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Statistical Interdependences Among Characteristics of Sunspot Groups

by

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ABSTRACT

This paper is a continuation of a former work by Jakimiec and Bartkowiak (1990). We consider the sunspot group daily characteristics which are used in short term predictions of flare activity. In our former paper we explained the interrelations among these characteristics using factor analysis. In this paper we present a graphic presentation of these interrelations in the form of dendrograms obtained from cluster analysis. We employ this method to investigate the intrinsic structure of 14 variables describing sunspot groups of D, E, F Zurich classes. The results obtained by use of cluster analysis and of common factor method are in a sense overlapping and in a sense they yield different information about the considered characteristics of sunspot groups.

1. Considered problem

We are concerned with the interrelation structure of sunspot group characteristics which are used as predictors in short-term predictions of flare activity. In a former paper (Jakimiec and Bartkowiak 1990) we have investigated the impact of outliers occurring in the data on the interdependency structure revealed by factor analysis. It was found that the interdependency structure is stable and not influenced by the few outliers revealed in the data.

In this paper we continue the investigations on the interdependency structure of the considered in mentioned paper predictors. There are very simple and very sophisticated methods of discovering the relationships between variables. One simple method consists in constructing a dendrogram visualizing similarities among the considered variables. The method is described and the dendrograms visualizing similarities among clusters of variables characterizing sunspot groups are presented in Section 3. More sophisticated methods allow to find out the dimensionality of the data and to reveal common factors hidden in the considered variables. These methods are described and the results are discussed in Sections 4 and 5. Moreover, in Section 5 we compare the results obtained by cluster analysis with those obtained formerly by use of factor analysis (Jakimiec and Bartkowiak 1990).

2. Data

We consider the same set of data which was previously analyzed by Jakimiec and Wanke-Jakubowska (1988) and by Jakimiec and Bartkowiak (1990). The data set comprises $p = 14$ characteristics of sunspot groups of D, E, F Zurich classes taken from Solar Geophysical Data (SGD, 1979) for 1979 year. The data set I ($n_I = 234$) contains characteristics of sunspot groups in the increase phase (IPh) of evolution while in the set II ($n_{II} = 139$) are included characteristics of sunspot groups in the decay phase (DPh).

We consider the following characteristics: 1. (McI) – McIntosh sunspot group class. It was calculated (Hirman *et al.* 1980) as a product of three McIntosh parameters of sunspot group (Zurich class, type of penumbra in the largest spot in group and density of the sunspot population in the group); 2. (A) – Sunspot group area in tens of millionths of the solar hemisphere; 3. (CaA) – Calcium plage area in tens of millionths of the solar hemisphere; 4. (CaI) – Calcium plage intensity in five-step scale; 5. (Mag) – Magnetic class in five-step scale; 6. (H) – Magnetic field strength in thousands of Gausses; 7. (MFI) – Magnetic field index introduced by Jakimiec and Wasiucionek (1980) as a product of the neutral line index (described in eight-step scale) and of strength of magnetic field divided by the distance between the magnetic strength maxima. The further seven variables 8 – 14 are described by Jakimiec and Wanke-Jakubowska (1988). They are as follows: 8. ($maxX$) – Maximum value of flare X-ray flux ($maxX$); 9. (NFF) – Number of faint flares; 10. (NSF) – Number of strong flares; 11. (Fs) – Daily sum of the flare X-ray fluxes in the wavelength range 1–8 Å(Fs); 12. ($HI = Fh/Fs$) – Hardness index; 13. (Fh) – Daily sum of the flare X-ray fluxes for the wavelengths 0.5–4 Å; 14. ($maxh$) – Daily maximum value of the six-hour hardness indices as defined by Jakimiec and Wanke-Jakubowska (1987). The variables $x8$, $x11$ and $x13$ are expressed in erg$\cdot$cm^{-2}s^{-1}.

We transformed the original values of the variables no. 1, 2, 3, 7, 8 and 11 taking log's of the values. The values of the variable no. 13 were transformed by the formualae $x' = \log(x) + 2$.

