THE ASTROPHYSICAL JOURNAL, 373:L59–L62, 1991 June 1

THE EXPANSION OF THE CRAB NEBULA

M. F. BIETENHOLZ AND P. P. KRONBERG[1]
Department of Astronomy, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S-1A7
D. E. HOGG
National Radio Astronomy Observatory, Edgemont Road, Charlottesville, VA 22901
AND
A. S. WILSON
Astronomy Program, University of Maryland, College Park, MD 20742
Received 1990 December 18; accepted 1991 March 13

ABSTRACT

Using high-resolution radio observations from 1982 and 1987 we have measured the expansion of the synchrotron component of the Crab Nebula, including a measurement of the expansion of the nebula's outer edge. Our measurements show a rate of expansion similar to that obtained from optical data for the line-emitting filaments. We show that the synchrotron component of the Crab expands homologously and that its rate of expansion has accelerated *since* the supernova explosion. The data further suggest that the acceleration of the synchrotron component may be larger than that of the emission-line filaments which, if confirmed by future observations, implies that the relativistic gas is currently "bursting through" the net of filaments. The absence of deceleration allows the establishment of stringent upper limits on the density of gas into which the observed nebula is expanding.

Subject headings: interstellar: matter — magnetic fields — nebulae: Crab Nebula — nebulae: supernova remnants — radio sources: general

1. INTRODUCTION

Among the many noteworthy features of the Crab Nebula is the fact that the filaments can be shown to be currently moving faster than their average speed since the supernova explosion in A.D. 1054 (Trimble 1968). In other words they have been accelerated *since* the supernova event. The observed nebula consists of two main components: the aforementioned filaments, which are dense condensations of thermal gas, and a bubble consisting of a light, relativistic gas and an embedded magnetic field, both of which presumably have their origin in the pulsar. The agents for the post-SN acceleration of the filaments are presumably the magnetic field in the pulsar bubble and the relativistic particles. In this *Letter*, we report the first measurement of the current expansion rate of this synchrotron-emitting bubble.

Two of us (A. S. W. and D. E. H.) observed the Crab Nebula using the NRAO[2] VLA in 1982 in both the 20 and 6 cm bands (see Wilson, Samarasinha, & Hogg 1985, hereafter referred to as WSH). The remaining two of us made similar observations in 1987 (see Bietenholz & Kronberg 1990, hereafter Paper I; Bientenholz 1990), using four frequencies, two each in the 20 cm (1410 and 1515 MHz) and 6 cm (4625 and 4885 MHz) bands. Both sets of observations used all four configurations of the VLA in the 20 cm band and the B, C, and D configurations in the 6 cm band. One of us (M. F. B.) has reduced and self-calibrated all the VLA data in a consistent way (as described in Bietenholz 1990 and Paper I). The raw images were cleaned using maximum entropy deconvolution to obtain images with a restored beam size of 1″.8 × 2″. The final images had peak fluxes of 0.11 and 0.09 Jy beam^{-1} with noise levels of 0.2 and 0.3 mJy beam^{-1} at 1.5 and 5 GHz, respectively, except for the epoch 1982 image at 1.5 GHz which had a noise level of 0.4 mJy beam^{-1} (for further details, see Bietenholz 1990).

[1] Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-5300, Bonn 1, Germany.

[2] The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under cooperative agreement with the National Science Foundation.

2. DETERMINING THE EXPANSION OF THE CRAB NEBULA

The Crab Nebula is expanding at roughly 1500 km s^{-1}, or 0″.15 per year at 2 kpc. This implies an angular change of about 1″, or about $\frac{2}{3}$ resolution element over the interval between our two sets of observations. While it is difficult to determine the motion of individual features, especially in the faint exterior regions, an averaging over many features should provide a precise expansion of the nebula from the radio data since the area covered by the nebula represents about 2.5 × 10^4 beam areas.

