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HUBBLE SPHERES AND PARTICLE HORIZONS

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ABSTRACT

Isotropy of the cosmic microwave background radiation highlights the horizon problem. Analysis of the horizon problem and its inflationary solution requires that we study the properties of the Hubble sphere and distinguish between the "Hubble surface" and the particle horizon. All regions in the visible universe become causally connected when inflation increased the distance of the particle horizon by a factor of 3. If exponential inflation occurs because of a phase transition at temperature $\sim 10^{15}$ GeV, the number of *e*-foldings to effect this threefold increase in the particle horizon distance is $N \simeq 60$. The properties of the Hubble sphere provide the answer to the question: How, in a universe of age *t*, can regions separated by distances much greater than *ct* be causally connected?

Subject headings: cosmology — relativity

1. INTRODUCTION

How can the global uniformity of the universe be explained when most regions now observed apparently lacked causal interconnection in the past? This is the horizon problem. Generally, in decelerating homogeneous and isotropic universes (such as the Friedmann versions), the range of causal interaction expands faster than the universe, and at zero time all parts of the universe were causally unconnected.

In the flat Friedmann (Einstein–de Sitter) universe, for example, two galaxies of redshifts greater than 1.25 in opposite directions of the sky were causally unconnected at the time they emitted the light that we now see; each existed outside the particle horizon of the other, and observers in both galaxies were unaware of the existence of the other galaxy. Yet both galaxies are composed of identical atoms in similar proportions and both occupy regions that exhibit similar astronomical structures. The remarkable isotropy of the 2.7 K cosmic microwave background radiation (Davies et al. 1987, Partridge 1988, and Smoot et al. 1991) highlights the horizon problem: regions separated more than ~ 1 arcdegree were causally unconnected at the time when the radiation decoupled at redshift $z_d \simeq 1000$. Cosmology offers two alternative solutions: the universe begins in a perfectly uniform state of preestablished harmony (made plausible by arguments such as the anthropic principle), or the universe evolved through an early period of accelerated expansion during which causal connections became greatly distended. The second solution is currently more popular in the form of the inflationary universe proposed by Guth in 1981. Ellis & Stoerger (1988) and Hübner & Ehlers (1991) have examined the horizon problem in homogeneous and isotropic inflationary universes and dispelled several previous misapprehensions.

In this paper I discuss the Hubble sphere and the observable universe and distinguish between the "Hubble surface" that bounds the Hubble sphere and the particle horizon that bounds the observable universe. I show that a causal explanation of the isotropy of the microwave background radiation requires at least a threefold enlargement in the particle horizon of the standard model.

If nothing travels through space faster than light, and the age of the universe is *t*, how can regions separated by distances much greater than *ct* have causal connection? This study shows that the causal mechanism depends on the properties of the Hubble sphere.

2. HUBBLE SPHERES

2.1. *Expanson of the Hubble Sphere*

The velocity-distance law:

$$V = H \times \text{distance}$$

where $H = \dot{R}/R$ is the Hubble term and $R(t)$ is the scale factor, follows from the Robertson-Walker metric and is the consequence of postulating time-invariant homogeneity.[1] The Hubble sphere in an expanding ($H > 0$) universe, with the observer at the center, at time *t* has radius

$$L_H = c/H \,, \tag{1}$$

and $L_H \simeq 10^{10}$ lt-yr at the present time t_0. This three-dimensional sphere contains all astronomical systems at the time of observation that, according to the velocity-distance law, recede from the observer at less than the velocity of light *c*. Recession inside the Hubble sphere is subluminal and outside is superluminal. The boundary of the Hubble sphere, the "Hubble surface," separates the subluminal inner sphere from the superluminal outer sphere. Light emitted toward the observer by galaxies inside the subluminal (or Hubble) sphere approaches the observer, whereas light emitted toward the observer by galaxies in the superluminal sphere (or outside the Hubble sphere) recedes (Harrison 1981).

In this representation of the Hubble sphere, spacetime is physically real with observable geometric and dynamic properties in accordance with general relativity. Cosmic symmetry decomposes spacetime into a uniformly curved space and an orthogonal common (or cosmic) time. Bodies move in this space locally no faster than *c* in accordance with special relativity, but space itself is dynamic locally and globally. Motion in the cosmic setting is absolute, and comoving bodies are stationary in a space that is itself nonstatic.