3. Similarities among the variables

Applying methods of cluster analysis we may construct dendrograms visualizing the similarities among the variables. The principles of cluster analysis may be found *e.g.* in Mardia *et al.* (1979).

3.1. Construction of a dendrogram

Let us consider p variables. First we establish a square matrix $S = (s_{ij})$ of "similarities" between the variables. We define the similarity s_{ij} (between the i-th and j-th variable, for each $i = j = 1, ..., p$) in such a manner that the similarity of the i-th variable to itself is equal to one.

One way to define the similarity of the i-th to j-th ($i = j$) variable (especially when the variables are valued on the real axis) is to calculate r_{ij}, the classical (Pearsonian) correlation coefficient and then to define the similarity as the absolute value of r_{ij}:

$$s_{ij} = |r_{ij}|, \quad i = 1, \ldots, p, \quad j = 1, \ldots, p \tag{1}$$

We see that with increasing values of s_{ij} (what means that the interdependency between these variables becomes more and more linear) the variables are becoming also more and more similar.

Then we may apply to the obtained matrix S of similarities the method of cluster analysis. We use an agglomerative method seeking for the nearest neighbor (single linkage method). At the start we have p clusters, each containing one of the considered variables. Next we seek for the pair of variables (clusters) which are the most similar. Suppose these are the variables no. k and l. We link these clusters together. After this fusion we have only $p - 1$ clusters. We repeat this process until all the variables are linked together forming one common cluster.

Now we define the similarity between clusters. Suppose we have v clusters ($v \leq p$). Let us denote the cluster no. k as $C_k^{(v)}$ and the cluster no. l as $C_l^{(v)}$ ($k = 1, 2, \ldots, v$, $l = 1, 2, \ldots, v$, $k = l$). Using the single linkage method the similarity $s_{kl}^{(v)}$ between the clusters is defined as follows:

$$s_{kl}^{(v)} = \begin{cases} 1 & \text{for } k = l \\ \max\limits_{t \in C_k, u \in C_l} S_{tu} & \text{for } k \neq l \end{cases} \tag{2}$$

Using the complete linkage method the similarity $s_{kl}^{(v)}$ is defined as follows:

$$s_{kl}^{(v)} = \begin{cases} 1 & \text{for } k = l \\ \min\limits_{t \in C_k, u \in C_l} S_{tu} & \text{for } k \neq l \end{cases} \tag{2a}$$

The process of the cluster fusion in subsequent steps may be visualized by a dendrogram.

3.2. *Results of cluster analysis*

The dendrograms obtained from cluster analysis are shown in Figs. 1 and 2. Figs. 1a and 1b show relationships of characteristics for sunspot groups in the increase phase of the evolution (IPh). Figs. 2a and 2b visualize analogous relationships for sunspot groups in the decay phase (DPh). The y-axis shows the values of similarity ($s_{kl}^{(v)}$) at which the respective clusters were linked together.