Previous determinations of the proper motions of different parts of the nebula (e.g., Trimble 1968) relied exclusively on the measurements of compact line-emitting optical filaments. Because the edge of the synchrotron nebula is more sharply defined in the radio than in the optical, our data allow a determination of the motion of the outer edge of the nebula, which also allows a determination of the expansion *outside* the region occupied by the filaments used in earlier proper motion studies.

At this point, we note that the optimal approach to determining the expansion from interferometric radio data might initially seem to be an analysis of the original Fourier transform plane (*u-v*) data and *not* the cleaned images. However, the two sets of *u-v* data consist of unevenly spaced samplings of the *u-v* plane, and interpolation would be required to compare them. The cleaning process uses information about physically plausible images to perform this interpolation—and thus is superior to a straightforward interpolation in the *u-v* plane.

The Crab shows relatively little radio structure on the smallest spatial scales (see WSH; Bietenholz & Kronberg 1991, hereafter Paper II). Furthermore the outermost intensity contours of the radio image are relatively smooth. Because of this lack of compact, discrete features, we chose to measure the expansion by determining the scaling parameters which make the image at the first epoch "look like" the image at the second epoch. Let the first image be $X(x, y)$ and the second image be $Y(x, y)$. Now let

$$X'(x, y) = AX(ex + x_0, ey + y_0) + b \ . \quad (1)$$

Here, (A, b, e, x_0, y_0) represent the parameters of the transformation used to make image X "look like" image Y. A and b are the flux scaling and offsets, e is the expansion factor, and (x_0, y_0) are the translation. We want to determine the parameter e, but because of the uncertain flux calibration and the positional offsets that can be caused by self-calibration, we also determine the parameters (A, b, x_0, y_0). An obvious way to estimate these five parameters is to minimize the sum of the squared differences between X' and Y. However, if noise is present in both X (and hence X') and Y, this method produces biased estimates, especially of A.

Tan & Gull (1985) have developed a maximum entropy method of minimizing the differences between the two images, which produces unbiased estimates of the parameters. They fit a template $F(x, y)$ to the data, we well as fitting the other parameters. The template represents the best estimate of the true image flux, and Tan & Gull's method simultaneously minimizes the sum of the squared differences both between X' and F and between Y and F. We used the program IMDIFF, coded in R. Sault's WERONG set of programs, to perform this optimization of the parameters (A, b, e, x_0, y_0) defined above.

The largest source of uncertainty in the expansion rate derived from our measurements will be the uncertainty in the epoch of the observations, since both the 1982 and 1987 measurements involved observations taken at several array configurations spaced over about one year. Unfortunately, the sequence of array configurations in 1981–1982 was different from that of 1987–1988, so that it is not simply a question of having two largely similar data sets each spaced over a year.

The obvious possibilities for weighting the u-v data to determine a representative epoch for the resulting image are (1) uniform weight, (2) weighting by flux, or (3) an assignment of higher weights to data at higher spatial frequencies (since the amount of expansion is small, and thus best resolved only on the longest baselines). If the emission consisted mostly of clearly distinguished, individual, compact features, then possibility (3) would be the optimum weighting for determining their motion. However, this is not the case (WSH; Paper I); it seems likely that the compact features are *not* what determine the overall expansion as calculated by IMDIFF. To determine the proper weighting, then, it is necessary to examine the effects of the expansion in the u-v plane.

This was done as follows: One of the later images (4885 MHz, 1987) was homologously contracted ("shrunk") by interpolation. This contracted image was subtracted from the original one (with the fluxes scaled so that the residuals would have a mean of zero). The resulting difference image was Fourier transformed, and the amplitude of this transformed image is the magnitude of the change in the u-v plane caused by the contraction. The image is circular to within 50%, and a plot of the radial dependence of the amplitude (V_R) in the u-v domain is shown in Figure 1. At the center of the u-v plane