The Hubble surface recedes at radial velocity

$$dL_H/dt = c(1 + q) \,, \tag{2}$$

[1] The redshift is $z = R_0/R - 1$, and the redshift-distance or Hubble law $zc = H \times$ distance, derived by Hubble from observations, follows from the velocity-distance law ($V = cz$) and is valid only for small redshifts.

where $q = -\ddot{R}R/\dot{R}^2$ is the deceleration term. The Hubble sphere contracts when $q < -1$, remains stationary when $q = -1$, and expands when $q > -1$. The Hubble sphere, despite its interesting properties and conceptual importance, has received scant attention in the literature.

2.2. *The Hubble Surface*

Galaxies at the Hubble surface recede at c, and the surface overtakes the galaxies at relative velocity cq according to equation (2). The Hubble surface is not a cosmological horizon except when it becomes degenerate with the particle horizon at $q = 1$ and with the event horizon at $q = -1$.

2.2.1. *Decelerating Expansion*

In decelerating universes ($q > 0$), the Hubble surface recedes faster than the galaxies and the baryonic mass of the Hubble sphere increases. A galaxy outside the Hubble sphere, receding at velocity greater than c, is overtaken in the course of time by the Hubble surface; the galaxy then lies inside the Hubble sphere and recedes at a velocity less than c. Galaxies at distances $L > L_H$ are later at $L < L_H$, and their superluminal recession in the course of time becomes subluminal. The light emitted toward the observer by a galaxy outside the Hubble surface recedes until overtaken by the Hubble surface, and only then can it begin to approach the observer,. Thus most events are in principle observable at some time, and all decelerating universes lack event horizons unless they terminate at some future time.

2.2.2. *Linear Expansion*

In linearly expanding universes ($q = 0$), the Hubble surface comoves and the baryonic mass of the Hubble sphere stays constant. These universes lack particle and event horizons.

2.2.3. *Accelerating Expansion*

In accelerating universes ($q < 0$), the galaxies recede faster than the Hubble surface and the baryonic mass of the Hubble sphere decreases. (In the steady state and inflationary universes, the continuous creation of energy with a McCrea-type equation of state [McCrea 1951] violates this rule.) All accelerating universes, including universes having only a limited period of acceleration, have the property that galaxies at distances $L < L_H$ are later at $L > L_H$, and their subluminal recession in the course of time becomes superluminal. Light emitted outside the Hubble sphere and traveling through space toward the observer recedes and can never enter the Hubble sphere and approach the observer. Clearly, there are events that can never be observed, and such universes have event horizons.

2.3. *Examples*

The power-law models, in which the scale factor $R(t)$ varies according to t^n, with n constant, illustrate the behavior of the Hubble sphere. We have

$$H = n/t , \tag{3}$$

$$q = (1 - n)/n , \tag{4}$$

and the Hubble sphere of radius $L_H = ct/n$ expands at velocity $dL_H/dt = c/n$. In the matter-dominated Einstein–de Sitter universe of $n = \frac{2}{3}$, we find $q = \frac{1}{2}$, the Hubble sphere expands at velocity $3c/2$, and its surface overtakes the comoving galaxies at relative velocity $c/2$. In the radiation-dominated version of the Einstein–de Sitter universe of $n = \frac{1}{2}$, we find $q = 1$, the Hubble sphere expands at velocity $2c$, and its surface overtakes all comoving regions at relative velocity c.

When the scale factor $R(t)$ increases exponentially (H is constant),

$$R = R_0 \exp H(t - t_0) , \tag{5}$$

as in the de Sitter, steady state, and inflationary universes, and $q = -1$, the Hubble sphere has a constant radius. Comoving bodies cross the Hubble surface at velocity c, and light emitted toward the observer by these bodies at the instant of crossing remains stationary at the Hubble surface; this light reaches the observer in the infinite future with infinite redshift. All events outside the Hubble sphere can never be observed, and the Hubble surface acts as an event horizon.