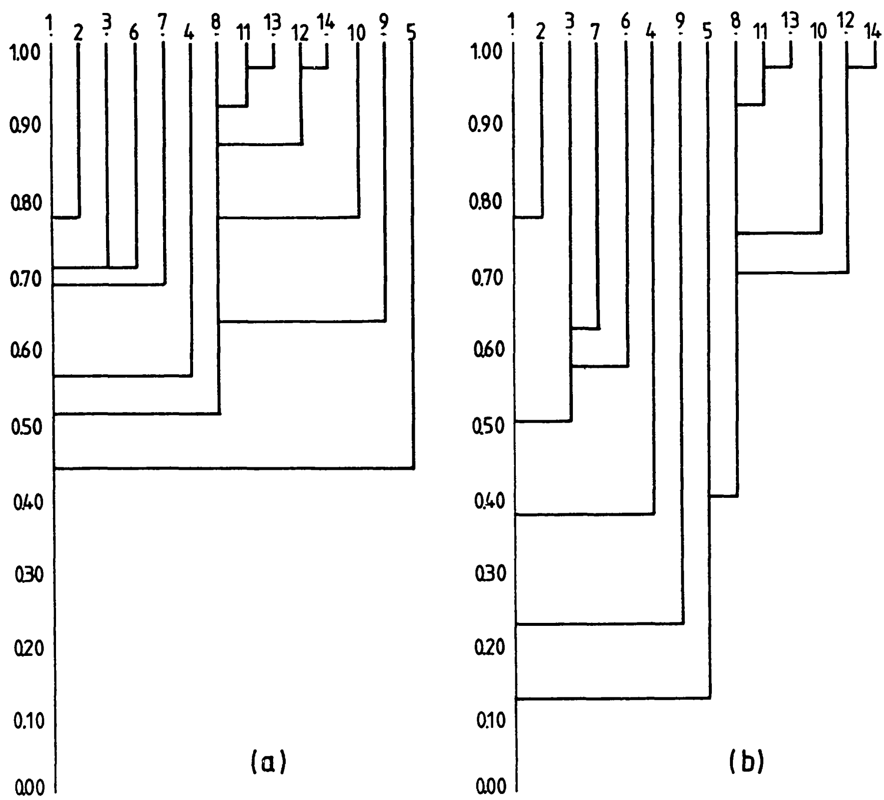

Fig. 1. The dendrograms obtained for the data set I (sunspot groups in the increase phase of the evolution) by use of (a) single linkage and (b) complete linkage method.

One can see that the pattern of Figs. 1a and 2a (single linkage) is very similar. So is the pattern of Figs. 1b and 2b (complete linkage). The differences between the patterns obtained by single linkage (Figs. 1a and 2a) and the patterns obtained by comlete linkage (Figs. 2a and 2b) are much larger. Let us remind that single linkage of two clusters C_k and C_l at the level $s_{kl}^{(v)}$ means that there exists at least one pair (x_i, x_j) of variables (with $x_i \in C_k$ and $x_j \in C_l$) such that the similarity between these variables equals $s_{kl}^{(v)}$. For other pairs of variables the similarity may be smaller. On the other hand, complete linkage of two clusters C_k and C_l at the

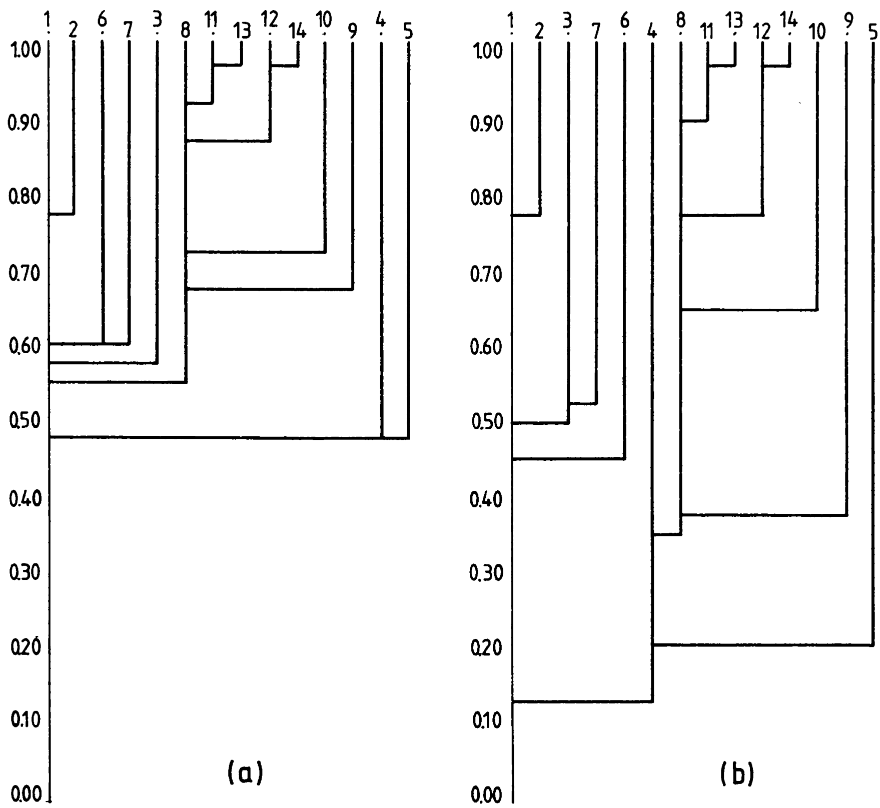

Fig. 2. The dendrograms obtained for the data set II (sunspot groups in the decay phase of evolution) by use of (a) single linkage and (b) complete linkage method.