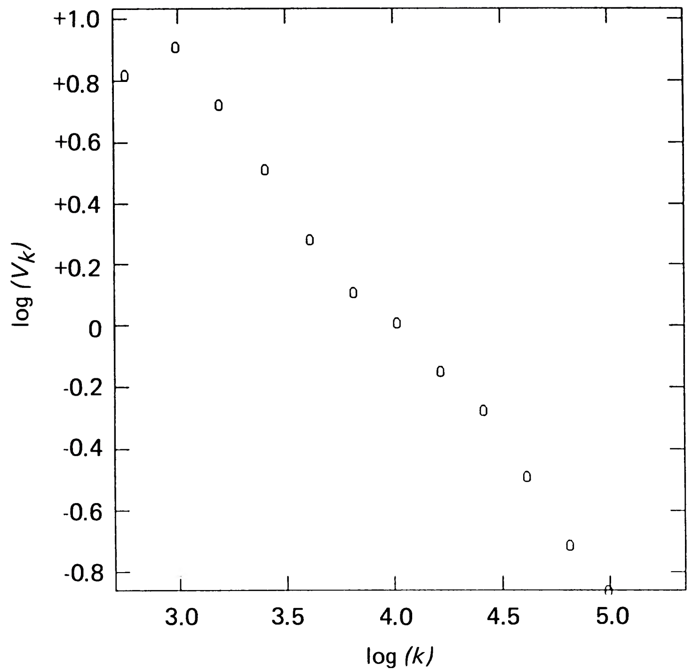

FIG. 1.—Logarithm of the amplitude (in janskys) of the changes in the u-v plane caused by expansion as a function of the logarithm of distance (in λ) from the center of the u-v plane (k).

($k = 0$, k being the u-v distance from the phase center) the amplitude is zero, which it must be because the residuals have a mean flux of zero. The approximate scaling is $V_R \propto 1/k$ for $k > 1\mathrm{K}\lambda$ (which corresponds approximately to the overall size of the Crab nebula).

In consequence, the weighting ($\mathscr{W}$) that was used was the following:

$$\mathscr{W}(k) \propto \begin{cases} k^{-2} & \text{for } k > 1, \text{ where } k \text{ is in K}\lambda \\ k^2 & \text{for } k < 1 \end{cases} \ . \quad (2)$$

The effect is to weight most strongly the spatial scales close to the overall size of the nebula, as is desirable for a measurement of its global expansion. The weighting was chosen to go to zero in the center of the u-v plane to reflect the fact that the amplitude scaling and offset are free parameters, so the points near $k = 0$ should have little effect on the determination of the expansion parameter, e. Table 1 presents the intervals in Julian days between the two sets of observations as determined by using various weights. The internal variances in these intervals are negligible compared to the overall difference between the various weighting schemes.

TABLE 1

THE EFFECTIVE INTERVALS BETWEEN THE OBSERVATIONS, IN DAYS

	Observations	
Weight	1.5 GHz	5 GHz
Uniform	1988	2016
Flux	1865	1814
$\mathscr{W}(k)$	1911	1813

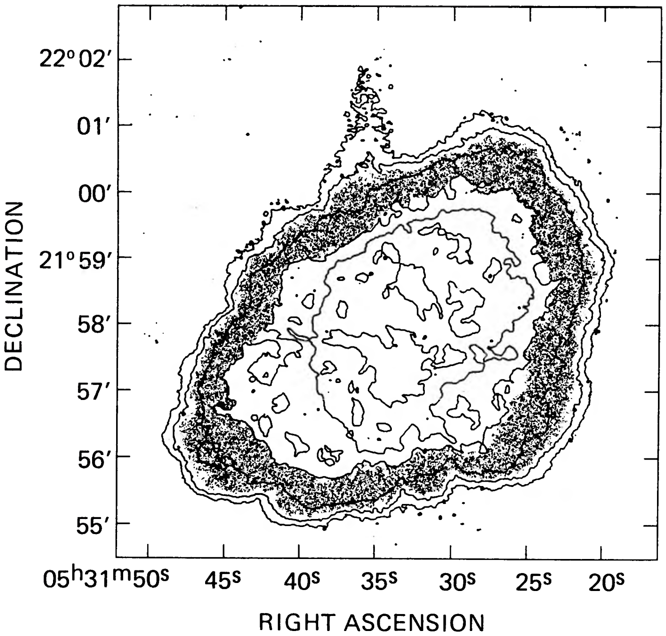

FIG. 2.—Area (*shaded*) used in the calculation of the expansion of the edge of the Crab Nebula. Contours are from the 1410 MHz image, with contour levels at 0.5, 3, 10, 30, 50, and 80 mJy beam^{-1}.