3. PARTICLE HORIZONS

The subject of cosmological horizons was confusing (North 1965) until Rindler (1956) in a classic paper distinguished between event and particle horizons. In this paper we comment on particle horizons in homogeneous and isotropic universes; a general discussion on horizons is given by Hawking & Ellis (1973). An observer's particle horizon (a more appropriate name would be "world-line horizon") divides all world lines into two classes at the instant of observation: those that intersect the observer's past light cone, and those that lie outside the reach of the observer's light cone, as shown in Figure 1. The particle horizon forms a spherical surface in space about the observer, enclosing the "observable universe" that consists of all world lines that in principle can be observed. The observable universe includes not only visible regions but also regions made invisible by obscuration and light scattering (Sato 1968) and also the early universe that exists before the background radiation decouples (or before the background neutrinos decouple when they become detectable). The "visible universe," as distinct from the observable universe, is the region about the observer extending out in space and back in time to the primordial plasma at the decoupling redshift z_d.

3.1. *Past Light Cone*

Conformal coordinates and diagrams illustrate clearly the properties of cosmological horizons (Penrose 1964; Centrella 1973; Hawking & Ellis 1973; Tipler, Clarke, & Ellis 1980; Harrison 1981; MacCallum 1983; Ellis & Stoergert 1988). The conformally flat homogeneous and isotropic Robertson-Walker metric has the form

$$ds^2 = -dt^2 + R^2(t)[dr^2 + f^2(r)(d\theta^2 + \sin^2\theta\, d\phi^2)] , \tag{6}$$

where r, θ, ϕ are comoving space coordinates and $f(r) = \sin r$, r, or $\sinh r$ correspond, respectively, to the curvature-constant values $k = 1$, 0, or -1, We assume that all rays travel at the vacuum speed of light c and ignore dispersion, scattering, deflection, and absorption (Harrison 1977). Null geodesics ($ds = 0$) radiating toward and away from the observer's world line at $r = 0$ (hence $d\theta = 0$, $d\phi = 0$) obey $dr = dt/R$ (the future light cone) and $dr = -dt/R$ (the past light cone). The observer's past light cone in conformal coordinates is

$$r = \tau_0 - \tau , \tag{7}$$

where the zero subscript denotes the present time, and the lower limit of the integral

$$\tau = \int dt/R(t) , \tag{8}$$

is either $t = 0$, say, for a universe of finite age, or $t = -\infty$ for infinite age. The proper distance ($R \times$ coordinate distance) from the observer's world line to the past light cone at the time τ_0 of observations is

$$L(\tau) = R_0(\tau_0 - \tau) . \quad (9)$$

In universes of constant $q > 0$, this equation has the useful redshift form

$$L(z) = L_H q^{-1}[1 - (1 + z)^{-q}] . \quad (10)$$

The distance from the observer's world line to a galaxy at the time it emits the light now observed is

$$l(\tau) = R(\tau_0 - \tau) , \quad (9a)$$

and this distance has a maximum value $l_{max} = cR/\dot{R}$ (from $dl/d\tau = 0$). Galaxies at l_{max}, at the time when they emit the light now seen (and the Hubble term is $H_{max} = c/l_{max}$), recede at the velocity of light. In universes of constant q, these maximum emission–distance galaxies have redshift

$$z_{max} = (1 + q)^{1/q} - 1 , \quad (10a)$$

and all other galaxies are closer at the time they emit the light now seen. For $q = \frac{1}{2}$, we find $z_{max} = 1.25$, and for $q = 1$, $z_{max} = 1$.

In a static universe (R constant), a particle horizon exists if the universe originates in the finite past. In conformal coordinates, horizons in static and nonstatic universes can be treated similarly; hence in a nonstatic universe, a particle horizon exists if the universe originates in the finite past in conformal time[2] ($\tau = 0$, say). No particle horizon exists in universes that originate in τ-time in the infinite past.

[2] The event horizon is also easily defined in conformal time. An observer's event horizon divides all events into two classes: those observed at some time, and those never observed. It is the terminal null geodesic on the observer's world line. An event horizon exists when conformal time τ has a finite upper bound, as in the $k = 1$ Friedmann and $n > 1$ universes; no event horizon exists when τ extends to $+\infty$, as in the $k = 0, -1$ Friedmann and $n < 1$ universes.