level $s_{kl}^{(v)}$ means that all pairs (x_i, x_j) of variables (with $x_i \in C_k$ and $x_j \in C_l$) are similar at least at the level $s_{kl}^{(v)}$, but for some of the pairs the similarity may be greater. It follows from these definitions of the single and complete linkage methods that, in principle, the considered characteristics are clustered easier by the simple than by the complete linkage method. We can see it clearly from Figs. 1a,b and 2a,b. All the p = 14 characteristics are clustered together into one common cluster by the single linkage method at the level $s_{kl}^{(v)} \simeq 0.46$ (Figs. 1a and 2a). However,when using the complete linkage method the last fusion occurs not before the level $s_{kl}^{(v)} = 0.13$ (Figs. 1b and 2b).

Looking at the process of clustering we find some distinctive patterns:
(a) The variables $x8$, $x11$ and $x13$ ($maxX$, Fs and Fh) were first linked together at a level $s_{kl}^{(v)} > 0.90$, independently of whether we have used single or the complete linkage method. The high similarities between the variables $x8$ and $x11$ or between the variables $x8$ and $x13$ reflect their relation connected with thc

definition of these variables: Fs and Fh are the daily sums of the X-ray flare fluxes. The high values of Fs and Fh are observed mainly for sunspot groups in which at least one strong X-ray flare occurred ($x10$). Only for very few cases high values of Fs and Fh are caused by very large number of faint flares ($x9$). The high similarity between the variables $x11$ and $x13$ means close physical interdependence between softer and harder X-ray fluxes of flares (see *e.g.* Sawyer *et al.* 1986 or Jakimiec and Wanke-Jakubowska 1987).
(b) The variables $x12$ and $x14$ (HI and $maxh$) are similar at a level $s_{kl}^{(v)} > 0.97$, independently whether we have used the single or the complete linkage method. This similarity also reflects the definitional relation between two variables characterizing the hardness of flare X-ray flux.
(c) The two clusters ($x8$, $x11$, $x13$) and ($x12$, $x14$) are linked together at the level $s_{kl}^{(v)} > 0.88$ when using the single linkage method and at much lower level $s_{kl}^{(v)} > 0.70$ when using the complete linkage method. This cluster fusion means that strong flare activity does imply also increased X-ray hardness of flares.
(d) The variable $x10$ (NSF) characterizing the number of strong flares is linked to the former clusters at the levels 0.75 (Fig. 1a, 1b), and 0.72 and 0.65 (Fig. 2a, 2b). This means a significant correlation between the variable $x10$ and the others characterizing strong flare activity. It is due not only to the definitional relation between $x10$ and ($x11$, $x13$), this is due also to the fact that strong flares occure very often in series.
(e) The variable $x9$ (NFF) characterizing the number of faint flares reveals specific behaviour. When using the single linkage method it clusters together with the other flare characteristics with a similarity coefficients 0.64 and 0.68 for the data sets I (Fig. 1a) and II (Fig. 2a), respectively. Conversely, when seeking for complete linkage, we find that $x9$ was clustered with variables describing sunspot groups features ($x1$, $x2$, $x3$, $x4$, $x6$ and $x7$) with the similarity coefficient $s_{kl}^{(v)} = 0.23$ (Fig. 1b), while in Figure 2b $\{x9\}$ was clustered with other flare characteristics but only at the level 0.38. So, we can say that the behavior of $x9$ (NFF) is different for the data sets I and II. When sunspot groups are in the increasing phase of evolution, their characteristics determine to some extent the number of faint flares independently on whether strong flares occure or not. However, the number of faint flares are more related to strong flare activity than to other photospheric characteristics of sunspot groups in the decay phase of evolution.
(f) The variables $x1$ and $x2$ (McI and A) were linked together with the similarity coefficient equal to 0.78. This similarity reflects the fact that all the three factors contributing to the variable $x1$ (McI) are related positively to the area of sunspot group (see *e.g.* SESC Glossary of Solar-Terrestrial Terms, 1989).
(g) The variables $x3$, $x6$ and $x7$ (characteristics CaA, H and MFI) cluster together at a fairly moderate level of similarity equal to 0.69, 0.58. 0.60 and 0.45, what can be seen in Figs. 1a, 1b, 2a and 2b, respectively.
(h) The variable $x5$ (Mag) reveals an outstanding behaviour. It is linked with