3. THE EXPANSION OF THE SYNCHROTRON COMPONENT

We determined expansion parameters, e, from the 1987 to the 1982 observations (i.e., e is less than 1). The expansion age is simply $\Delta t/(1 - e)$ where Δt is the interval between the two sets of observations. The Δt values are those obtained by using the weighting $\mathscr{W}(k)$ defined in equation (2) above (see also Table 1). We determined e separately for the whole nebula and for the outside edge. The area included in the edge calculations is shown in Figure 2. The edge region was chosen to exclude the very edge (low signal-to-noise ratio) and most of the filamentary emission. The same region was used for all images.

To reduce biases, the IMDIFF program was run in both directions, that is from the 1982 data to the 1987 data and vice versa (the latter determining $1/e$). The two values obtained for e typically agreed to less than 0.0003, and we use the average of the two determinations of e below. The resulting values for the expansion factor, e, are shown in Table 2.

The expansion ages obtained, which are with reference to 1987.4, give a convergence date of A.D. 1233 for the whole nebula and A.D. 1257 for the outer edge. This is somewhat later than the dates of A.D. 1120 ± 7 and A.D. 1140 obtained by Wyckoff & Murray (1977) and Trimble (1968), respectively. It is difficult to determine the uncertainty in our observations. The standard deviations (over four frequencies) quoted in Table 2 are likely lower limits. However, the uncertainty in the result for the outer edge should be larger than that for the whole nebula result (fewer pixels and lower signal-to-noise ratio in the former), so we will take the larger of the two deviations (± 92 yr) to represent the true uncertainty. Our results are then within 1.2 and 1.5 σ of those quoted by Trimble and Wyckoff & Murray, respectively.

A fit allowing differential expansion in two orthogonal directions was made, one along the major axis (P.A. = 131°) and one at right angles to this. *No significant difference was found.* Trimble also (1968) did not find any significant difference in the expansion factors for the optical filaments along the major and minor axes. Thus, both the radio and the optical measurements show that the expansion of the nebula seems to be homologous.

TABLE 2
EXPANSION FACTORS AND AGES FOR THE CRAB NEBULA

FREQUENCY (MHz)	WHOLE NEBULA e[a]	WHOLE NEBULA t_c (yr)[b]	OUTER EDGE e[a]	OUTER EDGE t_c (yr)[b]
1410.........	0.99359	817	0.99289	734
1515.........	0.99370	830	0.99297	744
4625.........	0.99213	630	0.99289	698
4887.........	0.99326	737	0.99333	745
mean		754 ± 92[c]		730 ± 22[c]

[a] Expansion factors were calculated from 1987 to 1982 as described in the text.

[b] Expansion ages are from epoch 1987.4.

[c] Quoted uncertainties are merely the standard deviations of the ages in the given column.

4. DISCUSSION

We find that the expansion of the pulsar bubble must have been accelerated since the supernova event. The amount of post-SN acceleration seems to be *slightly larger* than that observed for the line-emitting filaments by Trimble (1968; see also Wyckoff & Murray 1977). We emphasize that the previous results applied only to the (line-emitting) filamentary material, whereas our results apply to the more smoothly distributed (synchrotron-emitting) material, and to a larger volume. While our uncertainties are larger than those obtained by Trimble and Wyckoff & Murray, our data still clearly show that *post-SN acceleration must have taken place.* We note that there is no evidence of compact features moving more rapidly through the diffuse material, as is observed in Cassiopeia A (Braun, Gull, & Perley 1987). Also, there is no difference in the convergence dates between the major and the minor axis directions of the nebula. To the accuracy of the observations, the expansion of the Crab is homologous. Furthermore, *the outside edge* of the nebula has also undergone post-SN acceleration.