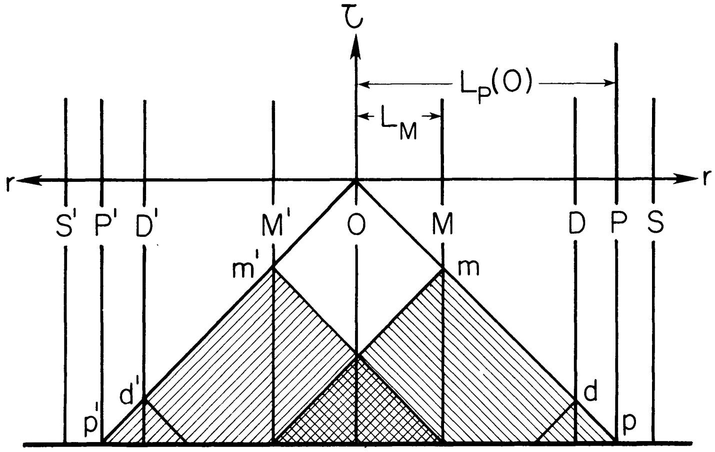

FIG. 1.—A conformal coordinate spacetime diagram of a standard Robertson-Walker universe that shows the observer's past light cone intersecting the vertical world lines of comoving bodies. Decoupling of the cosmic background radiation occurs on world lines, D, D′ at events d, d′. World lines D, D′ form the boundary of O's visible universe. The particle horizon $L_P(0)$ on world lines P, P′ forms the boundary of O's observable universe. World lines S, S′ lie beyond the particle horizon and outside the observable universe. Events m, m′ on world lines M, M′ are seen in opposite directions at distance L_M; when these world lines are at one-third the distance to the particle horizon, they are causally connected at maximum distance from the observer. Thus bodies in opposite directions at distances greater than $L_P(0)/3$ are causally unconnected.

3.2. *Recession of the Particle Horizon*

The particle horizon lies at the world lines intersecting the light cone at maximum coordinate distance $r_P = \tau_0$, given by $\tau = 0$ in equation (7), and has proper distance $L_P = R_0 r_P$, or

$$L_P = R_0 \tau_0 . \quad (11)$$

This is the radius, measured from the observer, of the observable universe; at its boundary (the particle horizon) the redshift is infinite if $R = 0$ at $\tau = 0$.

The particle horizon recedes at velocity

$$dL_P/dt = HL_P + c . \quad (12)$$

Comoving particles at the horizon recede from the observer at velocity HL_P, and the horizon itself recedes at velocity $HL_P + c$. The general rule is that in all homogeneous and isotropic universes, either expanding, static, or contracting, the particle horizon overtakes comoving bodies at velocity c. The mass of the observable universe always increases; in effect, as the universe ages, we see more of it. Only by reversing time can bodies move outward across the horizon and leave the observable universe. This is the principal difference between the particle horizon (which acts like a one-way membrane) and the Hubble surface (which acts like a two-way membrane). The not uncommon remark that inflation sweeps particles (for example, monopoles) beyond the horizon comes from a failure to distinguish between the different properties of the Hubble surface and the particle horizon.

3.3. *General Properties of the Particle Horizon*

For universes of constant q, equations (11) and (12) become

$$L_P = L_H/q , \quad (13)$$

$$dL_P/dt = c(1 + q^{-1}) . \quad (14)$$

When $q \leq 0$, no particle horizon exists, and the light cone extends to $\tau = -\infty$ and intercepts all world lines in the universe. When $0 < q < 1$, the Hubble sphere lies inside the observable universe ($L_H < L_P$) and bodies receding at velocity c at the Hubble surface have finite redshift. When $q > 1$, the observable universe lies inside the Hubble sphere ($L_H > L_P$) and bodies of infinite redshift recede at velocity less than c. This illustrates the important fact that the cosmological and Doppler redshifts are distinctly different (Harrison 1981).

