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# THE GALACTIC DISK SURFACE MASS DENSITY AND THE GALACTIC FORCE $K_z$ AT $z = 1.1$ KILOPARSECS

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## ABSTRACT

We rediscuss a set of distance and velocity data previously obtained and analyzed by ourselves to determine the surface mass density of the Galactic disk. We show that these data reliably determine the integral surface mass density of *all* (disk + halo) Galactic components within 1.1 kpc from the Galactic plane near the Sun to be $71 \pm 6\ M_\odot\ \mathrm{pc}^{-2}$, independent of the disk/halo ratio. We show that determination of the fraction of this total mass which is distributed in the Galactic disk and the fraction which is associated with an extended halo remains highly model-dependent. Attempts to use extant data for stars far from the Galactic plane to determine the disk/halo mass fraction in the local surface density are likely to be unreliable. The best available estimate of the relative contributions of disk mass and halo mass to the local integral surface density, obtained from modeling of the Galactic rotation curve, yields a surface mass density of $48 \pm 9\ M_\odot\ \mathrm{pc}^{-2}$ for mass associated with the Galactic disk near the Sun. The corresponding mass of identified disk matter is $48 \pm 8\ M_\odot\ \mathrm{pc}^{-2}$. There remains no evidence for any significant unidentified mass in the Galactic disk.
*Subject headings:* dark matter — galaxies: internal motions — galaxies: The Galaxy — stars: stellar dynamics

## 1. INTRODUCTION

In a recent set of papers, Kuijken & Gilmore (1989a, b, c) determined the vertical gravitational potential within about 1 kpc of the Sun and showed that their result was consistent with there being no dynamically significant unseen mass associated with the Galactic disk. Their result involved both a new data set and a new method of analysis. Subsequent reanalyses of their method and results have suggested somewhat paradoxical results. Gould (1990) suggested that the true surface mass density of the Galactic disk corresponding to the potential which best describes the Kuijken & Gilmore data is systematically 0.9 $\sigma$ greater than the Kuijken & Gilmore result. King (1989) suggested that the error bars resulting from current uncertainties in the determination of the stellar density profile far from the Galactic plane may be greater than Kuijken & Gilmore derived, but did not quantify a likely magnitude for the effect. Statler (1989) studied simulated data suggested to be similar to actual data of Kuijken & Gilmore and concluded that the formal random statistical errors appropriate to their data should be roughly twice as great as those determined by Kuijken & Gilmore (and by Gould). Additionally, the maximum likelihood analysis of their local stellar data by Kuijken & Gilmore apparently favored potentials with no significant extended dark matter distribution, in contradiction to the global solution for the combined local stellar data and the extended Galactic rotation curve adopted by these authors.

In this *Letter* we clarify these paradoxical results and emphasize which of the conclusions which can be drawn from the Kuijken & Gilmore data are robust and which are more model-dependent. It remains correct that available data suggest that no dynamically significant dark matter is associated with the Galactic disk.

## 2. THE ABEL TRANSFORM METHOD FOR DETERMINATION OF $K_z(z)$

The gravitational force towards the Galactic plane $K_z(z) = d\psi(z)/dz$ can be measured from data describing the vertical balance between the gravitational attraction toward the plane and the pressure (velocity dispersion) force away from the plane of a suitable stellar "tracer" population. Kuijken & Gilmore (1989a, b, hereafter together KG) used a new sample of 512 K dwarf stars toward the south Galactic pole. For their calculation, one needs to know both the space density and the velocity distribution as a function of distance from the Galactic plane of the tracer sample.

The basis of the KG analysis is the (standard) assumption that the $z$-motions of disk stars near the Sun can be described by the one-dimensional collisionless Boltzmann equation (the effect of failure of this assumption is discussed in KG and in § 6 below). Then the energy in the $z$-motion $E_z = \psi(z) + \frac{1}{2}v_z^2$ is an integral, and hence by Jeans's theorem the phase space distribution function $f_z(z, v_z)$ of any tracer population depends on $E_z$ only. Given such a distribution function $f_z(E_z)$ and a potential $\psi(z)$, one can calculate the density $\nu(z)$ as the zeroth velocity moment of $f_z$:

$$\nu(z) = \int_{-\infty}^{\infty} f_z(z, v_z) dv_z = 2 \int_{\psi}^{\infty} \frac{f_z(E_z)}{\sqrt{2(E_z - \psi)}} dE_z , \qquad (1)$$

where we have reparameterized the $z$-height in terms of the potential $\psi$. This equation is an Abel transform, which has the standard inversion

$$f_z(E_z) = \frac{1}{\pi} \int_{E_z}^{\infty} \frac{-d\nu/d\psi}{\sqrt{2(\psi - E_z)}} d\psi , \qquad (2)$$

so that there is a unique relation between $\nu(\psi)$ and $f_z(E_z)$. Because of this equivalence of $\nu(\psi)$ and $f_z(E_z)$, there is a triangular mathematical relationship between the three functions $\psi(z)$, $\nu(z)$, and $f_z(E_z)$: any one of them can be deduced from the other two. It is important to note that $f_z(E_z)$ depends on the density only at points where the potential exceeds $E_z$, i.e., beyond the point $z = \psi^{-1}(E_z)$. Therefore, one can derive the potential at large distances from the plane from high-$z$ data alone, without having to worry in detail about the shape of the potential nearer to the plane. The only requirement is a suitably general model potential and appropriate data.

The $z$-potential above the bulk of the Galactic disk can be adequately described by a linear plus a quadratic term in $z$, $\psi = Kz + Fz^2$. Here $K$ is proportional to the integral surface density of the disk $\Sigma$, while $F$ depends on the larger scale structure of the Galaxy, chiefly on the local density of the dark matter corona. $|K_z(z)| = d\psi/dz$ thus yields the surface density of all matter (disk + corona) within a distance $z$ of the Galactic plane. The space density profile of the tracer population was used by KG to predict its velocity distribution at different heights, assuming different model potentials. These were then compared to the data using a maximum likelihood technique in order to select the best-fitting potential. The effect of uncertain knowledge of the shape and orientation of the stellar velocity ellipsoid above the Galactic plane, where its components cannot all be measured directly, was evaluated by KG by assuming two plausibly limiting cases, namely those of cylindrical and spherical alignment of the velocity ellipsoid's axes. The method is described in detail by KG, who applied it to derive a surace mass density of mass associated with the local Galactic disk of $46 \pm 9\ M_\odot\ \mathrm{pc}^{-2}$.

## 3. WHAT DOES $K_z(z)$ REALLY MEASURE?

Separate measurement of the parameters describing the disk ($K$) and halo ($F$) potentials is complicated by the fact that they are highly correlated near the plane. The likelihood function of fits of model potentials to the velocity data is dominated by a long ridge of possible solutions, so that only a *linear combination* of $K$ and $F$ is measured. We can write any such linear combination as $K + 2z_0 F = \mathrm{const}$, with $z_0 = 1.1$ kpc for the KG data. From the definition of the potential, the constant is just $K_z(z_0)$, the force at height $z_0$. In other words, the *robust* number resulting from the KG analysis is the value of $K_z(z)$ at $z = 1.1$ kpc, which is related simply to the total surface density of mass below that height. The slope ($2F$) of $K_z(z)$, which is the halo contribution to the total potential, is not determined to great accuracy from the data. To illustrate this, Figure 1 presents the results of the maximum likelihood determination of the potential from the KG data in the (total surface mass density, halo contribution to the total), i.e., $[\Sigma(<1.1\ \mathrm{kpc}), F]$ plane. Clearly, there is effectively no correlation between the total force measured at $z = 1.1$ kpc and the halo contribution to that total in the model potential. Thus, we can conclude that, independent of the details of the Galactic model which produced the global gravitational field that was used to model the K dwarf kinematics, the surface mass density of *all* matter in whatever spatial distribution between $z = \pm 1.1$ kpc is

$$\Sigma_{1.1\ \mathrm{kpc}} = \frac{|K_z(1.1\ \mathrm{kpc})|}{2\pi G} = 71 \pm 6\ M_\odot\ \mathrm{pc}^{-2}\ . \qquad (3)$$

It is this number which is most reliably determined from the KG data with their analysis technique, even though KG do not mention it. We now consider the uncertainties in this value, and in its deconvolution into "disk" and "halo" contributions to the total potential.