other variables at similarity about 0.45 when using the single linkage method. When using the complete linkage method for the data set I (IPh) the cluster comprising variables characterizing flare activity is linked to the variable $x5$ with similarity about 0.40. However, for sunspot groups in the decay phase the flare characteristics are linked to the variable $x4$ (CaI) with the similarity equal to 0.33, and the variable $x5$ is linked to them at the similarity 0.20 only.

4. Revealing the dimensionality of the data

The dimensionality of the data is the object of investigation of principal component analysis. The principles of this method may be found *e.g.* in Mardia *et al.* (1979) and Morrison (1967).

4.1. Description of principal component method

Suppose we consider standardized variables (*i.e.* with expected values equal to zero and with unit variances) with a correlation matrix R. By a fundamental theorem of algebra the matrix R can be presented as follows:

$$R = A \cdot \Lambda \cdot A^T = \lambda_1 \mathbf{a}_1 \mathbf{a}_1^{\mathbf{T}} + \ldots + \lambda_p \mathbf{a}_\mathbf{p} \mathbf{a}_\mathbf{p}^{\mathbf{T}} \tag{3}$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_p \geq 0$ are the eigenvalues of R, and $\mathbf{a}_1, \ldots, \mathbf{a}_\mathbf{p}$ are the respective eigenvectors, $A = (\mathbf{a}_1, \ldots, \mathbf{a}_p)$ is the matrix comprising the eigenvectors columnwise. The sum of the diagonal elements of R is called the total inertia of the data. From the theorem on the spectral decomposition of R it follows that the total inertia I equals:

$$I = \mathrm{trace}(R) = \lambda_1 + \lambda_2 + \ldots + \lambda_p \tag{4}$$

If the last eigenvalues are small then the matrix R can be approximated by $R^{(r)}$, a matrix of lower rank, say r $(1 \leq r \leq p)$ such that :

$$\mathrm{tr}(R - R^{(r)}) = \lambda_{r+1} + \ldots + \lambda_p \tag{5}$$

On the basis of the eigenvectors $\mathbf{a}_1, \ldots, \mathbf{a}_\mathbf{p}$ some new, uncorrelated variables can be constructed. Say, we denote the observed standardized variables by $X_1, \ldots, X_p$. Then, using the eigenvectors $\mathbf{a}_1, \ldots, \mathbf{a}_p$ (evaluated from the correlation matrix of the observed variables) we can transform the variables $X_1, \ldots, X_p$ to uncorrelated variables $Y_1, \ldots, Y_p$ by the formula

$$Y_1 = (X_1, \ldots, X_p)\mathbf{a}_1, \quad \ldots \quad , Y_p = (X_1, \ldots, X_p)\mathbf{a}_\mathbf{p} \tag{6}$$

The covariance matrix of the new variables $Y_1, \ldots, Y_p$ equals to:

$$\mathrm{Cov}(Y_1, \ldots, Y_p) = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_p \end{bmatrix} \tag{7}$$

If we have decided to take only r transformed variables ($1 \leq r \leq p$), then they explain

$$\left(\sum_{i=1}^{r} \lambda_i \Big/ \sum_{i=1}^{p} \lambda_i\right) \cdot 100 \text{ per cent}$$

of the total inertia. If this part is great (*i.e.* near to one), we infer that the initial p variables $X_1, \ldots, X_p$ (determining an Euclidean space of p dimensions) may be replaced by r uncorrelated variables $Y_1, \ldots, Y_p$ reproducing a great part of the total inertia.