For constant q, the redshift of bodies at the Hubble distance is

$$z_H = (1 - q)^{-1/q} - 1 , \text{ for } 0 < q \leq 1 , \quad (15a)$$

$$z_H = e - 1 = 1.718 , \text{ for } q = 0 . \quad (15b)$$

For example, in the matter-dominated Einstein–de Sitter universe of $q = \frac{1}{2}$, equations (13)–(15) give $L_P = 2L_H$, $dL_P/dt = 3c$, and $z_H = 3$; and in the radiation-dominated version of $q = 1$, they give $L_P = L_H$, $dL_P/dt = 2c$, and $z_H = \infty$, and the particle horizon and Hubble surface are coincident and both recede at velocity $2c$.

The conformal-coordinate spacetime diagram Figure 1, which graphically resembles the Minkowski diagram, shows the cutoff nature of the particle horizon. The observer's past light cone terminates at world lines P, P′, of proper distance $L_P = R_0 r_P$, at events p, p′ of infinite redshift. Observed world lines M, M′ that intersect the light cone are inside the particle horizon, whereas the unobserved world lines S, S′ are outside the particle horizon. Any two world lines separated a coordinate distance less than r_P may causally interact: each may observe and influence the other.

Particle horizons exist in all decelerating (big bang, Friedmann, and $n < 1$) universes. No particle horizon exists in the accelerating (de Sitter steady state, and $n > 1$) universes of continual accelerated expansion. A particle horizon, however, once created, cannot be eliminated (except in a closed universe, see below), and a limited period of accelerated expansion, as in the inflationary universe, cannot eliminate a particle horizon created in a preinflationary period of deceleration.

3.4. *The Horizon Problem*

The horizon problem is the following: observer O at time t_0 sees in opposite directions astronomical systems M and M′, as in Figure 1, and attributes their similarity to a previous history of causal interaction. World lines M and M′, however, can have interacted with each other prior to observation only if their coordinate distances are less than $r_P/3$. Hence,

$$L_M = L_P/3 , \quad (16)$$

is the maximum distance for causal connection between bodies observed in opposite directions, where L_P is the particle horizon distance. Only 1/27 of the volume of the observable universe (if flat) contains the observer and bodies that have interacted with one another.

In constant q models, the redshift for maximum L_M given by equation (16) is

$$z_m = (3/2)^{1/q} - 1 , \quad (17)$$

and $z_m = 1.25$ when $q = \frac{1}{2}$, and $z_m = 0.5$ when $q = 1$. Galaxies in opposite directions at redshifts greater than z_m have not seen each other at the time they emit the light we now see. The cosmic background radiation decouples at d, d′ on world lines D, D′, and the number of horizon distances separating d, d′ is

$$2(\tau_0/\tau_d - 1) = 2[(1 + z_d)^q - 1] \simeq 60$$

for $q = 0.5$, showing clearly that events d, d′ are causally disconnected in the standard model.

A closed universe ($k = 1$) has the interesting property that a particle horizon can contract into the antipode and the entire universe then becomes observable. As the particle horizon, recedes from the observer, it approaches and contracts on the antipode at $r = \pi$. When $\tau_0 > \pi$, all world lines in the universe intersect at least once the observer's past light cone and the observable universe is unbounded.

4. INFLATION AND THE PARTICLE HORIZON

According to the inflationary hypothesis, exponential expansion occurs in the very early universe, beginning at a grand unified temperature $T_i \sim 10^{15}$ GeV at time $t_i \sim 10^{-35}$ s (Guth 1981). A false-vacuum equation of state that mimics the properties of a large-value cosmological constant drives the exponential expansion:

$$R = R_i \exp H_{\rm inf}(t - t_i) . \quad (18)$$

Inflation terminates at time t_f, after N e-folding periods:

$$R_f = R_i e^N , \quad (19)$$

and a phase transition in the distended supercooled false vacuum creates high entropy. At present, the parameter N is imprecisely determined (Kolb & Turner 1990). In this discussion the curvature k/R^2 is neglected; this will not affect our main conclusions.