## 4. STATISTICAL ERRORS IN THE STELLAR SPATIAL DENSITY DISTRIBUTION

Following a comment by King (1989) we have evaluated the statistical errors in the fit to the space density data of KG more generally than in our previous analysis. We varied the parameters of the fit to the observed K dwarf star counts and multiplied the likelihood of this fit (given by the Poissonian statistics of the predicted, error-convolved star counts in each bin) into the velocity data likelihoods. We allowed the scale height $H$ of the high-$z$ component in the fits to vary to the point where the likelihood of the fit to the density data was a factor of 30 lower than optimum, which corresponds to a $\sim 2.5\ \sigma$ offset. The analysis of KG was then repeated with these density laws, and the combined likelihood $L_{\mathrm{tot}}$ (now a function of the three parameters $[K, F, H]$) for the density and velocity data calculated. For each set of potential parameters $(K, F)$ the maximum of $L_{\mathrm{tot}}(K, F, H)$ over $H$, $L_{\max}(K, F)$, was then calculated. This is the likelihood of $(K, F)$, allowing directly for uncertainties in determining the disk scale height far from the plane. Contours of $L_{\max}$ are shown in Figure 2. These contours are slightly shifted (toward higher $K$, and more stretched toward high $F$, low $K$) relative to those published by KG. The width of the likelihood function orthogonal to the "ridge" is virtually unchanged. The effects of explicit inclusion of uncertainties in the high-$z$ density law are to weaken somewhat the constraint the data place on the large-scale Galactic potential and to increase systematically the best-fit value of the disk surface

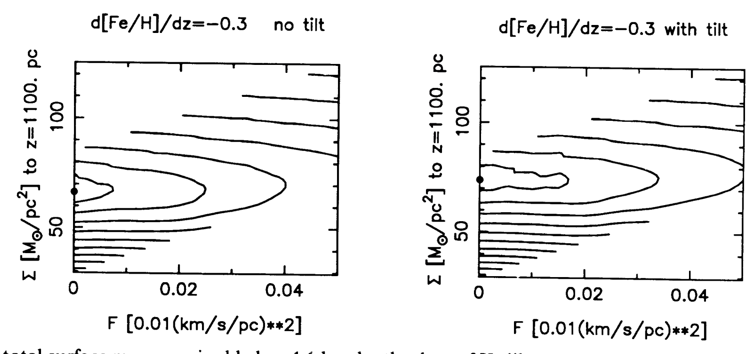


FIG. 1.—The independence of the total surface mass required below 1.1 kpc by the data of Kuijken and Gilmore (1989b; *vertical axis*) and the amount of the local potential provided by the halo (*horizontal axis*, using units defined in the text). Contours of likelihood (at intervals corresponding to 1 $\sigma$, 2 $\sigma$, ... in Gaussian statistics) of fits of different model potentials to the stellar data are shown. $\Sigma(1.1\ \mathrm{kpc})$ is determined independent of the numerical value of $F$.

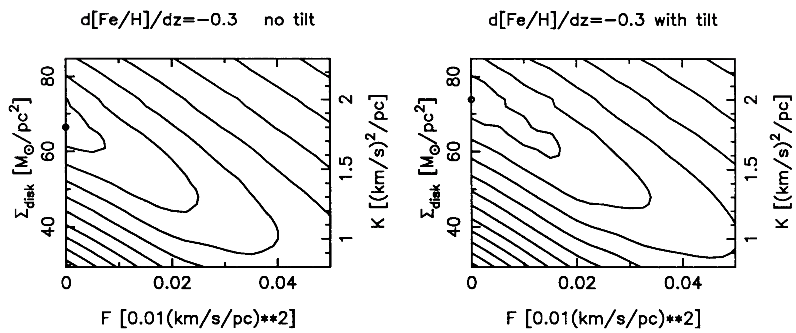


FIG. 2.—The likelihood contours of Fig. 1 plotted in the ($K$, $F$) plane, where $K$ and $F$ parameterize the disk and halo contributions to the local potential, respectively. These likelihoods incorporate the general density law fits described in § 3 of the text.

mass density. Note that this source of uncertainty is included in the determination of the total surface mass density below 1.1 kpc discussed in § 3 above.