In most models of the Crab Nebula (e.g., Rees & Gunn 1974; Kundt & Krotschek 1980; Kennel & Coroniti 1984; Reynolds & Chevalier 1984) the pulsar emits a wind consisting of relativistic particles, magnetic field, and possibly strong, low-frequency electromagnetic waves. This wind inflates a bubble inside the expanding supernova ejecta, which are assumed to form a continuous shell. The shell of filaments, however, is not complete, as can be seen in optical images (see also Paper II). The coupling mechanism between the filaments and the relativistic gas is unclear. In any case, the relativistic bubble is seen to extend significantly beyond the faintest optical filaments, especially in the northwest quadrant (WSH), so the filaments obviously do not completely contain the relativistic gas.

Our data suggest that the post-SN acceleration of the relativistic bubble may be larger than that of the line-emitting filaments. We think it very unlikely that the tentative difference in convergence dates between the present measurements of the relativistic bubble and the previous measurements of the line-emitting filaments can be ascribed to the different epochs at which they were measured, since a huge acceleration over the last $\simeq 50$ yr would be required to reconcile them. While the greater acceleration of the pulsar bubble is only marginally (1.2–1.5 σ) established in our data, and requires further work for confirmation, it is exactly what would be expected if the relativistic bubble is "bursting through" the filamentary shell.

The relativistic bubble is expected to accelerate under a range of circumstances (e.g., Chevalier 1984; Tenorio-Tagle, Bodenheimer, & Franco 1987), and a convergence date of A.D. 1233–1257 for the relativistic bubble (§ 3) would be entirely consistent with Chevalier's (1977) predicted convergence date for a pulsar braking index $n = 3.0$ (assumed constant). In our scenario, the filamentary shell is only partially successful in confining the bubble.

If the pulsar bubble has, indeed, "burst through" the filamentary shell, then the relativistic fluid must be expanding directly into the surrounding medium. Our finding that the relativistic bubble is accelerating implies that deceleration by the ram or thermal pressure of any surrounding medium is small, allowing upper limits to its density to be obtained, as we now demonstrate. The nebula must be expanding into one of the following three entities: (1) the undisturbed ISM, (2) the region occupied by the wind of the pre-SN Crab progenitor star, or (3) the postulated Crab "halo" (see Chevalier 1977), consisting of the outer, H-rich layers of the progenitor star, which were ejected in the SN event, but have not so far been unambiguously detected. The minimum pressure inside the bubble is several $\times 10^{-9}$ dyn cm^{-2}, which is higher than the thermal pressure expected from either the ISM or a pre-SN wind zone. Therefore, if the pulsar bubble is being decelerated by either of these components, ram pressure would have to provide the decelerating external force (as suggested by Kundt 1983). The absence of deceleration implies that the density in the surrounding medium must be less than about 0.05 cm^{-3}, which is not unreasonable given the distance of 180 pc above the Galactic plane. Because of the small swept-up mass, the X-ray emission from a shock at the interface would be too faint to be detectable.

If the bubble is expanding into the H-rich envelope ejected by the supernova (Chevalier 1977), then the speed of expansion *into* this "halo" is only the amount by which the bubble has been post-SN accelerated, that is, 200 km s^{-1}. If 8 $M_\odot$ were ejected at velocities up to 7000 km s^{-1}, the mean density of the envelope now is $\sim$0.3 cm^{-3}. The ram pressure at the boundary between the bubble and the envelope would be $\sim 2 \times 10^{-10}$ dyn cm^{-2}, too low to decelerate the bubble. The thermal pressure in the envelope would be $\sim 4 \times 10^{-10}(T/10^7 \text{ K})$. A gas capable of static pressure confinement, however, would be inconsistent with the upper limit on the amount of X-ray–emitting gas set by Schattenburg et al. (1980). Furthermore, the low speed of expansion into the halo would not generate a shock strong enough to heat the gas to 10^7 K. We conclude that the decelerating effect of any surrounding envelope ejected in the supernova explosion is small.