4.1. *The Particle Horizon during Inflation*

Generally, inflation of the scale factor is enormous, and some authors have erroneously remarked that monopoles are swept beyond the horizon. The monopoles are swept out of the Hubble sphere but always remain inside the particle horizon. After N e-foldings of inflation, the particle horizon lies at distance

$$L_P = L_H(2e^N - 1) , \quad (20)$$

where $L_H = c/H_{\rm inf} \sim 10^{-35}$ cm is the fixed radius of the Hubble sphere during inflation. The particle horizon recedes at velocity

$$dL_P/dt = 2ce^N , \quad (21)$$

whereas comoving particles at the horizon recede at velocity

$$H_{\rm inf} L_P = c(2e^N - 1) , \quad (22)$$

which is c less than the velocity of recession of the horizon, in agreement with the general rule. Clearly, the particle horizon outspaces everything and particles, once captive in the observable universe, never leave. A comoving particle at the horizon, when traced back to the commencement of inflation at $L_P = L_H$, lies at distance $L_H(2 - e^{-N})$, or $2 - e^{-N}$ times the distance of the horizon at that time.

4.2. *Solution of the Horizon Problem*

Let L_D denote the radius of the present visible universe out to the decoupling redshift z_d. A necessary condition that all particles inside the visible universe have interacted with one another because of inflation is

$$L_D < L_P(N)/3 , \quad (23)$$

where $L_P(N)$ is the distance of the particle horizon after inflation. But $L_D < L_P(0)$, wher $L_P(0)$ is the distance of the particle horizon when there is no inflation. Hence, a necessary and a sufficient condition for the solution of the horizon problem is

$$L_P(N) > 3L_P(0) , \quad (24)$$

and inflation must increase the distance of the particle horizon in the standard model at least threefold. This is the main conclusion of our discussion.

As an example, from equation (13) for a constant q model, $L_P(0) = L_H/q$, and a period of inflation must therefore make $L_P(N)$ equal to or greater than $3L_H/q$. If $q = \frac{1}{2}$, as in the Einstein–de Sitter universe, then $L_P(N) > 6L_H$ solves the horizon problem. Figure 2 illustrates how inflation greatly increases the age of the universe in conformal time τ and thereby extends the particle horizon.

4.3. *A Simplified Calculation*

Let the particle horizon in an inflationary universe be at distance

$$L_P(N) = R_0(I_1 + I_2 + I_3) , \quad (25)$$

and in the absence of inflation at distance

$$L_P(0) = R_0 I_4 , \quad (26)$$

In these equations I_1, I_2, and I_3 are the preinflation inflation, and postinflation contributions, respectively, and I_4 is the age of the universe in conformal time in the absence of inflation. Neglecting the curvature term, and using the conditions that R

and $\dot{R}$ are continuous we find

$$I_1 = \frac{1}{H_{\rm inf} R_i}\,, \qquad (27)$$

for a relativistic fluid ($R \propto t^{1/2}$);

$$I_2 = \frac{1}{H_{\rm inf}}\left(\frac{1}{R_i} - \frac{1}{R_f}\right), \qquad (28)$$

for inflation [$R \propto \exp H_{\rm inf}(t - t_0)$]; and

$$I_3 = \frac{1}{H_q R_q} - \frac{1}{H_{\rm inf} R_f} + 2\left(\frac{1}{H_0 R_0} - \frac{1}{H_q R_q}\right), \qquad (29)$$

for a relativistic fluid to the epoch t_q of equal radiation and matter densities, and a nonrelativistic fluid ($R \propto t^{2/3}$) from t_q to the present epoch t_0. Also, in the absence of inflation, we find

$$I_4 = \frac{1}{H_q R_q} + 2\left(\frac{1}{H_0 R_0} - \frac{1}{H_q R_q}\right). \qquad (30)$$

Because the ratio $H_0 R_0/H_q R_q = (1 + z_q)^{-1/2} \sim 10^{-2}$ is negligible, equations (25)–(30) yield

$$L_P(N)/L_P(0) = 1 + F^{-1}(e^N - 1)\,, \qquad (31)$$

where F has the value

$$F = H_{\rm inf} R_f/H_0 R_0 \sim (T_f^2/T_0\, T_q)^{1/2}\,. \qquad (32)$$

The solution to the horizon problem requires $L_P(N) > 3L_P(0)$ according to equation (24), and therefore, from equation (31),

$$e^N > 2F + 1\,, \qquad (33)$$

Assuming a grand unified temperature $T_f \sim 10^{15}$ GeV, and using $T_q \sim 10^4$ K, we find $F \sim 10^{26}$, and the horizon problem is solved for $N > 60$.