4.2. *Results of principal component analysis*

The dimensionality of the considered data was determined by evaluating eigenvalues of the correlation matrices R_I and R_{II} for the data set I and II, respectively, and by examination of the percentages of exhaustion of the total inertia. That is we examine the exhaustion of trace (R) by subsequent eigenvalues (see formula (4)). We show these percentages in Table 1. One can see that $r = 7$ eigenvalues reproduce the total inertia of data set I in 92.28% and of data set II in 90.64%, while about 10% of the total inertia remains to explain by the last seven eigenvalues. So, seven new variables $Y_1, \ldots, Y_7$ defined by Eq. (6) would approximate equivalently the original data matrix with $p = 14$ characteristics accounting for 90% of total variance of these data.

5. Revealing common factors hidden in the variables

Factor analysis is a more sophisticated mathematically method for investigating interrelations among variables. It seeks for common factors hidden in the variables. The principles of this method may be found also *e.g.* in Mardia *et al.* (1979) and Morrison (1967).

5.1. *Description of common factor method*

The relations between the variables and the common uncorrelated factors $F_1, \ldots, F_m$ $(m < p)$ of unit length are expressed by the equations:

$$\begin{aligned} X_1 &= l_{11}F_1 + \ldots + l_{1m}F_m + e_1 \\ &\cdots\cdots\cdots\cdots \\ X_p &= l_{p1}F_1 + \ldots + l_{pm}F_m + e_p \end{aligned} \tag{8}$$

with $L = (\mathrm{l}_1, \ldots, \mathrm{l}_\mathrm{m})$ the matrix of factor loadings and $e' = (e_1, \ldots, e_p)$ the vector of specificities of the variables. We consider R, the correlation matrix of the variables $X_1, \ldots, X_p$, seeking for its representation as:

$$R = A \cdot A^T + \psi = \sum_{i=1}^{m} \mathrm{l_i l_i^T} + \psi \tag{9}$$

where $\psi = \mathrm{diag}(\psi_1, \ldots, \psi_p)$, with $\psi_i = \mathrm{Var}(e_i)$.

Table 1

Exhaustion of the total inertia of the correlation matrix R by subsequent eigenvalues of this matrix

No. of the eigenvalue	1	2	3	4	5	6	7
The eigenvalue	6.990	2.601	0.956	0.792	0.645	0.519	0.417
% of exhaustion	49.93	68.50	75.33	80.99	85.60	89.30	92.28
	8	9	10	11	12	13	14
	0.378	0.318	0.164	0.129	0.056	0.019	0.016
	94.98	97.25	98.42	99.35	99.75	99.88	100

Correlation matrix obtained from $n = 234$ items - for sunspot groups in the increase phase of evolution

No. of the eigenvalue	1	2	3	4	5	6	7
The eigenvalue	7.334	2.018	0.897	0.819	0.718	0.463	0.441
% of exhaustion	52.39	66.80	73.21	79.06	84.18	87.49	90.64
	8	9	10	11	12	13	14
	0.413	0.372	0.248	0.200	0.048	0.018	0.011
	93.59	96.25	98.02	99.45	99.79	99.92	100

Correlation matrix obtained from $n = 149$ items - for sunspot groups in the decay phase of evolution

Equation (9) permits to approximate the correlation matrix R by a matrix of lower rank (m) given by the columns $(\mathrm{l}_1, \ldots, \mathrm{l}_p)$ in Eq. (8).

Application of factor analysis to the data describing sunspot groups characteristics may be found *e.g.* in paper by Jakimiec and Bartkowiak (1986).

5.2. *Results of factor analysis*

Detailed results of factor analysis performed for the data described in Section 2 are presented in a former paper (Jakimiec and Bartkowiak 1990, Fig. 3). Seven factors in the meaning of Eq. (8) explain only about 75% of the total inertia, while seven principal components can explain more than 90% of total inertia. The specific feature of principal component analysis is that it usually yields principal components which account for a higher amount of the total inertia than the factors obtained by factor analysis method do.