## 5. IS THERE DARK MATTER IN THE DISK?

An interesting question is the distribution of the mass generating the potential whose gradient at 1.1 kpc from the plane has been determined in equation (3). Crudely, is there a significant contribution from dark *disk* matter as well as from the dark halo matter required to support the extended rotation curve? The dominant contribution to the derived local surface mass density below 1.1 kpc is certainly the mass of the disk, but significant contributions do come from more spherically distributed components, parameterized in the model potentials by $F$.

These other contributions are not completely unconstrained. Together, all the mass in the Galaxy must satisfy the requirement that the potential determined locally be consistent with that required to support the Galactic rotation curve. From (model-dependent) fits to the Galactic rotation curve, KG deduced that this may be quantified by imposing the constraint $F = 0.041 - 0.0094K \pm 0.008$ on the maximum likelihood solutions for the disk and halo potential parameters $K$ and $F$. The corresponding local volume mass density in dark matter with an assumed spherical distribution is $\rho_{\rm dark\ halo} = 0.369F\ M_\odot\ {\rm pc}^{-3}$.

However, different ways of combining this constraint with the information provided by the K dwarf sample yield different solutions for the disk surface mass density near the Sun, quantified by the parameter $K$.

1. Adopting the constraint $F = 0.041 - 0.0094K \pm 0.008$ in conjunction with the derived $\Sigma_{1.1\ \rm kpc}$ from equation (3) to solve for $K$ and $F$ simultaneously, and allowing for uncertainties in Galactic length scales, one obtains a *disk* surface mass density of $36.9K$ in KG's units, or $48 \pm 9\ M_\odot\ {\rm pc}^{-2}$. The resulting value of the halo parameter in the potential is $F = 0.029$, corresponding to a local volume density of all more spherical components (luminous and dark) of $0.011\ M_\odot\ {\rm pc}^{-3}$. Note that this value for the surface mass density of the Galactic disk is greater (by $2\ M_\odot\ {\rm pc}^{-2}$, or $0.2\ \sigma$) than that derived by KG, because of the more general density fits adopted here. This new result supersedes the KG result. The *identified* surface mass density of the Galactic disk, $48 \pm 8\ M_\odot\ {\rm pc}^{-2}$ as determined by KG, is unaffected by these changes. Thus there remains no evidence for any significant dark matter component of the Galactic disk.

2. Alternatively, one can cast the requirement that the total potential reproduce the rotation curve as another likelihood function and incorporate it directly in the maximum likelihood solution for the local halo and disk potential parameters from the K dwarf data. This technique was discussed by Gould (1990), who showed that the answer exceeds that of KG by 8 $M_\odot\ {\rm pc}^{-2}$. Repeating Gould's analysis with the likelihood function shown in Figure 2, we obtain $55 \pm 9\ M_\odot\ {\rm pc}^{-2}$. We now consider the relative merits of the two approaches outlined above.

The KG analysis (1) combines the total force determined from the stellar data at $z = 1.1$ kpc with a rotation curve constraint determined near the plane. It explicitly discards any constraints directly from the K dwarf data regarding the relative disk and halo contributions to this total force. Gould's method (2), on the other hand, incorporates this extra information into the analysis. Although Gould's is apparently the more objective method, in that all constraints are included directly in the solution, this objectivity is in fact illusory, as the disk/halo potential constraint from the stellar data is model-dependent in a way which is not included in the likelihood function.

Figure 2 indicates that the data favor a low value of $F$, which parameterizes the large-scale (i.e., all except the local disk) potential. This information is carried mainly by those stars with greatest energy. Unfortunately, these same stars are the most difficult to model with the Abel transform method of § 2, since for such stars the assumption of separability of vertical and horizontal motions is least reliable. The observed radial velocities of stars at large $z$-distances are determined in part by an increase (if any) in the velocity dispersion of the stellar population, and in part by the projection effect of the changing orientation (if any) of the stellar velocity ellipsoid. The Abel inversion of equation (2) requires any hot component at high-$z$ to be present at low-$z$ with the same velocity dispersion and cannot allow correctly for the $z$-dependence of the projection effects. Thus the high-velocity wings of the data, which contain the stars that reach high-$z$ and hence those stars providing the strongest constraints on the large-scale potential, are in fact not modeled rigorously.