5. DISCUSSION

5.1. *Causal Connections*

In the study of causal connections, the Hubble sphere bounded by the Hubble surface is as important as the observable universe bounded by the particle horizon. The answer to the question, How, in a universe of age t can causally connected distances of $L \gg ct$ exist? draws on the properties of both the Hubble sphere and the observable universe.

Let two comoving bodies be separated by a distance L sufficiently small that each lies in the observable universe of the other. Each body remains thereafter permanently in the other's observable universe, and the ratio L/ct during expansion depends on the behavior of the Hubble sphere.

In a decelerating universe, the Hubble sphere expands faster than the universe, and a body at distance L either is inside or will soon be inside the Hubble sphere. Hence, any two bodies must eventually recede from each other at subluminal velocity, and the ratio L/ct will then decrease in time.

In an accelerating universe, the Hubble sphere expands slower than the universe, and a body at distance L either is outside or will soon be outside the Hubble sphere. Hence, any two bodies must eventually recede from each other at superluminal velocity, and the ratio L/ct will then increase in time.

How can causally connected distances of $L \gg ct$ exist? The answer is that the universe passes through a period of accelerated expansion, and causal connections of $L < ct$, established before acceleration, expand superluminally outside the Hubble sphere.

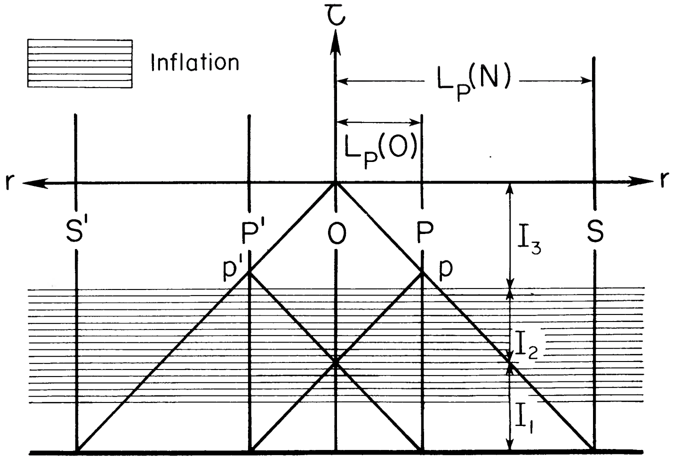

FIG. 2.—A conformal coordinate spacetime diagram of an inflationary Robertson-Walker universe. Inflation or any form of accelerated expansion (*shown striated*) in the early universe extends the range of conformal time prior to decoupling and transfers the particle horizon from world lines P, P′ at distance $L_P(0)$ to world lines S, S′ at distance $L_P(N)$. When world line S is at a distance greater than 3 times that of world line P (and S′ at a distance greater than 3 times that of P′), and hence $L_P(N) > 3L_P(0)$, the events p, p′ that formerly were at the particle horizon of the standard universe (as shown in Fig. 1) are causally connected in the inflationary universe. Thus $L_P(N) > 3L_P(0)$ is the sufficient condition that all events in the visible universe, such as d, d′ in Fig. 1, are causally connected.

5.2. *The Horizon Problem*

A period of acelerated expansion distends all previously established causal connections and increases the distance to the particle horizon. The entire visible universe is causally interconnected when the acceleration occurring in the extreme early universe increases the particle horizon by a factor of at least 3. A minimum threefold increase solves the horizon problem.

Some words of caution are necessary. First, although events p, p′ in Figure 2 are causally connected, each is affected by events that do not affect the other. Hence, p and p′ cannot possess identical histories, and presumably the universe must originate in some minimal state of homogeneity and isotropy. Second, satisfying the conditions for causal connectivity in a homogeneous and isotropic universe, as in this paper, cannot guarantee that homogeneity and isotropy will be achieved in a universe previously inhomogeneous and anisotropic. Last, solving the horizon problem does not solve the unity problem: why are the laws and constants of nature the same everywhere?

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