5.3. *Comparison of results obtained by common factor and cluster analysis*

Now we compare the results obtained from factor analysis Jakimiec and Bartkowiak (1990) with those obtained by use of cluster analysis (Section 3). The detailed features of this comparison are as follows:

(a) The first column of the factor loadings from Eq. (8) is composed mainly of flare characteristics (except $x9$). In this column there are also low values of factor loadings connected with the photospheric characteristics of sunspot groups. The first factor explains large part (about 35%) of the total inertia. There is no very clear correspondence between the branches of the dendrogram and the variables loaded in subsequent factors. Commenting the results of cluster analysis in Section 3 we stressed that all the flare characteristics (except $x9$) clustered at the similarity level 0.60. The revealed by cluster analysis definitional relations among flare characteristics may be found in the fourth common factor (explaining about 1% of total inertia).

(b) The second column of the loading matrix is composed mainly of the characteristic $x9$ which keeps an outstanding position also in the dendrograms shown in Figs. 1 and 2. The second factor (about 8% of total inertia) explains mainly the contribution of faint flares ($x9$) to the total flare fluxes ($x11$, $x13$) and this fact is not confirmed through cluster analysis.

(c) The third column of the loading matrix explaining about 20% of total inertia contains, in general, the variables describing the photospheric features of sunspot groups (from $x1$ to $x7$). The similarities obtained by cluster analysis give similar pattern of grouping of the variables.

(d) The fifth column is loaded mainly in $x1$ and $x2$ and explains the correlation between the McIntosh sunspot class and the sunspot group area. The relationship between these characteristics was revealed also by cluster analysis. These variables are linked together with high similarity coefficient but their common factor explains only 3–4% of the total inertia.

(e) The interrelation between the variables $x3$ and $x4$ characterizing calcium plage area and intensity are revealed in the sixth column (the sixth factor explain 5–7% of the total inertia). One can see that these characteristics ($x3$ and $x4$) appear together with flare activity characteristics of sunspot groups only in the decay phase of evolution. For sunspot groups in the increase phase the flare characteristics do not appear in the sixth factor. The cluster analysis shows that flare characteristics are linked to the variable $x4$ only for sunspot groups in the decay phase.

(f) There is also one column of loadings that reveals the correlation between the magnetic class ($x5$) and the characteristics of flare activity ($x10$, $x11$, $x13$). For sunspot groups in the decay phase this factor explains higher percent of total inertia (about 7%) than for sunspot groups in the increase phase (only 1%). The results of cluster analysis does not confirm this pattern. The similarities between the variable $x5$ and the variables characterizing flare activity are much higher (0.40) for sunspot groups in the increase phase than for sunspot groups in the decay phase (only 0.20). This characteristic was previously discussed in the comments concerning the results yielded by cluster analysis: it was found that this characteristic was located on an outstanding position in the dendrograms and did not cluster easy with other variables. Eventually it has clustered with $x4$ (CaI), a characteristic which is also badly explained by the introduced seven factors. Therefore, the variable $x5$

seems to be a specific variable. Really, the variable x5 play the most important role in the specificities (*i.e.* the part of the total inertia unexplained by the introduced factors).

(g) There is not such a separate column of factor loadings that could confirm the fact that variables ($x3$, $x6$ and $x7$) are clustered together at the similarity level about 0.60.

6. Remarks

The dendrograms obtained from cluster analysis visualize in a simple way interdependencies among the considered variables. The results obtained by clustering variables are easy to interpret. However, the results are only explorative – they do not point directly to a mathematical model explaining the observed phenomena.

The loadings matrix obtained from factor analysis permits to build a mathematical model. However, the factors obtained from factor analysis are assumed to be orthogonal – this is a serious restriction which is criticized : the "true" factors encounted in the practice are often interrelated. We found for our data that the results obtained by two used techniques, cluster and factor analysis, are in a sense overlapping and in a sense they yield different information about the considered data.

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