The effect of the changing orientation of the velocity ellipsoid can be corrected to first order by replacing $K_z$ with an "effective force" $K_{z,\rm eff}$ (KG). However, as KG emphasized, this is not a fully self-consistent description of the distribution function—in fact, in general no such description exists. If one approximates the Galactic potential by a Stäckel potential, analytic expressions can be derived (e.g., Statler 1989), while for general potentials, leading-order expressions obtained from an ordering of the Jeans equations and its higher order analogs in terms of the disk velocity dispersion are presented in Amendt

& Cuddeford (1991). This is a fundamental and under-appreciated limitation on further analysis of the potential derived by KG. Correction for the poorly determined effects of the orientation of the velocity ellipsoid, using the "effective force" technique of KG (see Fig. 14 of Kuijken & Gilmore 1989b), changes the value of the total surface mass density $K_z$(1.1 kpc) by $\sim$10%–20%, but changes the slope of the force law by factors of $\sim$2. Thus, while the approximate allowance for the orientation of the velocity ellipsoid adopted by KG *can* be used to measure $K_z$, derivation of the slope of $K_z$(i.e., $F$) is much less secure. Alternative guesses for the behavior of the velocity ellipsoid far from the Sun, for example changing the tilt angle, axis ratio, or radial gradients of the tracer population's phase-space density, will affect $K_z$(1.1 kpc) only slightly but will have a large effect on $F$. In other words, the constraint the K dwarf data place on the halo contribution to the local potential is much more model-dependent than that placed on the value of the total potential itself. *This uncertainty is not quantified in the likelihood function* shown in Figure 2, which is based on a single model of the very many possible for the behavior of the velocity ellipsoid far from the Sun.

Use of the constraint derived from the stellar data and our restricted model of the velocity ellipsoid tilt concerning the relative contributions of disk and halo to the local potential on an equal basis with the determination of the total local potential from those same data, as advocated by Gould (1990), is not a statistically objective procedure. Combination of a determination of the total local potential with an independent potential constraint from the rotation curve, as advocated by KG, remains the more objective method.

## 6. STÄCKEL ANALYSES AND MONTE CARLO SIMULATIONS

Statler (1989) reports simulations of the KG data set which suggest that the KG analysis substantially underestimates the statistical uncertainties in the disk contribution to the total surface mass density. KG derived an uncertainty of 20% in this value, whereas Statler concludes that the true error distribution of the $K$-measurement (before inclusion of other uncertainties due to tilt and global potential) is substantially broader than the likelihood function of the velocity data, by a factor of $n^{1/2}$, where $n \simeq 3$.

We show here that differences between the simulated sampling adopted by Statler and the actual sampling undertaken by KG are the origin of this discrepancy. Statler assumed that *all* stars of a tracer population within a fixed area on the sky were observed, irrespective of distance. Actual samples are magnitude-limited. His simulation contains many "data" at distances beyond 2.5 kpc from the plane, whereas the KG survey has very few stars at such large distances. This apparently minor point has very important consequences, since a measurement of $K_z(z)$ is essentially a comparison between the velocity distribution of a stellar population at a particular height $z$ and the space density of stars *above* that height, and hence the most distant velocity data carry little weight in the analysis. Inclusion of distant stars in a simulation of a sample of fixed size therefore effectively reduces the statistical weight of the velocity determination at smaller distances, which is the data set of relevance to the problem. Moreover, the more distant stars a sample contains, the greater is the effective height $z_0$ at which $K_z$ is constrained by the data. For models of the data consistent with the rotation curve constraint (see § 5; and note that all Statler's models have the same rotation curve, and hence correspond to measuring $K$ along this rotation curve constraint) $K_z(z_0) = K + 2Fz_0 = 0.082z_0 + (1 - 0.019z_0)K$, i.e., the greater $z_0$, the weaker the dependence of $K_z(z_0)$ on $K$. Since the former quantity is what is measured, with an error set by the sample size, the derived errors on $K$ increase if the sample is selected from stars at greater distances from the plane. We have performed a series of Monte Carlo simulations to quantify this.

