nuclei acquire a slightly eccentric orbit with a semimajor axis $a_e \approx 20$ AU is about the same for high and low initial eccentricities of the nuclei. Since the time for decrease of the eccentricity of the nucleus is nearly independent of the masses of the bodies, it could actually have been an order of magnitude less than the time for the change of a_e to 20 AU for the slightly eccentric orbit of the nucleus. The changes in the average eccentricity e_{av} of the orbits of the bodies through their interaction with the nucleus depend primarily on the total mass of the bodies. Thus, the nuclei make a much greater contribution to the rate of change of e_{av} for large N than for $N < 1000$. In model *D*, the masses of the initial bodies are comparable to the masses of the terrestrial planets, and with evolution, these planets acquire high eccentricities and may be ejected from the solar system. Bodies of small mass, in interacting with these planets, actually reduce their orbital eccentricities.

These results indicate that nuclei of Uranus and Neptune initially several Earth masses in size could have migrated from the zone of Saturn, moving all the while in slightly eccentric orbits. The nucleus of Saturn with an initial mass up to several dozen Earth masses could also have migrated from the zone of Jupiter. Besides the nuclei of Uranus and Neptune, other smaller bodies could have migrated from the zones of Jupiter and Saturn into the zones of Uranus and Neptune.

Our investigations do not rule out the possibility of the formation of the nuclei of Uranus and Neptune directly in the supply zones of these planets. For example, massive gas–dust aggregates later becoming the nuclei of Uranus and Neptune could have been formed in the zones of these planets.

V. N. Zharkov has suggested that the nuclei of Uranus and Neptune, after forming in the supply zones of these planets, could have acquired hydrogen envelopes by accumulation of bodies containing hydrogen. This suggestion is supported by our calculations showing that the total mass of the planetesimals from the zones of Jupiter and Saturn reaching the supply zones of Uranus and Neptune could have reached dozens of Earth masses. These planetesimals would also have affected the accumulation of bodies beyond the orbit of Saturn.

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Linear stability analysis of double-flow accretion

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Local analysis within the framework of linear stability theory shows that double-flow accretion can lead to the development in accretion disks of instability of the acoustic-resonance type.

Introduction. Studies of the possible development of various instabilities in accretion disks (AD) in close binary systems (CBS) constitute an urgent task, especially as applied to the problem of eddy viscosity and to ascertaining the nature of the wide spectrum of variabilities in AD. The overflow of matter in CBS from an optical star to a compact object can take place in the regime of double-flow accretion (Lipunov, 1980). In this regime the rotation velocity of the matter in the disk is many times greater than the radial velocity, whereas above the AD, matter in the form of a stellar wind has, on the other hand, a low orbital velocity compared to the radial velocity. Some situations leading to such a regime are considered by Kolykhalov and Syunyaev (1979) and by Lipunov (1987). For us, the determining factors here are, firstly, the presence at the transition from the disk to the wind region of a zone in which the gas velocity vector varies greatly and, secondly, the essentially supersonic nature of the flow. Note that a similar situation can also occur in supercritical disk accretion (Shakura and Syunyaev, 1973) – matter from the inner accretion zone of the system flows out as a quasispherical stream under the influence of radiation pressure.

In a model of two plane-parallel supersonic tangential discontinuities (TD) of velocity, instabilities of the acoustic-resonance type can develop (Ferrari et al., 1982; Hardee and Norman, 1988). Kolykhalov (1983) investigated an instability of this type using a model of TD and a rigid wall. However, it is impossible to apply directly to AD the results obtained in a model of two TD. This is because the width of the transition zone from disk to stellar wind is comparable to the thickness of the disk $2h$, and, consequently, the z-dependence of the velocity of matter v_0, its density ρ_0, and the vertical component of the force of gravity g of the central star may affect considerably the instability parameters, right up to stabilization (Chandrasekhar, 1962). Moreover, in a model of two TD is it essentially impossible to construct an equilibrium model correctly taking into account the force of gravity.

Here we will investigate the possibility of the development of instability of the acoustic-resonance type within the framework of the standard AD model of Shakura and Syunyaev (1973), allowing for vertical nonuniformity of the equilibrium parameters; viscosity, self-gravitation, and magnetic fields will be neglected.

Equilibrium model. Since according to the standard model the AD is thin ($h << r$), we can limit ourselves to a consideration of the region $|z| << r$. Let us transform to a local Cartesian coordinate system rotating with angular velocity $\Omega = (GM/r^3)^{1/2}$ at a distance r from a compact object, such that the plane $z = 0$ coincides with the symmetry plane of the system and the x axis is in the direction of the stellar-wind velocity v_w. In view of the condition $h << r$ we can assume that the equilibrium parameters of the system do not vary in x or y.

Let us consider a flow of compressible gas which is in hydrostatic equilibrium along the z coordinate:

$$\frac{1}{\rho}\frac{dP}{dz} = g(z) = -\frac{GM}{(r^2+z^2)^{3/2}}z. \tag{1}$$

The density distribution is taken to be

$$\rho_0(z) = \rho_0(0)\,[(1-\varepsilon)\exp(-z^2/h^2)+\varepsilon],$$

where $\varepsilon = \rho_0(\infty)/\rho_0(0) = \rho_w/\rho_d$ is in general small, and as a model for the distribution of the matter velocity $v_0(z)$ we take a continuous piecewise-linear profile (two-dimensional Couette flow). Figure 1 shows the density of matter and its velocity as functions of z. $L_v = 2d$ characterizes the width of the transition zone from disk to wind for the distribution $v_0(z)$.

Fundamental equations. Let us consider the dynamics of small perturbations against the background of our equilibrium model. Because of the uniformity of the equilibrium state in the x and y directions, each of the perturbed hydrodynamic quantities can be represented as a function $f(\mathbf{r}, t) = f(z)\cdot\exp(-i\omega t + ik_x x + ik_y y)$. The equations for the displacement of matter from the equilibrium position $\xi(z)$ and the pressure $p(z)$ have the form (Morozov et al., 1976):

$$d\xi/dz = Ap + B\xi, \tag{2}$$

$$dp/dz = C\xi - Bp, \tag{3}$$

where $A = (k^2/\hat{\omega}^2 - 1/c_s^2)/\rho_0$, $B = g/c_s^2$, $\hat{\omega} = \omega - \mathbf{k}\mathbf{v}_0 = \omega - kv_0\cos\alpha$, $C = \rho_0(\hat{\omega}^2 + g^2/c_s^2 + gd(\ln\rho_0)/dz)$, $k^2 = k_x^2 + k_y^2$ and c_s is the adiabatic velocity of sound. It should be noted that in system (2), (3), Coriolis forces are not taken into account. The correctness of this approach is based on the fact that the buildup of acoustic resonance takes place at short-wavelength ($kr >> 1$) high-frequency harmonics, so