Our conclusion that deceleration effects are small implies that the bubble is in almost free expansion. The Alfvén velocity near the outer edge of the bubble must then be on the order of the 1500 km s^{-1} expansion speed, considerably lower than the hydromagnetic speeds of $0.1c$ observed near the center of the nebula (Scargle 1969). Assuming a field of 2×10^{-4} G, such a velocity would correspond to a thermal density of 0.08 cm^{-3} near the periphery of the bubble. This value is in accord with the upper limit of 0.2 cm^{-3} for the thermal density obtained by Paper II from an analysis of the rotation measure distribution.

This discussion leaves unsettled the question of why the filaments and the synchrotron nebula are roughly in the same place. The expansion of the synchrotron bubble is largely determined by the parameters of the pulsar wind and of the external medium, while the expansion of the line-emitting filaments is largely determined by the supernova explosion (the post-SN acceleration is only $\sim$8%; Trimble 1968). Why are they largely coincident? This coincidence implies (Wilson 1972) a considerable, but not complete, coupling between the filaments and the bubble, and such a coupling is also needed in order to accelerate the filaments. If this coupling was stronger in the past, the synchrotron bubble and the filaments would have expanded together in the early phases of the evolution, and only relatively recently would the bubble have "burst through" the filamentary net, which would explain why the bubble is now almost coincident with, but somewhat larger than, the line-emitting filaments. This is in agreement with the scenario presented by Chevalier (1977), who shows that the growth time of the Rayleigh-Taylor instability in the shell of line-emitting material is comparable to the present age of the Crab.

M. F. B. and P. P. K. acknowledge financial support for this work from the Natural Sciences and Engineering Research Council of Canada. M. F. B. acknowledges further support from the University of Toronto and the government of Ontario. A. S. W. thanks the University of Maryland Graduate School for a Faculty Research Fellowship during the completion of this work. We thank the Ontario Centre for Large Scale Computation (OCLSC) for computing support.

REFERENCES

Bietenholz, M. F. 1990, Ph.D. thesis, University of Toronto
Bietenholz, M. F., & Kronberg, P. P. 1990, ApJ, 357, L13 (Paper I)
———. 1991, ApJ, 368, 231 (Paper II)
Braun, R., Gull, S. F., & Perley, R. A. 1987, Nature, 327, 395
Chevalier, R. A. 1977, in Supernovae, ed. D. N. Schramm (Dordrecht: Reidel), 53
———. 1984, ApJ, 280, 797
Kennel, C. F., & Coroniti, F. V. 1984, ApJ, 283, 694
Kundt, W. 1983, A&A, 121, L15
Kundt, W., & Krotscheck, E. 1980, A&A, 83, 1
Rees, M. J., & Gunn, J. E. 1974, MNRAS, 167, 1
Reynolds, S. P., & Chevalier, R. A. 1984, ApJ, 278, 630
Scargle, J. D. 1969, ApJ, 156, 401
Schattenberg, M. L., Canizares, C. R., Berg, C. J., Clark, G. W., Markert, T. H., & Winkler, P. F. 1980, ApJ, 241, L151
Tan, S. M., & Gull, S. F. 1985, MNRAS, 216, 949
Tenorio-Tagle, G., Bodenheimer, P., & Franco, J. 1987, in High Energy Astrophysics, ed. G. Börner (Berlin: Springer), 77
Trimble, V. L. 1968, AJ, 73, 535
Wilson, A. S. 1972, MNRAS, 160, 373
Wilson, A. S., Samarasinha, N. H., & Hogg, D. E. 1985, ApJ, 294, L121 (WSH)
Wyckoff, S., & Murray, C. A. 1977, MNRAS, 180, 717