We simulated star count and radial velocity data, from the spatial density profile found by KG to give the best fit to their data, in the potential of § 5 ($K = 1.30$, $F = 0.029$), using the same sampling (area surveyed as a function of magnitude, distance distribution of the velocity data) that was used in the actual survey of KG. Then adopting a range of model ("trial") potentials, we found that for all our model potentials, the density law of the simulated tracer population could be fitted well as the sum of three isothermal components $\nu_i(z) = \nu_{0,i} \exp(-\psi/\sigma_{zz,i}^2)$. For each trial potential, the velocity dispersion of the hottest component was fixed by first fitting the density data with the sum of two exponentials, and then using the scale height $H_3$ of this fit at $z = 3$ kpc to derive the dispersion from $\sigma^2 = H_3 K_z$(3 kpc). The normalization of this component, and the dispersion and normalization of the two other components, were then fitted by least-squares to the star counts, considering $N^{1/2}$ Poisson errors. The $\chi^2$ values obtained in this way indicated that the density profile was adequately fitted.

The velocity distribution at height $z$ of such a model is $\Sigma_i \nu_i(z)N(\sigma_{zz,i})$, where $N(\sigma)$ is a normal distribution of dispersion $\sigma$ and zero mean. Using these velocity distributions, we calculated the likelihood function of the velocity data as a function of the parameters $K$ and $F$ and found its maximum along the rotation curve constraint, as detailed in KG. From 1000 such simulations, we find a 1 $\sigma$ error of 0.20 in $K$, or an error of 15%. This agrees with the error estimated from the likelihood function itself. The error is largely dominated by the velocity sample size: using the "true" rather than the "measured" $\nu(z)$ still leaves an error on $K$ of 0.17. Thus the extra error associated with the star count sample size is small, and the width of the likelihood contours is a good estimate of the standard deviation in the derived value of $K$.

The sampling strategy employed by Statler was also simulated: with star count and velocity samples of 500 stars, $\sigma(K) = 0.33(=25\%)$ from sample size alone, in good agreement with his result of 25%. The average half-width of the $L > e^{-1/2}L_{max}$ region was 0.27. Uniformly sampling 500 stars in an area-limited survey limited to heights below 4 and 2 kpc gives an error of 0.28 and 0.24, respectively, demonstrating that among samples of fixed size, those containing stars closer to the plane constrain $K$ better.

Adding in further errors of 0.1 and 0.15 due to tilt term and rotation curve uncertainties yields a final error of 0.27 in $K$, or 20%. This error budget from realistic modeling of the real data set is in excellent agreement with the results calculated directly by KG but is substantially below the 30% obtained by Statler (1989). The conclusion of Statler (1989), that the width of the likelihood function derived by KG must be multiplied by a factor of $\sim$1.7, does not apply to the KG sample. The best estimate of the uncertainty in the result derived from the data of KG remains that calculated by KG.

## 7. CONCLUSIONS

In this *Letter* we have shown that analysis of the radial velocity and photometric survey of K dwarf stars described by Kuijken & Gilmore (1989a, b) determines the total surface mass density of all gravitating matter below $z = \pm 1.1$ kpc from the Galactic plane near the Sun to be

$$\frac{|K_z(1.1\ \mathrm{kpc})|}{2\pi G} = 71 \pm 6\ M_\odot\ \mathrm{pc}^{-2}\ . \tag{4}$$

This result is independent of the relative contributions of disk and halo to the local potential. We argue that any further inferences about the Galactic potential drawn from these data are inherently limited by current uncertainty in the behavior of the velocity ellipsoid of the K dwarf population beyond 1 kpc from the Galactic plane. The relation between the disk contribution to the local potential and the total potential below 1.1 kpc depends on the Galactic structure constants, particularly the local circular speed, galactocentric distance, and local Oort constants. Allowing for uncertainties in these constants, and by combining the result of equation (4) with the requirement that the disk and the round Galactic components should together produce the observed radial gravitational force, as measured by the Galactic rotation curve, we can, however, determine the best available estimate of the surface mass density which is associated with the Galactic disk to be

$$\Sigma_{\mathrm{disk,tot}} = 48 \pm 9\ M_\odot\ \mathrm{pc}^{-2}\ . \tag{5}$$

The corresponding best available estimate for the surface mass density of *identified* matter associated with the Galactic disk is

$$\Sigma_{\mathrm{disk,tot}} = 48 \pm 8\ M_\odot\ \mathrm{pc}^{-2}\ . \tag{6}$$

There remains no significant evidence for any unidentified matter associated with the Galactic disk near the Sun.

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